TD 5. CALCUL NATUREL

Exercice 1: Prouvez que :

1. $a \wedge b \models_{N} a \vee b$

2. $a \vee a \models_{N} a$

3. $x \models_{N} x \vee (x \wedge y)$

4. $x \vee (x \wedge y) \models_{N} x$

5. $a \wedge (b \wedge c) \models_{N} (a \wedge b) \wedge c$

6. $a \vee (b \vee c) \models_{N} (a \vee b) \vee c$

7. $a \vee (b \wedge c) \models_{N} (a \vee b) \wedge (a \vee c)$

8. $(a \vee b) \wedge (a \vee c) \models_{N} a \vee (b \wedge c)$

9. $\models_{N} (a \Rightarrow (b \Rightarrow c)) \Rightarrow ((a \Rightarrow b) \Rightarrow (a \Rightarrow c))$

10. $\models_{N} ((a \Rightarrow c) \wedge (b \Rightarrow c)) \Rightarrow ((a \vee b) \Rightarrow c)$

11. $b \models_{N} a \Rightarrow b$

Exercice 2: Prouvez que :

1. $A \Rightarrow (B \Rightarrow C) \models_{N} A \wedge B \Rightarrow C$

2. $A \wedge (B \vee C) \models_{N} (A \wedge B) \vee (A \wedge C)$

3. $A \Rightarrow B \models_{N} \neg A \vee B$

4. $A \Rightarrow B \models_{N} \neg B \Rightarrow \neg A$

5. $\neg A, A \vee B \models_{N} B$

6. $\neg A \wedge \neg B \models_{N} \neg(A \vee B)$

7. $\models_{N} (A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C))$

Exercice 2: Prouvez que :

1. $a \models_{N} \neg\neg a$

2. $\models_{N} a \vee \neg a$