TD 5

1 Calcul naturel – Cas positifs

Rappel. Rappelons que *positif* signifie *sans* $\neg$, par consquent intuitionniste.

Exercice 1 *Prouvez que :*

$$a \wedge b \vdash_N a \vee b$$

Exercice 2 *Prouvez que :*

$$a \vee a \vdash_N a$$

Exercice 3 *Prouvez que :*

$$x \vdash_N x \vee (x \wedge y)$$

Exercice 4 *Prouvez que :*

$$x \vee (x \wedge y) \vdash_N x$$

Exercice 5 *Prouvez que :*

$$a \wedge (b \wedge c) \vdash_N (a \wedge b) \wedge c$$

Exercice 6 *Prouvez que :*

$$a \vee (b \vee c) \vdash_N (a \vee b) \vee c$$

Exercice 7 *Prouvez que :*

$$a \vee (b \wedge c) \vdash_N (a \vee b) \wedge (a \vee c)$$

Exercice 8 *Prouvez que :*

$$(a \vee b) \wedge (a \vee c) \vdash_N a \vee (b \wedge c)$$

Exercice 9 *Prouvez que :*

$$\vdash_N (a \to (b \to c)) \to ((a \to b) \to (a \to c))$$

Exercice 10 *Prouvez que :*

$$\vdash_N (a \to c) \wedge (b \to c) \to ((a \vee b) \to c)$$

Exercice 11 *Prouvez que :*

$$b \vdash_N a \to b$$

TD 5 Bis

1) $A \to (B \to C) \vdash_N A \wedge B \to C$

2) $A \wedge (B \vee C) \vdash_N (A \wedge B) \vee (A \wedge C)$

3) $A \to B \vdash_N \neg A \vee B$

4) $A \to B \vdash_N \neg B \to \neg A$

5) $\neg A, A \vee B \vdash_N B$

6) $\neg A \wedge \neg B \vdash_N \neg (A \vee B)$

7) $\vdash_N (A \to (B \to C)) \to ((A \to B) \to (A \to C))$

$\alpha \vdash_N \neg\neg\alpha$

$\vdash_N \alpha \vee \neg\alpha$