Problem Set 5: Multilevel Models

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Preliminaries

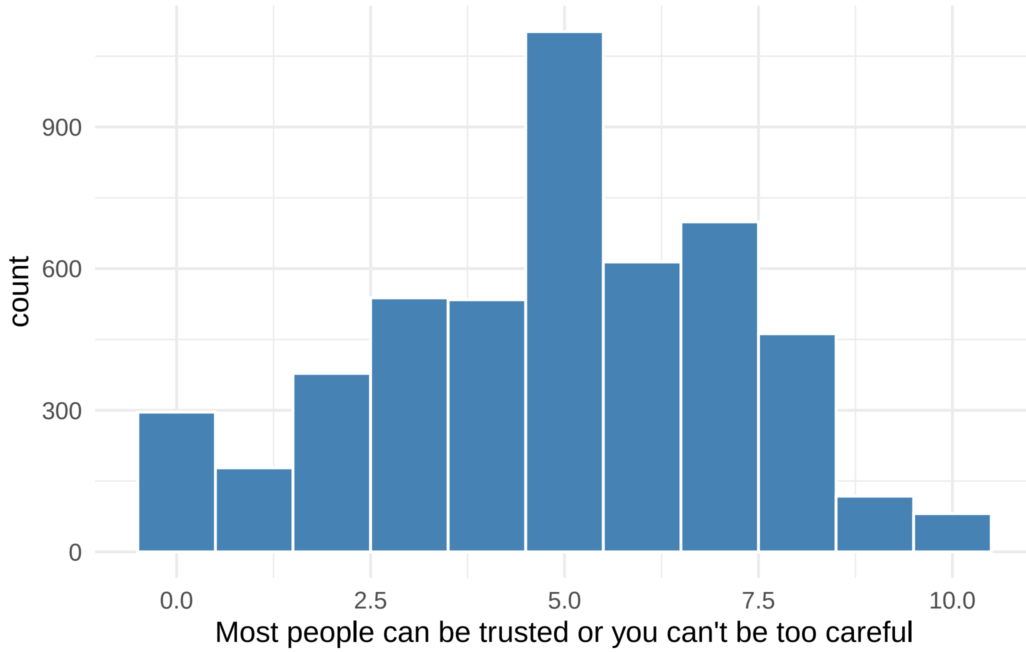
1.

```
psych::describe(d) |>
  filter(vars == 3) |>
  mutate(variance = sd^2) |>
  select(mean, variance)
```

```
        mean variance
ppltrst 4.87     5.55
```

2.

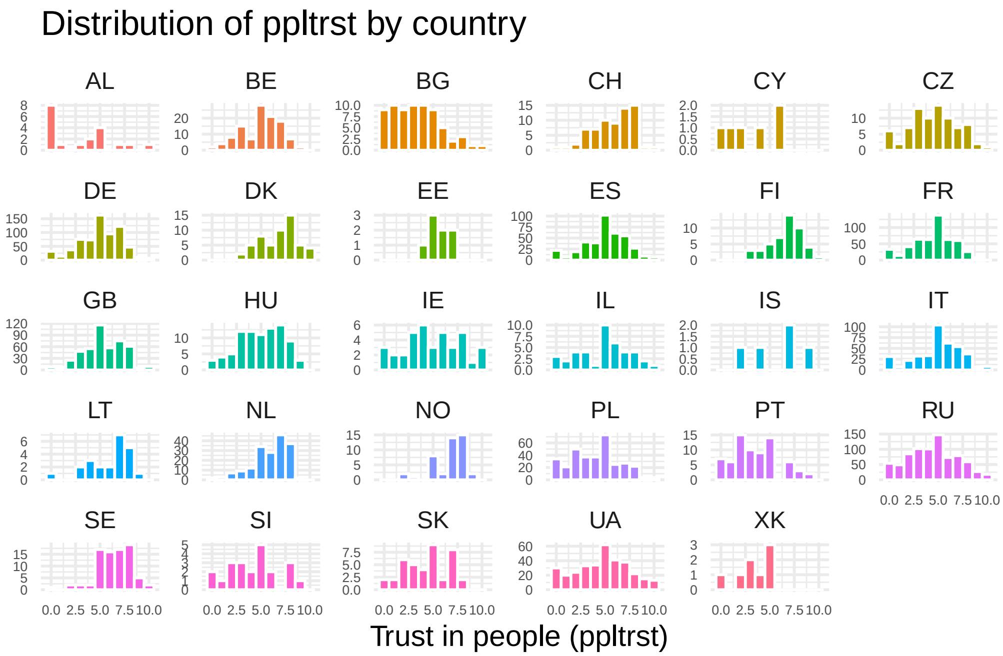
```
d |>
  ggplot(aes(x=ppltrst)) +
  geom_histogram(binwidth = 1, fill="steelblue", color="white") +
  theme_minimal()
```



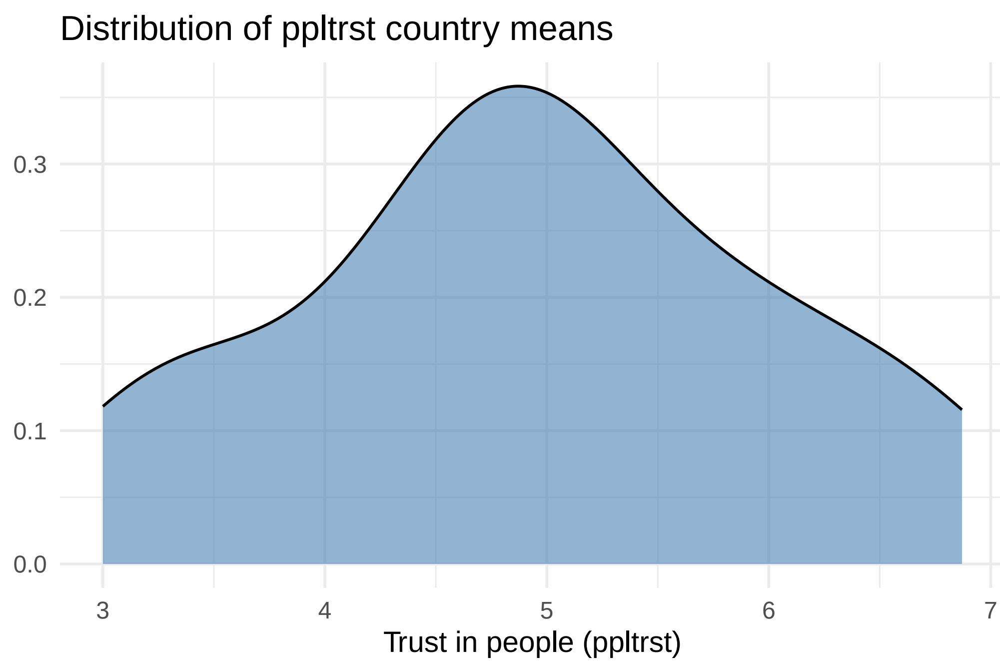
The modal value is 5.

3.

```
d |>
  ggplot(aes(x = ppltrst, fill = cntry)) +
  geom_histogram(binwidth = 1, color = "white") +
  facet_wrap(~cntry, scales = "free_y") +
  theme_minimal() +
  labs(
    title = "Distribution of ppltrst by country",
    x = "Trust in people (ppltrst)",
    y = ""
  ) +
  theme(legend.position = "none",
        axis.text = element_text(size=5))
```




```
d |>
  group_by(cntry) |>
  summarise(mean = mean(ppltrst)) |>
  ggplot(aes(x=mean)) +
  geom_density(fill="steelblue", color="black", alpha=0.6) +
  theme_minimal() +
  labs(
    title = "Distribution of ppltrst country means",
    x = "Trust in people (ppltrst)",
    y = ""
  )
```



If we estimate the country means using partial pooling, the estimate of the means of countries with fewer people in the sample will be closer to the overall mean (~5). In some countries, this wont have much of an effect, but in cases like Albania, Cyprus and Iceland the estimated mean will be quite different from the sample mean.

Single Dimension

4.

```r
m1 <- lmer(ppltrst ~ 1 + (1 | cntry),
           data = d,
           REML = FALSE)
summary(m1)
```

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: ppltrst ~ 1 + (1 | cntry)
   Data: d

      AIC       BIC    logLik -2*log(L)  df.resid
  22490.9   22510.4  -11242.4   22484.9      4997
```

```
Scaled residuals:
     Min       1Q   Median       3Q      Max 
-2.79077 -0.66911  0.00127  0.67580  2.87402 

Random effects:
 Groups   Name        Variance Std.Dev.
 cntry    (Intercept) 0.8744   0.9351  
 Residual             5.1737   2.2746  
Number of obs: 5000, groups:  cntry, 29

Fixed effects:
            Estimate Std. Error t value
(Intercept)   4.9732     0.1866   26.66
```

5.

```r
icc <- 0.8744 / (0.8744 + 5.1737)
icc
```

```
[1] 0.1445743
```

6.

```r
performance::icc(m1)
```

```
# Intraclass Correlation Coefficient

    Adjusted ICC: 0.145
  Unadjusted ICC: 0.145
```

7.

An ICC of 0.145 can be interpreted as saying that 14.5% percent of the variance in ppltrst is explained by variation between countries, meaning that the resulting 76.5% of variance is explained by within country variation.

8.

```r
m2 <- lm(ppltrst ~ 1,
         data = d)
summary(m2)
```

```

Call:
lm(formula = ppltrst ~ 1, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-4.8678 -1.8678  0.1322  2.1322  5.1322 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  4.86780    0.03331   146.1   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.355 on 4999 degrees of freedom
```

The mean estimate for the partially pooled model (4.9732) is not significantly different from the completely pooled model (4.8678). Suggesting that the complexity of random intercepts is not necessary in this case.

9.

```r
global_mean <- fixef(m1)["(Intercept)"]

eb <- ranef(m1)$cntry |>
  rownames_to_column(var="cntry") |>
  rename(u_j = `(Intercept)`) |>
  mutate(partial_mean = global_mean + u_j)

#highest mean
eb |>
  arrange(desc(partial_mean)) |>
  slice(1)
```

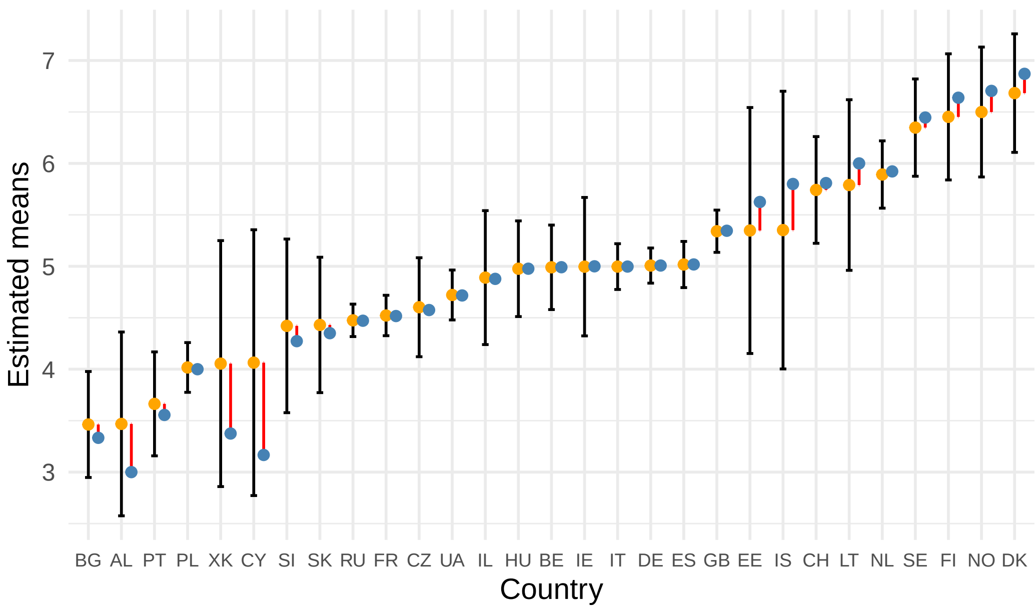
```
  cntry      u_j partial_mean
1    DK 1.709806     6.683016
```

```
#lowest mean
eb |>
  arrange(partial_mean) |>
  slice(1)
```

```
  cntry       u_j partial_mean
1    BG -1.510355     3.462854
```

```
se <- attr(ranef(m1, condVar=TRUE)[[1]], "postVar")
eb$se <- sqrt(se[1,1,])

d |>
  group_by(cntry) |>
  summarise(no_pool_mean = mean(ppltrst)) |>
  inner_join(eb, by=join_by(cntry)) |>
  ggplot(aes(x=reorder(cntry, partial_mean))) +
  geom_errorbar(aes(ymin = partial_mean - 1.96*se,
                    ymax = partial_mean + 1.96*se),
                width = 0.2) +
  geom_point(aes(y=partial_mean),
             color="orange") +
  geom_errorbar(aes(ymin = partial_mean,
                    ymax = no_pool_mean),
                width = 0.01,
                position = position_nudge(0.3),
                color="red") +
  geom_point(aes(y=no_pool_mean),
             color="steelblue",
             position=position_nudge(0.3)) +
  theme_minimal() +
  labs(
    title = "Partial means v. No-pooled means",
    x = "Country",
    y = "Estimated means"
  ) +
  theme(axis.text.x = element_text(size=7))
```

Partial means v. No−pooled means



It was the case that the partial means of countries with a low n differed the most from the their no pooled estimates as they were estimated to be closer to the overall mean.

Two Dimensions

10.

```r
d <- d |>
  mutate(eduyrs_scaled = rescale(eduyrs))

m3 <- lmer(ppltrst ~ eduyrs_scaled + (1 | cntry),
           data = d,
           REML = FALSE)

summary(m3)
```

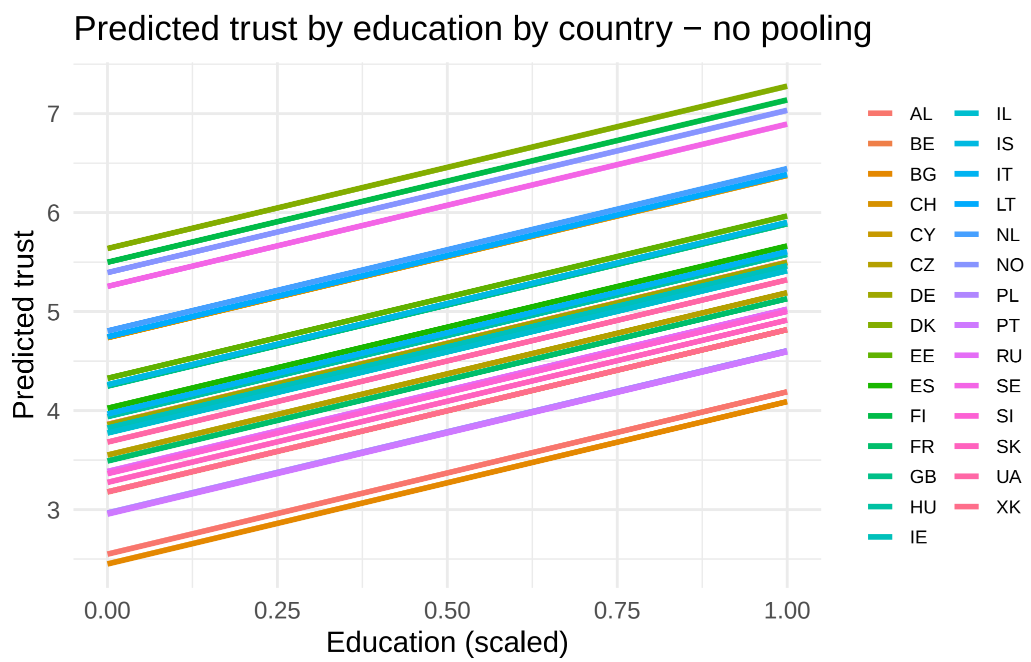
```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: ppltrst ~ eduyrs_scaled + (1 | cntry)
   Data: d

      AIC       BIC    logLik -2*log(L)  df.resid
```

```
  22409.6   22435.6  -11200.8   22401.6      4996

Scaled residuals:
     Min       1Q   Median       3Q      Max
-2.80179 -0.67975  0.07135  0.68638  3.01157

Random effects:
 Groups   Name        Variance Std.Dev.
 cntry    (Intercept) 0.8144   0.9024
 Residual             5.0896   2.2560
Number of obs: 5000, groups:  cntry, 29

Fixed effects:
              Estimate Std. Error t value
(Intercept)     3.9368     0.2131  18.471
eduyrs_scaled   1.6403     0.1789   9.167

Correlation of Fixed Effects:
            (Intr)
eduyrs_scld -0.531
```

The maximum effect for education on ppltrst is 1.6403. That means that, for the average country, the most that education can increase trust is 1.6403.

11.

```r
ggpredict(m3, terms=c("eduyrs_scaled", "cntry"), type="random") |>
  ggplot(aes(x = x , y = predicted , group = group, color = group )) +
  geom_smooth(formula= y ~ x,
              method = "lm",
              alpha = .5, se=FALSE) +
  theme_minimal() +
  labs(title = "Predicted trust by education by country - no pooling",
       y = "Predicted trust",
       x = "Education (scaled)",
       color = NULL) +
  theme(
    legend.position = "right",
    legend.text  = element_text(size = 7),
    legend.key.size = unit(0.4, "cm")
  )
```



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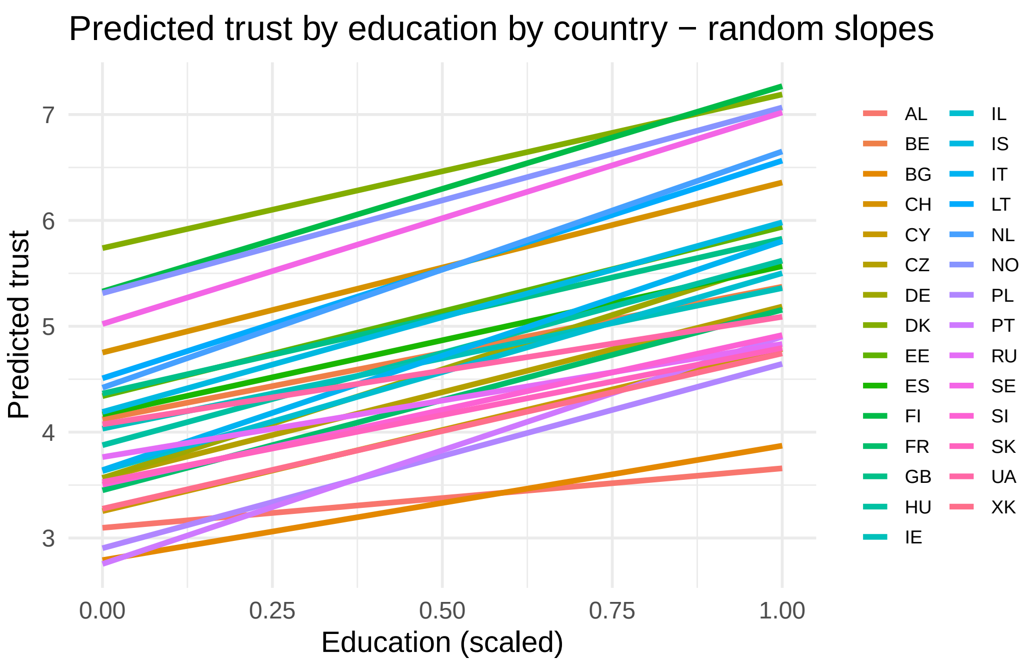
```
m4 <- lmer(ppltrst ~ eduyrs_scaled + (eduyrs_scaled | cntry),
           data = d,
           REML = FALSE)

ggpredict(m4, terms=c("eduyrs_scaled", "cntry"), type="random") |>
  ggplot(aes(x = x , y = predicted , group = group, color = group )) +
  geom_smooth(formula= y ~ x,
              method = "lm",
              alpha = .5, se=FALSE)  +
  theme_minimal() +
  labs(title = "Predicted trust by education by country - random slopes",
       y = "Predicted trust",
       x = "Education (scaled)",
       color = NULL) +
  theme(
    legend.position = "right",
    legend.text   = element_text(size = 7),
    legend.key.size = unit(0.4, "cm")
  )
```



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```r
compare_performance(m3, m4, metrics=c("BIC", "RMSE", "R2"))
```

```
Some of the nested models seem to be identical and probably only vary in
  their random effects.

# Comparison of Model Performance Indices

Name |   Model |   BIC (weights) | R2 (cond.) | R2 (marg.) |  RMSE
-------------------------------------------------------------------
m3   | lmerMod | 22435.6 (0.999) |      0.151 |      0.015 | 2.250
m4   | lmerMod | 22450.6 (<.001) |      0.155 |      0.014 | 2.246
```

The model without random slopes performs better. This means that the effect of education on trust does not vary significantly across countries.

14

```r
d <- d |>
  mutate(agea_scaled = rescale(agea),
         attend_scaled = rescale(attend),
         health_scaled = rescale(health))

m5 <- lmer(ppltrst ~ eduyrs_scaled + agea_scaled +
             I(agea_scaled^2) + attend_scaled +
             health_scaled + female + (1 | cntry),
           data = d,
           REML = FALSE)

ggeffect(m5, terms="agea_scaled") |>
  plot() +
  labs(title = "Effect of age on trust",
       y = "Predicted trust",
       x = "Age (scaled)")
```