FI7015-1 Cosmología
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Tarea 1

Fecha entrega: 30 de septiembre.

Perform the full calculations with all the steps; the numerical parts should be written in python and the code uploaded to your personal Github repository.

P1. Galaxies typically have some random peculiar velocity relative to the overall Hubble expansion. Assuming galaxy distance and redshift can be measured exactly, and the typical (root mean square) peculiar velocity is 600 km s^{-1}, how far away would a galaxy have to be before it could be used to determine H_0 to within 10 % accuracy? (consider the two cases where the true Hubble constant is $H_0 = 100$ km s^{-1}Mpc^{-1} and $H_0 = 70$ km s^{-1}Mpc^{-1}).

P2. You are expected to provide detailed and formal responses to the following concepts and questions, using your own words.

- **a)** The Cosmological Principle
- **b)** The different distances (physical and comoving, luminosity and angular). Why does the angular diameter distance decays for higher redshift?
- **c)** Observational constraints on the baryon density are often expressed in terms of $\omega_{\rm b} = \Omega_{\rm b} h^2$ instead of $\Omega_{\rm b}$. Why is this better?

P3. Derive the continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{1}$$

from the covariant derivative of the energy momentum tensor for a perfect fluid.

P4. Let us consider the entropy conservation before and after annihilation of electrons and positrons when the cosmic temperature was of the order of the electron mass. The entropy density for a particle with density ρ, pressure p, and temperature T is defined by $s = (\rho + p)/T$. Before the annihilation there were photons, neutrinos, anti-neutrinos, electrons, and positrons with the same temperatures, whereas after the annihilation there were photons, neutrinos, and anti-neutrinos with different temperatures. By using the entropy conservation as well as the fact that the neutrino temperature scales as $T_\nu \propto a^{-1}$, show that the relation between the photon temperature t_γ and the neutrinos temperatures T_ν is given by

$$\frac{T_\gamma}{T_\nu} = \left(\frac{11}{4}\right)^{1/3} \tag{2}$$

P5. Obtain the Hubble parameter in a Universe filled with radiation, matter, curvature and a general dark energy fluid with an equation of state parameter $w_{DE}(z)$. Evaluate the effective equation of state parameter of the dark energy fluid using the Hubble parameter.

P6. Using `CAMB`, plot the evolution of the density parameters for matter, radiation, dark energy and curvature.

P7. The distance duality relation gives

$$D_L(z) = (1+z)^2 D_A(z), \tag{3}$$

where $D_L(z)$ and $D_A(z)$ are the luminosity distance and angular diameter distance, respectively, as a function of the redshift z. Using `Python`, Plot the lines of constant $f_K(\chi)$ in the plane $\Omega_{\mathrm{m},0} - \Omega_{\Lambda,0}$, where

$$f_K(\chi) = \begin{cases} \dfrac{c}{H_0\sqrt{\Omega_{K,0}}} \sinh\left(\sqrt{\Omega_{K,0}} \displaystyle\int_0^z \frac{dx}{E(x)}\right) & \Omega_{k,0} > 0 \\ \dfrac{c}{H_0} \displaystyle\int_0^z \frac{dx}{E(x)} & \Omega_{k,0} = 0 \\ \dfrac{c}{H_0\sqrt{-\Omega_{K,0}}} \sin\left(\sqrt{-\Omega_{K,0}} \displaystyle\int_0^z \frac{dx}{E(x)}\right) & \Omega_{k,0} < 0. \end{cases} \tag{4}$$