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20.677 580-7

P1) $ds^2 = a^2(\tau)\left[-(1+2\psi(\vec{x},t))d\tau^2 + (1-2\phi(\vec{x},t))\delta_{ij}dx^i dx^j\right]$

$$\Rightarrow \quad g_{\mu\nu} = \begin{pmatrix} -a^2 - a^2 2\psi & 0 & 0 & 0 \\ 0 & a^2 - a^2 2\phi & 0 & 0 \\ 0 & 0 & a^2 - a^2 2\phi & 0 \\ 0 & 0 & 0 & a^2 - a^2 2\phi \end{pmatrix}$$

Nota: $dt = a(\tau)\,d\tau$, t cósmico, τ conforme.

aquí $\boxed{\bar{g}_{\mu\nu} = a^2 \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \text{ y } \delta g_{\mu\nu} = a^2 \begin{pmatrix} -2\psi & & & \\ & -2\phi & & \\ & & -2\phi & \\ & & & -2\phi \end{pmatrix}}$

queremos: $\delta G_{\mu\nu} = 8\pi G\, \delta T_{\mu\nu}$ $\quad c=1$

veamos los Christoffel

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\rho}\left(g_{\rho\mu,\nu} + g_{\nu\rho,\mu} - g_{\mu\nu,\rho}\right) \quad / \delta \quad \text{perturbando y despreciando términos } \delta^2$$

$$\delta\Gamma^\alpha_{\mu\nu} = \frac{1}{2}\delta g^{\alpha\rho}\left(\bar{g}_{\rho\mu,\nu} + \bar{g}_{\nu\rho,\mu} - \bar{g}_{\mu\nu,\rho}\right) + \frac{1}{2}\bar{g}^{\alpha\rho}\left(\delta g_{\rho\mu,\nu} + \delta g_{\nu\rho,\mu} - \delta g_{\mu\nu,\rho}\right)$$

Nota: g es diagonal $\Rightarrow \boxed{\rho = \alpha} \Rightarrow$ en la $\sum_\rho$ solo sobrevive 1 término, α $\Rightarrow$ No es necesario sumar.

Además, teorema del aux:

M invertible $\Rightarrow M^{-1}M = \mathbb{1}$

$\delta(M^{-1}M) = 0$

$\delta M^{-1}\cdot M + M^{-1}\delta M = 0$

$\delta M^{-1} = -M^{-1}\delta M \cdot M^{-1}$

$\Rightarrow \boxed{\delta g^{\mu\nu} = -g^{\mu\alpha} g^{\nu\beta}\delta g_{\alpha\beta}}$

será $\bar{g}$ por despreciar órdenes superiores

$$\delta g^{00} = -(-a^{-2})(-a^{-2})(-a^2 2\psi) = \frac{2\psi}{a^2}$$

$$\delta g^{0i} = 0$$

$$\delta g^{ij} = -(\bar{g}^{i\alpha})(\bar{g}^{j\beta})\delta g_{\alpha\beta} = -(a^{-2}\delta^{ia})(a^{-2}\delta^{jb})(-2\phi a^2 \delta_{ab})$$

$$= 2\phi a^{-2}\delta^{ia}\underbrace{\delta^{jb}\delta_{ab}}_{\delta^j_a}$$

$$= 2\phi a^{-2}\delta^{ij}$$

y sabiendo que

$$\boxed{\bar{g}^{\mu\nu} = a^{-2}\begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}}$$

$\Downarrow$

$$\boxed{\delta g^{\mu\nu} = a^{-2}\begin{pmatrix} 2\psi & & & \\ & 2\phi & & \\ & & 2\phi & \\ & & & 2\phi \end{pmatrix}}$$

Con esto:

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$$\delta\Gamma^0_{00} = \frac{1}{2}\delta g^{00}\left(\bar{g}_{00,0} + \bar{g}_{00,0} - \bar{g}_{00,0}\right) + \frac{1}{2}\bar{g}^{00}\left(\delta g_{00,0} + \delta g_{00,0} - \delta g_{00,0}\right)$$

$$= \frac{1}{2}\delta g^{00}\left(\bar{g}_{00,0}\right) + \frac{1}{2}\bar{g}^{00}\left(\delta g_{00,0}\right)$$

$$= \frac{1}{2}\left(a^{-2}2\psi\right)\cdot\left(-2a\dot{a}\right) + \frac{1}{2}\left(-a^{-2}\right)\cdot\underbrace{\left(-a^2 2\psi\right)^{\cdot}}_{\left(-2a\dot{a}2\psi - a^2 2\dot{\psi}\right)}$$

$$= \cancel{-a^{-1}2\dot{a}\psi} + \left(\cancel{+a^{-1}\dot{a}2\psi} + \dot{\psi}\right) = \boxed{\dot{\psi}}$$

$$\delta\Gamma^0_{ij} = \frac{1}{2}\delta g^{00}\left(\bar{g}_{0i,j} + \bar{g}_{j0,i} - \bar{g}_{ij,0}\right) + \frac{1}{2}\bar{g}^{00}\left(\delta g_{0i,j} + \delta g_{j0,i} - \delta g_{ij,0}\right)$$

$$= \frac{1}{2}\left(a^{-2}2\psi\right)\cdot\left(0 + 0 - 2a\dot{a}\delta_{ij}\right) + \frac{1}{2}\left(-a^{-2}\right)\cdot\left(0 + 0 - \left(-2a\dot{a}2\phi - a^2 2\dot{\phi}\right)\delta_{ij}\right)$$

$$= -a^{-2}\psi\cdot 2a\dot{a}\delta_{ij} + \frac{1}{2}a^{-2}\left(-2a\dot{a}\cancel{\phi 2} - a^2 2\dot{\phi}\right)\delta_{ij}$$

$$= -\frac{\dot{a}}{a}2\psi\delta_{ij} - \frac{\dot{a}}{a}2\phi\delta_{ij} - \dot{\phi}\delta_{ij}$$

$$= \boxed{-\delta_{ij}\left[2(\phi+\psi)\mathcal{H} + \dot{\phi}\right]}$$

recordar

$$\mathcal{H} = \frac{\dot{a}}{a} = \frac{1}{a}\frac{da}{d\tau}$$

$$H = \frac{1}{a}\cdot\frac{da}{dt}$$

$$\Rightarrow \boxed{\mathcal{H} = a\cdot H}$$

$$\delta\Gamma^0_{0i} = \delta\Gamma^0_{i0} = \frac{1}{2}\delta g^{00}\left(\bar{g}_{00,i} + \cancel{\bar{g}_{i0,0}} - \cancel{\bar{g}_{i0,0}}\right) + \frac{1}{2}\bar{g}^{00}\left(\delta g_{00,i} + \cancel{\delta g_{i0,0}} - \cancel{\delta g_{i0,0}}\right)$$

$$= \frac{1}{2}\left(a^{-2}2\psi\right)\cdot\underbrace{\partial_i\left(-a^2\right)}_{0} + \frac{1}{2}\left(-a^{-2}\right)\cdot\partial_i\left(-a^2 2\psi\right)$$

$$= \boxed{\partial_i\psi = \psi_{,i}}$$

¡No sumado

$$\delta\Gamma^i_{00} = \frac{1}{2}\delta g^{ii}\left(\overset{0}{\cancel{\bar{g}_{i0,0}}} + \overset{0}{\bar{g}_{i0,0}} - \bar{g}_{00,i}\right) + \frac{1}{2}\bar{g}^{ii}\left(\overset{0}{\cancel{\delta g_{i0,0}}} + \overset{0}{\cancel{\delta g_{i0,0}}} - \delta g_{00,i}\right)$$

¡ No sumado

$$= \frac{1}{2}\left(-a^2 2\phi\right)\cdot\left(-\underbrace{\left(-\partial_i a^2\right)}_{0}\right) + \frac{1}{2}\left(a^{-2}\right)\cdot\left(-\left(-a^2 2\psi_{,i}\right)\right) = \boxed{\psi_{,i}}$$

7.

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$$\delta\Gamma^i_{j0} = \delta\Gamma^i_{0j} = \frac{1}{2}\delta g^{ii}(\bar g_{ij,0} + \bar g_{i0,j} - \bar g_{j0,i}) + \frac{1}{2}\bar g^{ii}(\delta g_{ij,0} + \delta g_{i0,j} - \delta g_{j0,i})$$

i No sumado

$0 \quad 0 \quad 0 \quad 0$

$$= \frac{1}{2}(2\phi a^{-2})\cdot(2a\dot a\,\delta_{ij}) + \frac{1}{2}(a^{-2})\cdot\underbrace{(-a^2 2\phi)_{,0}}_{(-2a\dot a 2\phi - a^2 2\dot\phi)}\delta_{ij}$$

$$= \frac{\dot a}{a}\phi\, 2\delta_{ij} - \frac{\dot a}{a}2\phi\delta_{ij} - \dot\phi\,\delta_{ij} = \boxed{-\dot\phi\,\delta_{ij}}$$

$$\delta\Gamma^i_{jk} = \frac{1}{2}\delta g^{ii}(\bar g_{ij,k} + \bar g_{ik,j} - \bar g_{jk,i}) + \frac{1}{2}\bar g^{ii}(\delta g_{ij,k} + \delta g_{ik,j} - \delta g_{jk,i})$$

i No sumado

$$= \frac{1}{2}(a^{-2}\cdot 2\phi)\left[(a^2\delta_{ij})_{,k} + (a^2\delta_{ik})_{,j} - (a^2\delta_{jk})_{,i}\right]$$

$0 \quad 0 \quad 0$

$$+ \frac{1}{2}(+a^{-2})\left[(-a^2 2\phi\,\delta_{ij})_{,k} + (-a^2 2\phi\,\delta_{ik})_{,j} - (-a^2 2\phi\,\delta_{jk})_{,i}\right]$$

$$= -\phi_{,k}\delta_{ij} - \phi_{,j}\delta_{ik} + \phi_{,i}\delta_{jk}$$

$$= \boxed{(\delta_{jk}\partial_i - \delta_{ij}\partial_k - \delta_{ik}\partial_j)\phi}$$

Notamos que también requerimos los christoffel no perturbados de $\bar{g}_{\mu\nu} = \begin{pmatrix} -a^2 & & & \\ & a^2 & & \\ & & a^2 & \\ & & & a^2 \end{pmatrix}$

y con la coordenada temporal τ $\Rightarrow$ difieren un poco c/r a los de la tarea 1

$$\bar{\Gamma}^0_{00} = \frac{1}{2}\bar{g}^{00}(\bar{g}_{00,0}) = \frac{1}{2}(-a^{-2})\cdot(-2a\dot{a}) = \frac{\dot{a}}{a} = \mathcal{H}$$

$$\bar{\Gamma}^0_{ij} = \frac{1}{2}\bar{g}^{00}(\cancel{\bar{g}_{0i,j}} + \cancel{\bar{g}_{0j,i}} - \bar{g}_{ij,0}) = \frac{1}{2}(-a^{-2})\left(-(2a\dot{a})\delta_{ij}\right) = \mathcal{H}\delta_{ij}$$

$$\bar{\Gamma}^0_{0i} = \frac{1}{2}\bar{g}^{00}(\bar{g}_{00,i} + \cancel{\bar{g}_{0i,0}} - \cancel{\bar{g}_{0i,0}}) = \frac{1}{2}(-a^{-2})\cdot \underbrace{-a^2_{\ ,i}}_{0} = 0$$

$$\bar{\Gamma}^i_{00} = \frac{1}{2}\bar{g}^{ii}(\cancel{\bar{g}_{i0,0}} + \cancel{\bar{g}_{i0,0}} - \bar{g}_{00,i}) = \frac{1}{2}(a^{-2})\cdot(-\underbrace{(-a^2)_{,i}}_{0}) = 0$$

i no sumado

$$\bar{\Gamma}^i_{j0} = \frac{1}{2}\bar{g}^{ii}(\bar{g}_{ij,0} + \cancel{\bar{g}_{i0,j}} - \cancel{\bar{g}_{j0,i}}) = \frac{1}{2}(-a^{-2})\cdot\underbrace{(a^2\delta_{ij})_{,0}}_{2a\dot{a}\delta_{ij}} = \mathcal{H}\delta_{ij}$$

$$\bar{\Gamma}^i_{jk} = \frac{1}{2}\bar{g}^{ii}(\bar{g}_{ij,k} + \bar{g}_{ik,j} - \bar{g}_{jk,i}) = \frac{1}{2}(-a^{-2})\cdot 0 = 0$$

i no sumado

funciones de a, pero a no depende de coordenadas espaciales

$\Downarrow$

En resumen con todo:

y recordar que $\Gamma^\alpha_{\mu\nu} = \Gamma^\alpha_{\nu\mu}$

$$\boxed{\begin{aligned}
\Gamma^0_{00} &= \mathcal{H} + \dot{\psi} \\
\Gamma^0_{ij} &= \mathcal{H}\delta_{ij} - \delta_{ij}[2(\phi+\psi)\mathcal{H} + \dot{\phi}] \\
\Gamma^0_{0i} &= \partial_i\psi \\
\Gamma^i_{00} &= \partial_i\psi \\
\Gamma^i_{j0} &= \mathcal{H}\delta_{ij} - \dot{\phi}\delta_{ij} \\
\Gamma^i_{jk} &= (\delta_{jk}\partial_i - \delta_{ij}\partial_k - \delta_{ik}\partial_j)\phi
\end{aligned}}$$

ahora el tensor del Ricci

$$R_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\mu\alpha,\nu} + \Gamma^{\alpha}_{\beta\alpha}\Gamma^{\beta}_{\mu\nu} - \Gamma^{\alpha}_{\beta\nu}\Gamma^{\beta}_{\mu\alpha}$$

$$\Downarrow$$

$$\delta R_{\mu\nu} = \delta\Gamma^{\alpha}_{\mu\nu,\alpha} - \delta\Gamma^{\alpha}_{\mu\alpha,\nu} + \delta\Gamma^{\alpha}_{\beta\alpha}\Gamma^{\beta}_{\mu\nu} + \Gamma^{\alpha}_{\beta\alpha}\delta\Gamma^{\beta}_{\mu\nu} - \delta\Gamma^{\alpha}_{\beta\nu}\Gamma^{\beta}_{\mu\alpha} - \Gamma^{\alpha}_{\beta\nu}\delta\Gamma^{\beta}_{\mu\alpha}$$

$$\delta R_{00} = \delta\Gamma^{\alpha}_{00,\alpha} - \delta\Gamma^{\alpha}_{0\alpha,0} + \delta\Gamma^{\alpha}_{\beta\alpha}\Gamma^{\beta}_{00} + \Gamma^{\alpha}_{\beta\alpha}\delta\Gamma^{\beta}_{00} - \delta\Gamma^{\alpha}_{\beta 0}\Gamma^{\beta}_{0\alpha} - \Gamma^{\alpha}_{\beta 0}\delta\Gamma^{\beta}_{0\alpha}$$

$$= \cancel{\delta\Gamma^{0}_{00,0}} + \delta\Gamma^{i}_{00,i} - \cancel{\delta\Gamma^{0}_{00,0}} - \delta\Gamma^{i}_{0i,0}$$

$$+ \cancel{\delta\Gamma^{0}_{\beta 0}\Gamma^{\beta}_{00}} + \delta\Gamma^{i}_{\beta i}\Gamma^{\beta}_{00}$$

$$+ \cancel{\Gamma^{0}_{\beta 0}\delta\Gamma^{\beta}_{00}} + \Gamma^{i}_{\beta i}\delta\Gamma^{\beta}_{00}$$

$$- \cancel{\delta\Gamma^{0}_{\beta 0}\Gamma^{\beta}_{00}} - \delta\Gamma^{i}_{\beta 0}\Gamma^{\beta}_{0i}$$

$$- \cancel{\Gamma^{0}_{\beta 0}\delta\Gamma^{\beta}_{00}} - \Gamma^{i}_{\beta 0}\delta\Gamma^{\beta}_{0i}$$

$$= \delta\Gamma^{i}_{00,i} - \delta\Gamma^{i}_{0i,0}$$

$$+ \delta\Gamma^{i}_{0i}\Gamma^{0}_{00} + \cancel{\underbrace{\delta\Gamma^{i}_{ji}}_{\sim\phi}\underbrace{\Gamma^{j}_{00}}_{\sim\psi}} \quad \text{orden 2}$$

$$+ \Gamma^{i}_{0i}\delta\Gamma^{0}_{00} + \cancel{\underbrace{\Gamma^{i}_{ji}}_{\sim\phi}\underbrace{\delta\Gamma^{j}_{00}}_{\sim\psi}} \rightarrow \text{orden 2}$$

$$- \cancel{\underbrace{\delta\Gamma^{i}_{00}}_{\sim\psi}\underbrace{\Gamma^{0}_{0i}}_{\sim\psi}} \; \text{orden 2} - \delta\Gamma^{i}_{j0}\Gamma^{j}_{0i}$$

$$- \cancel{\underbrace{\Gamma^{i}_{00}}_{\sim\psi}\underbrace{\delta\Gamma^{0}_{0i}}_{\psi}} \; \text{orden 2} - \Gamma^{i}_{j0}\delta\Gamma^{j}_{0i}$$

$$\Rightarrow \delta R_{00} = \delta\Gamma^{i}_{00,i} - \delta\Gamma^{i}_{0i,0}$$

$$+ \delta\Gamma^{i}_{0i}\Gamma^{0}_{00}$$

$$+ \Gamma^{i}_{0i}\delta\Gamma^{0}_{00}$$

$$- \delta\Gamma^{i}_{j0}\Gamma^{j}_{0i}$$

$$- \Gamma^{i}_{j0}\delta\Gamma^{j}_{0i}$$

$$\delta R_{00} = (\partial_i \psi)_{,i} - (-\dot{\phi}\cdot 3)_{,0} + (-3\dot{\phi})\cdot(\mathcal{H}+\dot{\psi}) + (3(\mathcal{H}-\dot{\phi})\cdot\dot{\psi})$$

$$- \underbrace{(-\dot{\phi}\,\delta_{ij})\cdot(\mathcal{H}-\dot{\phi})\cdot\delta_{ji}}_{-3\dot{\phi}(\mathcal{H}-\dot{\phi})} - \underbrace{(\mathcal{H}-\dot{\phi})\delta_{ij}\cdot(-\dot{\phi}\,\delta_{ji})}_{-3\dot{\phi}(\mathcal{H}-\dot{\phi})}$$

son 2 Σ pero viven solo $i=j$

$$= \nabla^2\psi + 3\ddot{\phi} - 3\dot{\phi}(\mathcal{H}+\overset{\text{orden 2}}{\cancel{\dot{\psi}}}) + 3\dot{\psi}(\mathcal{H}-\overset{\text{orden 2}}{\cancel{\dot{\phi}}}) + 6\dot{\phi}(\mathcal{H}-\overset{\text{orden 2}}{\cancel{\dot{\phi}}})$$

$$= \nabla^2\psi + 3\ddot{\phi} - 3\dot{\phi}\mathcal{H} + 3\dot{\psi}\mathcal{H} + 6\dot{\phi}\mathcal{H}$$

$$= \nabla^2\psi + 3\ddot{\phi} + 3\dot{\phi}\mathcal{H} + 3\dot{\psi}\mathcal{H}$$

$$\Rightarrow \boxed{\delta R_{00} = \nabla^2\psi + 3\ddot{\phi} + 3\mathcal{H}(\dot{\phi}+\dot{\psi})}$$

$$\delta R_{0i} = \delta R_{i0} = \delta\Gamma^{\alpha}_{i0,\alpha} - \delta\Gamma^{\alpha}_{i\alpha,0} + \delta\Gamma^{\alpha}_{\beta\alpha}\Gamma^{\beta}_{i0} + \Gamma^{\alpha}_{\beta\alpha}\delta\Gamma^{\beta}_{i0} - \delta\Gamma^{\alpha}_{\beta 0}\Gamma^{\beta}_{i\alpha} - \Gamma^{\alpha}_{\beta 0}\delta\Gamma^{\beta}_{i\alpha}$$

$$= \cancel{\delta\Gamma^{0}_{i0,0}} + \delta\Gamma^{j}_{i0,j} - \cancel{\delta\Gamma^{0}_{i0,0}} - \delta\Gamma^{j}_{ij,0}$$

$$+ \cancel{\delta\Gamma^{0}_{\beta 0}}\Gamma^{\beta}_{i0} + \delta\Gamma^{j}_{\beta j}\Gamma^{\beta}_{i0}$$

$$+ \Gamma^{0}_{\beta 0}\cancel{\delta\Gamma^{\beta}_{i0}} + \Gamma^{j}_{\beta j}\delta\Gamma^{\beta}_{i0}$$

$$- \cancel{\delta\Gamma^{0}_{\beta 0}}\Gamma^{\beta}_{i0} - \delta\Gamma^{j}_{\beta 0}\Gamma^{\beta}_{ij}$$

$$- \Gamma^{0}_{\beta 0}\cancel{\delta\Gamma^{\beta}_{i0}} - \Gamma^{j}_{\beta 0}\delta\Gamma^{\beta}_{ij}$$

$\Downarrow$

$$\delta R_{i0} = \delta\Gamma^{j}_{i0,j} - \delta\Gamma^{j}_{ij,0}$$

$$+ \underbrace{\cancel{\delta\Gamma^{j}_{0j}}}_{\sim\phi}\underbrace{\Gamma^{0}_{i0}}_{\mathcal{H}} + \delta\Gamma^{j}_{kj}\Gamma^{k}_{i0}$$

orden 2

$$+ \Gamma^{j}_{0j}\delta\Gamma^{0}_{i0} + \underbrace{\cancel{\Gamma^{j}_{kj}}}_{\phi}\underbrace{\cancel{\delta\Gamma^{k}_{i0}}}_{\sim\phi}$$

orden 2

$$- \delta\Gamma^{j}_{00}\Gamma^{0}_{ij} - \cancel{\delta\Gamma^{j}_{k0}\Gamma^{k}_{ij}}$$

orden 2

$$- \cancel{\Gamma^{j}_{00}\delta\Gamma^{0}_{ij}} - \Gamma^{j}_{k0}\delta\Gamma^{k}_{ij}$$

orden 2

$$\delta R_{i0} = \delta\Gamma^{j}_{i0,j} - \delta\Gamma^{j}_{ij,0} + \delta\Gamma^{j}_{kj}\Gamma^{k}_{i0} + \Gamma^{j}_{0j}\delta\Gamma^{0}_{i0} - \delta\Gamma^{j}_{00}\Gamma^{0}_{ij} - \Gamma^{j}_{k0}\delta\Gamma^{k}_{ij}$$

$$= (-\dot\phi\,\delta_{ji})_{,j} - (-\partial_i\phi - 2\partial_i\phi)_{,0}$$

orden 2

$$+ (\delta_{kj}\partial_j - \delta_{jk}\partial_j - \delta_{ij}\partial_k)\phi\cdot(\mathcal{H}\underbrace{\delta_{ki}}_{\Rightarrow k=i} - \cancel{\dot\phi\,\delta_{ki}})$$

$\Downarrow$

$$\boxed{\delta\Gamma^{j}_{ij} = -3\partial_i\phi}$$

$$+ 3\mathcal{H}\cdot\partial_i\psi$$

$$- \partial_j\psi\cdot\mathcal{H}\delta_{ij}$$

$$- \mathcal{H}\underbrace{\delta_{jk}}_{j=k}\cdot(\delta_{ij}\partial_k - \delta_{ki}\partial_j - \delta_{kj}\partial_i)\phi$$

$$= -\dot\phi_{,i} + 3\,\dot\phi_{,i}$$

i no sumado

$$+ \mathcal{H}\cdot(\delta_{ij}\partial_j - \delta_{ji}\partial_j - \delta_{jj}\partial_i)\phi$$

$$+ 3\mathcal{H}\partial_i\psi$$

$$- \mathcal{H}\partial_i\psi$$

$$- \mathcal{H}(\delta_{ij}\partial_j - \delta_{ji}\partial_j - \delta_{jj}\partial_i)\phi$$

i no sumado

$$\Rightarrow = 2\dot\phi_{,i} + \mathcal{H}\cdot\cancel{(-\partial_i\phi - 2\partial_i\phi)}$$

$$+ 2\mathcal{H}\partial_i\psi$$

$$- \mathcal{H}\cancel{(-\partial_i\phi - 2\partial_i\phi)}$$

$$= 2(\dot\phi_{,i} + \mathcal{H}\psi_{,i})$$

$\mathcal{H}$ no depende del espacio

$$\boxed{\delta R_{i0} = 2(\dot\phi + \mathcal{H}\psi)_{,i}}$$

$$\delta R_{ij} = \delta\Gamma^{\alpha}_{ij,\alpha} - \delta\Gamma^{\alpha}_{i\alpha,j} + \delta\Gamma^{\alpha}_{\beta\alpha}\Gamma^{\beta}_{ij} + \Gamma^{\alpha}_{\beta\alpha}\delta\Gamma^{\beta}_{ij} - \delta\Gamma^{\alpha}_{\beta j}\Gamma^{\beta}_{i\alpha} - \Gamma^{\alpha}_{\beta j}\delta\Gamma^{\beta}_{i\alpha}$$

$$= \delta\Gamma^{0}_{ij,0} + \delta\Gamma^{k}_{ij,k} - \delta\Gamma^{0}_{i0,j} - \delta\Gamma^{k}_{ik,j}$$

$$+ \delta\Gamma^{0}_{\beta 0}\Gamma^{\beta}_{ij} + \delta\Gamma^{k}_{\beta k}\Gamma^{\beta}_{ij}$$

$$+ \Gamma^{0}_{\beta 0}\delta\Gamma^{\beta}_{ij} + \Gamma^{k}_{\beta k}\delta\Gamma^{\beta}_{ij}$$

$$- \delta\Gamma^{0}_{\beta j}\Gamma^{\beta}_{i0} - \delta\Gamma^{k}_{\beta j}\Gamma^{\beta}_{ik}$$

$$- \Gamma^{0}_{\beta j}\delta\Gamma^{\beta}_{i0} - \Gamma^{k}_{\beta j}\delta\Gamma^{\beta}_{ik}$$

$$= \delta\Gamma^{0}_{ij,0} + \delta\Gamma^{k}_{ij,k} - \delta\Gamma^{0}_{i0,j} - \delta\Gamma^{k}_{ik,j}$$

$$+ \delta\Gamma^{0}_{00}\Gamma^{0}_{ij} + \overset{\text{orden 2}}{\cancel{\delta\Gamma^{0}_{k0}\Gamma^{k}_{ij}}} + \delta\Gamma^{k}_{0k}\Gamma^{0}_{ij} + \overset{\text{orden 2}}{\cancel{\delta\Gamma^{k}_{lk}\Gamma^{l}_{ij}}}$$

$$+ \Gamma^{0}_{00}\delta\Gamma^{0}_{ij} + \overset{\text{orden 2}}{\cancel{\Gamma^{0}_{k0}\delta\Gamma^{k}_{ij}}} + \Gamma^{k}_{0k}\delta\Gamma^{0}_{ij} + \overset{\text{orden 2}}{\cancel{\Gamma^{k}_{lk}\delta\Gamma^{l}_{ij}}}$$

$$- \overset{\text{orden 2}}{\cancel{\delta\Gamma^{0}_{0j}\Gamma^{0}_{i0}}} - \underbrace{\delta\Gamma^{0}_{kj}}_{k=j}\Gamma^{k}_{i0} - \underbrace{\delta\Gamma^{k}_{0j}}_{k=j}\Gamma^{0}_{ik} - \overset{\text{orden 2}}{\cancel{\delta\Gamma^{k}_{lj}\Gamma^{l}_{ik}}}$$

$$- \overset{\text{orden 2}}{\cancel{\Gamma^{0}_{0j}\delta\Gamma^{0}_{i0}}} - \underbrace{\Gamma^{0}_{kj}}_{k=j}\delta\Gamma^{k}_{i0} - \Gamma^{k}_{0j}\underbrace{\delta\Gamma^{0}_{ik}}_{k=i} - \overset{\text{orden 2}}{\cancel{\Gamma^{k}_{lj}\delta\Gamma^{l}_{ik}}}$$

$$= \left(-\delta_{ij}\left[2(\dot{\phi}+\dot{\psi})\mathcal{H} + 2(\phi+\psi)\dot{\mathcal{H}} + \ddot{\phi}\right] + \left(\delta_{ij}\phi_{,kk} - \delta_{ki}\phi_{,jk} - \delta_{kj}\phi_{,ik}\right)\right.$$

$$- \psi_{,ij} - (-3\phi_{,ij}) + \dot{\psi}\mathcal{H}\delta_{ij} + (-\dot{\phi}3)\cdot\mathcal{H}\delta_{ij}$$

$$+ \mathcal{H}\cdot(-\delta_{ij})(2(\phi+\psi)\mathcal{H}+\dot{\phi}) + 3\mathcal{H}\cdot(-\delta_{ij}[2(\phi+\psi)\mathcal{H}+\dot{\phi}])$$

$$- (-(2(\phi+\psi)\mathcal{H}+\dot{\phi}))\cdot\mathcal{H}\delta_{ji} - (-\dot{\phi})\cdot(\mathcal{H}\delta_{ij})$$

$$- (\mathcal{H})\cdot(-\dot{\phi}\,\delta_{ji}) - \mathcal{H}\delta_{ij}\cdot[-2(\phi+\psi)\mathcal{H}-\dot{\phi}]$$

$\Downarrow$

$$\delta R_{ij} = -2(\dot\phi+\dot\psi)\mathcal{H}\delta_{ij} - 2(\phi+\psi)\dot{\mathcal{H}}\delta_{ij} - \ddot\phi\delta_{ij}$$
$$+\delta_{ij}\nabla^2\phi - \phi_{,ji} - \phi_{,ij} - \psi_{,ij} + 3\phi_{,ij}$$
$$+\dot\psi\mathcal{H}\delta_{ij} - \dot\phi\mathcal{H}\delta_{ij}\cdot 3$$
$$-2\cancel{(\phi+\psi)\mathcal{H}^2\delta_{ij}} - \mathcal{H}\cancel{\dot\phi\delta_{ij}} - \overset{4}{\cancel{6(\phi+\psi)\mathcal{H}^2}}\delta_{ij} - \cancel{\mathcal{H}\dot\phi}\delta_{ij}3$$
$$+2\cancel{(\phi+\psi)}\mathcal{H}^2\delta_{ij} + \dot\phi\cancel{\mathcal{H}\delta_{ij}} + \dot\phi\cancel{\mathcal{H}}\delta_{ij}$$
$$+\mathcal{H}\cancel{\dot\phi}\delta_{ij} + 2\cancel{(\phi+\psi)\mathcal{H}^2}\delta_{ij} + \dot\phi\cancel{\mathcal{H}}\delta_{ij}$$

$$= -2\dot\phi\mathcal{H}\delta_{ij} - 2\dot\psi\mathcal{H}\delta_{ij} - 2\phi\dot{\mathcal{H}}\delta_{ij} - 2\psi\dot{\mathcal{H}}\delta_{ij} - \ddot\phi\delta_{ij}$$
$$+\delta_{ij}\nabla^2\phi - \phi_{,ji} - \phi_{,ij} - \psi_{,ij} + 3\phi_{,ij}$$
$$+\dot\psi\mathcal{H}\delta_{ij} - 3\dot\phi\mathcal{H}\delta_{ij} - 4(\phi+\psi)\mathcal{H}^2\delta_{ij}$$

$$= \nabla^2\phi\,\delta_{ij} + \phi_{,ij} - \psi_{,ij} - \ddot\phi\,\delta_{ij} - 2\dot{\mathcal{H}}\delta_{ij}(\phi+\psi)$$
$$- \mathcal{H}\delta_{ij}(5\dot\phi+\dot\psi) - 4\mathcal{H}^2\delta_{ij}(\phi+\psi)$$

$$\boxed{\delta R_{ij} = \left[-\ddot\phi + \nabla^2\phi - \mathcal{H}(\dot\psi+5\dot\phi) - (2\dot{\mathcal{H}}+4\mathcal{H}^2)(\phi+\psi)\right]\delta_{ij} + (\phi-\psi)_{,ij}}$$

Faltan los no perturbados:

$$\bar R_{\mu\nu} = \bar\Gamma^{\alpha}_{\mu\nu,\alpha} - \bar\Gamma^{\alpha}_{\mu\alpha,\nu} + \bar\Gamma^{\alpha}_{\beta\alpha}\bar\Gamma^{\beta}_{\mu\nu} - \bar\Gamma^{\alpha}_{\beta\nu}\bar\Gamma^{\beta}_{\mu\alpha}$$

$$\bar R_{00} = \bar\Gamma^{\alpha}_{00,\alpha} - \bar\Gamma^{\alpha}_{0\alpha,0} + \bar\Gamma^{\alpha}_{\beta\alpha}\bar\Gamma^{\beta}_{00} - \bar\Gamma^{\alpha}_{\beta0}\bar\Gamma^{\beta}_{0\alpha}$$

$$= \cancel{\bar\Gamma^{0}_{00,0}} + \cancelto{0}{\bar\Gamma^{i}_{00,i}} - \cancel{\bar\Gamma^{0}_{00,0}} - \bar\Gamma^{i}_{0i,0}$$

$$+ \cancel{\bar\Gamma^{0}_{\beta0}\bar\Gamma^{\beta}_{00}} + \bar\Gamma^{i}_{\beta i}\bar\Gamma^{\beta}_{00} - \cancel{\bar\Gamma^{0}_{\beta0}\bar\Gamma^{\beta}_{00}} - \bar\Gamma^{i}_{\beta0}\bar\Gamma^{\beta}_{0i}$$

$$= -\bar\Gamma^{i}_{0i,0} + \bar\Gamma^{i}_{0i}\bar\Gamma^{0}_{00} + \bar\Gamma^{i}_{ji}\cancelto{0}{\bar\Gamma^{j}_{00}} - \cancelto{0}{\bar\Gamma^{i}_{00}}\bar\Gamma^{0}_{0i} - \underbrace{\bar\Gamma^{i}_{l0}}_{i=l}\bar\Gamma^{l}_{0i}$$

$$= -3\dot{\mathcal{H}} + 3\mathcal{H}^2 - \mathcal{H}^2\cdot 3$$

$$= \boxed{-3\dot{\mathcal{H}}}$$

$$\bar R_{i0} = \bar\Gamma^{\alpha}_{i0,\alpha} - \bar\Gamma^{\alpha}_{i\alpha,0} + \bar\Gamma^{\alpha}_{\beta\alpha}\bar\Gamma^{\beta}_{i0} - \bar\Gamma^{\alpha}_{\beta0}\bar\Gamma^{\beta}_{i\alpha}$$

$$= \cancelto{0}{\bar\Gamma^{0}_{i0,0}} + \underbrace{\bar\Gamma^{j}_{i0,j}}_{\mathcal{H}_{,j}=0} - \cancelto{0}{\bar\Gamma^{0}_{i0,0}} - \cancelto{0}{\bar\Gamma^{j}_{ij,0}} + \cancel{\bar\Gamma^{0}_{\beta0}\bar\Gamma^{\beta}_{i0}} + \bar\Gamma^{j}_{\beta j}\bar\Gamma^{\beta}_{i0} - \cancel{\bar\Gamma^{0}_{\beta0}\bar\Gamma^{\beta}_{i0}} - \bar\Gamma^{j}_{\beta0}\bar\Gamma^{\beta}_{ij}$$

$$= \bar\Gamma^{j}_{0j}\cancelto{0}{\bar\Gamma^{0}_{i0}} + \cancelto{0}{\bar\Gamma^{j}_{lj}}\bar\Gamma^{l}_{i0} - \cancelto{0}{\bar\Gamma^{j}_{00}}\bar\Gamma^{0}_{ij} - \bar\Gamma^{j}_{l0}\cancelto{0}{\bar\Gamma^{l}_{ij}} = 0$$

$$\bar R_{ij} = \bar\Gamma^{\alpha}_{ij,\alpha} - \bar\Gamma^{\alpha}_{i\alpha,j} + \bar\Gamma^{\alpha}_{\beta\alpha}\bar\Gamma^{\beta}_{ij} - \bar\Gamma^{\alpha}_{\beta j}\bar\Gamma^{\beta}_{i\alpha}$$

$$= \bar\Gamma^{0}_{ij,0} + \cancelto{0}{\bar\Gamma^{k}_{ij,k}} - \cancelto{0}{\bar\Gamma^{0}_{i0,j}} - \cancelto{0}{\bar\Gamma^{k}_{ik,j}} + \bar\Gamma^{0}_{\beta0}\bar\Gamma^{\beta}_{ij} + \bar\Gamma^{k}_{\beta k}\bar\Gamma^{\beta}_{ij} - \bar\Gamma^{0}_{\beta j}\bar\Gamma^{\beta}_{i0} - \bar\Gamma^{k}_{\beta j}\bar\Gamma^{\beta}_{ik}$$

$$= \bar\Gamma^{0}_{ij,0} + \bar\Gamma^{0}_{00}\bar\Gamma^{0}_{ij} + \cancelto{0}{\bar\Gamma^{0}_{k0}}\bar\Gamma^{k}_{ij} + \bar\Gamma^{k}_{0k}\bar\Gamma^{0}_{ij} + \cancelto{0}{\bar\Gamma^{k}_{lk}}\bar\Gamma^{l}_{ij} - \cancelto{0}{\bar\Gamma^{0}_{0j}}\bar\Gamma^{0}_{i0} - \underbrace{\bar\Gamma^{0}_{kj}}_{k=j}\underbrace{\bar\Gamma^{k}_{i0}}_{k=i}$$

$$- \bar\Gamma^{k}_{0j}\bar\Gamma^{0}_{ik} - \cancelto{0}{\bar\Gamma^{k}_{lj}}\bar\Gamma^{l}_{ik}$$

$$\Downarrow$$

$$\bar{R}_{ij} = \dot{\mathcal{H}}\delta_{ij} + \cancel{\mathcal{H}\cdot\mathcal{H}\delta_{ij}} + 3\mathcal{H}\cdot\mathcal{H}\delta_{ij} - \cancel{(\mathcal{H}\cdot\mathcal{H}\delta_{ij})} - (\mathcal{H}\mathcal{H}\delta_{ij})$$

$$= \dot{\mathcal{H}}\delta_{ij} + 2\mathcal{H}^2\delta_{ij}$$

así

$$\boxed{\begin{aligned} R_{00} &= -3\dot{\mathcal{H}} + \nabla^2\psi + 3\ddot{\phi} + 3\mathcal{H}(\dot{\phi}+\dot{\psi}) \\ R_{0i} = R_{i0} &= 2(\dot{\phi} + \psi\mathcal{H})_{,i} \\ R_{ij} &= (\dot{\mathcal{H}} + 2\mathcal{H}^2)\delta_{ij} + [-\ddot{\phi} + \nabla^2\phi - \mathcal{H}(\dot{\psi} + 5\dot{\phi}) - (2\dot{\mathcal{H}} + 4\mathcal{H}^2)(\phi+\psi)]\delta_{ij} + (\phi-\psi)_{,ij} \end{aligned}}$$

ahora el escalar de Ricci

$$R = g^{\mu\nu}R_{\mu\nu} \Rightarrow (\bar{g}^{\mu\nu} + \delta g^{\mu\nu})(\bar{R}_{\mu\nu} + \delta R_{\mu\nu})$$

$$= \underbrace{\bar{g}^{\mu\nu}\bar{R}_{\mu\nu}}_{\bar{R}} + \underbrace{\bar{g}^{\mu\nu}\delta R_{\mu\nu} + \delta g^{\mu\nu}\bar{R}_{\mu\nu}}_{\delta R}$$

$$\Rightarrow \delta R = \bar{g}^{0\nu}\delta R_{0\nu} + \bar{g}^{i\nu}\delta R_{i\nu} + \delta g^{0\nu}\bar{R}_{0\nu} + \delta g^{i\nu}\bar{R}_{i\nu}$$

$$= \bar{g}^{00}\delta R_{00} + \underbrace{\cancel{\bar{g}^{0i}}}_{0}\delta R_{0i} + \underbrace{\cancel{\bar{g}^{i0}}}_{0}\delta R_{i0} + \underbrace{\bar{g}^{ij}}_{i=j}\delta R_{ij} + \delta g^{00}\bar{R}_{00} + \underbrace{\cancel{\delta g^{0i}}}_{0}\bar{R}_{0i} + \underbrace{\cancel{\delta g^{i0}}}_{0}\bar{R}_{i0} + \overbrace{\delta g^{ij}}^{i=j}\bar{R}_{ij}$$

$$= (-a^{-2})\cdot(\nabla^2\psi + 3\ddot{\phi} + 3\mathcal{H}(\dot{\phi}+\dot{\psi})) + (a^{-2})\cdot\{3[-\ddot{\phi} + \nabla^2\phi - \mathcal{H}(\dot{\psi}+5\dot{\phi}) - (2\dot{\mathcal{H}} + 4\mathcal{H}^2)(\phi+\psi)] + (\phi-\psi)_{,ii}\}$$

$$+ (2\psi a^{-2})\cdot(-3\dot{\mathcal{H}}) + 3\cdot(a^{-2}2\phi)(\dot{\mathcal{H}} + 2\mathcal{H}^2)$$

al sumarlos 3 veces da ∇^2

$$= -a^{-2}\nabla^2\psi - a^{-2}3\ddot{\phi} - a^{-2}3\mathcal{H}(\dot{\phi}+\dot{\psi}) - a^{-2}3\ddot{\phi} + a^{-2}3\nabla^2\phi - 3a^{-2}\mathcal{H}(\dot{\psi}+5\dot{\phi})$$

$$-3a^{-2}(2\dot{\mathcal{H}} + 4\mathcal{H}^2)(\phi+\psi) + a^{-2}(\nabla^2\phi - \nabla^2\psi) - 3\dot{\mathcal{H}}a^{-2}2\psi + \cancel{6a^{-2}\phi\dot{\mathcal{H}}} + \cancel{12a^{-2}\phi\mathcal{H}^2}$$

$$\downarrow$$

$$-6a^{-2}\dot{\mathcal{H}}(\cancel{\phi}+\psi) - 12a^{-2}\mathcal{H}^2(\cancel{\phi}+\psi)$$

Escaneado con CamScanner

$$\delta R = -2a^{-2}\nabla^2\psi - 6a^{-2}\ddot{\phi} - 18\,a^{-2}\mathcal{H}\dot{\phi} - 6a^{-2}\mathcal{H}\dot{\psi}$$
$$+4a^{-2}\nabla^2\phi - 12a^{-2}\dot{\mathcal{H}}\psi - 12a^{-2}\mathcal{H}^2\psi$$

$$\boxed{\delta R = a^{-2}\left[-6\ddot{\phi} - 2\nabla^2\psi + 4\nabla^2\phi - \mathcal{H}(18\dot{\phi} + 6\dot{\psi}) - 12(\dot{\mathcal{H}} + \mathcal{H}^2)\psi\right]}$$

$$\bar{R} = \bar{g}^{0\nu}\bar{R}_{0\nu} + \bar{g}^{i\nu}\cdot\bar{R}_{i\nu} = \bar{g}^{00}\bar{R}_{00} + \overset{0}{\bar{g}^{0i}}\bar{R}_{0i} + \overset{0}{\bar{g}^{i0}}\bar{R}_{i0} + \bar{g}^{ij}\bar{R}_{ij}$$

$$= \underbrace{-a^{-2}\cdot(-3\dot{\mathcal{H}})}_{3\dot{\mathcal{H}}a^{-2}} + 3\cdot a^{-2}\cdot(\dot{\mathcal{H}} + 2\mathcal{H}^2)$$

$$\boxed{\bar{R} = 6\,a^{-2}(\dot{\mathcal{H}} + \mathcal{H}^2)}$$

ahora $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \bar{R}_{\mu\nu} + \delta R_{\mu\nu} - \frac{1}{2}(\bar{g}_{\mu\nu} + \delta g_{\mu\nu})(\bar{R} + \delta R)$

$$= \bar{R}_{\mu\nu} + \delta R_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} - \frac{1}{2}\bar{g}_{\mu\nu}\cdot\delta R - \frac{1}{2}\delta g_{\mu\nu}\bar{R}$$

$$= \bar{G}_{\mu\nu} + \underbrace{\delta R_{\mu\nu} - \frac{1}{2}(\bar{g}_{\mu\nu}\delta R + \delta g_{\mu\nu}\bar{R})}_{\delta G_{\mu\nu}}$$

$$\Rightarrow \quad \bar{G}_{00} = \bar{R}_{00} - \frac{1}{2}\bar{g}_{00}\bar{R} = -3\dot{\mathcal{H}} + \frac{1}{2}(+\cancel{a^2})\cdot 6\cancel{a^{-2}}(\dot{\mathcal{H}} + \mathcal{H}^2)$$

$$\boxed{\bar{G}_{00} = 3\mathcal{H}^2}$$

$$\bar{G}_{0i} = \bar{G}_{i0} = \bar{R}_{i0} - \frac{1}{2}\bar{g}_{i0}\bar{R} = \boxed{0}$$

$$\bar{G}_{ij} = \bar{R}_{ij} - \frac{1}{2}\bar{g}_{ij}\bar{R} = (\dot{\mathcal{H}} + 2\mathcal{H}^2)\delta_{ij} - \frac{1}{2}\cdot a^2\cdot 6a^{-2}(\dot{\mathcal{H}} + \mathcal{H}^2)\,\delta_{ij}$$

$$= \dot{\mathcal{H}}\delta_{ij} + 2\mathcal{H}^2\delta_{ij} - 3\dot{\mathcal{H}}\delta_{ij} - 3\mathcal{H}^2\delta_{ij} = \boxed{(-2\dot{\mathcal{H}} - \mathcal{H}^2)\delta_{ij}}$$

$$\delta G_{00} = \delta R_{00} - \frac{1}{2}\bar{g}_{00}\delta R - \frac{1}{2}\delta g_{00}\bar{R}$$

$$= \cancel{\nabla^2\psi} + \cancel{3\ddot{\phi}} + 3\mathcal{H}(\dot{\phi} + \cancel{\dot{\psi}}) + \frac{a^2}{2}\cdot a^{-2}\left(-6\cancel{\ddot{\phi}} - 2\cancel{\nabla^2\psi} + 4\nabla^2\phi - \mathcal{H}(18\dot{\phi} + \cancel{6\dot{\psi}}) - 12(\dot{\mathcal{H}} + \mathcal{H}^2)\psi\right)$$

$$+ \frac{a^2}{\cancel{2}}\cdot \cancel{2}\psi \cdot 6a^{-2}(\dot{\mathcal{H}} + \mathcal{H}^2)$$

$$\boxed{\delta G_{00} = -6\mathcal{H}\dot{\phi} + 2\nabla^2\phi}$$

$g_{i0} = 0$

$$\delta G_{0i} = \delta G_{i0} = \delta R_{i0} - 0$$

$$\boxed{\delta G_{i0} = \delta R_{i0}}$$

$$\delta G_{ij} = \delta R_{ij} - \frac{1}{2}\bar{g}_{ij}\delta R - \frac{1}{2}\delta g_{ij}\bar{R}$$

$$= \left[-\ddot{\phi} + \nabla^2\phi - \mathcal{H}(\dot{\psi} + 5\dot{\phi}) - (2\dot{\mathcal{H}} + 4\mathcal{H}^2)(\phi + \psi)\right]\delta_{ij} + (\phi - \psi)_{,ij}$$

$$+ \frac{1}{2}a^2\cdot a^{-2}\left[+6\ddot{\phi} + 2\nabla^2\psi - 4\nabla^2\phi + \mathcal{H}(18\dot{\phi} + 6\dot{\psi}) + 12(\dot{\mathcal{H}} + \mathcal{H}^2)\psi\right]\delta_{ij}$$

$$+ \frac{1}{\cancel{2}}\cdot\cancel{2}\phi\, a^2 \cdot 6\cdot a^{-2}(\dot{\mathcal{H}} + \mathcal{H}^2)\,\delta_{ij}$$

$$= \left(2\ddot{\phi} - \nabla^2\phi + 2\mathcal{H}\dot{\psi} + 4\mathcal{H}\dot{\phi} + (4\dot{\mathcal{H}} + 2\mathcal{H}^2)(\psi + \phi) + \nabla^2\psi\right)\delta_{ij} + (\phi - \psi)_{,ij}$$

$$\star \Rightarrow -2\dot{\mathcal{H}}\phi - 2\dot{\mathcal{H}}\psi - 4\mathcal{H}^2\phi - 4\mathcal{H}^2\psi + 6\dot{\mathcal{H}}\psi + 6\mathcal{H}^2\psi + 6\dot{\mathcal{H}}\phi + 6\mathcal{H}^2\phi$$

$$= 4\dot{\mathcal{H}}\phi + 4\dot{\mathcal{H}}\psi + 2\mathcal{H}^2\phi + 2\mathcal{H}^2\psi$$

$$= (4\dot{\mathcal{H}} + 2\mathcal{H}^2)(\psi + \phi)$$

$$\boxed{\delta G_{ij} = \left(2\ddot{\phi} + \nabla^2\psi - \nabla^2\phi + 2\mathcal{H}\dot{\psi} + 4\mathcal{H}\dot{\phi} + (4\dot{\mathcal{H}} + 2\mathcal{H}^2)(\psi + \phi)\right)\delta_{ij} + (\phi - \psi)_{,ij}}$$

ahora los $G^{\mu}{}_{\nu} = g^{\alpha\mu} G_{\alpha\nu} = (\bar{g}^{\alpha\mu} + \delta g^{\alpha\mu})(\bar{G}_{\alpha\nu} + \delta G_{\alpha\nu})$

$$= \bar{g}^{\alpha\mu}\bar{G}_{\alpha\nu} + \bar{g}^{\alpha\mu}\delta G_{\alpha\nu} + \delta g^{\alpha\mu}\bar{G}_{\alpha\nu}$$

los $\alpha = i$ son 0 pues g diagonal

$$G^0{}_0 = \bar{g}^{00}\bar{G}_{00} + \bar{g}^{00}\delta G_{00} + \delta g^{00}\bar{G}_{00}$$

$$= -a^{-2}\cdot 3\mathcal{H}^2 - a^{-2}\cdot(-6\mathcal{H}\dot{\phi} + 2\nabla^2\phi) + a^{-2}2\psi\cdot 3\mathcal{H}^2$$

$$\boxed{G^0{}_0 = a^{-2}(-3\mathcal{H}^2 + 6\mathcal{H}\dot{\phi} - 2\nabla^2\phi + 6\mathcal{H}^2\psi)}$$

$$G^i{}_0 = \bar{g}^{ji}\cancel{\bar{G}_{j0}}^{\,0} + \underbrace{\bar{g}^{ji}}_{j=i}\underbrace{\delta G_{j0}}_{=\delta R_{j0}} + \delta g^{ji}\cancel{\bar{G}_{j0}}^{\,0}$$

$$\boxed{G^i{}_0 = a^{-2}\cdot 2(\dot{\phi} + \psi\mathcal{H})_{,i}}$$

$$G^0{}_i = \bar{g}^{00}\cancel{\bar{G}_{0i}}^{\,0} + \bar{g}^{00}\overset{\delta R_{0i}}{\delta G_{0i}} + \delta g^{00}\cancel{\bar{G}_{0i}}^{\,0}$$

$$= -a^{-2}\cdot 2(\dot{\phi} + \psi\mathcal{H})_{,i}$$

$$\boxed{G^0{}_i = -G^i{}_0}$$

$$G^i{}_j = \bar{g}^{ii}\bar{G}_{ij} + \bar{g}^{ii}\delta G_{ij} + \delta g^{ii}\bar{G}_{ij}$$

$$= -a^{-2}\cdot(2\dot{\mathcal{H}} - \mathcal{H}^2)\delta_{ij} + a^{-2}\left[\left(2\ddot{\phi} + \nabla^2\psi - \nabla^2\phi + 2\mathcal{H}\dot{\psi} + 4\mathcal{H}\dot{\phi} + (4\dot{\mathcal{H}} + 2\mathcal{H}^2)(\psi + \cancel{\phi})\right)\delta_{ij} + (\phi-\psi)_{,ij}\right]$$

$$- a^{-2}\,2\cancel{\phi}\cdot(2\dot{\mathcal{H}} + \mathcal{H}^2)\delta_{ij}$$

$$\Downarrow$$

$$\boxed{G^i{}_j = a^{-2}\cdot(\phi-\psi)_{,ij} \quad \text{si } i \neq j}$$

$$\boxed{G^i{}_i = \bar{G}^i{}_i + a^{-2}\cdot\left[2\ddot{\phi} + \nabla^2(\psi-\phi) + 2\mathcal{H}(\dot{\psi} + 2\dot{\phi}) + (4\dot{\mathcal{H}} + 2\mathcal{H}^2)\psi + (\phi-\psi)_{,ii}\right]}$$

así

$$\boxed{\begin{aligned}
\delta G^0{}_0 &= a^{-2}\left[-2\nabla^2\phi + 6\mathcal{H}(\dot{\phi}+\mathcal{H}\psi)\right]\\
\delta G^i{}_0 &= 2a^{-2}(\dot{\phi}+\psi\mathcal{H})_{,i}\\
\delta G^0{}_i &= -\delta G^i{}_0\\
\delta G^i{}_j &= a^{-2}(\phi-\psi)_{,ij} \qquad i\neq j\\
\delta G^i{}_i &= a^{-2}\left[2\ddot{\phi}+\nabla^2(\psi-\phi)+2\mathcal{H}(\dot{\psi}+2\dot{\phi})+(4\dot{\mathcal{H}}+2\mathcal{H}^2)\psi+(\phi-\psi)_{,ii}\right]
\end{aligned}}$$

Ahora $T_{\mu\nu}$ (para fluido perfecto)

$$T_{\mu\nu} = (\rho+P)u_\mu u_\nu + P g_{\mu\nu} \qquad / g^{\alpha\mu}.$$ y cambiando $\alpha \to \mu$ después

$$T^\mu{}_\nu = (\rho+P)u^\mu u_\nu + P g^\mu{}_\nu \qquad /\delta$$

$$\delta T^\mu{}_\nu = (\delta\rho+\delta P)u^\mu u_\nu + (\rho+P)(\delta u^\mu u_\nu + u^\mu \delta u_\nu) + \delta P \underbrace{g^\mu{}_\nu}_{=\delta^\mu{}_\nu} + P\delta \underbrace{g^\mu{}_\nu}_{=\delta\delta^\mu{}_\nu = 0}$$

aquí para un fluido barotrópico

$$\delta P = c_s^2 \cdot \delta\rho = \frac{dP}{d\rho}\delta\rho = \frac{\dot{P}}{\dot{\rho}}\delta\rho = \omega\cdot\delta\rho, \quad \text{con } \omega \text{ constante}$$

$$\delta T^\mu{}_\nu = (1+\omega)\delta\rho\cdot u^\mu\cdot u_\nu + (\rho+\overset{\omega\rho}{P})(\delta u^\mu\cdot u_\nu + u^\mu\delta u_\nu) + \omega\delta\rho\,\delta^\mu{}_\nu$$

Analicemos u^μ, para un observador comóvil $\bar{u}^i = 0$ y $\bar{g}_{\mu\nu}\bar{u}^\mu\bar{u}^\nu = -1$

$$\Rightarrow -a^2\cdot\bar{u}^{0\,2} = -1$$

$$\Rightarrow \bar{u}^0 = 1/a$$

así $\bar{u}^\mu = \left(\frac{1}{a}, 0, 0, 0\right)$

ahora perturbo $\bar{u}^\mu$

$$\Rightarrow u^\mu = \left(\frac{1}{a} + \delta u^0, \delta u^i\right)$$

como $g_{\mu\nu} u^\mu u^\nu = -1 \Rightarrow -a^2(1+2\psi)\cdot\underbrace{\left(\frac{1}{a}+\delta u^0\right)^2}_{\text{desprecio } (\delta u^0)^2} + \underbrace{g_{ii}(\delta u^i)^2}_{\text{orden 2 } \sim 0} = -1$

$$\Rightarrow -a^2(1+2\psi)\cdot\left(\frac{1}{a^2} + 2\frac{\delta u^0}{a}\right) = -1$$

$$\Rightarrow -(1+2\psi) - a^2\cdot 2\frac{\delta u^0}{a} = -1$$

$$\Rightarrow \delta u^0 = \frac{1-1-2\psi}{2a} = -\frac{\psi}{a} \Rightarrow u^0 = \frac{1-\psi}{a}$$

Por otro lado vamos $\delta u^i = \frac{v^i}{a}$ → velocidad peculiar

pues se tiene $\underset{\text{distancia física}}{x_f} = a\cdot \underset{\text{comóvil}}{x_c} \Rightarrow \dot{x}_f = \underbrace{\dot{a}\, x_c}_{\text{expansión Universo}} + \underset{v \text{ peculiar}}{a\,\dot{x}_c}$

$$\Rightarrow \dot{x}_c = \frac{\dot{x}_f}{a} \Rightarrow \underset{\text{comóvil}}{\delta u^i} = \frac{v^i}{a}$$ → velocidad peculiar, física

v^i ya es orden δ

$$\Rightarrow u^\mu = \left(\frac{1}{a}(1-\psi), \frac{v^i}{a}\right)$$

$$\Rightarrow u_\mu = g_{\mu\nu}u^\nu = \left(-a^2\frac{(1+2\psi)(1-\psi)}{a},\ a^2(1-2\phi)\frac{v^i}{a}\right)$$

$$= \left(-a(1+2\psi-\psi), a^2\frac{v^i}{a}\right) = \left(-a(1+\psi), a v^i\right)$$

así $u_\mu u^\mu = -1$ //

$$\delta T^0{}_0 = (1+w)\delta\rho \cdot u^0 u_0 + (\rho+P)(\delta u^0 \cdot u_0 + u^0 \delta u_0) + w\delta\rho$$

$$= (1+w)\delta\rho \cdot \underbrace{\frac{1}{a}(1-\psi)\cdot(-a)(1+\psi)}_{\substack{-(1-\psi)(1+\psi) \\ -1-\psi+\psi \\ -1}} + (\rho+P)\underbrace{\left(-\frac{\psi}{a}\cdot(-a) + \frac{1}{a}\cdot(-a)\psi\right)}_{0} + w\delta\rho$$

$$= -(1+w)\delta\rho + w\delta\rho = -\delta\rho$$

$$\delta T^0{}_i = (1+w)\delta\rho u^0 u_i + (\rho+P)(\cancel{\delta u^0 u_i}^{\,0\ \text{orden } 2} + u^0\delta u_i) + w\delta\rho\cancel{\delta^0{}_i}^{\,0}$$

$$= (1+w)\delta\rho \cdot \frac{1}{\cancel{a}}(1-\cancel{\psi})\cdot\cancel{a}v_i + (\rho+P)\cdot\left(\frac{1}{a}\cdot a\cdot v_i\right)$$

$$= \underbrace{(1+w)\delta\rho\, v_i}_{\text{orden } 2} + (\rho+P)\,v_i = (\rho+P)\cdot v_i$$

$$= (1+w)\rho v_i$$

Como $\delta G^0{}_i = -G^i{}_0$ y las ecuaciones de Einstein son invariantes $\Rightarrow \delta T^i{}_0 = -\delta T^i{}_0$

↳ por el v^i que está arriba

$$\delta T^i{}_0 = -(1+w)\rho v^i$$

$$\delta T^i{}_j = (1+w)\delta\rho\cdot \cancel{u^i}^{\,\text{orden } 2} u_j + (1+w)\rho\cdot(\cancel{\delta u^i u_j}^{\,\text{orden } 2} + \cancel{u^i \delta u_j}^{\,\text{orden } 2}) + w\,\delta\rho\,\delta^i{}_j$$

si $i \neq j \Rightarrow \delta T^i{}_j = 0$

$i = j \Rightarrow \delta T^i{}_i = w\delta\rho = c_s^2\,\delta\rho$

Así

$$\boxed{\begin{aligned} &\delta T^0{}_0 = -\delta\rho \\ &\delta T^0{}_i = (1+w)\rho v_i \\ &\delta T^i{}_0 = -(1+w)\rho v^i \\ &\delta T^i{}_j = 0 \qquad i\neq j \\ &\delta T^i{}_i = c_s^2\delta\rho \end{aligned}}$$

ϕ en Fourier / $x \to k$

$\phi(x,\tau) \to e^{ikx}\phi_k(\tau)$

$\nabla\phi(x,\tau) \to i e^{ikx} k\phi_k(\tau)$

$\nabla^2\phi$ y $\nabla_i\nabla_j\phi \longrightarrow -e^{ikx}k^2\phi_k(\tau)$

además v_i es un campo vectorial y como $\vec{F} = -\nabla V$

la potencial escalar

así $\delta G^\mu{}_\nu = 8\pi G\,\delta T^\mu{}_\nu$

$\underline{0-0}$ $-\nabla^2\phi + 3\mathcal{H}(\dot\phi + \mathcal{H}\psi) = -4\pi G\delta\rho a^2$

En Fourier: $\boxed{k^2\phi + 3\mathcal{H}(\dot\phi + \mathcal{H}\psi) = -4\pi G a^2\delta\rho}$

campo de velocidad

$\underline{0-i}$ $\underline{i-0}$ $(\dot\phi + \psi\mathcal{H})_{,i} = -4\pi G a^2\cdot(1+w)\rho v^i$ / ∂_i y $\partial_i v^i = \theta$

En Fourier: $\boxed{k^2(\dot\phi + \psi\mathcal{H}) = 4\pi G a^2(1+w)\rho\theta}$

$\underline{i-i}$ $\ddot\phi + \nabla^2(\psi-\phi)\frac{1}{2} + \mathcal{H}(\dot\psi + 2\dot\phi) + (2\dot{\mathcal{H}} + \mathcal{H}^2)\psi + \frac{1}{2}(\phi-\psi)_{,ii} = 4\pi G a^2\cdot c_s^2\delta\rho$

En Fourier $\ddot\phi + \frac{k^2}{2}(\phi-\psi) + \mathcal{H}(\dot\psi + 2\dot\phi) + \psi(2\dot{\mathcal{H}} + \mathcal{H}^2) - \frac{k_i^2}{2}(\phi-\psi) = 4\pi G a^2 c_s^2\delta\rho$

Aquí sumamos los 3 i es decir $x-x + y-y + z-z$ y tendremos $k_x^2 + k_y^2 + k_z^2 = k^2$

$\Rightarrow$ $\ddot\phi + \frac{k^2}{2}(\phi-\psi) + \mathcal{H}(\dot\psi + 2\dot\phi) + \psi(2\dot{\mathcal{H}} + \mathcal{H}^2) - \frac{k^2}{\underbrace{3\cdot 2}_{k^2/6}}(\phi-\psi) = 4\pi G a^2 c_s^2\delta\rho$

y $\frac{1}{2} - \frac{1}{6} = \frac{1}{3}$

$\Rightarrow$ $\boxed{\ddot\phi + \frac{k^2}{3}(\phi-\psi) + \mathcal{H}(\dot\psi + 2\dot\phi) + \psi(2\dot{\mathcal{H}} + \mathcal{H}^2) = 4\pi G a^2 c_s^2\delta\rho}$

Por último $i-j$ con $i\neq j$ $\Rightarrow$ $(\phi-\psi)_{,ij} = 0$ $\Rightarrow$ $\boxed{\phi = \psi}$ //

Escaneado con CamScanner

Para la P2 es conveniente tener a mano:

$$\bar{T}^0{}_0 = (\rho+P)\cdot(-1) + P = -\rho$$

$$\bar{T}^i{}_i = (\rho+P)\cdot 0 + P = P = \omega\rho$$

0 otro caso

$\delta T^0{}_0 = -\delta\rho$

$\delta T^0{}_i = +(1+\omega)\,\rho\, v_i$

$\delta T^i{}_0 = -(1+\omega)\,\rho\, v^i$

$\delta T^i{}_j = 0 \quad i\neq j$

$\delta T^i{}_i = c_s^2\,\delta\rho = \omega\,\delta\rho = \delta P$

$\Gamma^0_{00} = \mathcal{H} + \dot{\psi}$

$\Gamma^0_{ij} = \mathcal{H}\delta_{ij} - \delta_{ij}\left[2(\phi+\psi)\mathcal{H} + \dot{\phi}\right]$

$\Gamma^0_{0i} = \partial_i \psi$

$\Gamma^i_{00} = \partial_i \psi$

$\Gamma^i_{j0} = \mathcal{H}\delta_{ij} - \dot{\phi}\,\delta_{ij}$

$\Gamma^i_{jk} = (\delta_{jk}\partial_i - \delta_{ij}\partial_k - \delta_{ik}\partial_j)\phi$

$\hookrightarrow \Gamma^i_{ji} = -3\,\partial_j\phi$

Por otro lado vamos la ecuación no perturbada: $\nabla_\mu \bar{T}^\mu{}_\nu = 0 \Rightarrow \bar{T}^\mu{}_{\nu,\mu} + \bar{T}^\alpha{}_\nu \bar{\Gamma}^\mu_{\alpha\mu} - \bar{T}^\mu{}_\alpha \bar{\Gamma}^\alpha_{\nu\mu} = 0$

$\underline{\nu = 0}$

$$\Rightarrow \bar{T}^0{}_{0,0} + \cancel{\bar{T}^i{}_{0,i}} + \bar{T}^0{}_0 \bar{\Gamma}^\mu_{0\mu} + \cancel{\bar{T}^i{}_0}\bar{\Gamma}^\mu_{i\mu} - \bar{T}^\mu{}_0 \bar{\Gamma}^0_{0\mu} - \bar{T}^\mu{}_i \bar{\Gamma}^i_{0\mu} = 0$$

$$\bar{T}^0{}_{0,0} + \bar{T}^0{}_0(\bar{\Gamma}^0_{00} + \bar{\Gamma}^i_{0i}) - \bar{T}^0{}_0 \Gamma^0_{00} - \bar{T}^i{}_i \Gamma^i_{0i} = 0$$

$$-\dot{\rho} - \rho\cdot(\mathcal{H} + 3\mathcal{H}) + \rho\mathcal{H} - 3P\cdot\mathcal{H} = 0$$

$$\boxed{\dot{\rho} + 3\mathcal{H}(P+\rho) = 0}$$

P2 $\nabla_\mu T^\mu{}_\nu = 0$

$$\Rightarrow \quad T^\mu{}_{\nu;\mu} = T^\mu{}_{\nu,\mu} + T^\alpha{}_\nu \Gamma^\mu{}_{\alpha\mu} - T^\mu{}_\alpha \Gamma^\alpha{}_{\nu\mu} = 0 \quad /\delta$$

$$\delta T^\mu{}_{\nu,\mu} + \delta T^\alpha{}_\nu \bar{\Gamma}^\mu{}_{\alpha\mu} + \bar{T}^\alpha{}_\nu \delta\Gamma^\mu{}_{\alpha\mu} - \delta T^\mu{}_\alpha \bar{\Gamma}^\alpha{}_{\nu\mu} - \bar{T}^\mu{}_\alpha \delta\Gamma^\alpha{}_{\nu\mu} = 0$$

Co. al despreciar términos de 2do orden

$\nu = 0$

$$\delta T^0{}_{0,0} + \delta T^i{}_{0,i} + \delta T^0{}_0 \bar{\Gamma}^\mu{}_{0\mu} + \delta T^i{}_0 \bar{\Gamma}^\mu{}_{i\mu} + \bar{T}^0{}_0 \delta\Gamma^\mu{}_{0\mu} + \bar{T}^i{}_0 \delta\Gamma^\mu{}_{i\mu}$$
$$- \delta T^\mu{}_0 \bar{\Gamma}^0{}_{0\mu} - \delta T^\mu{}_i \bar{\Gamma}^i{}_{0\mu} - \bar{T}^\mu{}_0 \delta\Gamma^0{}_{0\mu} - \bar{T}^\mu{}_i \delta\Gamma^i{}_{0\mu} = 0$$

$$\Rightarrow \delta T^0{}_{0,0} + \delta T^i{}_{0,i} + \delta T^0{}_0 (\bar{\Gamma}^0{}_{00} + \bar{\Gamma}^i{}_{0i}) + \delta T^i{}_0 (\cancel{\bar{\Gamma}^0{}_{i0}}^{\,0} + \cancel{\bar{\Gamma}^j{}_{ij}}^{\,0}) + \bar{T}^0{}_0 (\delta\Gamma^0{}_{00} + \delta\Gamma^i{}_{0i})$$
$$+ \cancel{\bar{T}^i{}_0}^{\,0} (\delta\Gamma^0{}_{i0} + \delta\Gamma^j{}_{ij}) - \delta T^0{}_0 \bar{\Gamma}^0{}_{00} - \delta T^i{}_0 \cancel{\bar{\Gamma}^0{}_{0i}}^{\,0} - \delta T^0{}_i \cancel{\bar{\Gamma}^i{}_{00}}^{\,0} - \underbrace{\delta T^j{}_i}_{i=j} \bar{\Gamma}^i{}_{0j}$$
$$- \bar{T}^0{}_0 \delta\Gamma^0{}_{00} - \cancel{\bar{T}^j{}_0}^{\,0} \delta\Gamma^0{}_{0j} - \cancel{\bar{T}^0{}_i}^{\,0} \delta\Gamma^i{}_{00} - \underbrace{\bar{T}^j{}_i}_{j=i} \delta\Gamma^i{}_{0j} = 0$$

$$\Rightarrow \delta T^0{}_{0,0} + \delta T^i{}_{0,i} + \delta T^0{}_0 (\bar{\Gamma}^0{}_{00} + \bar{\Gamma}^i{}_{0i}) + \bar{T}^0{}_0 (\delta\Gamma^0{}_{00} + \delta\Gamma^i{}_{0i}) - \delta T^0{}_0 \bar{\Gamma}^0{}_{00} - \delta T^k{}_i \bar{\Gamma}^i{}_{0k} - \bar{T}^0{}_0 \delta\Gamma^0{}_{00}$$
$$- \bar{T}^i{}_i \delta\Gamma^i{}_{0i} = 0$$

$$\Rightarrow -\delta\dot{\rho} - (1+w)\rho \underbrace{\partial_i v^i}_{\theta} - \delta\rho \cdot (\cancel{\mathcal{H}} + 3\mathcal{H}) - \rho(\cancel{\dot{\psi}} - 3\dot{\phi}) + \cancel{\delta\rho \cdot \mathcal{H}} - 3\,\delta P \cdot \mathcal{H}$$
$$+ \cancel{\rho \cdot \dot{\psi}} + 3w\rho \cdot \dot{\phi}$$

$\Downarrow$

$$\delta\dot{\rho} = -(1+w)\rho\theta - 3\delta\rho\mathcal{H} + \underline{\rho 3\dot{\phi}} - 3\delta P\mathcal{H} + \underline{3w\rho\dot{\phi}}$$

$$\delta\dot{\rho} = -(1+w)(\rho\theta - 3\rho\dot{\phi}) - 3\mathcal{H}(\delta\rho + \delta P) \qquad / \tfrac{1}{\rho}$$

$$\frac{\delta\dot{\rho}}{\rho} = -(1+w)(\theta - 3\dot{\phi}) - 3\mathcal{H}\frac{1}{\rho}(\delta\rho + \delta P)$$

aquí $\frac{\delta\rho}{\rho} \equiv \delta$ y vemos que $\dot{\delta} = \frac{\delta\dot{\rho}}{\rho} - \frac{\delta\rho}{\rho^2}\cdot\dot{\rho} \Rightarrow \frac{\delta\dot{\rho}}{\rho} = \dot{\delta} + \underbrace{\frac{\delta\rho}{\rho^2}\dot{\rho}}_{\delta\cdot\frac{\dot{\rho}}{\rho}}$

$$\Rightarrow \dot{\delta} = -(1+w)(\theta - 3\dot{\phi}) \underbrace{- 3\mathcal{H}\frac{1}{\rho}(\delta\rho + \delta P) - \delta\cdot\frac{\dot{\rho}}{\rho}}$$

uso $\boxed{\dot{\rho} = -3\mathcal{H}(P+\rho)}$

$$-3\mathcal{H}\delta - 3\mathcal{H}\frac{\delta P}{\rho} + \delta\cdot\frac{1}{\rho}3\mathcal{H}(P+\rho)$$

$$\cancel{-3\mathcal{H}\delta} - 3\mathcal{H}\frac{\delta P}{\rho} + 3\mathcal{H}\delta\cdot w + \cancel{3\mathcal{H}\delta}$$

$$\Rightarrow \dot{\delta} = -(1+w)(\theta - 3\dot{\phi}) + 3\mathcal{H}\left(-\frac{\delta P}{\rho} + \delta w\right)$$

$$\Rightarrow \boxed{\dot{\delta} = -(1+w)(\theta - 3\dot{\phi}) - 3\mathcal{H}\left(\frac{\delta P}{\delta\rho} - w\right)\delta}$$

//

Ahora $\nu = i$, primero no perturbado:

$$\bar T^{\mu}{}_{\nu;\mu} + \bar T^{\alpha}{}_{\nu} \bar\Gamma^{\mu}{}_{\alpha\mu} - \bar T^{\mu}{}_{\alpha} \bar\Gamma^{\alpha}{}_{\nu\mu} = 0$$

→ no considero los $\bar T^{\mu}{}_{\nu}$ con $\mu \neq \nu$ porque $\bar T^{\mu}{}_{\nu}$ es diagonal

$$\Rightarrow \bar T^{i}{}_{i,i} + \bar T^{i}{}_{i} \cancel{\bar\Gamma^{\mu}{}_{i\mu}}^{0} - \bar T^{\mu}{}_{0} \bar\Gamma^{0}{}_{i\mu} - \bar T^{\mu}{}_{j} \bar\Gamma^{j}{}_{i\mu} = 0$$

$$\Rightarrow \bar T^{i}{}_{i,i} - \bar T^{0}{}_{0} \cancel{\bar\Gamma^{0}{}_{i0}}^{0} - \bar T^{k}{}_{j} \cancel{\bar\Gamma^{j}{}_{ik}}^{0} = 0$$

$$\dot P = 0 \Rightarrow P = \text{constante en el } t$$

Ahora el perturbado

$$\Rightarrow \delta T^{\mu}{}_{i,\mu} + \delta T^{\alpha}{}_{i} \bar\Gamma^{\mu}{}_{\alpha\mu} + \bar T^{\alpha}{}_{i} \delta\Gamma^{\mu}{}_{\alpha\mu} - \delta T^{\mu}{}_{\alpha} \bar\Gamma^{\alpha}{}_{i\mu} - \bar T^{\mu}{}_{\alpha} \delta\Gamma^{\alpha}{}_{i\mu} = 0$$

$$\Rightarrow \delta T^{0}{}_{i,0} + \delta T^{j}{}_{i,j} + \delta T^{0}{}_{i} \bar\Gamma^{\mu}{}_{0\mu} + \delta T^{j}{}_{i} \bar\Gamma^{\mu}{}_{j\mu} + \cancel{\bar T^{0}{}_{i}}^{0} \delta\Gamma^{\mu}{}_{0\mu} + \bar T^{j}{}_{i} \delta\Gamma^{\mu}{}_{j\mu}$$
$$- \delta T^{\mu}{}_{0} \bar\Gamma^{0}{}_{i\mu} - \delta T^{\mu}{}_{j} \bar\Gamma^{j}{}_{i\mu} - \bar T^{\mu}{}_{0} \delta\Gamma^{0}{}_{i\mu} - \bar T^{\mu}{}_{j} \delta\Gamma^{j}{}_{i\mu} = 0$$

$$\Rightarrow \delta T^{0}{}_{i,0} + \underbrace{\delta T^{j}{}_{i,j}}_{j=i} + \delta T^{0}{}_{i} \left(\bar\Gamma^{0}{}_{00} + \bar\Gamma^{j}{}_{0j}\right) + \delta T^{j}{}_{i} \left(\cancel{\bar\Gamma^{0}{}_{j0}}^{0} + \cancel{\bar\Gamma^{k}{}_{jk}}^{0}\right)$$
$$+ \underbrace{\bar T^{j}{}_{i}}_{j=i} \left(\delta\Gamma^{0}{}_{j0} + \delta\Gamma^{k}{}_{jk}\right) - \delta T^{0}{}_{0} \cancel{\bar\Gamma^{0}{}_{i0}}^{0} - \delta T^{j}{}_{0} \underbrace{\bar\Gamma^{0}{}_{ij}}_{j=i} - \delta T^{0}{}_{j} \underbrace{\bar\Gamma^{j}{}_{i0}}_{j=i} - \delta T^{k}{}_{j} \underbrace{\bar\Gamma^{j}{}_{ik}}_{k=j}$$
$$- \bar T^{0}{}_{0} \delta\Gamma^{0}{}_{i0} - \underbrace{\bar T^{k}{}_{j}}_{k=j} \delta\Gamma^{j}{}_{ik} = 0$$

$$\Rightarrow \delta T^{0}{}_{i,0} + \delta T^{i}{}_{i,i} + \delta T^{0}{}_{i} \left(\bar\Gamma^{0}{}_{00} + \bar\Gamma^{j}{}_{0j}\right) + \bar T^{i}{}_{i} \left(\delta\Gamma^{0}{}_{i0} + \delta\Gamma^{k}{}_{ik}\right) - \delta T^{i}{}_{0} \bar\Gamma^{0}{}_{ii}$$
$$- \delta T^{0}{}_{i} \bar\Gamma^{i}{}_{i0} - \delta T^{j}{}_{j} \cancel{\Gamma^{j}{}_{ij}}^{0} - \bar T^{0}{}_{0} \delta\Gamma^{0}{}_{i0} - \bar T^{j}{}_{j} \delta\Gamma^{j}{}_{ij} = 0$$

$\Downarrow$

$$(1+w)\rho v_{i,0} + \delta P_{,i} + (1+w)\rho v_i(\cancel{\mathcal{H}}+3\mathcal{H}) + P(\partial_i\Psi - \cancel{3\partial_i\phi}) + (1+w)\rho v^i \cdot \mathcal{H}$$

$$-(1+w)\rho v_i \cdot \cancel{\mathcal{H}} + \rho\,\partial_i\Psi - P\cdot(\cancel{-3\partial_i\phi}) = 0$$

$$(1+w)\rho v_{i,0} + \delta P_{,i} + (1+w)\rho v_i 3\mathcal{H} + \partial_i\Psi(P+\rho) + (1+w)\rho v^i\mathcal{H} = 0$$

$/\partial_i$ y sumando las 3 ecuaciones en cada i

$$(1+w)(\dot{\rho}\theta + \rho\dot{\theta}) + \nabla^2\delta P + (1+w)4\mathcal{H}\rho\theta + \nabla^2\Psi\cdot(P+\rho) = 0$$

$$\dot{\rho}\theta + \rho\dot{\theta} + \frac{\nabla^2(\delta P)}{1+w} + 4\mathcal{H}\rho\theta + \nabla^2\Psi\frac{\overset{w\rho}{(P+\rho)}}{1+w} = 0 \qquad / \frac{1}{\rho} \text{ y } \theta \text{ ...}$$

$$\dot{\theta} = -\underbrace{\frac{\dot{\rho}}{\rho}}\theta + \frac{k^2\,\delta P}{\rho(1+w)} - 4\mathcal{H}\theta + k^2\Psi$$

$$\underbrace{-3\mathcal{H}\frac{(P+\rho)}{\rho}}$$

$$3\mathcal{H}(w+1)\theta$$

$$\Rightarrow \quad \dot{\theta} = \mathcal{H}(3w-1)\theta + \frac{k^2\delta P}{(1+w)\rho} + k^2\Psi$$

$$\boxed{\dot{\theta} = -\mathcal{H}(1-3w)\theta + \frac{k^2\delta P}{(1+w)\rho} + k^2\Psi}$$

$$\frac{1}{\rho} = \frac{\delta}{\delta\rho}$$

Resolvamos

$$\dot{\delta} = -(1+w)(\theta - 3\dot{\phi}) - 3\mathcal{H}\left(\frac{\delta P}{\rho} - w\delta\right)$$

$\delta\rho = \, ?$ $\delta = \frac{\delta\rho}{\rho}$

$$\dot{\theta} = -\mathcal{H}(1-3w)\theta + \frac{1}{(1+w)}\frac{k^2\delta P}{\rho} + k^2\psi$$

Caso que la perturbación es mayor al horizonte: $k \ll a H = \mathcal{H}$, $\delta P = w\rho$ (horizonte causal)

$\Downarrow$ k^2 despreciable

$\Downarrow$ $\frac{\delta P}{\rho} - w\delta = 0$

$\Downarrow$

$$\dot{\delta} = -(1+w)(\theta - 3\dot{\phi}) \overset{\phi \text{ constante}}{=} -(1+w)\theta \xRightarrow{d/d\tau} \ddot{\delta} = -(1+w)\dot{\theta} \quad (1)$$

$$\dot{\theta} = -\mathcal{H}(1-3w)\theta$$

remplazo $\dot{\theta}$ en (1) — lo saco de

$$\ddot{\delta} = -(1+w)\cdot(-\mathcal{H})(1-3w)\theta = -(1+w)(-\mathcal{H})(1-3w)\cdot\frac{\dot{\delta}}{-(1+w)}$$

$$\ddot{\delta} = -\mathcal{H}(1-3w)\dot{\delta}$$

$$\boxed{\ddot{\delta} = -\mathcal{H}\underbrace{(1-3w)}_{\equiv b}\dot{\delta}}$$

$\rho_{mat} \Rightarrow w = 0 \Rightarrow b = 1$

$\rho_r \Rightarrow w = 1/3 \Rightarrow b = 0$

$\rho_\Lambda \Rightarrow w = -1 \Rightarrow b = 4$

con $\mathcal{H} = a\cdot H$

$$\boxed{\ddot{\delta} + \mathcal{H}\cdot b\dot{\delta} = 0}$$

si $b > 0 \Rightarrow \delta = \int_1^t e^{-\int_1^{t'} \mathcal{H} b\, dt'} \cdot c_1\, dt + c_2$ (variable; solución que decrece; constante)

$b = 0 \Rightarrow \delta = c_2\cdot\tau + c_1$

$\Rightarrow$ Las perturbaciones de materia no rel y Λ no crecen si están fuera del horizonte causal. Para rel se requiere $c_2 = 0$ para que la solución no crezca

Caso de perturbación menor al horizonte $k >> aH = \mathcal{H}$

Para materia ($w=0$), pero con c_s

$$\dot{\delta} = -(1+w)(\theta - 3\dot{\phi}) - 3\mathcal{H}\left(\frac{\delta P}{\rho} - w\delta\right)$$

$$\dot{\theta} = -\mathcal{H}(1-3w)\theta + \frac{1}{(1+w)} k^2 \underbrace{\frac{\delta P}{\rho}}_{\frac{\delta P}{\delta \rho}\cdot\delta = c_s^2\delta} + k^2\psi$$

despreciable pues $k >> \mathcal{H}$ $\quad \nabla\phi \sim \frac{\delta}{k^2}$

$$\Rightarrow \boxed{\begin{aligned}\dot{\delta} &= -\theta + 3\dot{\phi} - 3\mathcal{H} c_s^2 \delta \\ \dot{\theta} &= -\mathcal{H}\theta + c_s^2 k^2 \delta + k^2 \phi\end{aligned}}$$

$$\Downarrow$$

$$\ddot{\delta} = -\dot{\theta}$$

reemplazo

$$\Rightarrow \ddot{\delta} - \mathcal{H}\underset{-\dot{\delta}}{\theta} + c_s^2 k^2 \delta + k^2\phi = 0$$

y usando: $k^2\phi = -4\pi G a^2 \underbrace{\delta\rho}_{=\rho\cdot\delta}$

ec. Einstein, se desprecia el $\mathcal{H}$

$$\Rightarrow \ddot{\delta} + \mathcal{H}\dot{\delta} + \underbrace{(c_s^2 k^2 - 4\pi G a^2 \rho)}_{\text{escala de Jeans} \equiv B(\tau)}\delta = 0$$

$$c_s^2 \frac{(2\pi)^2}{\lambda_J^2} = 4\pi G a^2 \rho \Rightarrow \lambda_J^2 = c_s^2 \cdot \frac{\pi}{G a^2 \rho} \Rightarrow \lambda_J = c_s\sqrt{\frac{\pi}{\rho G a}}$$

$$\Rightarrow \boxed{\ddot{\delta} + \mathcal{H}\dot{\delta} + B\delta = 0}$$

→ es como oscilador, si asumimos $\mathcal{H}$ y B constantes por simplicidad

$\Rightarrow$ ansatz: $e^{\lambda\tau}$

$$\Rightarrow \lambda^2 + \mathcal{H}\lambda + B = 0 \Rightarrow \lambda = \frac{-\mathcal{H} \pm \sqrt{\mathcal{H}^2 - 4B}}{2}$$

→ si $B \geq 0 \Rightarrow$ oscila ($\lambda \in \mathbb{C}$)

$B < 0 \Rightarrow$ crece exponencialmente ($\lambda \in \mathbb{R}$)

$\boxed{\text{⇒ La perturbación puede crecer u oscilar según el } c_s \text{, pues } B \text{ depende de este parámetro}}$

Para CDM: $c_s = w = 0$ y $k \gg \mathcal{H}$

$\Downarrow$

Es igual que antes pero con $c_s = 0 \Rightarrow B = -4\pi G a^2 \rho \delta$

$$\ddot{\delta} + \mathcal{H}\dot{\delta} - 4\pi G a^2 \rho \delta = 0$$

Tendremos una solución que puede crecer exponencialmente, pues la perturbación está fuera del horizonte de sonido pero dentro del de masas.

<u>Para radiación $w = c_s = 1/3$</u>

$$\Rightarrow \dot{\delta} = -\frac{4}{3}(\theta - 3\underbrace{\dot{\phi}}_{\sim 0}) \qquad \dot{\delta} = -\frac{4}{3}\theta \longrightarrow \ddot{\delta} = -\frac{4}{3}\ddot{\theta}$$

$$\dot{\theta} = \frac{3}{4}k^2 \cdot \frac{1}{3}\cdot\delta + k^2 \underbrace{\psi}_{=\phi} \quad \Rightarrow \quad \dot{\theta} = \frac{k^2}{4}\delta + k^2\phi \quad \text{reemplazo}$$

$$\Rightarrow \boxed{\ddot{\delta} + \frac{k^2}{3}\delta + \frac{4}{3}k^2\phi = 0}$$

La oscilador

$$\delta = -\frac{4}{3}\frac{k^2}{k^2}\cdot 3 + A\sin\left(\frac{k}{\sqrt{3}}\tau\right) + B\cos\left(\frac{k}{\sqrt{3}}\tau\right)$$

$\downarrow$

uniform

$$\boxed{\delta = -4 + A\sin\left(\frac{k}{\sqrt{3}}\tau\right) + B\cos\left(\frac{k}{\sqrt{3}}\tau\right)}$$

P3 $\frac{\partial f}{\partial t} + \frac{\vec{p}}{ma^2}\frac{\partial f}{\partial \vec{x}} - m\nabla\Phi\frac{\partial f}{\partial \vec{p}} = 0$

$$\rho(\vec{x},t) = \frac{m}{a^3}\int \frac{d^3p}{(2\pi)^3} f(\vec{x},\vec{p}) \qquad \sigma^{ij}(\vec{x},t) = \frac{1}{ma^5}\int\frac{d^3p}{(2\pi)^3} p^i p^j f(\vec{x},\vec{p})$$

$$\pi^i(\vec{x},t) = \frac{1}{a^4}\int\frac{d^3p}{(2\pi)^3} p^i f(\vec{x},\vec{p})$$

vamos el 1er momento, integramos en $\frac{d^3p}{(2\pi)^3}$ y multiplicamos por m

$$m\int\frac{d^3p}{(2\pi)^3}\cdot\frac{\partial f}{\partial t} + m\int\frac{d^3p}{ma^2}\underbrace{\frac{p^i}{(2\pi)^3}\partial_i f}_{\vec{p}\cdot\frac{\partial}{\partial \vec{x}}} - m^2\underbrace{\nabla\Phi}_{\text{no depende del } \vec{p}}\underbrace{\int\frac{d^3p}{(2\pi)^3}\cdot\frac{\partial f}{\partial\vec{p}}}_{=\frac{1}{(2\pi)^3}\cdot f\Big|^{\text{limites}}} = 0$$

el ∂_i sale

aquí sabemos que la función distribución cae a 0 en infinito $\Rightarrow$ este termino $\to 0$

$$\Rightarrow m\underbrace{\int\frac{d^3p}{(2\pi)^3}\frac{\partial f}{\partial t}}_{=\partial_t\int\frac{d^3p}{(2\pi)^3}f} + \frac{1}{a^2}\partial_i\int\frac{d^3p}{(2\pi)^3}\cdot p^i f = 0 \qquad /\cdot\frac{1}{a^3}$$

$$\Rightarrow \underbrace{\frac{m}{a^3}\partial_t\int\frac{d^3p\, f}{(2\pi)^3}}_{?} + \frac{1}{a}\partial_i\underbrace{\left(\frac{1}{a^4}\int\frac{d^3p}{(2\pi)^3}p^i f\right)}_{\pi^i} = 0$$

$$\partial_t\left(\frac{f}{a^3}\right) = \frac{1}{a^3}\partial_t f - 3f\frac{1}{a^4}\cdot\dot{a}$$

$$\Rightarrow \frac{1}{a^3}\partial_t f = \partial_t\left(\frac{f}{a^3}\right) + 3f\frac{1}{a^3}\cdot H$$

$\Downarrow$

$\Downarrow$

$$\partial_t\Big(\underbrace{\frac{m}{a^3}\int\frac{d^3p}{(2\pi)^3}f}_{\rho}\Big)+3\frac{H}{a^3}m\int\frac{d^3p}{(2\pi)^3}f+\frac{1}{a}\partial_i\pi^i$$

$$\Rightarrow \boxed{\dot{\rho}+3H\rho+\frac{1}{a}\vec{\nabla}\vec{\pi}=0} \quad //$$

Para el segundo momento: se integra por: $p^i\frac{d^3p}{(2\pi)^3}$

$$\int\frac{d^3p}{(2\pi)^3}p^i\partial_t f+\int\frac{d^3p}{(2\pi)^3}\frac{p^ip^j}{ma^2}\partial_j f-m\nabla\Phi\underbrace{\int\frac{d^3p}{(2\pi)^3}p^i\frac{\partial f}{\partial p}}=0 \quad /\cdot\frac{1}{a^3}$$

integrar por partes $= pf\Big|^{\text{limits}}_{\to 0}-\int f$

$$\Rightarrow \int\frac{d^3p}{(2\pi)^3}p^i\underbrace{\frac{1}{a^3}\partial_t f}_{\partial_t\left(\frac{f}{a^3}\right)+3\frac{H}{a^3}f}+\partial_j\underbrace{\int\frac{d^3p}{(2\pi)^3}\frac{p^ip^jf}{ma^5}}_{\sigma^{ij}}+m\nabla\Phi\underbrace{\int\frac{d^3p}{(2\pi)^3}\frac{f}{a^3}}_{\rho}=0$$

$$\Rightarrow \partial_t\left(\int\frac{d^3p}{(2\pi)^3}p^i\frac{f}{a^3}\right)+\underbrace{\int\frac{d^3p}{(2\pi)^3}p^i\,3\frac{H}{a^3}f}_{=3H\,\pi^i\cdot a}+\partial_j\sigma^{ij}+\nabla\Phi\cdot\rho=0 \quad /\frac{1}{a}$$

$$\frac{1}{a}\partial_t\left(\int\frac{d^3p}{(2\pi)^3}p^i\frac{f}{a^3}\right)+3H\,\pi^i+\frac{1}{a}\partial_j\sigma^{ij}+\frac{1}{a}\nabla\Phi\rho=0$$

$\downarrow$

$$\partial_t\left(\frac{f}{a^4}\right)=\frac{1}{a^4}\cdot\dot{f}-4\frac{f}{a^5}\dot{a}$$

$$=\frac{1}{a^4}\dot{f}-4H\frac{f}{a^4}$$

$\Downarrow$

$$\partial_t\Big(\underbrace{\int\frac{d^3p}{(2\pi)^3}\,p^i\frac{f}{a^4}}_{\Pi^i}\Big)+4H\underbrace{\int\frac{d^3p}{(2\pi)^3}\,p^i\frac{f}{a^4}}_{\Pi^i}+3H\Pi^i+\frac{1}{a}\partial_j\sigma^{ij}+\frac{1}{a}\nabla\Phi\rho=0$$

$$\boxed{\dot{\Pi}^i+H\Pi^i+\frac{1}{a}\partial_j\sigma^{ij}+\frac{1}{a}\nabla\Phi\rho=0}$$

/ usando los pasos extra del aux:

aquí $\Pi^i=\rho v^i$

$\partial_t\Pi^i=\dot{\Pi}^i=\dot{\rho}v^i+\rho\dot{v}^i$ y $\sigma^{ij}=\rho v^i v^j$

$$\Rightarrow\quad \dot{\rho}v^i+\rho\dot{v}^i+H\rho v^i+\frac{1}{a}v^i\rho\partial_j v^j+\frac{1}{a}\nabla\Phi\rho=0 \qquad /\frac{1}{\rho}$$

$$v^i\frac{\dot{\rho}}{\rho}+\dot{v}^i+Hv^i+\frac{1}{a}v^i\partial_j v^j+\frac{1}{a}\nabla\Phi=0$$

y eso $\dfrac{\dot{\rho}}{\rho}=-3H-\dfrac{1}{\rho a}\vec{\nabla}\vec{\Pi}=-3H-\dfrac{1}{a}\partial_j v^j$

$$\Rightarrow\quad v^i\cdot\Big(-3H-\cancel{\frac{1}{a}\partial_j v^j}\Big)+\dot{v}^i+Hv^i+\cancel{\frac{1}{a}v^i\partial_j v^j}+\frac{1}{a}\nabla\Phi=0$$

$$\Rightarrow\quad\boxed{\dot{v}^i-2Hv^i+\frac{1}{a}\nabla\Phi=0}$$

P4) Con las consideraciones dadas tenemos: $w=\delta P=\sigma=0$ y $\Psi=\phi$

$$\Rightarrow \begin{cases} \dot\delta = 3\dot\phi - \theta \\ \dot\theta = -\mathcal{H}\theta + k^2\phi \end{cases} \Rightarrow \ddot\delta = 3\ddot\phi - \dot\theta = 3\ddot\phi + \mathcal{H}\theta - k^2\phi$$

reemplazo de la misma ecuación de $\dot\delta$

$$\Rightarrow \ddot\delta = 3\ddot\phi + \mathcal{H}(3\dot\phi - \dot\delta) - k^2\phi$$

$$\ddot\delta + \mathcal{H}\dot\delta = 3\ddot\phi + 3\mathcal{H}\dot\phi - k^2\phi$$

Aquí usamos que $\frac{d}{d\tau} = a^2H\frac{d}{da}$ y $\frac{d}{d\tau} = ()^\circ$ $\frac{d}{da} = ()'$
y $\mathcal{H} = aH$

$$\Rightarrow (a^2H\delta')^\circ + aH\cdot a^2H\delta' = 3(a^2H\phi')^\circ + 3aHa^2H\phi' - k^2\phi$$

$$\Rightarrow a^2H\cdot(2aH\delta' + a^2H'\delta' + a^2H\delta'') + a^3H^2\delta' = 3a^2H\cdot(2aH\phi' + a^2H'\phi' + a^2H\phi'') + 3a^3H^2\phi' - k^2\phi$$

$$\Rightarrow \delta''\cdot a^4H^2 + \delta'(3a^3H^2 + a^4HH') = 3a^4H^2\phi'' + \phi'(9a^3H^2 + 3a^4HH') - k^2\phi$$

$$\Rightarrow \delta'' + \delta'\left(\frac{3}{a} + \frac{H'}{H}\right) = 3\phi'' + \phi'\left(\frac{9}{a} + 3\frac{H'}{H}\right) - \frac{k^2\phi}{a^4H^2}$$

ahora falta ϕ, de la ecuación de Einstein (con $w=0$ y $\phi=\Psi$)

$$k^2\phi = -4\pi Ga^2\rho\cdot\delta - 3\mathcal{H}\underbrace{(\dot\phi + \mathcal{H}\phi)}_{=\frac{4\pi Ga^2\rho\theta}{k^2}}$$

$$\Rightarrow k^2\phi = -4\pi Ga^2\rho\left(\delta - 3\frac{\mathcal{H}\theta}{k^2}\right)$$

$$k^2\phi = -4\pi Ga^2\rho\left(\delta - \frac{3aH}{k^2}\cdot\theta\right)$$ → para hacer el sistema simple, aproximaré

$$k^2\phi = -4\pi Ga^2\rho\delta$$

$\Downarrow$

$k^2 \phi = -4\pi G a^2 \rho \delta$

aquí recordamos que $\rho = \rho_0 \left(\frac{a}{a_0}\right)^{-3} = \rho_0 a^{-3} \cdot \frac{\rho_{c,0}}{\rho_{c,0}}$ $\quad$ $w=0$ y $a_0 = 1$ $\quad$ densidad crítica $\rho_{c,0} = \frac{3H_0^2}{8\pi G}$

$$\rho = \Omega_{m,0} \cdot a^{-3} \cdot \frac{3H_0^2}{8\pi G}$$

$$\Rightarrow k^2 \phi = -\cancel{4\pi G} a^2 \, \Omega_{m,0} a^{-3} \cdot \frac{3H_0^2}{\cancel{8\pi G}_{2}} \cdot \delta$$

$$k^2 \phi = -\frac{3}{2} \frac{1}{a} \Omega_{m,0} H_0^2 \delta$$

$$\boxed{\begin{aligned} &\Rightarrow \phi' = -\frac{3}{2k^2} \Omega_{m,0} H_0^2 \left(\frac{\delta'}{a} - \frac{\delta}{a^2}\right) \\ &\Rightarrow \phi'' = -\frac{3}{2k^2} \Omega_{m,0} H_0^2 \left(\frac{\delta''}{a} - \frac{2\delta'}{a^2} + 2\frac{\delta}{a^3}\right) \end{aligned}}$$

y

$$\boxed{\delta'' + \delta'\left(\frac{3}{a} + \frac{H'}{H}\right) = 3\phi'' + \phi'\left(\frac{9}{a} + 3\frac{H'}{H}\right) + \frac{3}{2}\frac{1}{a}\Omega_{m,0} H_0^2 \delta \cdot \frac{1}{a^4 H^2}}$$ //

↳ con $\delta' = \frac{d\delta}{da}$ $\quad$ es la ecuación a resolver

aquí

$$H^2 = H_0^2 \left(\Omega_{r,0} a^{-4} + \Omega_{m,0} a^{-3} + \underset{\Omega_{k,0}=0}{0} + \Omega_{DE,0} a^{-3}\right)$$

$\Omega_{DE,0} = 1 - \Omega_{m,0} - \Omega_{r,0}$

$\Omega_{r,0} = 10^{-4}$ $\quad$ y graficaremos en $a \in [10^{-4}, 1]$

$\Omega_{m,0} = 0{,}3$ $\quad$ con $k \in \{H_0, 5H_0, 20H_0, 200H_0\}$

$H_0 = 67 \frac{Km}{s\,Mpc}$

* Para la solución numérica, ahora nos damos cuenta que era mejor dejar el sistema como un 2×2

$$\Rightarrow \boxed{\begin{aligned} \dot{\delta} &= 3\dot{\phi} - \theta \\ \dot{\theta} &= -\mathcal{H}\theta + k^2\phi \end{aligned}}$$

Con lo de antes vemos que al usar $\frac{d}{d\tilde{\tau}} = a^2 H \frac{d}{da}$

$$\Rightarrow \boxed{\begin{aligned} a^2 H \delta' &= 3a^2 H \phi' - \theta \\ a^2 H \theta' &= -aH\theta + k^2\phi \end{aligned}}$$

$$\Rightarrow \boxed{\begin{aligned} \delta' &= 3\phi' - \frac{\theta}{a^2 H} \\ \theta' &= -\frac{\theta}{a} + \frac{k^2\phi}{a^2 H} \end{aligned}}$$

Pero $\phi = -\frac{3}{2k^2}\frac{1}{a}\Omega_{m,0}H_0^2\delta$

$$\phi' = -\frac{3}{2k^2}\Omega_{m,0}H_0^2\left(\frac{\delta'}{a} - \frac{\delta}{a^2}\right)$$

$$\Rightarrow \boxed{\begin{aligned} \delta' &= 3\left(-\frac{3}{2k^2}\Omega_{m,0}H_0^2\left(\frac{\delta'}{a} - \frac{\delta}{a^2}\right)\right) - \frac{\theta}{a^2 H} \\ \theta' &= -\frac{\theta}{a} - \frac{3}{2}\frac{1}{a^3 H}\Omega_{m,0}H_0^2\delta \end{aligned}}$$

$$\Rightarrow \boxed{\begin{aligned} \delta'\left(1 + \frac{9}{2k^2}\frac{\Omega_{m,0}H_0^2}{a}\right) &= \frac{9}{2k^2}\Omega_{m,0}H_0^2\frac{\delta}{a^2} - \frac{\theta}{a^2 H} \\ \theta' &= -\frac{\theta}{a} - \frac{3}{2}\frac{1}{a^3 H}\Omega_{m,0}H_0^2\delta \end{aligned}}$$

↳ Con Solve_ivp de Scipy esto sale más sencillo, considerando lo ya mencionado.