FI7015-1 Cosmología
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Tarea 2

Fecha entrega: 21 de octubre.

Perform the full calculations with all the steps; the numerical parts should be written in python and the code uploaded to your personal Github repository.

P1. Starting from the metric of the conformal Newtonian Gauge

$$ds^2 = a^2(\tau)[-(1 + 2\psi(\vec{x}, t)d\tau^2 + (1 - 2\phi(\vec{x}, t)\delta_{ij}dx^i dx^j], \tag{1}$$

evaluate everything needed for finding the perturbed Einstein equations.

P2. The energy momentum tensor T^u_ν for a single fluid satisfies the equation $\nabla_\mu T^\mu_\nu$. Solve the first order part of this equation for a general perfect fluid, defined by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} \tag{2}$$

P3. Derive the conservation equation of energy and momentum by taking moments of the collisionless Boltzmann equation. Consider the Newtonian limit, giving

$$\frac{Df}{Dt} = 0 \iff \frac{\partial f}{\partial t} + \frac{\vec{p}}{ma^2} \cdot \frac{\partial f}{\partial \vec{x}} - m\nabla\Phi \cdot \frac{\partial f}{\partial \vec{p}} = 0, \tag{3}$$

where $\Phi = \Phi(\vec{x}, t)$ is the gravitational potential and it satisfies the Poisson equation

$$\nabla^2\Phi = 4\pi G a^2 \rho. \tag{4}$$

Hint: use the following definitions for the energy density ρ, momentum density π^i and kinetic tensor σ^{ij}

$$\rho(\vec{x}, t) = \frac{m}{a^3}\int \frac{d^3p}{(2\pi)^3}\, f(\vec{x}, \vec{p}) \tag{5}$$

$$\pi^i(\vec{x}, t) = \frac{1}{a^4}\int \frac{d^3p}{(2\pi)^3}\, p^i f(\vec{x}, \vec{p}) \tag{6}$$

$$\sigma^{ij}(\vec{x}, t) = \frac{1}{ma^5}\int \frac{d^3p}{(2\pi)^3}\, p^i p^j f(\vec{x}, \vec{p}) \tag{7}$$

P4. Numerically solve the perturbation equations for a matter component with $w = \delta p = \sigma = 0$, where w is the equation of state parameter, δp the pressure perturbation and σ the anistropic stress. Assume a ΛCDM model with $\Omega_{\mathrm{r},0} = 10^{-4}$ and $\Omega_{\mathrm{m},0} = 0.3$. Plot your results for $10^{-4} \le a \le 1$ and four different scales: $k = H_0, k = 5H_0, k = 20H_0$ and $k = 200H_0$. Consider $H_0 = 67$ Km/(sMpc).

P5. Using `CAMB` compute the matter power spectrum at different redshift. Then, manually set the density of massive neutrinos to zero. Comment the differences and justify the results.

P6. Using `CAMB` compute the TT angular power spectrum. Then, explore different cosmological parameters and explain the differences in the results. Give at least one compelling example and its justification.