



Asignatura:

MÉTODOS COMPUTACIONALES - MÉTODOS NUMÉRICOS

Tema 7 Ejercicios resueltos

2020





1. Resolver la ecuación diferencial de primer orden y primer grado

$$y' = \frac{6x^2}{y}$$

con la condición inicial $\,y_0=16,\;x_0=4;\,{\rm h=0.1}\,$

- a) Con **Euler** en (4.0; 4.4)
- b) Con Euler Modificado en (4.0; 4.4)



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Con el **Método de Euler**, conociendo h=0.1 y $y' = \frac{6x^2}{y}$



$$x_0 = 4$$

$$y_0 = 16$$

$$y'_0 = f(x_0; y_0) = \frac{6x_0^2}{y_0} = 6 * \frac{4^2}{16} = 6$$

ℳMétodo de Euler

 $y_{i+1} = y_i + y_i'h$

$$x_{i+1} = x_i + h$$

$$x_1 = x_0 + h = 4 + 0.1 = 4.1$$

 $y_1 = y_0 + y'_0 * h = 16 + 6 * 0.1 = 16.6$

$$y'_1 = f(x_1; y_1) = \frac{6x_1^2}{y_1} = 6 * \frac{4.1^2}{16.6} = 6.07590$$

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$$x_2 = x_1 + h = 4.1 + 0.1 = 4.2$$

$$x_2 = x_1 + h = 4.1 + 0.1 = 4.2$$

 $y_2 = y_1 + y'_1 * h = 16.6 + 6.07590 * 0.1 = 17.20759$

$$y'_2 = f(x_2; y_2) = \frac{6x_2^2}{y_2} = 6 * \frac{4.2^2}{17.20759} = 6.15077$$

$$x_3 = x_2 + h = 4.2 + 0.1 = 4.3$$

$$x_3 = x_2 + h = 4.2 + 0.1 = 4.3$$

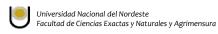
 $y_3 = y_2 + y'_2 * h = 17.20759 + 6.15077 * 0.1 = 17.82267$

$$y'_3 = f(x_3; y_3) = \frac{6x_3^2}{y_3} = 6 * \frac{4.3^2}{17.82267} = 6.22466$$

$$x_4 = x_2 + h = 4.3 + 0.1 = 4.4$$

$$x_4 = x_3 + h = 4.3 + 0.1 = 4.4$$

 $y_4 = y_3 + y'_3 * h = 17.82267 + 6.22466 * 0.1 = 18.44514$





Se pueden organizar los datos obtenidos en una tabla como la siguiente:

i	x_i	y_i	y'_i
0	4	16	6
1	4.1	16.6	6.07590
2	4.2	17.20759	6.15077
3	4.3	17.82267	6.22466
4	4.4	18.44514	

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$$x_0 = 4$$

$$x_0 = 4$$

$$y_0 = 16$$

$$y'_0 = f(x_0; y_0) = \frac{6x_0^2}{y_0} = 6 * \frac{4^2}{16} = 6$$

Para calcular el predicho de y₁

$$x_1 = x_0 + h = 4 + 0.1 = 4.1$$

 $P(y_1) = y_0 + y'_0 * h = 16 + 6 * 0.1 = 16.6$

$$P(y'_1) = f(x_1; P(y_1)) = \frac{6x_1^2}{y_1} = 6 * \frac{4.1^2}{16.6} = 6.07590$$

Método Modificado de Euler

$$P(y_{i+1}) = y_i + y_i' h$$

$$C(y_{i+1}) = y_i + \left[\frac{y'_i + P(y'_{i+1})}{2}\right]h$$



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Para calcular el corregido de y_1 :



$$C(y_1) = y_0 + \frac{y'_0 + P(y'_1)}{2} * h = 16 + \frac{6 + 6.07590}{2} * 0.1 = 16.60380$$

$$|C(y_1) - P(y_1)| = |16.60380 - 16.6| = 0.0038 > E$$

$$C(y'_1) = f(x_1; C(y_1)) = \frac{6x_1^2}{C(y_1)} = 6 * \frac{4.1^2}{16.60380} = 6.07451$$

$$C^{1}(y_{1}) = y_{0} + \frac{y'_{0} + C(y'_{1})}{2} * h = 16 + \frac{6 + 6.07451}{2} * 0.1 = 16.60373$$

$$C^{1}(y'_{1}) = f(x_{1}; C^{1}(y_{1})) = \frac{6x_{1}^{2}}{C^{1}(y_{1})} = 6 * \frac{4.1^{2}}{16.60373} = 6.07454$$

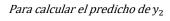
$$|\mathcal{C}^1(y_1) - \mathcal{C}(y_1)| = |16.60373 - 16.60380| = 0.00007 < E$$

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$$x_2 = x_1 + h = 4.1 + 0.1 = 4.2$$

$$P(y_2) = y_1 + y'_1 * h = 16.60373 + 6.07454 * 0.1 = 17.21118$$

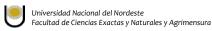
$$P(y'_2) = f(x_2; P(y_2)) = \frac{6x_2^2}{P(y_2)} = 6 * \frac{4.2^2}{17.21118} = 6.14949$$

Para calcular el corregido de y2:

$$C(y_2) = y_1 + \frac{y'_1 + P(y'_2)}{2} * h = 16.60373 + \frac{6.07454 + 6.14949}{2} * 0.1 = 17.21493$$

 $|C(y_2) - P(y_2)| = |17.21493 - 17.21118| = 0.00375 > E$

$$C(y'_2) = f(x_2; C(y_2)) = \frac{6x_2^2}{C(y_2)} = 6 * \frac{4.2^2}{17.21493} = 6.14815$$





$$C^{1}(y_{2}) = y_{1} + \frac{y'_{1} + C(y'_{2})}{2} * h = 16.60373 + \frac{6.07454 + 6.14815}{2} * 0.1 = 17.21486$$

$$C^{1}(y'_{2}) = f(x_{2}; C^{1}(y_{2})) = \frac{6x_{2}^{2}}{C^{1}(y_{2})} = 6 * \frac{4.2^{2}}{17.21486} = 6.14818$$

$$|\mathcal{C}^1(y_2) - \mathcal{C}(y_2)| = |17.21486 - 17.21493| = 0.00007 < E$$

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Para calcular el predicho de y_3 $x_3 = x_2 + h = 4.2 + 0.1 = 4.3$



$$P(y_3) = y_2 + y'_2 * h = 17.21486 + 6.14818 * 0.1 = 17.82968$$

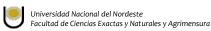
$$P(y'_3) = f(x_3; P(y_3)) = \frac{6x_3^2}{P(y_3)} = 6 * \frac{4.3^2}{17.82968} = 6.22221$$

Para calcular el corregido de y₃:

$$C(y_3) = y_2 + \frac{{y'}_2 + P({y'}_3)}{2} * h = 17.21486 + \frac{6.14818 + 6.22221}{2} * 0.1 = 17.83338$$

 $|C(y_3) - P(y_3)| = |17.83338 - 17.82968| = 0.0037 > E$

$$C(y'_3) = f(x_3; C(y_3)) = \frac{6x_3^2}{C(y_3)} = 6 * \frac{4.3^2}{17.83338} = 6.22092$$





$$C^{1}(y_{3}) = y_{2} + \frac{y'_{2} + C(y'_{3})}{2} * h = 17.21486 + \frac{6.14818 + 6.22092}{2} * 0.1 = 17.83332$$

$$C^{1}(y'_{3}) = f(x_{3}; C^{1}(y_{3})) = \frac{6x_{3}^{2}}{C^{1}(y_{3})} = 6 * \frac{4.3^{2}}{17.83332} = 6.22094$$

$$|C^{1}(y_{2}) - C(y_{2})| = |17.83332 - 17.83338| = 0.00006 < E$$

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Para calcular el predicho de y_4 $x_4 = x_3 + h = 4.3 + 0.1 = 4.4$

$$P(y_4) = y_3 + y'_3 * h = 17.83332 + 6.22094 * 0.1 = 18.45541$$

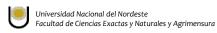
$$P(y'_4) = f(x_4; P(y_4)) = \frac{6x_4^2}{P(y_4)} = 6 * \frac{4.4^2}{18.45541} = 6.29409$$

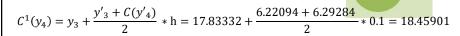
Para calcular el corregido de y₄:

$$C(y_4) = y_3 + \frac{y'_3 + P(y'_4)}{2} * h = 17.83332 + \frac{6.22094 + 6.29409}{2} * 0.1 = 18.45907$$

 $|C(y_4) - P(y_4)| = |18.45907 - 18.45541| = 0.00366 > E$

$$C(y'_4) = f(x_4; C(y_4)) = \frac{6x_4^2}{C(y_4)} = 6 * \frac{4.4^2}{18.45907} = 6.29284$$





 $|C^{1}(y_{4}) - C(y_{4})| = |18.45901 - 18.45907| = 0.00006 < E$

Se pueden organizar los datos obtenidos en una tabla como la siguiente

i	x_i	y_i	y'_i	
0	4	16	6	
1	4.1	16.60373	6.07454	
2	4.2	17.21486	6.14818	
3	4.3	17.83332	6.22094	
4	4.4	18.45901		

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Sabiendo que $y=2x\sqrt{x}$ es la solución exacta de la ecuación diferencial dada, podemos observar los errores cometidos en los métodos vistos:

i	x_i	y_i (Exacto)	y _i (M. Euler)	y_i (M. modif Euler)	Error (M. Euler)	Error (M. modif. Euler)
0	4	16	16	16		
1	4.1	16.60373452	16.6	16.60373	0.00373452	0.00000452
2	4.2	17.21487729	17.20759	17.21486	0.00728729	0.00001729
3	4.3	17.83333956	17.82267	17.83333	0.01066956	0.00000956
4	4.4	18.45903573	18.44514	18.45902	0.01389573	0.00001573

