SECOND ORDER SUBSPACE STATISTICS FOR ADAPTIVE STATE-SPACE PARTITIONING IN MULTIPLE PARTICLE FILTERING

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ABSTRACT

One of the main challenges in nonlinear Bayesian filtering is the so-called curse of dimensionality, that is, the computational complexity increase and associated performance degradation in high-dimensional systems. In the context of particle filtering (PF), a possible solution to mitigate such performance loss is the multiple PF (MPF) approach, where the original state is partitioned into several lower dimensional subspaces, and a set of interconnected PFs are used to characterize the marginal subspace posteriors. Two key issues are: i) how to partition the state, which is application dependent, and ii) how to let the filters (i.e., subspaces) fuse or merge depending on the time-varying conditions of the system, in order to improve the overall estimation performance. We propose a probabilistic approach to the adaptive state-partitioning problem within the MPF, which is based on the computation of subspace second order statistics. An illustrative multiple target tracking example is considered to support the discussion.

Index Terms— Adaptive state-space partitioning, multiple particle filtering, second order subspace statistics.

1. INTRODUCTION & BACKGROUND ON MPF

This paper deals with the problem of performing tracking of potentially large state-space models that, in turn, can be nonlinear and non-Gaussian. Whereas for linear systems the Kalman filter (KF) is a standard solution [1], its performance is degraded in nonlinear/non-Gaussian setups where an appealing solution is that of particle filters (PFs) due to their versatility in adapting to a large variety of probabilistic models [2]. However, PFs are known to suffer from the *curse of dimensionality* [3], which points out that their performance will degrade dramatically as the dimension of the state-space grows. Even in the context of nonlinear/Gaussian systems, where more efficient approaches based on Gaussian filtering [4, 5] can be considered, one also encounters the so-called curse of dimensionality [6].

In the context of PFs, a mitigation strategy was initially proposed in [7, 8]. The solution was named multiple particle filter (or MPF for short) and consists in splitting the state-space into subspaces that are tracked by separate filters, thus reducing the dimensionality at each of those filters. Those filters composing the MPF need to interact since, typically, subspaces cannot be fully decoupled (in which case independent PFs could be used). Several strategies can be considered to implement such interaction including passing point estimates, a set of particles, or a statistical summary of the other filters' marginal posteriors. On the other hand, within nonlinear/Gaussian systems, an alternative is to apply such partitioning to Gaussian filters. The approach was named, analogously, multiple Gaussian filtering (MGF) [9–12] and is conceptually similar.

An important component of MPFs (and MGFs) is the ability to decide which subspaces to split or merge such that the approximations made by state-partitioning schemes are not critically violated. That is, these methods should be able to learn the correlations among states (ideally on-line) such that a bank of coupled filters could be put in place. This issue was first addressed in [13, 14], in the context of multiple target tracking (MTT), where different filters merge or split depending on the distance between targets based on a predetermined threshold. More recently, the authors investigated probabilistic state-partitioning in MGFs [15] based on the estimated cross-correlation among subspaces. In this contribution, we propose merging and splitting methods based on second order statistics of the distributions approximated by the PFs composing the MPF.

The remainder of the paper is organized as follows. Section 2 details the proposed probabilistic method to split and merge subspaces based on second order statistics. Section 3 validates the methodology in a MTT problem, where the MPF is used to compute the split/merge metrics. Section 4 concludes the paper with final remarks.

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2. ADAPTIVE STATE-SPACE PARTITIONING

Standard MGFs [6, 9–11] and MPFs [7, 8, 16] consider a pre-established state-partitioning, which is application-dependent, and in general does not take into account the possible time-varying correlation among subspaces. Mathematically, this implies to assume that the joint posterior distribution of states $\mathbf{x}_t$ given data $\mathbf{y}_{1:t}$ is expressed as[1]

$$p(\mathbf{x}_t|\,\mathbf{y}_{1:t}) = p(\mathbf{x}_t^{(s)}|\,\mathbf{y}_{1:t})p(\mathbf{x}_t^{(-s)}|\,\mathbf{y}_{1:t}) = \prod_{i=1}^{S} p(\mathbf{x}_t^{(i)}|\,\mathbf{y}_{1:t}).$$

Therefore, for a general applicability of such multiple state-partitioning strategies a crucial step involves how to: i) partition the state, and ii) let the different filters (i.e., subspaces) fuse or merge depending on the time-varying conditions of the system (i.e., taking into account possible correlations). In other words, to obtain a good overall estimation performance, uncorrelated subspaces should be decoupled to reduce the dimension that each filter has to explore, but correlated subspaces should be kept in a single filter. A possible solution for target tracking based on an inter-target distance threshold, is to consider that targets which are close in distance are merged into a single subspace, and targets which are far apart can be tracked with different filters [13, 14]. In the Gaussian case, some preliminary results have been proposed in [15], where the cross-correlation (i.e., via the cross-subspace covariance) among subspaces is used to decide which subspaces must merge into a single filter. Let's recall that we are interested in state-space models with states, $\mathbf{x}_t$, which can be separated in the process equation, but not in the observation equation,

$$\mathbf{x}_t = \mathbf{f}_{t-1}(\mathbf{x}_{t-1}, \boldsymbol{\nu}_{t-1}) = \mathbf{f}_{t-1}(\mathbf{x}_{t-1}^{(1)}, \ldots, \mathbf{x}_{t-1}^{(S)}, \boldsymbol{\nu}_{t-1}), \quad (1)$$

$$\mathbf{x}_t^{(i)} = \mathbf{f}_{t-1}^{(i)}(\mathbf{x}_{t-1}^{(i)}, \mathbf{x}_{t-1}^{(-i)}, \boldsymbol{\nu}_{t-1}^{(i)}), \text{ for } i = 1, \ldots, S, \quad (2)$$

$$\mathbf{y}_t = \mathbf{h}_t(\mathbf{x}_t, \mathbf{n}_t) = \mathbf{h}_t(\mathbf{x}_t^{(1)}, \ldots, \mathbf{x}_t^{(S)}, \mathbf{n}_t) \quad (3)$$

which is typically the case for multiple target tracking applications (i.e., each target evolves independently from the others, $\mathbf{x}_t^{(i)} = \mathbf{f}_{t-1}^{(i)}(\mathbf{x}_{t-1}^{(i)}, \mathbf{x}_{t-1}^{(-i)}, \boldsymbol{\nu}_{t-1}^{(i)}) = \mathbf{f}_{t-1}^{(i)}(\mathbf{x}_{t-1}^{(i)}, \boldsymbol{\nu}_{t-1}^{(i)})$), where position-related measurements arise from the superposition of individual target contributions (i.e., received signal strength observations). $\boldsymbol{\nu}_t$ and $\mathbf{n}_t$ are random noise terms.

For the sake of simplicity, but without loss of generality, let's consider that the state at t is partitioned into three subspaces, $\mathbf{x}_t = [\mathbf{x}_t^{(1)}, \mathbf{x}_t^{(2)}, \mathbf{x}_t^{(3)}]$. The MPF is build up from two PFs running in parallel, the first one approximating the marginal subspace posterior $p(\mathbf{x}_t^{(1)}, \mathbf{x}_t^{(2)}|\,\mathbf{y}_{1:t})$, and the second one approximating $p(\mathbf{x}_t^{(3)}|\,\mathbf{y}_{1:t})$. The goal is to design a probabilistic methodology to decide if at time $t+1$ the partitioning must be:

- $\{\mathbf{x}_t^{(1)}, \mathbf{x}_t^{(2)}\}$, $\{\mathbf{x}_t^{(3)}\}$ (the same 2 filters as at time t)
- $\{\mathbf{x}_t^{(1)}\}$, $\{\mathbf{x}_t^{(2)}\}$, $\{\mathbf{x}_t^{(3)}\}$ (3 separate filters)
- $\{\mathbf{x}_t^{(1)}, \mathbf{x}_t^{(2)}, \mathbf{x}_t^{(3)}\}$ (1 filter)
- $\{\mathbf{x}_t^{(1)}\}$, $\{\mathbf{x}_t^{(2)}, \mathbf{x}_t^{(3)}\}$ or $\{\mathbf{x}_t^{(2)}\}$, $\{\mathbf{x}_t^{(1)}, \mathbf{x}_t^{(3)}\}$ (2 filters but different combination of subspaces)

To decide whether the partitioning from t to $t+1$ must change or remain the same, we have to characterize the possible correlation among subspaces. We propose to exploit the subspace second order statistics. Note that the merge/split does not need to be done at every time step.

2.1. Subspace Splitting by Second Order Statistics

We are here interested in the case of a PF within the MPF tracking several subspaces (i.e., targets) and deciding whether to continue in that way or split into several PFs operating on smaller subspaces. We propose to use the estimation of the covariance matrix and determine whether the cross-correlation terms between subspaces are negligible (then decide to split) or not (then decide to keep the joint filter).

Each PF within the MPF approximates the marginal subspace posterior, $p(\mathbf{x}_t^{(s)}|\,\mathbf{y}_{1:t})$. Considering that the PF uses M_s particles and weights, $\{\mathbf{x}_{t,s}^i, w_{t,s}^i\}_{i=1}^{M_s}$, the second order statistics can be approximated as

$$\boldsymbol{\Sigma}_{t,x}^{(s)} \approx \sum_{i=1}^{M_s} w_{t,s}^i \left(\mathbf{x}_{t,s}^i - \hat{\mathbf{x}}_t^{(s)}\right)\left(\mathbf{x}_{t,s}^i - \hat{\mathbf{x}}_t^{(s)}\right)^\top, \quad (4)$$

where $p(\mathbf{x}_t^{(s)}|\,\mathbf{y}_{1:t}) \approx \sum_{i=1}^{M_s} w_{t,s}^i \delta(\mathbf{x}_t^{(s)} - \mathbf{x}_{t,s}^i)$ and the subspace MMSE estimate is

$$\hat{\mathbf{x}}_t^{(s)} = \sum_{i=1}^{M_s} w_{t,s}^i \mathbf{x}_{t,s}^i \,. \quad (5)$$

Going back to the running example with three targets, let's consider the PF which tracks $\{\mathbf{x}_t^{(1)}, \mathbf{x}_t^{(2)}\}$. Notice that (4) in this case is arranged as

$$\boldsymbol{\Sigma}_{t,x}^{(\{1,2\})} = \left(\begin{array}{c|c} \boldsymbol{\Sigma}_{t,x}^{(1)} & \boldsymbol{\Sigma}_{t,x}^{(1,2)} \\ \hline \boldsymbol{\Sigma}_{t,x}^{(2,1)} & \boldsymbol{\Sigma}_{t,x}^{(2)} \end{array}\right), \quad (6)$$

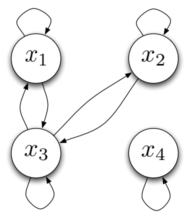
and we are interested in determining whether $\boldsymbol{\Sigma}_{t,x}^{(1,2)}$ is $\mathbf{0}$ in order to decide splitting subspaces 1 and 2 into separate PFs. We explain the proposed strategy first assuming scalar subspaces, in which case the above reduces to

$$\boldsymbol{\Sigma}_{t,x}^{(\{1,2\})} = \left(\begin{array}{c|c} \sigma_{x1}^2 & \rho_{12}\sigma_{x1}\sigma_{x2} \\ \hline \rho_{12}\sigma_{x1}\sigma_{x2} & \sigma_{x2}^2 \end{array}\right), \quad (7)$$

[1] $\mathbf{x}^{(s)}$ denotes the s-th element (possibly a vector) in a vector $\mathbf{x}$ and $\mathbf{x}^{(-s)}$ is the vector of all elements in $\mathbf{x}$ except for $\mathbf{x}^{(s)}$. The dimension of each subspace $n_x^{(s)} = \dim\{\mathbf{x}_t^{(s)}\}$ is defined such that $\sum_{s=1}^{S} n_x^{(s)} = n_x$, $s \in \mathcal{S} = \{1, \ldots, S\}$, and $n_x^{(-s)} = \dim\{\mathbf{x}_t^{(-s)}\}$.

with $|\rho_{12}| \in [0, 1]$ the correlation coefficient between state variables x_1 and x_2. Given a correlation threshold β_1 (i.e., $\beta_1 = 0.5$), a binary correlation matrix $\mathbf{B}_{t,x}^{(\{1,2\})}$ can be constructed by thresholding (7), that is, if $|\rho_{ij}| > \beta_1 \to 1$ or 0 otherwise. For subspaces with $n_x^{(i)} > 1$, the correlation coefficients ρ_{ij} can be computed by pairs of variables. The binary correlation matrix $\mathbf{B}_{t,x}^{(\{1,2\})}$ can be analyzed via a graph approach to decide which state variables must be kept together and the ones which can be split. Such concept is easy to understand graphically, for instance, consider the following correlation matrix for four state variables ($\{x_1, x_2, x_3, x_4\}$) and its associated graph,

$$\mathbf{B}_{t,x}^{(\{1:4\})} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow$$



where it is clear that we must group $\{x_1, x_2, x_3\}$, but x_4 can be separated from the original set. In the multiple target tracking case of interest, we must take into account a minimum subspace dimension constraint, that is, we might want to keep together the state variables of a given target. In that case, from the pairwise binary correlation matrix we must construct an equivalent group subspace binary matrix. If connections exist between variables among groups, then we assume that the two groups are correlated. In the previous example, if we define a group being $g_1 = \{x_1, x_2\}$, and the other one $g_2 = \{x_3, x_4\}$, the group binary matrix reads,

$$\mathbf{B}_{t,x}^{(\{g_1,g_2\})} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},$$

then, both groups are kept together. If the off-diagonal elements were equal to 0 the filter should decide to split in two.

2.2. Subspace Merging by KL Divergence

The problem of merging subspaces from two separate filters into a joint filter is slightly different. In this case, we do not have direct access to the cross-correlation term in (6), since the filters are only characterizing the diagonal elements. We propose instead to compare the marginal distributions of the two filters, $p(\mathbf{x}_t^{(s)}|\,\mathbf{y}_{1:t})$ and $p(\mathbf{x}_t^{(p)}|\,\mathbf{y}_{1:t})$, to determine if the subspaces are statistically *close*. We suggest to summarize the particle approximation with its mean and covariance, and then compare those Gaussian approximations using the Kullback-Leibler (KL) divergence metric. More precisely, we approximate the marginal of the s-th subspace as

$$p(\mathbf{x}_t^{(s)}|\,\mathbf{y}_{1:t}) \approx \mathcal{N}\left(\hat{\mathbf{x}}_t^{(s)}, \mathbf{\Sigma}_{t,x}^{(s)}\right) \tag{8}$$

using the summaries provided by (4) and (5). We proceed similarly for $p(\mathbf{x}_t^{(p)}|\,\mathbf{y}_{1:t})$. Then, we compute the KL divergence, assessing if two marginals are statistically different:

$$\mathcal{D}_{KL}^{(\{s,p\})} = \frac{1}{2}\left(\mathrm{tr}\left(\left(\mathbf{\Sigma}_{t,x}^{(s)}\right)^{-1}\mathbf{\Sigma}_{t,x}^{(p)}\right) + \ln\left(\frac{\det \mathbf{\Sigma}_{t,x}^{(s)}}{\det \mathbf{\Sigma}_{t,x}^{(p)}}\right) + \right.$$
$$\left.(\hat{\mathbf{x}}_t^{(s)} - \hat{\mathbf{x}}_t^{(p)})^\top \left(\mathbf{\Sigma}_{t,x}^{(s)}\right)^{-1}(\hat{\mathbf{x}}_t^{(s)} - \hat{\mathbf{x}}_t^{(p)}) - k\right),$$

where k is the dimension of the subspaces.

3. COMPUTER SIMULATIONS

The validity of the proposed adaptive MPF approach is illustrated through a simple MTT example, but notice that it could be applied to other contexts. A received signal strength (RSS) 2D MTT scenario is considered, where K targets are tracked using a set of N sensors, uniformly distributed in a grid. For each target i, both 2D position and velocity are to be estimated, $\mathbf{x}_t^{(i)} = [p_{x,t}^{(i)}, p_{y,t}^{(i)}, v_{x,t}^{(i)}, v_{y,t}^{(i)}]^\top$. The m-th sensor, at time t, receives the superposition of the different RSS contributions from the targets present in the area,

$$y_{m,t} = \sum_{i=1}^{K} 10\log_{10}\left(\frac{1}{|\mathbf{r}_m - \mathbf{l}_{i,t}|^2}\right) + n_{m,t},$$

with $n_{m,t} \sim \mathcal{N}(0, \sigma_m^2)$ being the sensor noise related to the RSS measurement, $\mathbf{l}_{i,t} = [p_{x,t}^{(i)}, p_{y,t}^{(i)}]^\top$ the i-th target position, and $\mathbf{r}_m$ the known 2D grid sensor position. The K targets follow a constant velocity dynamic model,

$$\mathbf{x}_t^{(i)} = \begin{pmatrix} \mathbf{I}_2 & T_s \cdot \mathbf{I}_2 \\ \mathbf{0} & \mathbf{I}_2 \end{pmatrix} \mathbf{x}_{t-1}^{(i)} + \boldsymbol{\nu}_{t-1}^{(i)},\ \boldsymbol{\nu}_{t-1}^{(i)} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}),$$

with $\mathbf{Q} = \mathrm{diag}(\sigma_{p_x}^2, \sigma_{p_y}^2, \sigma_{v_x}^2, \sigma_{v_y}^2)$ accounting for a possible dynamic model error, T_s the sampling period and $\mathbf{I}_2$ the 2×2 identity matrix. The full state to be estimated is the concatenation of the set of target's states $\mathbf{x}_t^\top = [(\mathbf{x}_t^{(1)})^\top, \cdots, (\mathbf{x}_t^{(K)})^\top]$. Only the $K = 2$ targets case ($n_x = 8$) is analyzed in the simulations for the sake of simplicity. We consider $N = 100$ sensors in a 1000×1000 m^2 grid and the same noise statistics (σ_m^2 and $\mathbf{Q}$) for all sensors and targets.

3.1. Case 1: Splitting Strategy

First, we analyze the validity of the splitting strategy proposed in Section 2.1 based on the estimated subspace second order statistics (4). We consider one of the PFs within the MPF, using $M = 2000$ particles and tracking the two targets $\mathbf{x}_t^\top = [(\mathbf{x}_t^{(1)})^\top, (\mathbf{x}_t^{(2)})^\top]$. In the top plot of Fig. 1 we show the trajectory of both targets and the corresponding PF estimate, where we can see that the PF is performing well. In the bottom plot of Fig. 1 we show: i) the normalized distance between targets, and ii) the off-diagonal value of the group subspace binary matrix, where we considered a correlation threshold $\beta_1 = 0.4$. A value $\mathbf{B}_{t,x}^{(g)}(2, 1)$ equal to 0 implies that both subspaces can be split (no correlation), otherwise they should be kept together. In the figure we can see that this value is only equal to 1 when both targets are close in distance, which confirms the validity of the proposed approach based on the subspace covariance. To provide an insight of the valuable information in the cross-covariance regarding the correlation between subspaces, we also plot: iii) the normalized trace of the cross-covariance between targets, and iv) the corresponding normalized determinant. We can see that their value increase only when both targets are close.

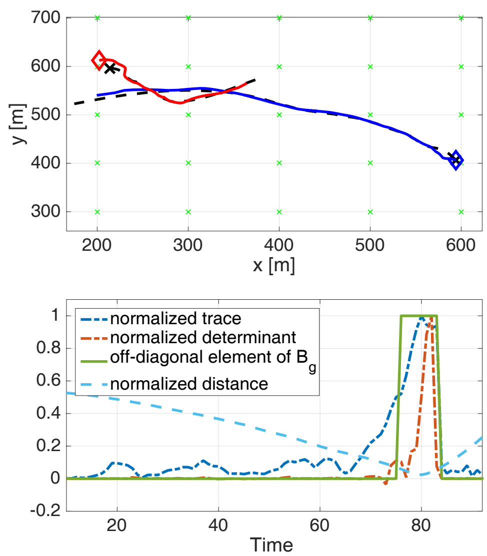


Fig. 1. (Top) True trajectories and PF estimate; (Bottom) Trace, determinant, off-diagonal element of the group subspace binary matrix and normalized distance between targets.

3.2. Case 2: Merging Strategy

To assess the validity of the merging strategy based on the KL divergence between marginal subspace posteriors, we consider a MPF with two filters, each one tracking one target, and $M_1 = M_2 = 200$ particles. In the left plots of Fig. 2 we show the true and estimated MPF trajectories for two different examples, and the corresponding relative inter-target distance and KL metric is shown in the right plots. In the top left plot, the estimated trajectories are miss-associated after the targets get close in distance. In the left bottom plot, the estimated trajectories degrade after the targets get close. In both cases, to avoid those issues, the two filters within the MPF should have been merged in a single filter. This is clear in the right plots, where the KL divergence drops when both targets get close in distance, which confirms the validity of the metric.

In conclusion, it was shown that the MPF has to take into account the correlation among subspaces: i) splitting subspaces to avoid the curse of dimensionality and improve the overall computational complexity, and ii) merging subspaces to avoid performance degradation. The proposed strategies based on the estimated second order subspace statistics provide a probabilistic solution to the problem.

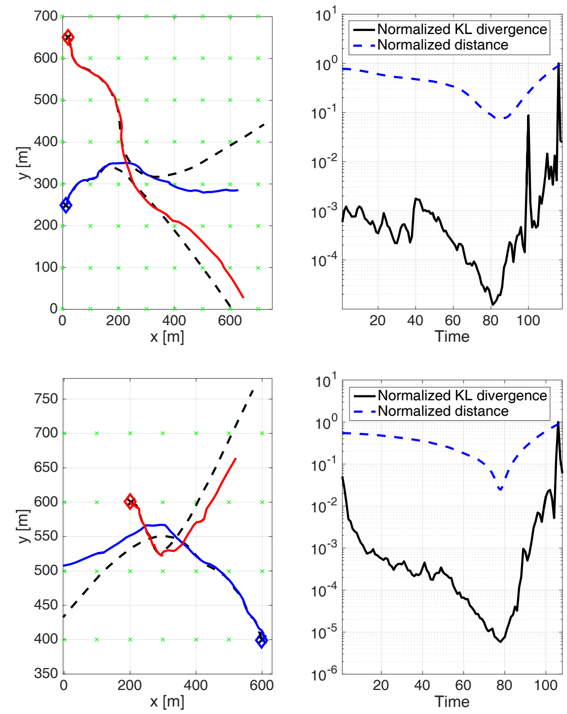


Fig. 2. (Left) Two examples of true and MPF estimated trajectories; (Right) Normalized distance between targets and KL divergence metric for both examples.

4. CONCLUSIONS

Multiple particle filtering (MPF) takes a divide-and-conquer approach to deal with the curse of dimensionality of standard particle filters. A critical step is to decide how the full state-space is to be splitted into multiple subspaces, potentially doing so online as the filter operates. We have seen in previous work that it is important to tackle subspaces jointly when the states become correlated. In this paper we propose a methodology for splitting/merging subspaces that accounts for the correlation among the states associated to each target. The method is based on the estimated second order subspace statistics, which are obtained from the outputs provided by the different PFs associated to the overall MPF. For splitting, we operate directly on the cross-subspace covariance, while for merging the KL divergence between the marginals is exploited since the cross-covariance is not available. The results show promising performance of both processes in a multiple target tracking case, being able to discriminate when to split or merge the different subspace tracking filters.

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