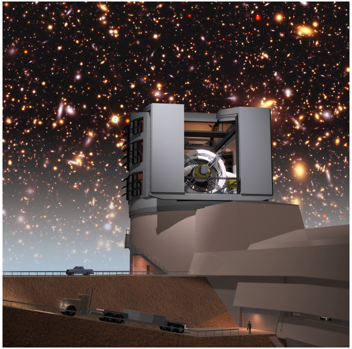
Weak Lensing: theory & estimators

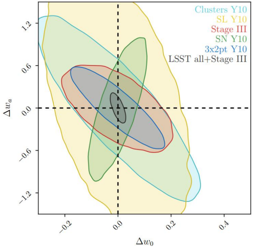
Alejandro Avilés & Sofía Samario
sabiduria_sofy@hotmail.com, avilescervantes@gmail.com

MACSS. June 19, 2023

León, Guanajuato

Motivation: Rubin Observatory LSST – DESC





LSST DESC SRD v1 (1809.01669)

Science cases:

1. Dark Energy: Dark Energy Task Force report (Albrecht et al. 2006).
2. Constrain neutrino mass
3. Test gravity
4. Halo-Galaxy connection

Photometric redshift

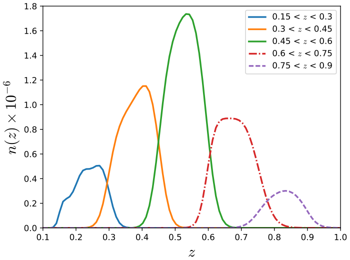
We do not always have access to accurate redshift measurements of astronomical objects. This is the case of, for example, photometric surveys. However, we do have high-quality images in several wavelength bands. These colors can be converted into rough estimates of redshift, so-called photometric redshifts, which can be used as proxies (with significant spread) for true redshifts.

More precisely, we can infer distributions of number of galaxies,

$$W_g(z) = \frac{1}{N_g}\frac{dN_g}{dz},$$

and depending on the features of each galaxy decide to which distribution it belongs.

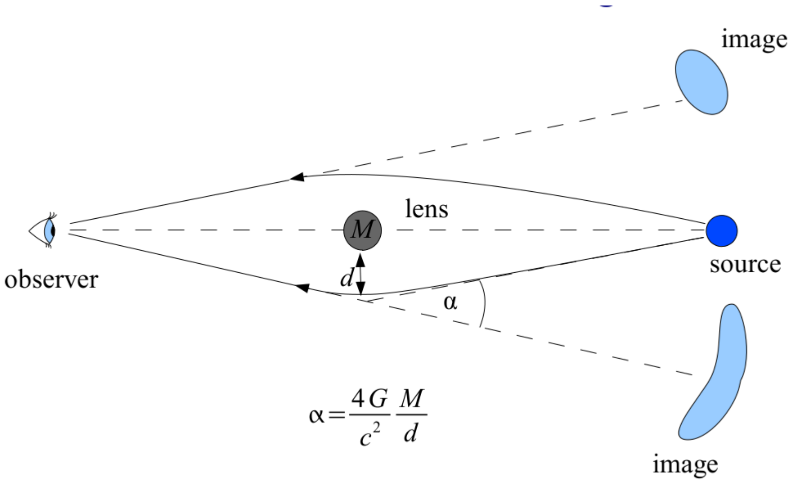
DES 1 yr results (1708.01536)



Participación mexicana (in-kind contributions)

1. Strong Lensing: contact Alma or Luis
2. Photo-z and blinding: contact Sebastien, Josué or Alberto
3. 3-point correlation functions: contact Alejandro or Gustavo
4. Stars, Milky Way and Local Volume.
5. Galaxy classification with Machine Learning: contact Gibran

Convocatoria: lsst.mx/convocatorias



Credit: Stefan Hilbert (MPA)

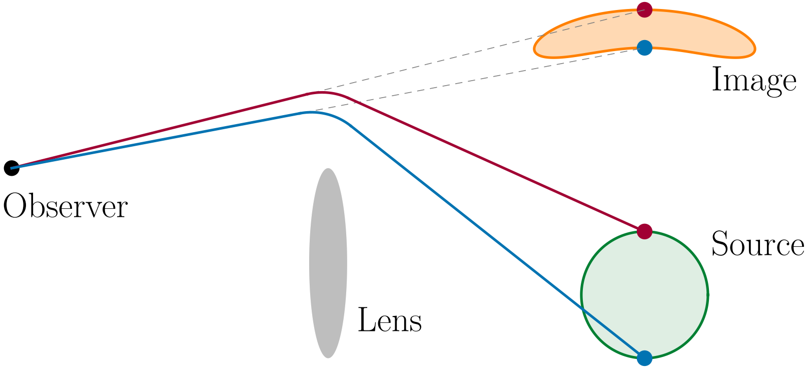
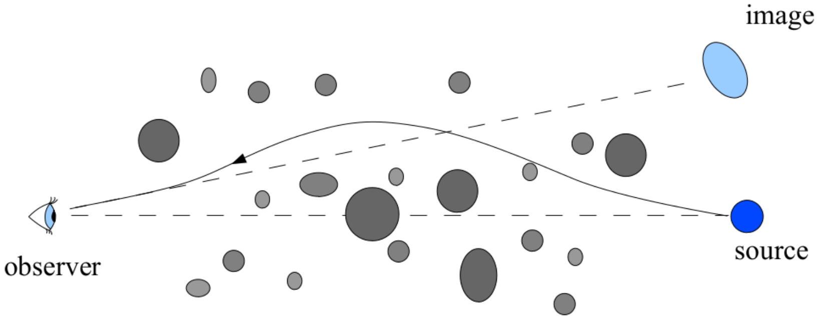


Image
Observer
Lens
Source



Credit: Stefan Hilbert (MPA)

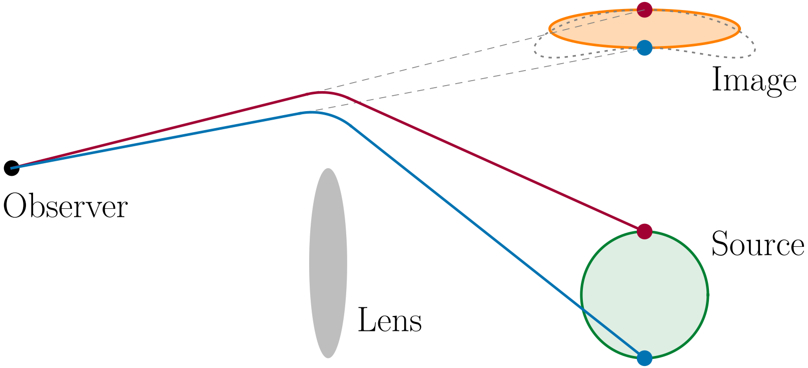
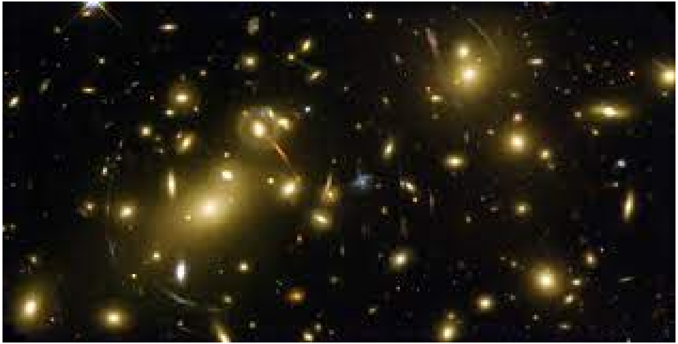
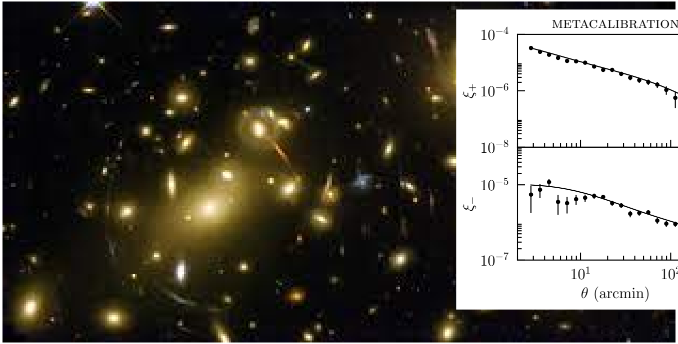


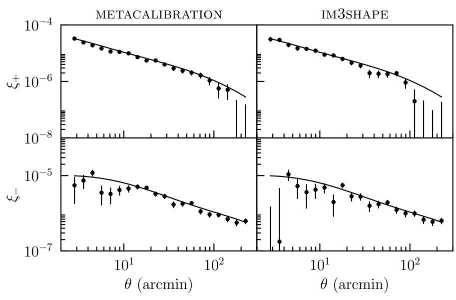
Image
Observer
Source
Lens

The statistical properties of the cosmic shear are directly linked to the statistical properties of density fluctuations (that is, to the total matter power spectrum). Hence, contrary to other Cosmological tests, WL is probing the dark matter itself

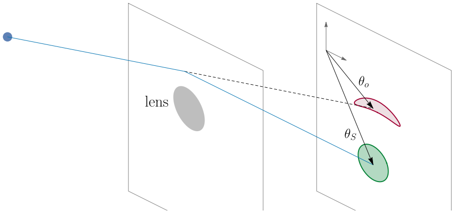


The statistical properties of the cosmic shear are directly linked to the statistical properties of density fluctuations (that is, to the total matter power spectrum). Hence, contrary to other Cosmological tests, WL is probing the dark matter itself





DES y1 results (1708.01538)



Geodesic equation for photons $\quad \theta_m^{\text{Source}} = \theta_m^O + \dfrac{\partial \psi(\boldsymbol{\theta}, \chi)}{\partial \theta^m}$

with lensing potential $\quad \psi(\boldsymbol{\theta}, \chi) = -\int_0^{\chi} \frac{d\chi'}{\chi'} \frac{\chi - \chi'}{\chi} \Big[\Psi(\boldsymbol{x}(\boldsymbol{\theta}, \chi')) + \Phi(\boldsymbol{x}(\boldsymbol{\theta}, \chi')) \Big]$

(linearize: $\quad \theta \to \theta^O + \cdots$)

where $\quad \chi(z) = \int_0^z \frac{dz'}{H(z')} = \eta_0 - \eta(z)$ $\quad$ is the radial comoving distance

For small deflections of path light rays:

$$A_{mn} \equiv \frac{\partial \theta_m^S}{\partial \theta^n} = I_{mn} + \partial_m \partial_n \psi \equiv \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}.$$

Assuming small deflection angles we expand

$$\theta_m^S(\theta) = \underbrace{A_{mn}\theta_n}_{\text{Convergence + Shear}} + \underbrace{\frac{1}{2} D_{mns}\theta_n\theta_s}_{\text{Flexion}} + \cdots$$

$$\text{Weak lensing:} \qquad \boxed{\theta_m^S = A_{mn}\theta_n.}$$

That is, weak lensing is described by a linear map relating the observed and true positions of the sources.

Weak lensing assumes that the value of the derivatives of the lensing potential $\partial_m \partial_n \psi$ do not change across the source surface (e.g., through a galaxy subtended solid angle).

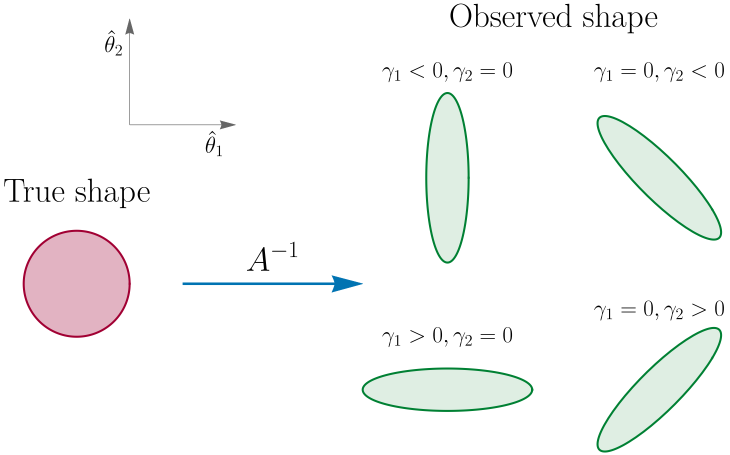
Weak lensing:

$$\begin{pmatrix} \theta_1^S \\ \theta_2^S \end{pmatrix} = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} + \begin{pmatrix} -\kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & -\kappa + \gamma_1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}.$$

Using $\theta_i^S = \theta_i + (\partial_i \partial_j \psi)\,\theta^j + \cdots$, with ψ the lensing potential

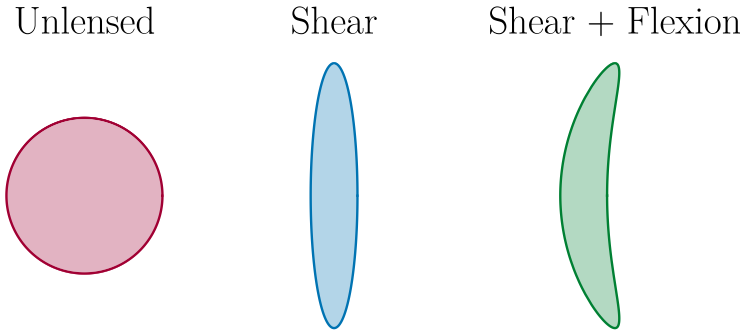
convergence: $\kappa = -\frac{1}{2}(\partial_1\partial_1 + \partial_2\partial_2)\psi = -\frac{1}{2}\partial^2\psi$

shear: $\gamma_1 = -\frac{1}{2}(\partial_1\partial_1 - \partial_2\partial_2)\psi, \qquad \gamma_2 = -\partial_1\partial_2\psi$



$\hat{\theta}_2$
$\hat{\theta}_1$
Observed shape
$\gamma_1 < 0, \gamma_2 = 0$
$\gamma_1 = 0, \gamma_2 < 0$
True shape
A^{-1}
$\gamma_1 > 0, \gamma_2 = 0$
$\gamma_1 = 0, \gamma_2 > 0$

$$\boldsymbol{\theta}_s = \boldsymbol{A}\,\boldsymbol{\theta} + \frac{1}{2}\boldsymbol{\theta}^T \boldsymbol{D}\,\boldsymbol{\theta}$$



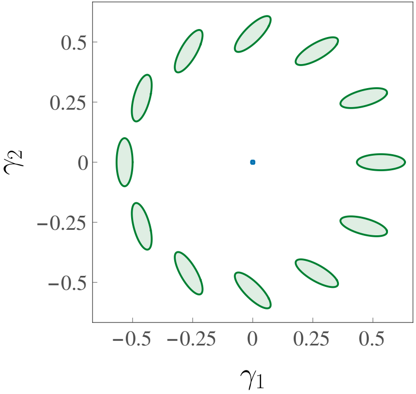
$$A_{mn} = \begin{pmatrix} 1-\kappa & 0 \\ 0 & 1-\kappa \end{pmatrix} + \begin{pmatrix} -\gamma_1 & -\gamma_2 \\ -\gamma_2 & \gamma_1 \end{pmatrix} \equiv (1-\kappa) I_{mn} + \Gamma_{mn}.$$

Against a rotation, the convergence κ does not change (spin-0 component).

The shear components γ_1 and γ_2 transform between themselves as

$$\begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} \to \begin{pmatrix} \gamma_1' \\ \gamma_2' \end{pmatrix} = \begin{pmatrix} \gamma_1 \cos(2\varphi) - \gamma_2 \sin(2\varphi) \\ \gamma_2 \cos(2\varphi) + \gamma_1 \sin(2\varphi) \end{pmatrix},$$

that is, they transform with the double angle (spin-2 components).

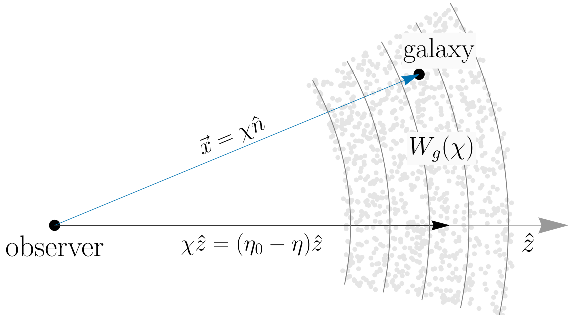


Convergence as a projected density

$$\kappa(\boldsymbol{\theta},\chi) = -\frac{1}{2}\partial^2\psi = \frac{3}{2}\Omega_m H_0^2 \int_0^{\chi} \frac{d\chi'}{a(\chi')}\chi'\frac{\chi-\chi'}{\chi}\delta(\chi'\boldsymbol{\theta},\chi'),$$

where we used $\partial^2 = \chi^2\nabla^2_{\boldsymbol{x}}$ and the Poisson equation.

Notice that the geometrical factor $\chi'(\chi-\chi')$ is a parabola with maximum at $\chi' = \chi/2$. Hence, structures at half the distance between the source and the observers are more efficient to produce lensing distortions.

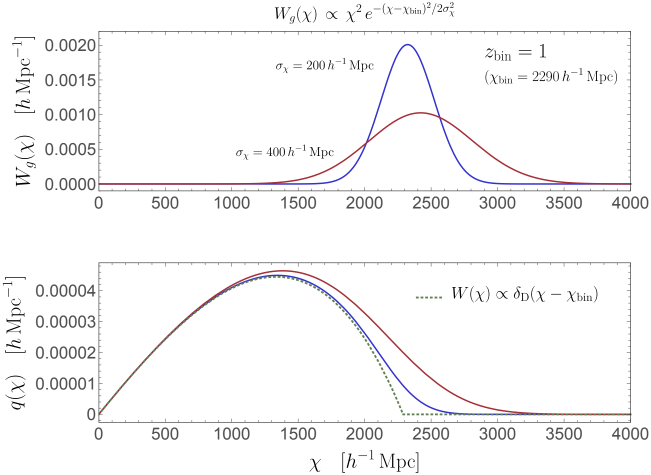


The total convergence from a population of source galaxies is obtained by weighting $\kappa(\theta, \chi)$ with the galaxy probability distribution $W_g(\chi)$:

$$\kappa(\boldsymbol{\theta}) = \int_0^\infty d\chi W_g(\chi)\kappa(\boldsymbol{\theta}, \chi) = \int_0^\infty d\chi' q(\chi')\delta(\chi'\boldsymbol{\theta}, \chi')$$

with the lens efficiency q defined as $\quad q(\chi) = \frac{3}{2}\Omega_m H_0^2 \frac{\chi}{a(\chi)} \int_\chi^\infty d\tilde{\chi}\, W_g(\tilde{\chi}) \frac{\tilde{\chi} - \chi}{\tilde{\chi}}$

The convergence becomes a linear measure of the total matter density, projected along the line of sight and weighted by the source galaxy distribution W_g.



$W_g(\chi) \propto \chi^2 e^{-(\chi-\chi_{\rm bin})^2/2\sigma_\chi^2}$
$W_g(\chi) \quad [h\,{\rm Mpc}^{-1}]$
0.0020
0.0015
0.0010
0.0005
0.0000
$\sigma_\chi = 200\,h^{-1}\,{\rm Mpc}$
$\sigma_\chi = 400\,h^{-1}\,{\rm Mpc}$
$z_{\rm bin} = 1$
$(\chi_{\rm bin} = 2290\,h^{-1}\,{\rm Mpc})$
0
500
1000
1500
2000
2500
3000
3500
4000
$q(\chi) \quad [h\,{\rm Mpc}^{-1}]$
0.00004
0.00003
0.00002
0.00001
0
$W(\chi) \propto \delta_{\rm D}(\chi - \chi_{\rm bin})$
$\chi \quad [h^{-1}\,{\rm Mpc}]$

Convergence angular power spectrum

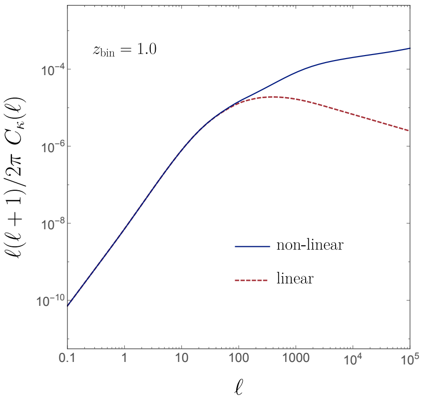
FT:

$$\kappa(\boldsymbol{\ell}) = \int d^2\theta \, e^{-i\boldsymbol{\theta}\cdot\boldsymbol{\ell}} \kappa(\boldsymbol{\theta})$$

PS def:

$$\langle \kappa(\boldsymbol{\ell})\kappa(\boldsymbol{\ell}') \rangle = (2\pi)^2 \delta_{\rm D}(\boldsymbol{\ell}+\boldsymbol{\ell}') C_{\mathcal{E}}(\ell)$$

$$C_\kappa(\ell) = \int_0^\infty \frac{d\chi}{\chi^2} q^2(\chi) P_\delta\left(k = \frac{\ell + 1/2}{\chi}, \chi\right).$$

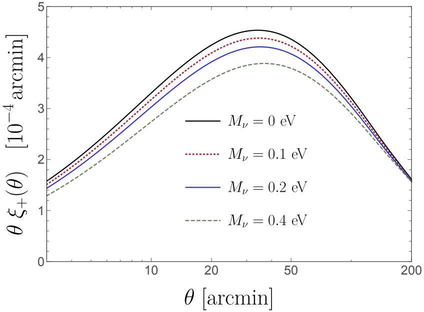


Convergence correlation function

2pcf def:

$$\xi(\theta) = \langle \kappa(\boldsymbol{\nu})\kappa(\boldsymbol{\nu}+\boldsymbol{\theta})\rangle$$

$$\xi(\theta) = \int \frac{d^2\ell}{(2\pi)^2} e^{i\boldsymbol{\theta}\cdot\boldsymbol{\ell}} C_\kappa(\ell)$$
$$= \int_0^\infty \frac{d\ell}{2\pi}\, \ell\, C_\kappa(\ell) J_0(\theta\ell)$$



Credit:Rafael Morales (github.com/rafaelmoramore/WLnu)

Weak Lensing @ MACSS 2023

Project session 1:

Construct an estimator for the weak lensing convergence field correlation function

github.com/alejandroaviles/WL_MACSS-2023

k : spin-0 field in 2D.
Its 2PCF is

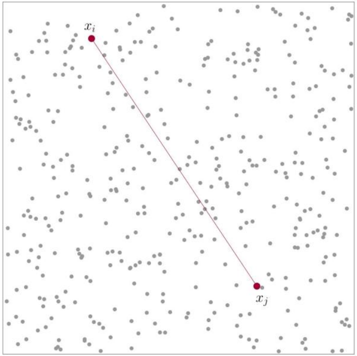
$$\xi(\theta) = \langle \kappa(\boldsymbol{\nu})\kappa(\boldsymbol{\nu} + \boldsymbol{\theta}) \rangle$$

An unbiased estimator is given by

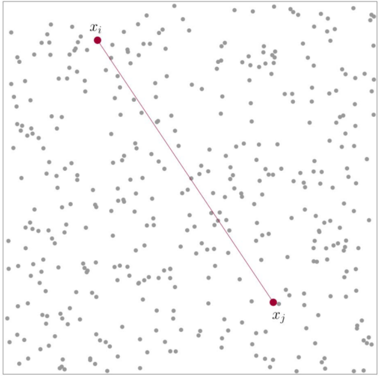
$$\hat{\xi}(\theta) = \int \frac{d^2\nu}{A} \kappa(\boldsymbol{\nu})\kappa(\boldsymbol{\nu} + \boldsymbol{\theta})$$

which has computational complexity O(N^2)
(not considering tree methods)

$$\kappa(\boldsymbol{\theta}) = \sum_{i=1}^{N} \kappa(\boldsymbol{\theta}_i)\delta_{\mathrm{D}}(\boldsymbol{\theta} - \boldsymbol{\theta}_i)$$



$$\hat{\xi} = \frac{1}{\sum_{ij} w_i w_j} \sum_{ij} w_i w_j \kappa_i \kappa_j$$



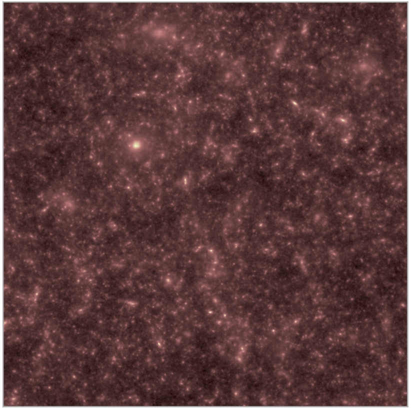
Data: N number of particles with positions $\boldsymbol{x}_i$ and convergence values κ_i.

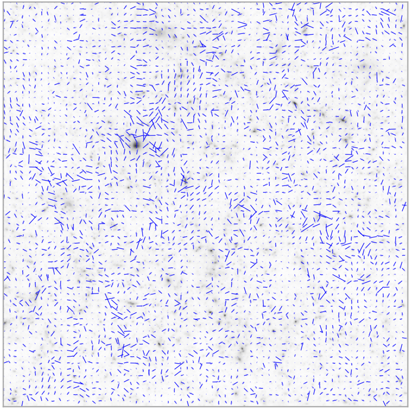
Result: $\xi = \frac{1}{N_{bin}} \sum_{pairs} \kappa_i \kappa_j$

for i in N **do**

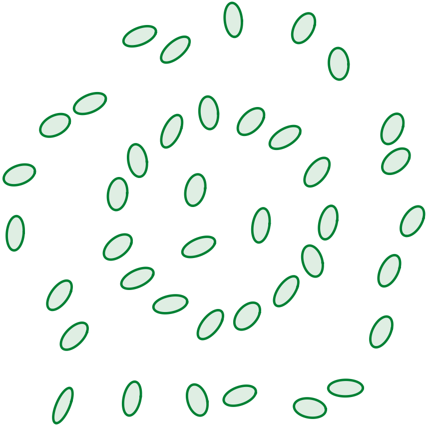
```
    kappai = kappa(x_i);
    for j in N do
        distance = |x_i - x_j|;
        kappaj = kappa(x_j);
        ibin;    # e.g. ibin=distance/Maxdist
        Bins[ibin] += kappai*kappaj;
        NumberInBins[ibin] += 1;
    end
end
histogram[ibin] = Bins[ibin]/NumberInBins[ibin];
```

Galaxy shear

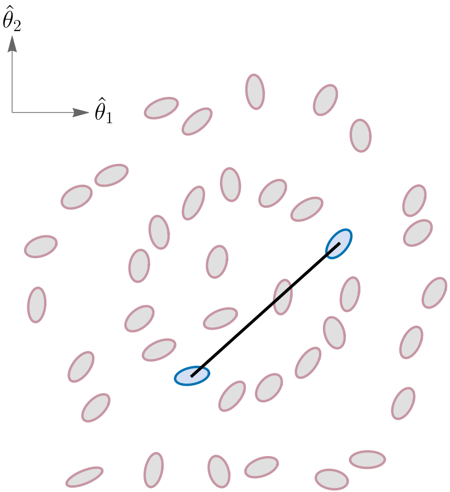


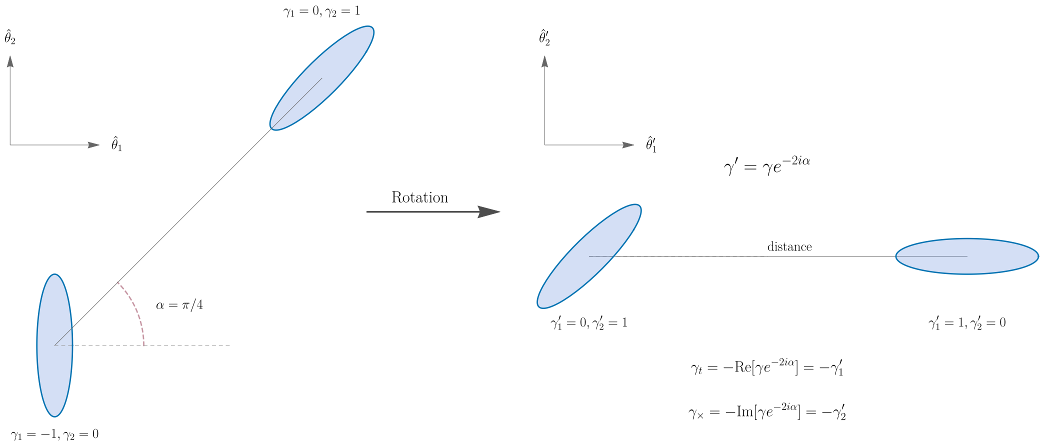


How can we construct a correlation function for galaxy shear $\xi_\gamma(\theta)$?



How can we construct a correlation function for galaxy shear $\xi_\gamma(\theta)$?





$\gamma_1 = 0, \gamma_2 = 1$
$\hat{\theta}_2$
$\hat{\theta}_1$
$\alpha = \pi/4$
$\gamma_1 = -1, \gamma_2 = 0$
Rotation
$\hat{\theta}'_2$
$\hat{\theta}'_1$
$\gamma' = \gamma e^{-2i\alpha}$
distance
$\gamma'_1 = 0, \gamma'_2 = 1$
$\gamma'_1 = 1, \gamma'_2 = 0$
$\gamma_t = -\mathrm{Re}[\gamma e^{-2i\alpha}] = -\gamma'_1$
$\gamma_\times = -\mathrm{Im}[\gamma e^{-2i\alpha}] = -\gamma'_2$

The shear correlation functions are defined as

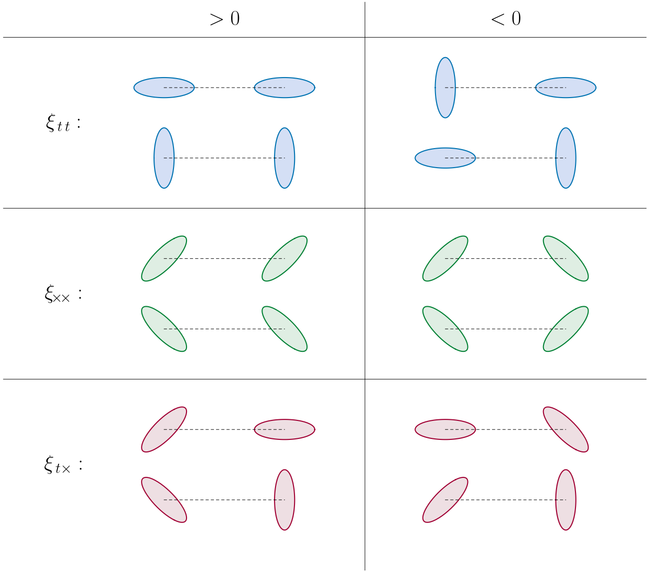
$$\xi_{tt}(\theta) = \langle \gamma_t(\boldsymbol{\nu}) \gamma_t(\boldsymbol{\nu} + \boldsymbol{\theta}) \rangle$$
$$\xi_{\times\times}(\theta) = \langle \gamma_\times(\boldsymbol{\nu}) \gamma_\times(\boldsymbol{\nu} + \boldsymbol{\theta}) \rangle$$
$$\xi_{t\times}(\theta) = \langle \gamma_t(\boldsymbol{\nu}) \gamma_\times(\boldsymbol{\nu} + \boldsymbol{\theta}) \rangle$$

$$\xi_+(\theta) = \xi_{tt}(\theta) + \xi_{\times\times}(\theta)$$
$$\xi_-(\theta) = \xi_{tt}(\theta) - \xi_{\times\times}(\theta)$$

$\xi_{t\times} = 0$ for shear produced by weak lensing

Discrete Estimators:

$$\hat{\xi}_{tt}(\theta) = \frac{1}{N_{\text{pairs}}} \sum_{\text{pairs } i,j} \gamma_{i,t}(\boldsymbol{\theta}_1) \gamma_{j,t}(\boldsymbol{\theta}_2) \Delta_\theta(|\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2|)$$
$$\hat{\xi}_{\times\times}(\theta) = \frac{1}{N_{\text{pairs}}} \sum_{\text{pairs } i,j} \gamma_{i,\times}(\boldsymbol{\theta}_1) \gamma_{j,\times}(\boldsymbol{\theta}_2) \Delta_\theta(|\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2|)$$



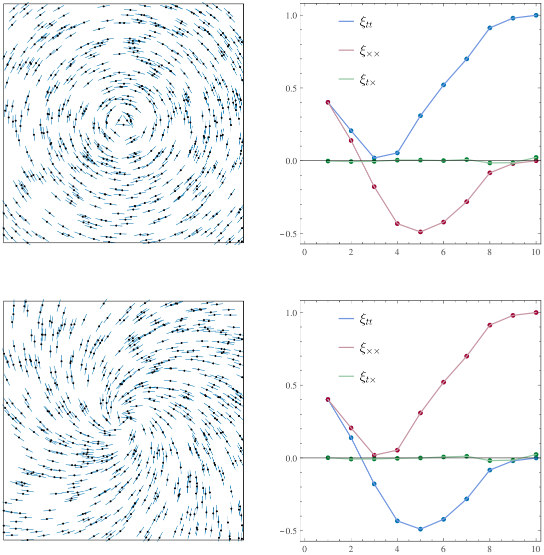
> 0
< 0
ξ_{tt} :
$\xi_{\times\times}$:
$\xi_{t\times}$:

Weak Lensing @ MACSS 2023

Project session 2:

Construct an estimator for the weak lensing galaxy shear field correlation function

github.com/alejandroaviles/WL_MACSS-2023



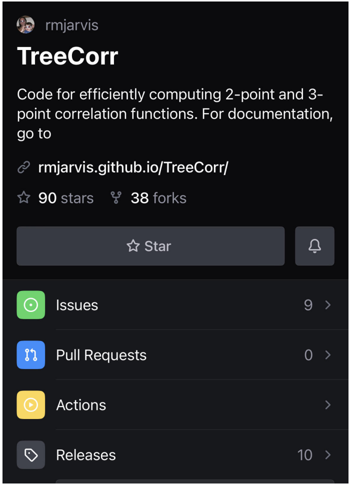
ξ_{tt}
$\xi_{\times\times}$
$\xi_{t\times}$
ξ_{tt}
$\xi_{\times\times}$
$\xi_{t\times}$

Weak Lensing @ MACSS 2023

Project session 3:

Use treecorr to compute correlation function from realistic simulations/real surveys

github.com/alejandroaviles/WL_MACSS-2023



Pass the catalogue to treecorr

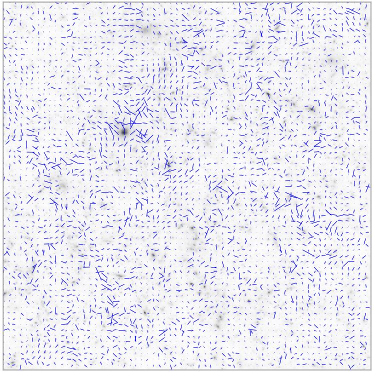
```
[ ] #cat=treecorr.Catalog(ra=xT,dec=yT,g1=A0gamma1,g2=A0gamma2,ra_units="rad",dec_units="rad")
    catalogue=treecorr.Catalog(x=xT,y=yT,g1=gamma1,g2=gamma2,x_units="rad",y_units="rad")
```

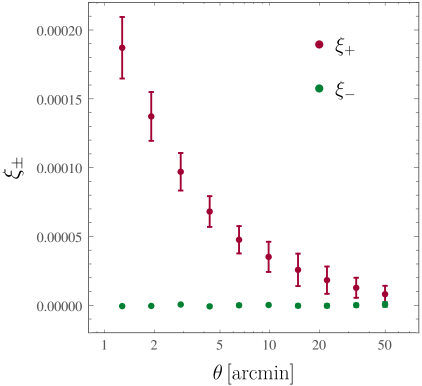
Run treecorr

```
[ ] GG=treecorr.GGCorrelation(min_sep=minRad,max_sep=maxRad,verbose=2,nbins=nbins)
    GG.process(catalogue)
```

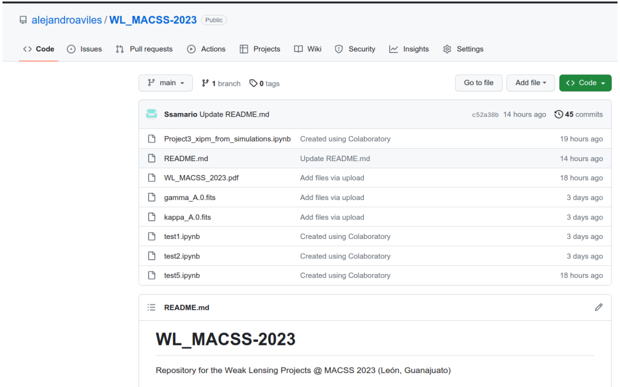
```
nbins = 10, min,max sep = 0.000290888..0.0174533, bin_size = 0.409434
INFO:treecorr:nbins = 10, min,max sep = 0.000290888..0.0174533, bin_size = 0.409434
Starting process GG auto-correlations
INFO:treecorr:Starting process GG auto-correlations
Using 2 threads.
INFO:treecorr:Using 2 threads.
Building GField
INFO:treecorr:Building GField
Starting 256 jobs.
INFO:treecorr:Starting 256 jobs.
varg = 0.000354: sig_sn (per component) = 0.018827
INFO:treecorr:varg = 0.000354: sig_sn (per component) = 0.018827
```

From shear to statistics

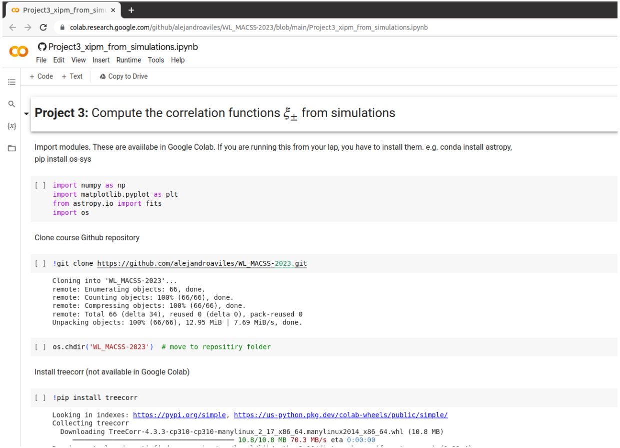




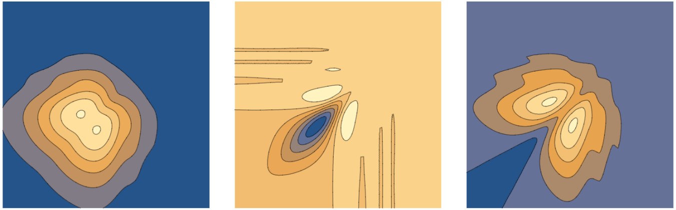
github.com/alejandroaviles/WL_MACSS-2023



We will use google colab



If you prefer, you can use your own laptop to run the projects, and download the notebooks and data from the github and google drive. But you will need to install `numpy`, `matplotlib`, `astropy` and `TreeCorr`.



https://github.com/alejandroaviles/WL_MACSS-2023

Sesiones/Proyecto B: miércoles 9:00-11:00, jueves 11:30-13:00, viernes 9:00-11:00.

Bib: WL_MACSS-2023_notes.pdf (GitHub repo). Kilbinger (1411.0115). Dodelson & Schmidt chapter 13.