

# **Weak Lensing: theory & estimators**

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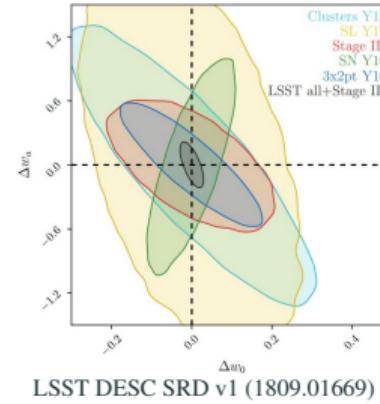
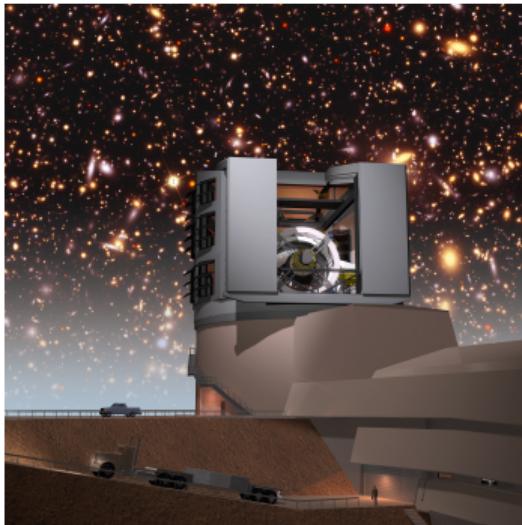
Alejandro Avilés & Sofía Samario

sabiduria\_sofy@hotmail.com, avilescervantes@gmail.com

MACSS. June 19, 2023

León, Guanajuato

# Motivation: Rubin Observatory LSST – DESC



Science cases:

1. Dark Energy: Dark Energy Task Force report (Albrecht et al. 2006).
2. Constrain neutrino mass
3. Test gravity
4. Halo-Galaxy connection

## Photometric redshift

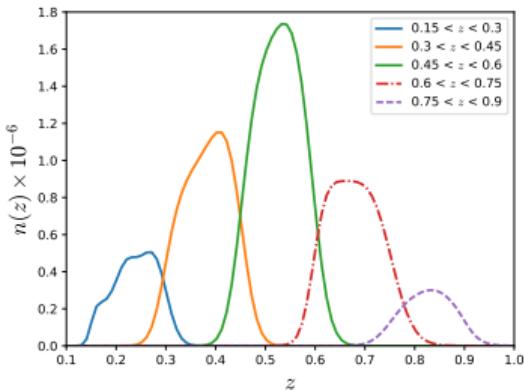
We do not always have access to accurate redshift measurements of astronomical objects. This is the case of, for example, photometric surveys. However, we do have high-quality images in several wavelength bands. These colors can be converted into rough estimates of redshift, so-called photometric redshifts, which can be used as proxies (with significant spread) for true redshifts.

More precisely, we can infer distributions of number of galaxies,

$$W_g(z) = \frac{1}{N_g} \frac{dN_g}{dz},$$

and depending on the features of each galaxy decide to which distribution it belongs.

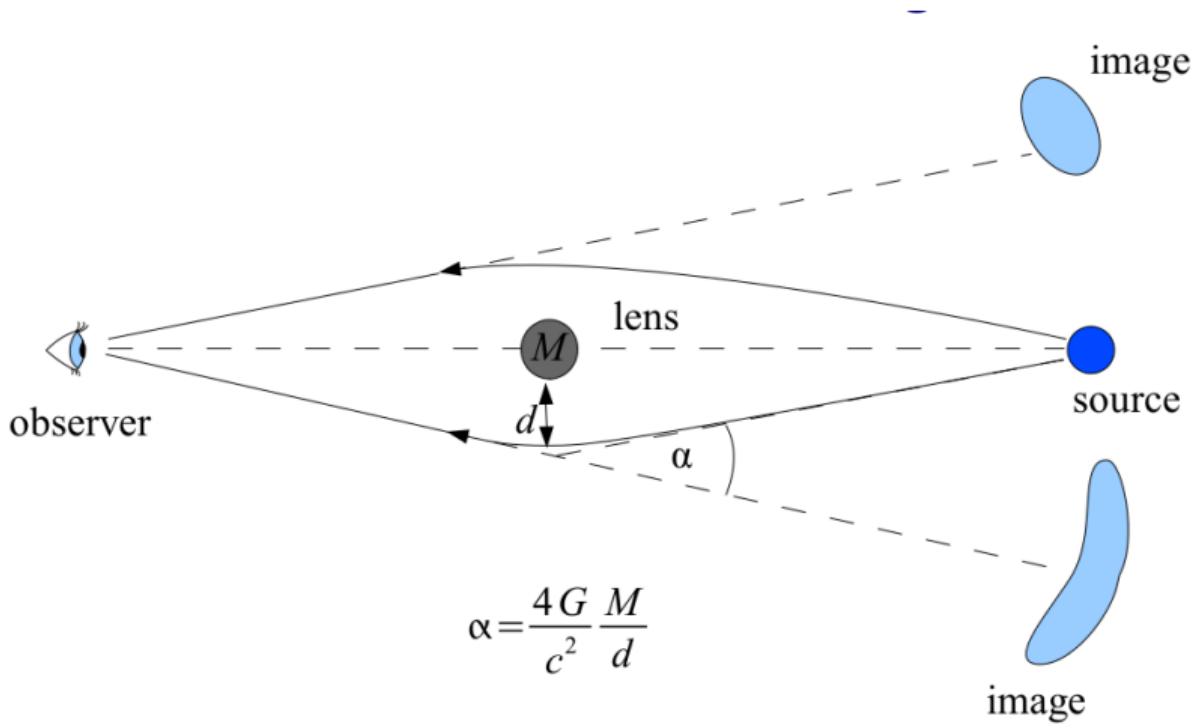
DES 1 yr results (1708.01536)



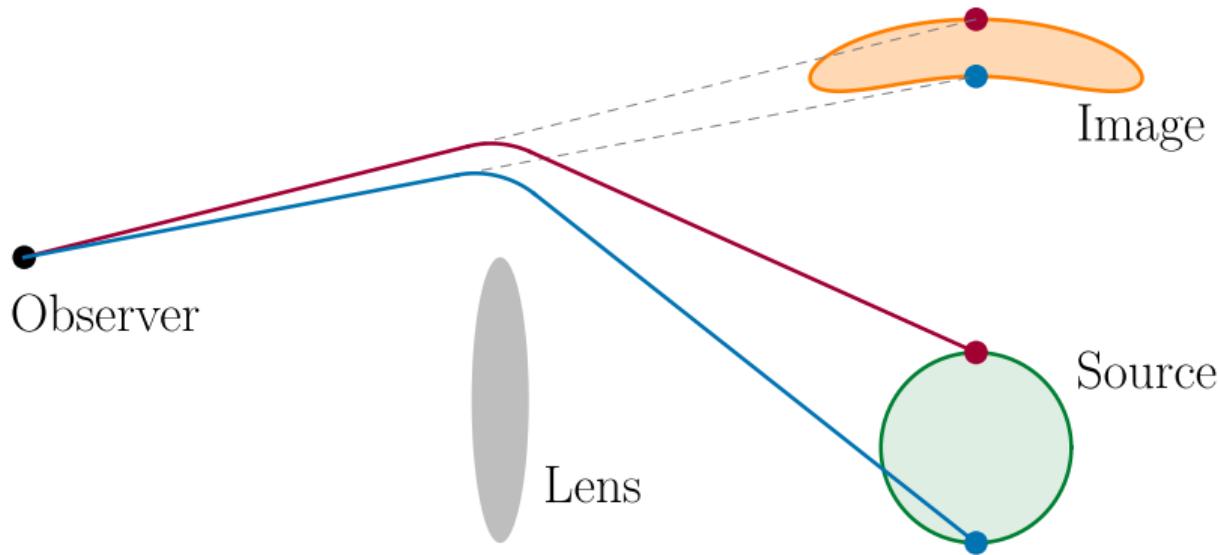
Participación mexicana (in-kind contributions)

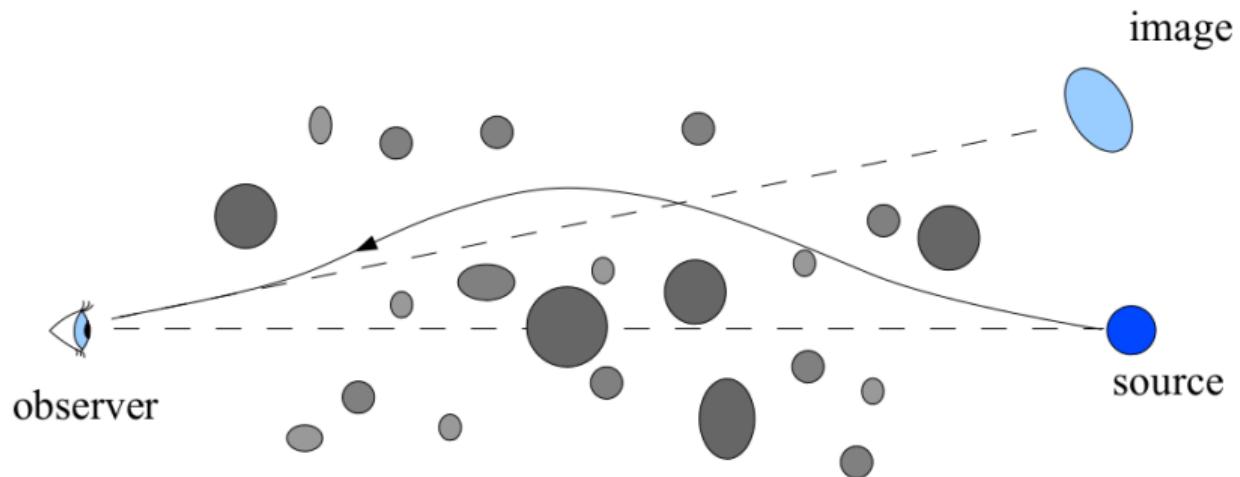
1. Strong Lensing: contact Alma or Luis
2. Photo-z and blinding: contact Sébastien, Josué or Alberto
3. 3-point correlation functions: contact Alejandro or Gustavo
4. Stars, Milky Way and Local Volume.
5. Galaxy classification with Machine Learning: contact Gibran

Convocatoria: lsst.mx/convocatorias

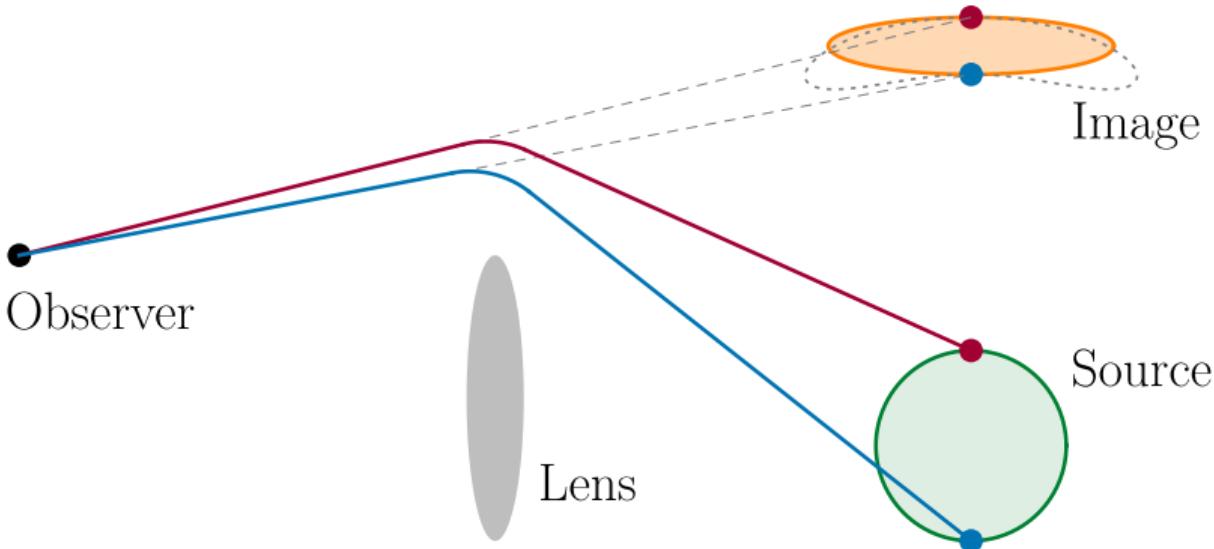


Credit: Stefan Hilbert (MPA)





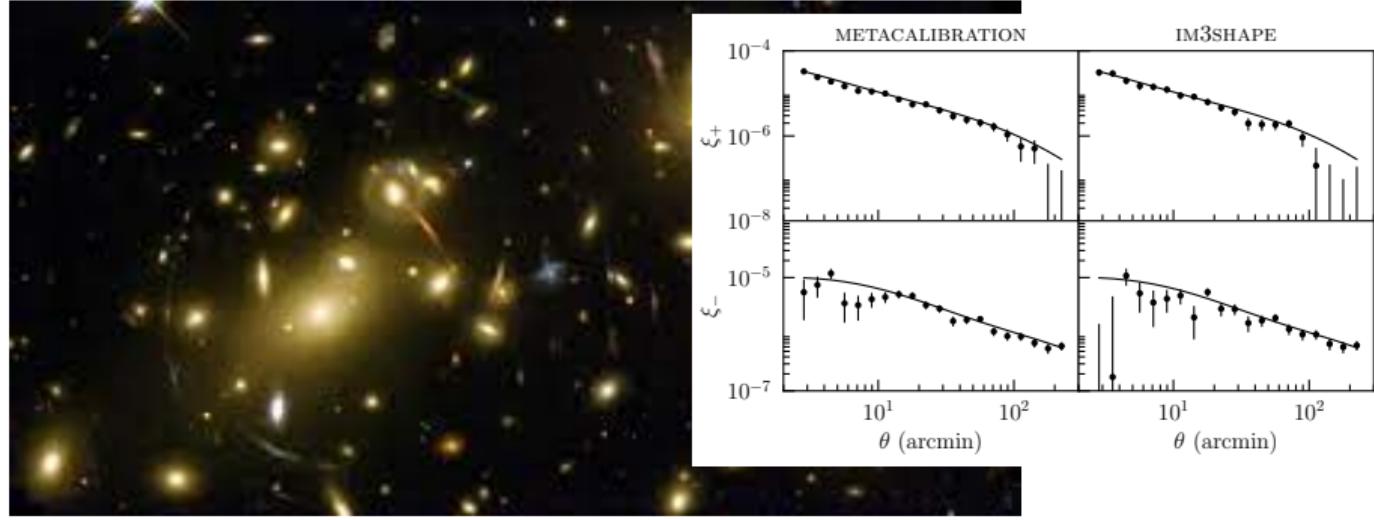
Credit: Stefan Hilbert (MPA)



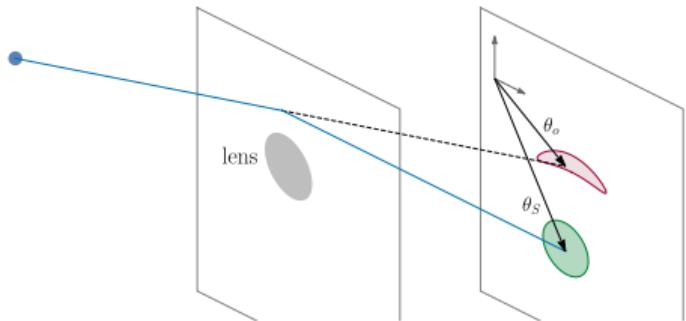
The statistical properties of the cosmic shear are directly linked to the statistical properties of density fluctuations (that is, to the total matter power spectrum). Hence, contrary to other Cosmological tests, WL is probing the dark matter itself



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DES y1 results (1708.01538)



Geodesic equation for photons  $\theta_m^{\text{Source}} = \theta_m^O + \frac{\partial \psi(\boldsymbol{\theta}, \chi)}{\partial \theta^m}$

with lensing potential  $\psi(\boldsymbol{\theta}, \chi) = - \int_0^\chi \frac{d\chi'}{\chi'} \frac{\chi - \chi'}{\chi} [\Psi(\mathbf{x}(\boldsymbol{\theta}, \chi')) + \Phi(\mathbf{x}(\boldsymbol{\theta}, \chi'))]$   
 (linearize:  $\theta \rightarrow \theta^O + \dots$ )

where  $\chi(z) = \int_0^z \frac{dz'}{H(z')} = \eta_0 - \eta(z)$  is the radial comoving distance

For small deflections of path light rays:

$$A_{mn} \equiv \frac{\partial \theta_m^S}{\partial \theta^n} = I_{mn} + \partial_m \partial_n \psi \equiv \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}.$$

Assuming small deflection angles we expand

$$\theta_m^S(\theta) = \underbrace{A_{mn} \theta_n}_{\text{Convergence + Shear}} + \underbrace{\frac{1}{2} D_{mns} \theta_n \theta_s}_{\text{Flexion}} + \dots$$

Weak lensing:  $\boxed{\theta_m^S = A_{mn} \theta_n.}$

That is, weak lensing is described by a linear map relating the observed and true positions of the sources.

Weak lensing assumes that the value of the derivatives of the lensing potential  $\partial_m \partial_n \psi$  do not change across the source surface (e.g., through a galaxy subtended solid angle).

Weak lensing:

$$\begin{pmatrix} \theta_1^S \\ \theta_2^S \end{pmatrix} = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} + \begin{pmatrix} -\kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & -\kappa + \gamma_1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}.$$

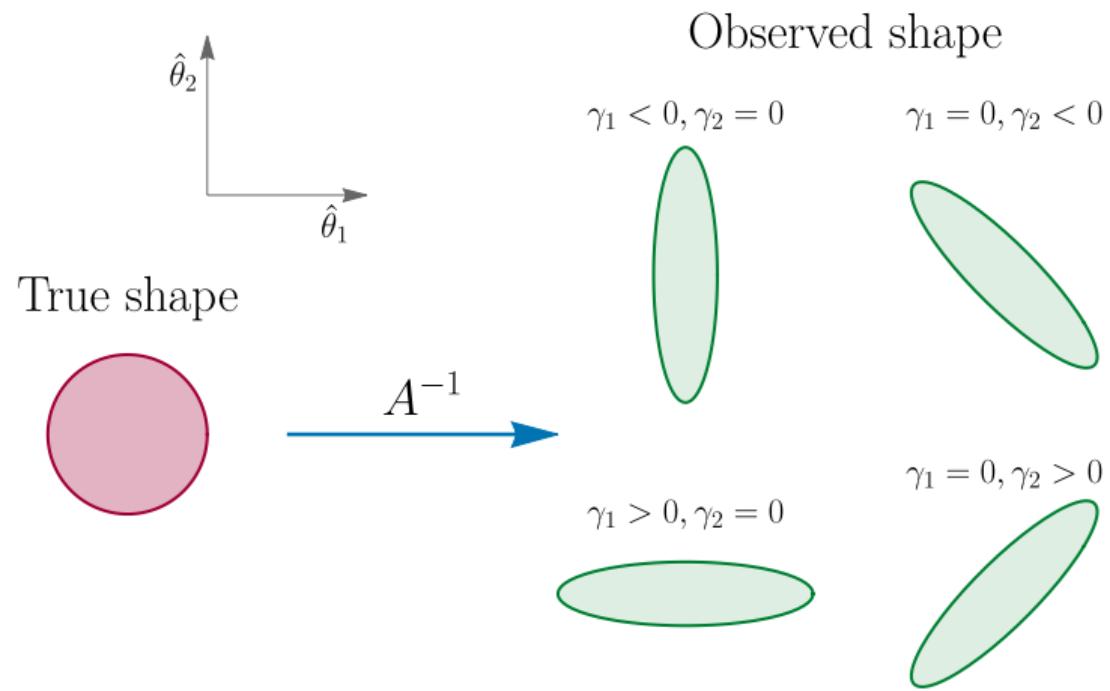
Using  $\theta_i^S = \theta_i + (\partial_i \partial_j \psi) \theta^j + \dots$ , with  $\psi$  the lensing potential

convergence:

$$\kappa = -\frac{1}{2}(\partial_1 \partial_1 + \partial_2 \partial_2) \psi = -\frac{1}{2} \partial^2 \psi$$

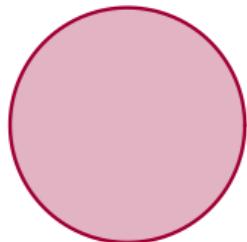
shear:

$$\gamma_1 = -\frac{1}{2}(\partial_1 \partial_1 - \partial_2 \partial_2) \psi, \quad \gamma_2 = -\partial_1 \partial_2 \psi$$



$$\boldsymbol{\theta}_s = \mathbf{A} \boldsymbol{\theta} + \frac{1}{2} \boldsymbol{\theta}^T \mathbf{D} \boldsymbol{\theta}$$

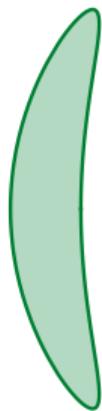
Unlensed



Shear



Shear + Flexion



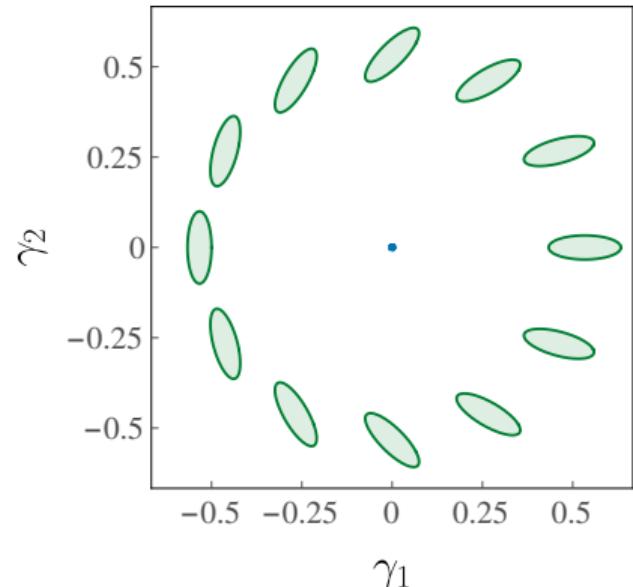
$$A_{mn} = \begin{pmatrix} 1 - \kappa & 0 \\ 0 & 1 - \kappa \end{pmatrix} + \begin{pmatrix} -\gamma_1 & -\gamma_2 \\ -\gamma_2 & \gamma_1 \end{pmatrix} \equiv (1 - \kappa)I_{mn} + \Gamma_{mn}.$$

Against a rotation, the convergence  $\kappa$  does not change (spin-0 component).

The shear components  $\gamma_1$  and  $\gamma_2$  transform between themselves as

$$\begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} \rightarrow \begin{pmatrix} \gamma'_1 \\ \gamma'_2 \end{pmatrix} = \begin{pmatrix} \gamma_1 \cos(2\varphi) - \gamma_2 \sin(2\varphi) \\ \gamma_2 \cos(2\varphi) + \gamma_1 \sin(2\varphi) \end{pmatrix},$$

that is, they transform with the double angle (spin-2 components).

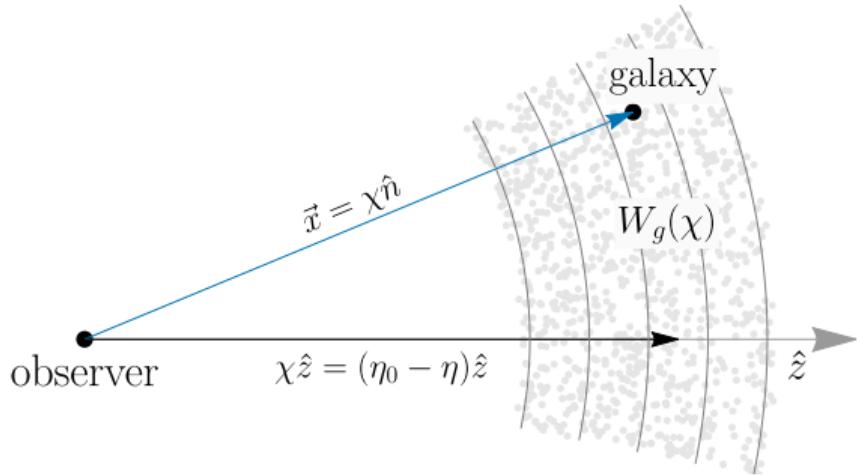


## Convergence as a projected density

$$\kappa(\boldsymbol{\theta}, \chi) = -\frac{1}{2} \partial^2 \psi = \frac{3}{2} \Omega_m H_0^2 \int_0^\chi \frac{d\chi'}{a(\chi')} \chi' \frac{\chi - \chi'}{\chi} \delta(\chi' \boldsymbol{\theta}, \chi'),$$

where we used  $\partial^2 = \chi^2 \nabla_x^2$  and the Poisson equation.

Notice that the geometrical factor  $\chi'(\chi - \chi')$  is a parabola with maximum at  $\chi' = \chi/2$ . Hence, structures at half the distance between the source and the observers are more efficient to produce lensing distortions.

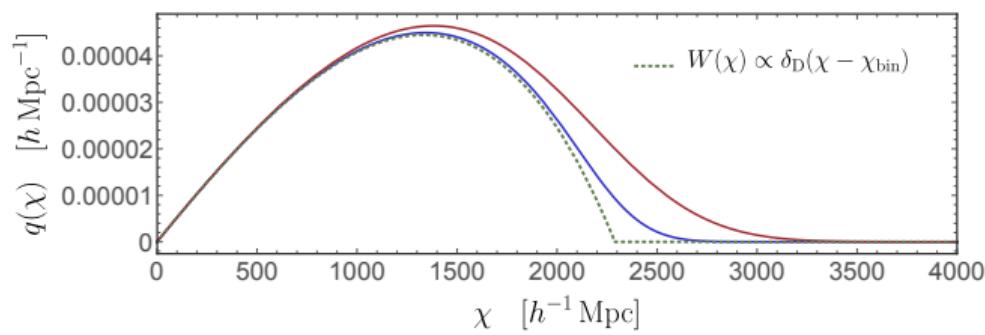
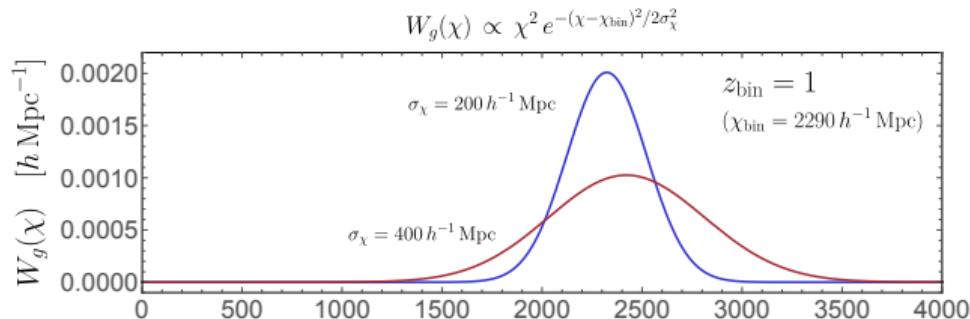


The total convergence from a population of source galaxies is obtained by weighting  $\kappa(\boldsymbol{\theta}, \chi)$  with the galaxy probability distribution  $W_g(\chi)$ :

$$\kappa(\boldsymbol{\theta}) = \int_0^\infty d\chi W_g(\chi) \kappa(\boldsymbol{\theta}, \chi) = \int_0^\infty d\chi' q(\chi') \delta(\chi' \boldsymbol{\theta}, \chi')$$

with the **lens efficiency**  $q$  defined as  $q(\chi) = \frac{3}{2} \Omega_m H_0^2 \frac{\chi}{a(\chi)} \int_\chi^\infty d\tilde{\chi} W_g(\tilde{\chi}) \frac{\tilde{\chi} - \chi}{\tilde{\chi}}$

The convergence becomes a linear measure of the total matter density, projected along the line of sight and weighted by the source galaxy distribution  $W_g$ .



## Convergence angular power spectrum

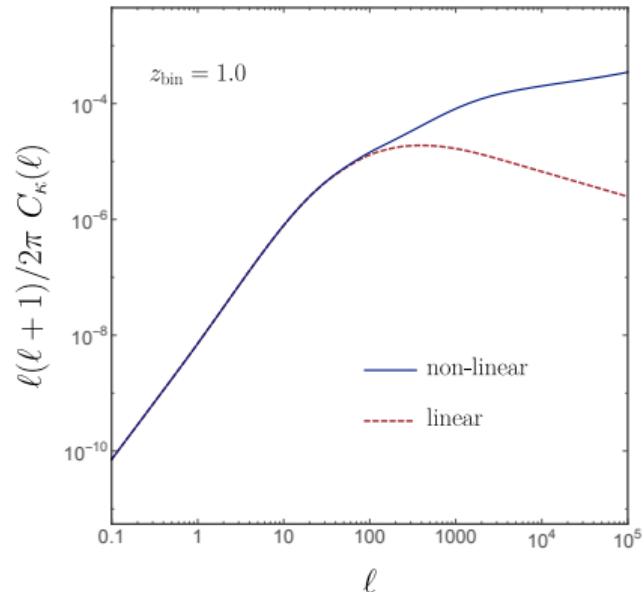
FT:

$$\kappa(\ell) = \int d^2\theta e^{-i\theta \cdot \ell} \kappa(\theta)$$

PS def:

$$\langle \kappa(\ell) \kappa(\ell') \rangle = (2\pi)^2 \delta_D(\ell + \ell') C_\kappa(\ell)$$

$$C_\kappa(\ell) = \int_0^\infty \frac{d\chi}{\chi^2} q^2(\chi) P_\delta\left(k = \frac{\ell + 1/2}{\chi}, \chi\right).$$

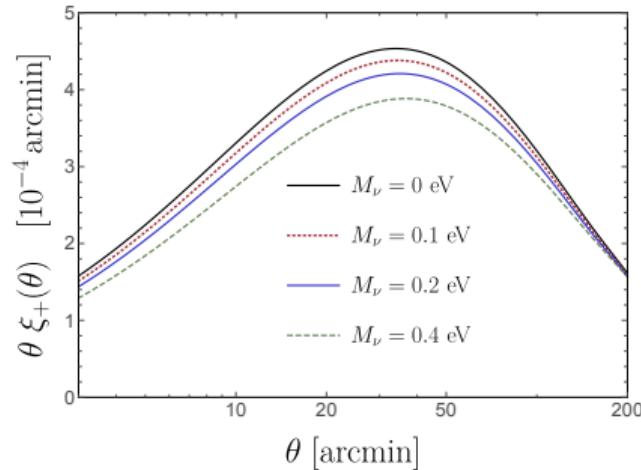


## Convergence correlation function

2pcf def:

$$\xi(\theta) = \langle \kappa(\nu) \kappa(\nu + \theta) \rangle$$

$$\begin{aligned}\xi(\theta) &= \int \frac{d^2\ell}{(2\pi)^2} e^{i\theta \cdot \ell} C_\kappa(\ell) \\ &= \int_0^\infty \frac{d\ell}{2\pi} \ell C_\kappa(\ell) J_0(\theta\ell)\end{aligned}$$



Credit: Rafael Morales ([github.com/rafaelmoramore/WLnu](https://github.com/rafaelmoramore/WLnu))

# Weak Lensing @ MACSS 2023

## Project session 1:

Construct an estimator for the weak lensing convergence field correlation function

[github.com/alejandroaviles/WL\\_MACSS-2023](https://github.com/alejandroaviles/WL_MACSS-2023)

$k$  : spin-0 field in 2D.

Its 2PCF is

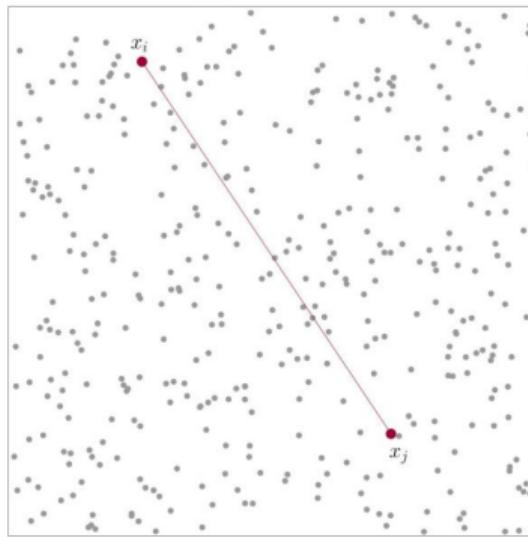
$$\xi(\theta) = \langle \kappa(\boldsymbol{\nu}) \kappa(\boldsymbol{\nu} + \boldsymbol{\theta}) \rangle$$

An unbiased estimator is given by

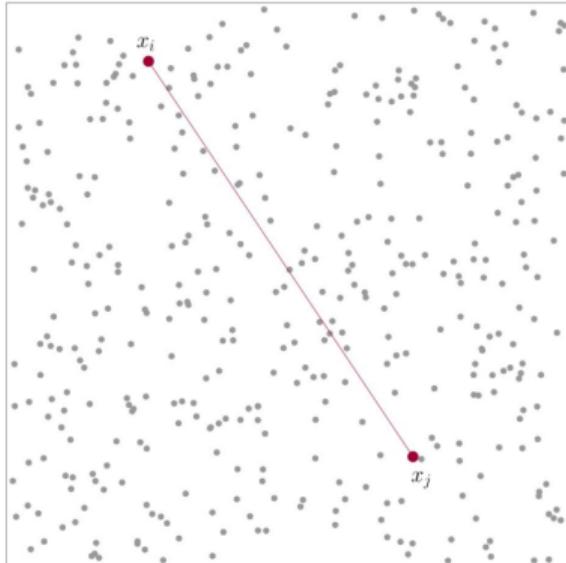
$$\hat{\xi}(\theta) = \int \frac{d^2\nu}{A} \kappa(\boldsymbol{\nu}) \kappa(\boldsymbol{\nu} + \boldsymbol{\theta})$$

which has computational complexity  $O(N^2)$   
(not considering tree methods)

$$\kappa(\boldsymbol{\theta}) = \sum_{i=1}^N \kappa(\boldsymbol{\theta}_i) \delta_D(\boldsymbol{\theta} - \boldsymbol{\theta}_i)$$



$$\hat{\xi} = \frac{1}{\sum_{ij} w_i w_j} \sum_{ij} w_i w_j \kappa_i \kappa_j$$

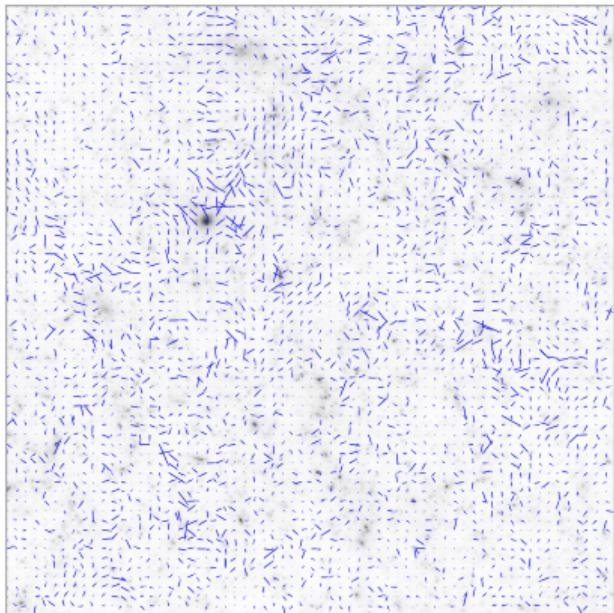
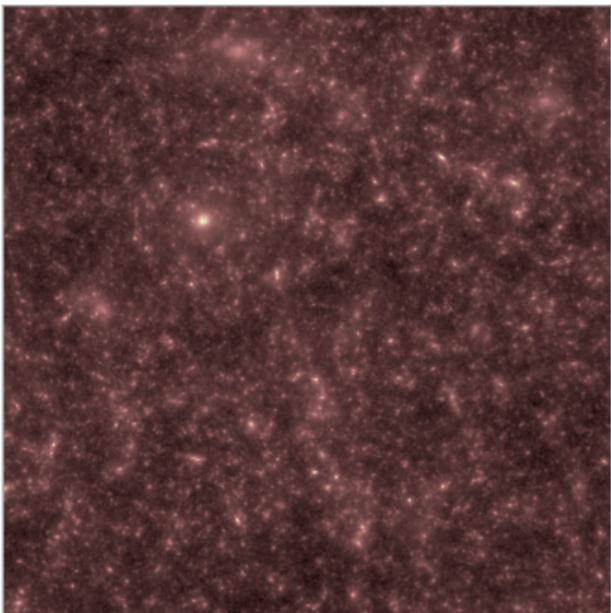


**Data:**  $N$  number of particles with positions  $\boldsymbol{x}_i$   
and convergence values  $\kappa_i$ .

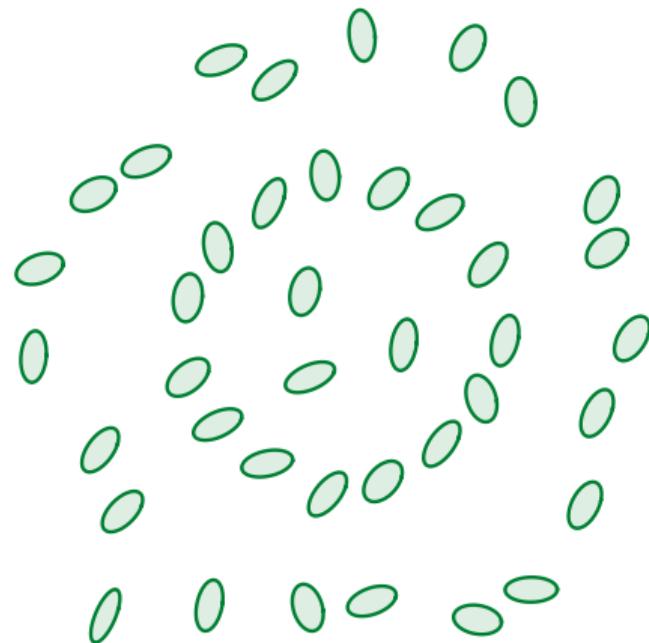
**Result:**  $\xi = \frac{1}{N_{bin}} \sum_{pairs} \kappa_i \kappa_j$

```
for i in N do
    kappai = kappa( $\boldsymbol{x}_i$ );
    for j in N do
        distance =  $|\boldsymbol{x}_i - \boldsymbol{x}_j|$ ;
        kappaj = kappa( $\boldsymbol{x}_j$ );
        ibin;      # e.g. ibin=distance/Maxdist
        Bins[ibin] += kappai*kappaj;
        NumberInBins[ibin] += 1;
    end
end
histogram[ibin] = Bins[ibin]/NumberInBins[ibin];
```

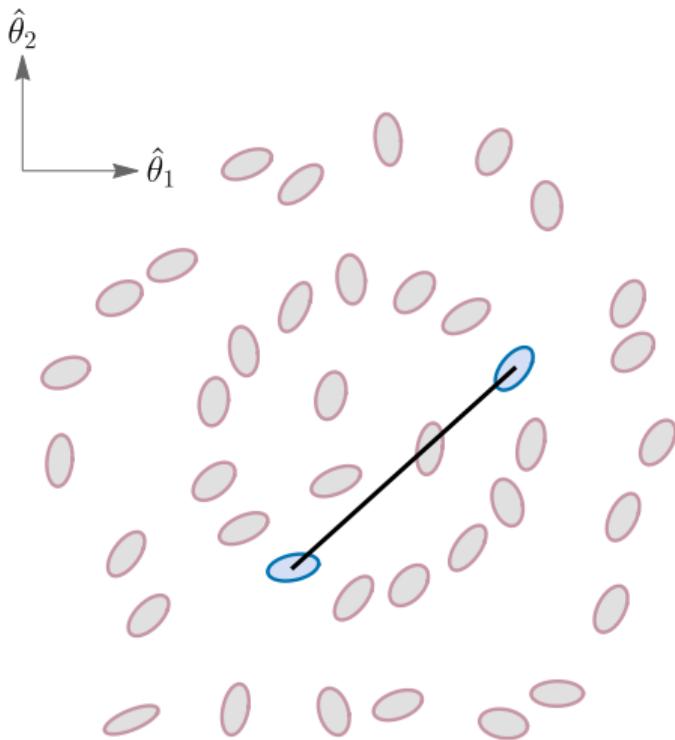
## Galaxy shear

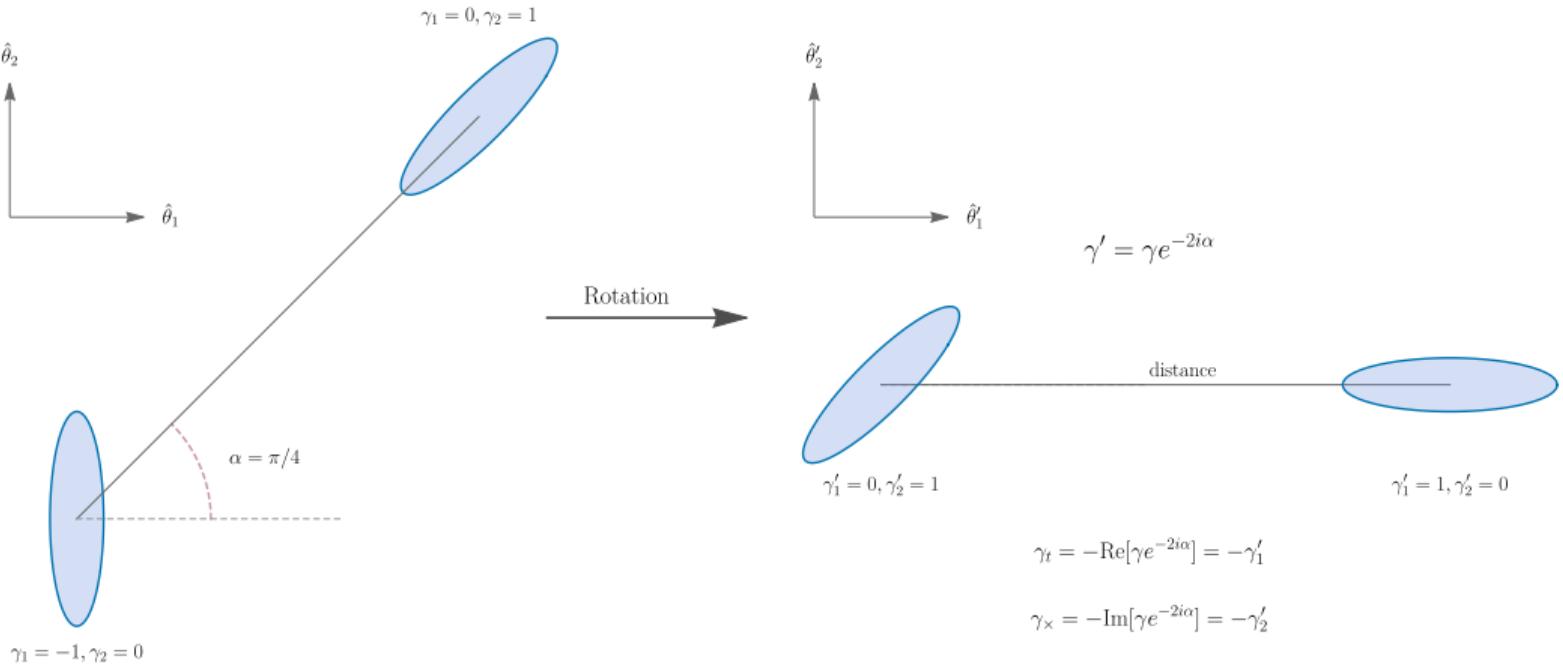


How can we construct a correlation function for galaxy shear  $\xi_\gamma(\theta)$ ?



How can we construct a correlation function for galaxy shear  $\xi_\gamma(\theta)$ ?





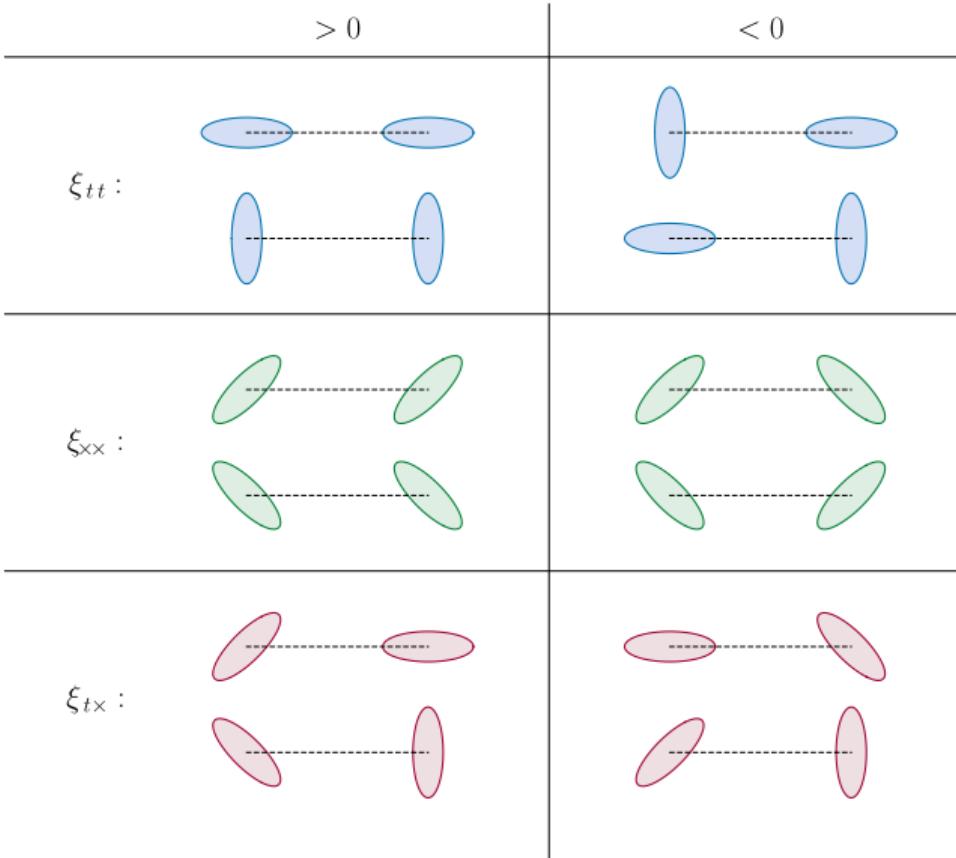
The shear correlation functions are defined as

$$\begin{aligned}\xi_{tt}(\theta) &= \langle \gamma_t(\boldsymbol{\nu}) \gamma_t(\boldsymbol{\nu} + \boldsymbol{\theta}) \rangle & \xi_+(\theta) &= \xi_{tt}(\theta) + \xi_{\times\times}(\theta) \\ \xi_{\times\times}(\theta) &= \langle \gamma_\times(\boldsymbol{\nu}) \gamma_\times(\boldsymbol{\nu} + \boldsymbol{\theta}) \rangle & \xi_-(\theta) &= \xi_{tt}(\theta) - \xi_{\times\times}(\theta) \\ \xi_{t\times}(\theta) &= \langle \gamma_t(\boldsymbol{\nu}) \gamma_\times(\boldsymbol{\nu} + \boldsymbol{\theta}) \rangle\end{aligned}$$

$$\xi_{t\times} = 0 \text{ for shear produced by weak lensing}$$

Discrete Estimators:

$$\begin{aligned}\hat{\xi}_{tt}(\theta) &= \frac{1}{N_{\text{pairs}}} \sum_{\text{pairs } i,j} \gamma_{i,t}(\boldsymbol{\theta}_1) \gamma_{j,t}(\boldsymbol{\theta}_2) \Delta_\theta(|\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2|) \\ \hat{\xi}_{\times\times}(\theta) &= \frac{1}{N_{\text{pairs}}} \sum_{\text{pairs } i,j} \gamma_{i,\times}(\boldsymbol{\theta}_1) \gamma_{j,\times}(\boldsymbol{\theta}_2) \Delta_\theta(|\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2|)\end{aligned}$$

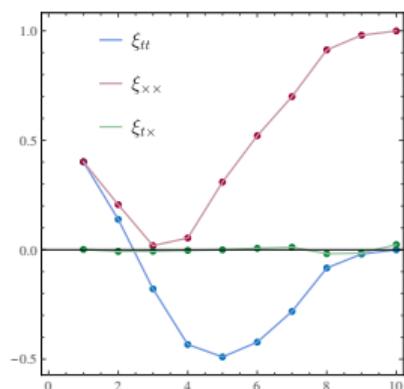
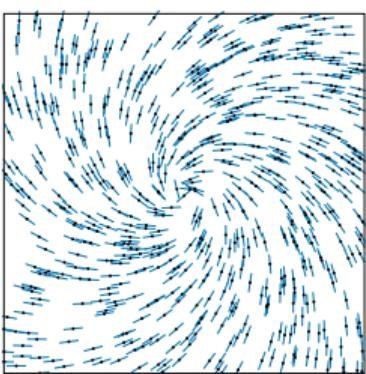
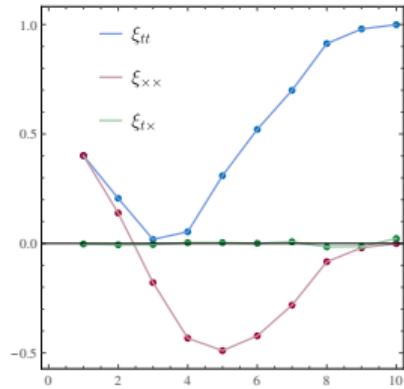
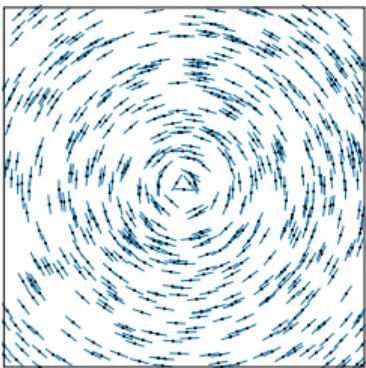


# Weak Lensing @ MACSS 2023

## Project session 2:

Construct an estimator for the weak lensing galaxy shear field correlation function

[github.com/alejandroaviles/WL\\_MACSS-2023](https://github.com/alejandroaviles/WL_MACSS-2023)

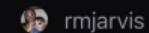


# Weak Lensing @ MACSS 2023

## Project session 3:

Use treecorr to compute correlation function from  
realistic simulations/real surveys

[github.com/alejandroaviles/WL\\_MACSS-2023](https://github.com/alejandroaviles/WL_MACSS-2023)



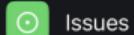
## TreeCorr

Code for efficiently computing 2-point and 3-point correlation functions. For documentation, go to

[rmjarvis.github.io/TreeCorr/](https://rmjarvis.github.io/TreeCorr/)

☆ 90 stars ⚡ 38 forks

★ Star



Issues

9 >



Pull Requests

0 >



Actions

>



Releases

10 >

Pass the catalogue to treecorr

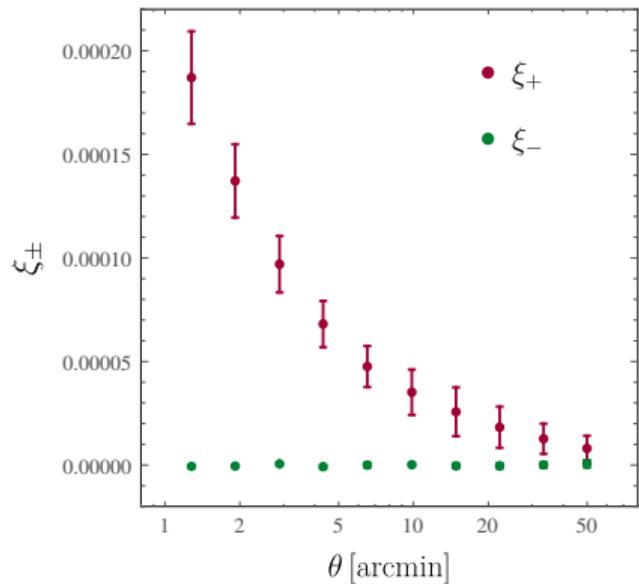
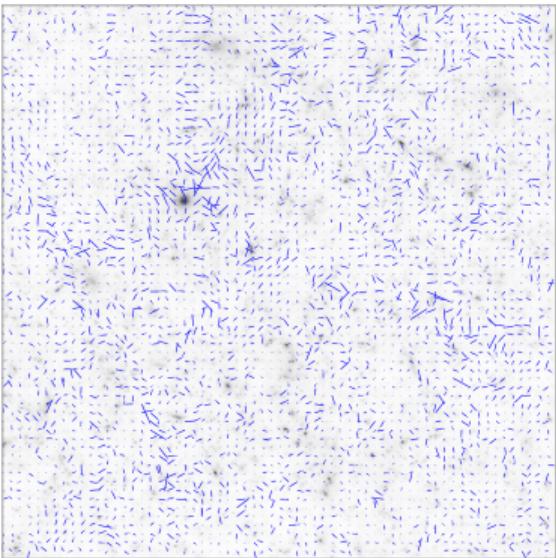
```
[ ] #cat=treecorr.Catalog(ra=xT,dec=yT,g1=A0gamma1,g2=A0gamma2,ra_units="rad",dec_units="rad")
      catalogue=treecorr.Catalog(x=xT,y=yT,g1=gamma1,g2=gamma2,x_units="rad",y_units="rad")
```

Run treecorr

```
[ ] GG=treecorr.GGCorrelation(min_sep=minRad,max_sep=maxRad,verbose=2,nbins=nbins)
      GG.process(catalogue)
```

```
nbins = 10, min,max sep = 0.000290888..0.0174533, bin size = 0.409434
INFO:treecorr:nbins = 10, min,max sep = 0.000290888..0.0174533, bin_size = 0.409434
Starting process GG auto-correlations
INFO:treecorr:Starting process GG auto-correlations
Using 2 threads.
INFO:treecorr:Using 2 threads.
Building GField
INFO:treecorr:Building GField
Starting 256 jobs.
INFO:treecorr:Starting 256 jobs.
varg = 0.000354: sig_sn (per component) = 0.018827
INFO:treecorr:varg = 0.000354: sig_sn (per component) = 0.018827
```

# From shear to statistics



[github.com/alejandroaviles/WL\\_MACSS-2023](https://github.com/alejandroaviles/WL_MACSS-2023)

alejandroaviles / WL\_MACSS-2023 Public

Code Issues Pull requests Actions Projects Wiki Security Insights Settings

main · 1 branch · 0 tags Go to file Add file · Code

Ssamarrio Update README.md c52a38b · 14 hours ago 45 commits

Project3\_xipm\_from\_simulations.ipynb · Created using Colaboratory 19 hours ago

README.md · Update README.md 14 hours ago

WL\_MACSS\_2023.pdf · Add files via upload 18 hours ago

gamma\_A.0.fits · Add files via upload 3 days ago

kappa\_A.0.fits · Add files via upload 3 days ago

test1.ipynb · Created using Colaboratory 3 days ago

test2.ipynb · Created using Colaboratory 3 days ago

test5.ipynb · Created using Colaboratory 18 hours ago

README.md

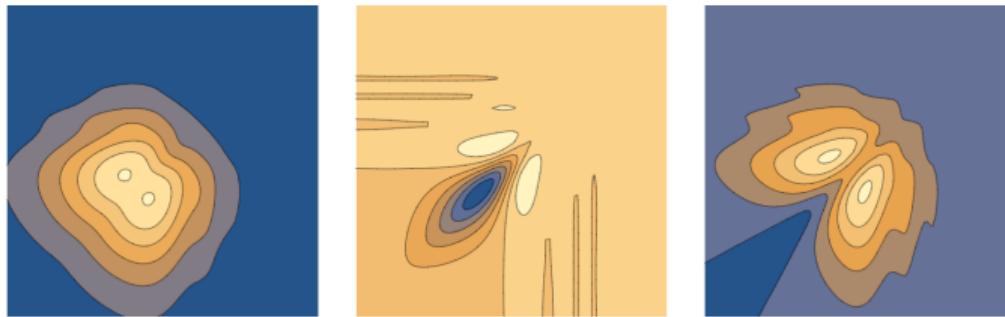
**WL\_MACSS-2023**

Repository for the Weak Lensing Projects @ MACSS 2023 (León, Guanajuato)

We will use google colab

```
Project3_xipm_from_simulations.ipynb
File Edit View Insert Runtime Tools Help
+ Code + Text ⚡ Copy to Drive
Project3: Compute the correlation functions  $\xi_{\pm}$  from simulations
Import modules. These are available in Google Colab. If you are running this from your lap, you have to install them. e.g. conda install astropy, pip install os-sys
[ ] import numpy as np
import matplotlib.pyplot as plt
from astropy.io import fits
import os
Clone course Github repository
[ ] !git clone https://github.com/alejandroaviles/WL_MACSS-2023.git
Cloning into 'WL_MACSS-2023'...
remote: Enumerating objects: 66, done.
remote: Counting objects: 100% (66/66), done.
remote: Compressing objects: 100% (66/66), done.
remote: Total 66 (delta 34), reused 8 (delta 0), pack-reused 0
Unpacking objects: 100% (66/66), 12.95 MiB | 7.69 MiB/s, done.
[ ] os.chdir('WL_MACSS-2023') # move to repository folder
Install treecorr (not available in Google Colab)
[ ] !pip install treecorr
Looking in indexes: https://pypi.org/simple, https://us-python.pkg.dev/colab-wheels/public/simple/
Collecting treecorr
  Downloading Treecorr-4.3.3-cp310-cp310-manylinux_2_17_x86_64_manylinux2014_x86_64.whl (10.8 MB)
    10.8/10.8 MB 70.3 MB/s eta 0:00:00
```

If you prefer, you can use your own laptop to run the projects, and download the notebooks and data from the github and google drive. But you will need to install numpy, matplotlib, astropy and TreeCorr.



[https://github.com/alejandroaviles/WL\\_MACSS-2023](https://github.com/alejandroaviles/WL_MACSS-2023)

Sesiones/Proyecto B: miércoles 9:00-11:00, jueves 11:30-13:00, viernes 9:00-11:00.

Bib: WL\_MACSS-2023\_notes.pdf (GitHub repo). Kilbinger (1411.0115). Dodelson & Schmidt chapter 13.