5: Setting the Stage

Exercise 1

1. **Prove that $\emptyset$ is an inicial object in Sets.**
2. **Prove that any one-point set $\Omega = \{x_0\}$ is a terminal object in Sets. In particular what is the function $\emptyset \to \Omega$?**

Proof. Let A be a set.

1. Any function $f : \emptyset \to A$ must be $f = \emptyset$ becuase these functions are formally subsets of $\emptyset \times A = \emptyset$. Since $\emptyset$ satisfies the definition of function, we have $\{\emptyset\} \in \operatorname{Hom}(\emptyset, A) \subseteq \mathcal{P}(\emptyset)$.
2. Observe that $a \mapsto x_0$ is a well defined function $c_0 : A \to \Omega$. As a subset of $A \times \Omega$, $c_0 = A \times \Omega$. Any other such function $f : A \to \Omega$ must be a subset of $A \times \Omega$. Any element of $A \times \Omega$ is of the form (a, x_0), furthermore, for $f \subset A \times \Omega$ to be a function, every ordered pair of the same form has to be an element of f so that $A \times \{x_0\} \subseteq f$ and therefore $f = A \times \Omega = c_0$.

$\square$