Definition 1. A *group* is a set G together with a function $\mu : G \times G \to G$ called a *binary operation*, defined as $(x, y) \mapsto xy$, that satisfies the following properties:

1. (Associativity) For all $x, y, z \in G$, $(xy)z = x(yz)$.
2. (Identity element) There exists an element $1 \in G$, called the *identity*, such that $1x = x = x1$ for all $x \in G$.
3. (Existence of Inverses) For all $x \in G$ there exists an element $y \in G$ (denoted by x^{-1}, see property 2) such that $xy = 1 = yx$.

A group is called an *abelian group* if furthermore the operation is commutative, that is $xy = yx$ for all $x, y \in G$. In this case the notation becomes additive: $xy \leftrightarrow x + y$, $x^{-1} \leftrightarrow -x$ and for $x^n := x \cdots x$ we denote $E \leftrightarrow nx$.

Properties.

1. Identity elements are necesarily unique: if $1, 1' \in G$ are identity elements then $1 = 1'1 = 11' = 1'$ where the first two equalities hold becuase $1'$ is an identity element and the third equality holds because 1 is an identity.
2. If $y, z \in G$ are inverses of x then $xy = 1 = yx$