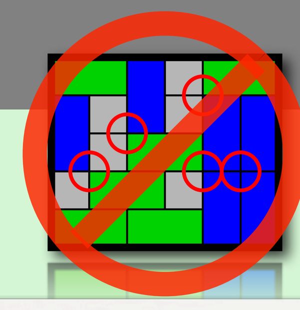
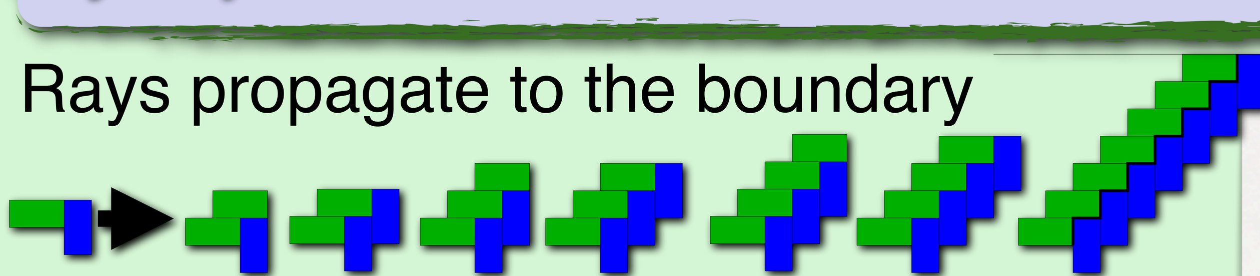
Tatami Tilings: No four tiles meet

by Alejandro Erickson



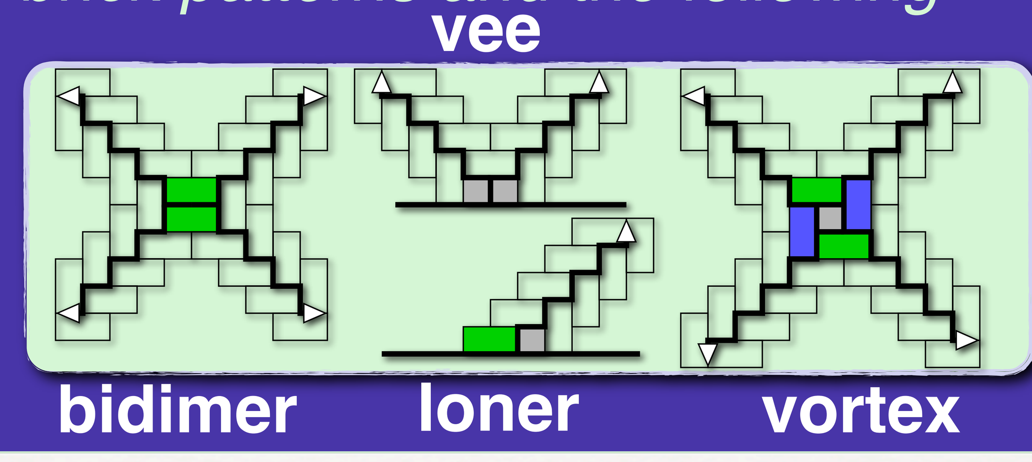
Rays propagate to the boundary



How do rays start ?

1. **Bidimer**: Occurs anywhere.
2. A □ at beginning.
   (a) **Loner**: Only on boundary.
   (b) **Vortex**: Not on boundary.
   (c) **Vee**: Only on boundary.

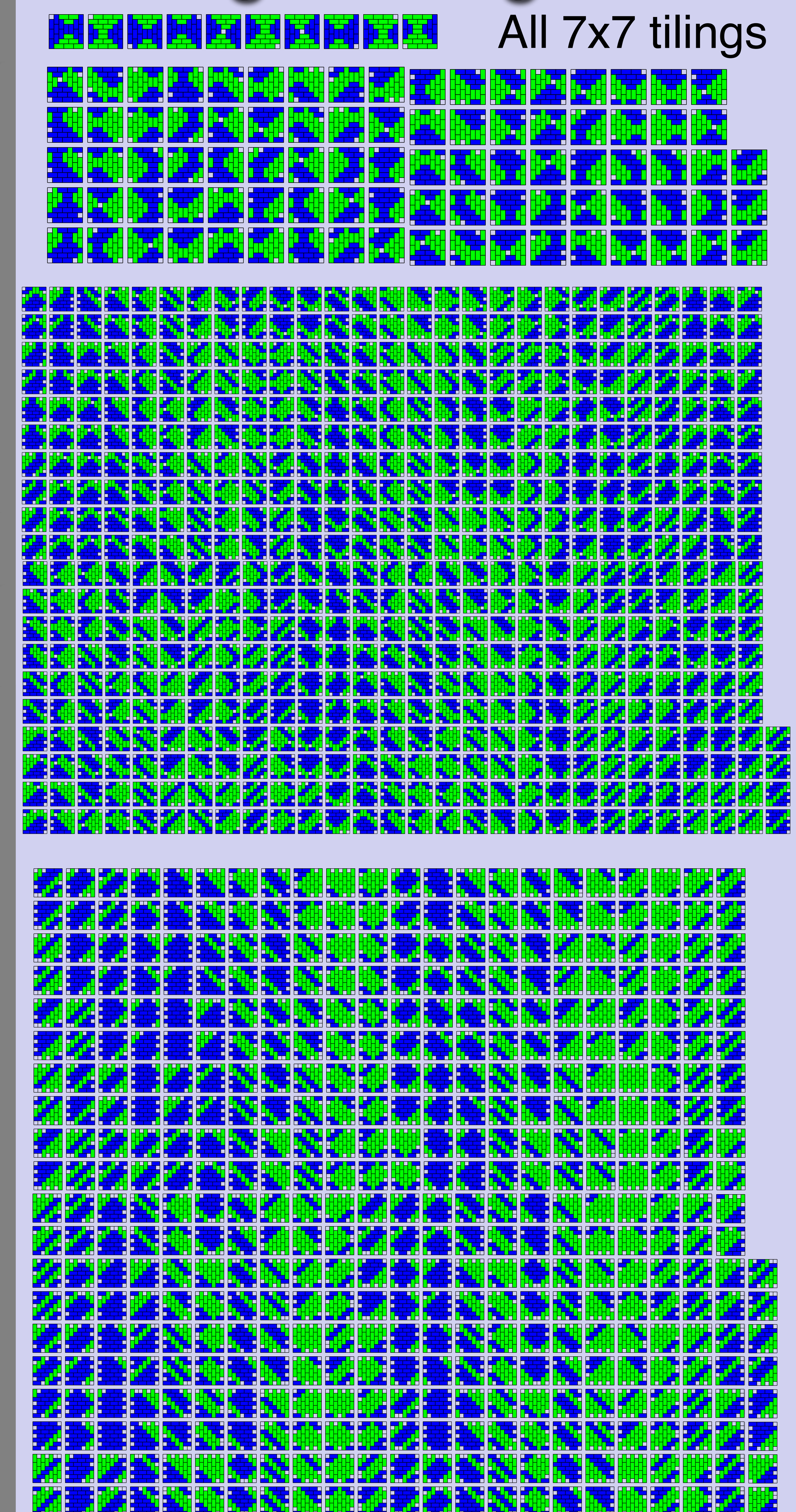
Theorem: *Configurations in a rectangular tatami tiling are restricted to brick patterns and the following*



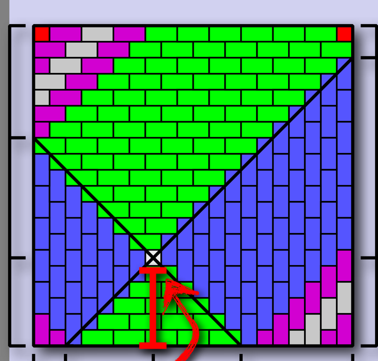
Theorem: *A rectangular tatami tiling is determined by the tiles on its boundary.*

Listing all tilings

All 7x7 tilings



Enumerating tilings



The number of □s in a square tiling is determined by the shortest distance from a ▯, ▭, ⧉ or ⧉ to the boundary.

Example: A ⧉ centered at one of these squares is 5 units from the nearest boundary. Diagonals are flipped independently to obtain different tilings.

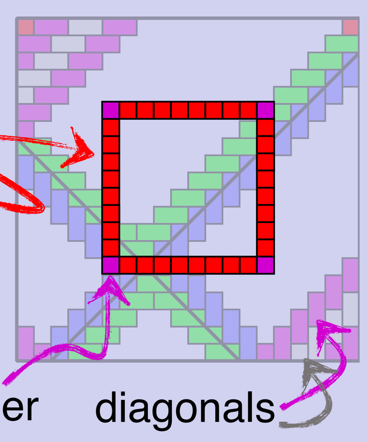


Table: Classifying tilings with m □s

Position	Type of Feature	Number of Sides	Positions per Side	Flippable diagonals
corner	▯ or ▭	4	1	m-1
corner	⧉ or ⧉	4	1	m-2
not a corner	▯ or ▭	4	m-1	m-2
not a corner	⧉ or ⧉	4	m-2	m-3

Each row contributes a term to

$$2\cdot 4\cdot 1\cdot 2^{m-1}+2\cdot 4\cdot 1\cdot 2^{m-2}$$
$$+2\cdot 4\cdot (m-1)\cdot 2^{m-2}+2\cdot 4\cdot (m-2)\cdot 2^{m-3}$$
$$=2\cdot 2^{m+1}+2\cdot 2^{m}+(m-1)2^{m+1}+(m-2)2^{m}$$

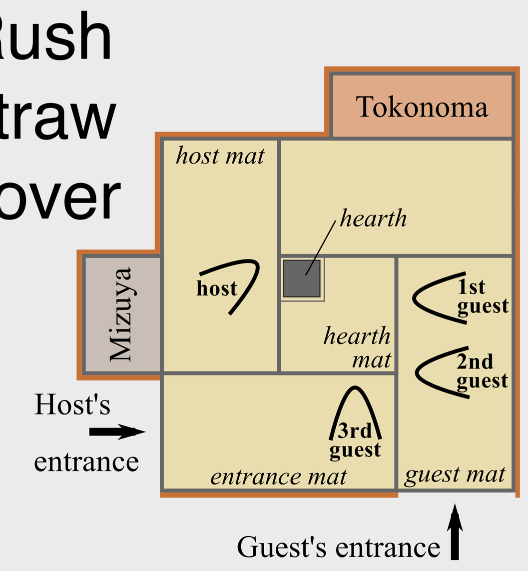
Theorem: *The number of n x n tilings with m<n □s is* $m2^m+(m+1)2^{m+1}$

Theorem: *The number of n × n tilings is equal to the sums of squares of all parts of all compositions of n.*

Japanese Tatami Mats

Rice straw core

Rush straw cover

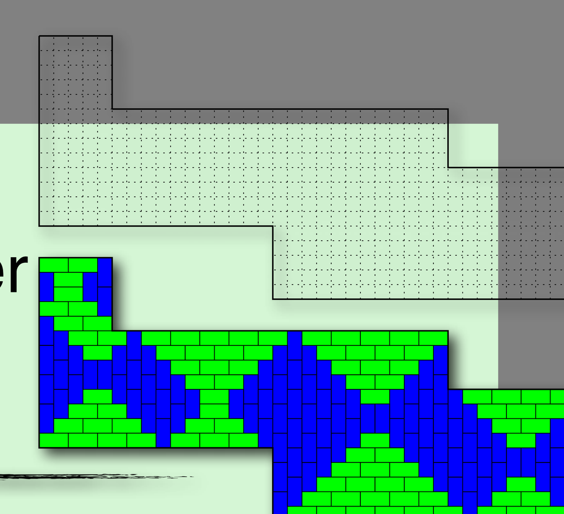


NO FOUR MATS MEET

A medley...

Complexity

What is the complexity of determining the smallest number of □s in a tiling of a given rectilinear region?



Algebraic Enumeration

How many tilings are there with a fixed number of ▯s?

$2.99

Conjecture: For n x n tilings the generating polynomial is $P(n,z)\prod_{j\geq 1} S_{\left\lfloor \frac{n-1}{2^j}\right\rfloor}(z)$, where $P(n,z)$ is an irreducible polynomial and $S_n(z)=\prod_{j=1}^{n}\left(\phi_{2j}(z)\right)^{\left\lfloor \frac{n+j}{2j}\right\rfloor}$, for all $n\geq 1$, where $\phi_n(z)$ is the nth cyclotomic polynomial.

Combinatorial Generation

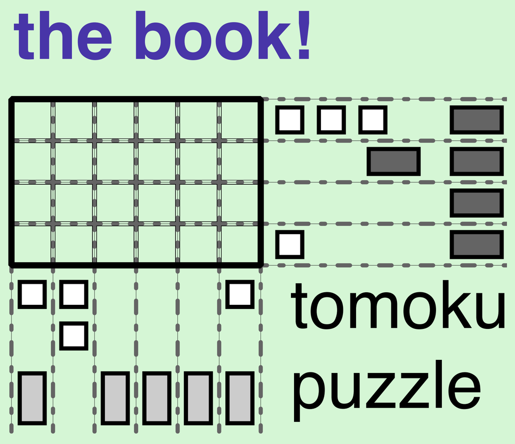
Can rectangular tilings be generated in *O(1)* time per tiling? Do they form a Gray code? What representation should we use?

Tomoku!

Game: Reconstruct a tiling from its tomographic row and column projections. What is the complexity of this?

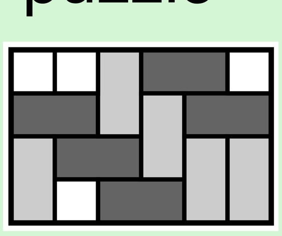
Web-game playable now at alejandroerickson.com

Pencil-and-paper version. **Ask me for a preview of the book!**



tomoku puzzle

solution



Another game: **Tatami Dots and Boxes**. Just like dots and boxes but you can't put four lines at one dot. Try playing it on a non-orientable surface!

Scoreless game on the Klein bottle. Can a player force a scoreless game?

Credits

Submit tatami related items to the **Tatami blog at alejandroerickson.com**

Funding: University of Victoria NSERC CRSNG

Tatami tilings are joint work with Frank Ruskey, Mark Schurch and Jennifer Woodcock.

Artwork credits (own work unless otherwise noted):
Rice harvester by E., Yangshuo, China, 2010.
Rush straw (Juncus effuses) from Meggar(en.wikipedia).
Tatami room from http://ristomarijanikointokyo.blogspot.com/.
Tatami tea room floor layout from Bamse(commons.wikipedia).
Some tiling diagrams by Frank Ruskey.