If the method can be used for forecasts of several years' duration, it may be desirable or even necessary to correct all actual readings to "standard" or "normal" weather conditions before forecasting. Regression analysis has been used to define the load-to-weather relationship [17]. Projections might then be made on the basis of normal weather and also on the basis of record weather. Both projections would be of value, and the spread between the two would be of particular interest and significance.

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[17] G. T. Heinemann, D. A. Nordman, and E. C. Plant, "The relationship between summer weather and summer loads—a regression analysis," *IEEE Trans. Power Apparatus and Systems*, vol. PAS-85, pp. 1144–1154, November 1966.

Junichi Toyoda, Mo-Shing Chen, and **Yukiyoshi Inoue:** The authors wish to express their appreciation to all those who participated in the discussions of this paper and of [11]. We believe they contributed a great deal to the papers.

Manuscript received March 27, 1970.

In reply to the discussion by Dr. Guy, Mr. Platts, and Mr. Rennie, the authors want to point out that the papers are devoted to very short term load forecasting. This type of study is somewhat new to the utility industry. Therefore, we had difficulty in finding available data for comparison.

The suggestions by Dr. Mehra and Mr. Jazwinski are interesting. The authors have no experience in the method of using "innovation sequence." Equations (10) and (11) are one feasible solution to this kind of problem. Any better method of identifying Q and R will be of great interest to the authors.

We want to thank Mr. Brownlee for his discussion. The field of electrical engineering has been growing so rapidly that the introduction of new definitions and terminology is sometimes necessary. The word state is not new. It has been defined and used in the field of control systems for many years. In general we do agree that all authors should be careful in notations and terminology. The authors also wish to point out that very short term load forecasting is mainly for the computer control of power systems. It is different from the regular load forecasting in the utilities.

Mr. Parks presented a very interesting discussion. We agree with him heartily. The relationship between the forecasting model and the forecasting period may have been determined by experience. We successfully used the basic method with modification to obtain results in long-term load forecasting.

Transmission Network Estimation Using Linear Programming

LEN L. GARVER, MEMBER, IEEE

***Abstract*—One aspect of long-range planning of electric power systems involves the exploration of various designs for the bulk power transmission network. The use of linear programming for network analysis to determine where capacity shortages exist and, most importantly, where to add new circuits to relieve the shortages is presented. The new method of network estimation produces a feasible transmission network with near-minimum circuit miles using as input any existing network plus a load and generation schedule. An example is used to present the two steps of the method: 1) linear flow estimation and 2) new circuit selection. The method has become a fundamental part of computer programs for transmission network synthesis.**

INTRODUCTION

SUPPOSE we wish to develop a transmission network capable of meeting the needs of future generation and load conditions. The locations and magnitudes of the loads and generation are known and a network of circuits linking these locations must be designed. Computer programs to aid in this design have been developed [1], [2] using the network estimation method to be presented. The steps taken in estimating a transmission network are as follows.

1) Formulate the power flow equations as a linear minimization problem.
2) Use linear programming to solve the minimization problem for the needed power movements. This result is called a linear flow estimate.
3) Select a circuit addition based on the location of the largest overload in this flow estimate.
4) Repeat the flow-estimation and circuit-selection steps until no overloads remain.

The most important new feature of the flow-estimation technique is the fact that overloads will not appear on circuits but on a new type of network link called an overload path. Overload paths exist between every bus in the network. Therefore, overloads will not necessarily occur parallel to existing circuits, and the addition of circuits based on the location of overloads will do more than just parallel an existing network. The overload-path method easily handles the study of buses not included in an existing network.

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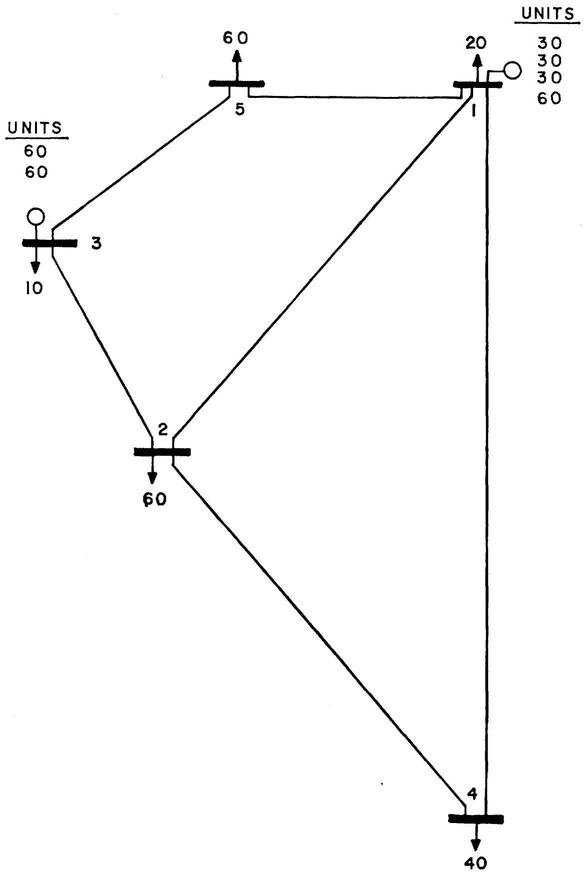

Fig. 1. Existing generation and transmission system.

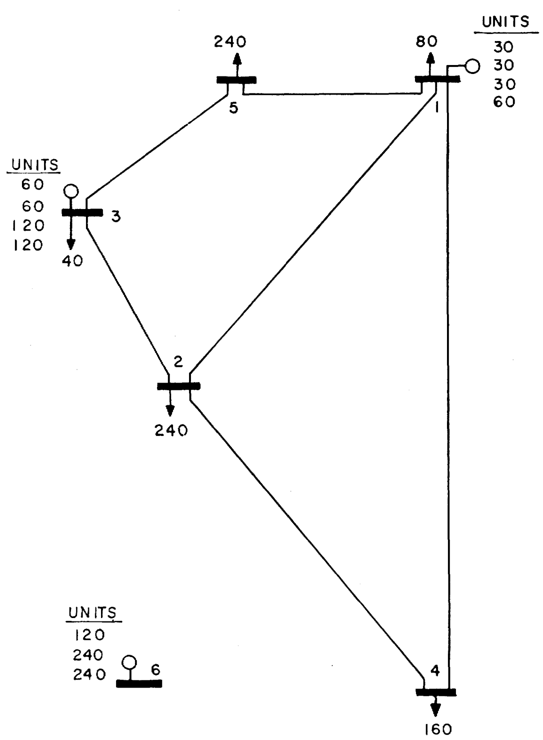

Fig. 2. Existing network with future loads and generating units.

TABLE I
DATA REQUIREMENTS FOR NETWORK ESTIMATION

Bus Characteristics
Net generation minus load
Circuit Characteristics
Bus terminals
Guide number
Power capacity
Overload-Path Characteristics
Bus terminals
Guide number
New Circuit Characteristics
Guide number as a function of length
Power capacity as a function of length

TABLE II
NET-GENERATION-MINUS-LOAD DATA

Bus	Generation	Load	Net
1	50	80	−30
2	0	240	−240
3	165	40	125
4	0	160	−160
5	0	240	−240
6	545	0	545
Totals	760	760	0

NETWORK ESTIMATION

Example Problem

The existing power system shown in Fig. 1 is to be expanded to a future condition with the loads four times their present values. (The existing system is taken from Stevenson's presentation of the ac load flow problem [3].) Fig. 2 presents the future condition with the higher loads and new generation. Two 120-MW units have been added at bus 3 and a new bus 6 with three new units has been established. An adequate network with a minimum of additional circuit miles is desired. An ac load flow will be used as the final test of the adequacy of any new network.

An ac load flow or any other electrical circuit analysis method cannot aid in this initial planning task because one bus is not yet connected into the network.

Network Estimation Data

Estimating the network for a future load level begins with the specification of the quantities listed in Table I.

The net generation minus load is computed from the load forecast at each bus and the generation scheduled to exactly match the total load. Power losses are not included in this analysis. For the example in Fig. 2 the generation was scheduled as shown in Table II.

The circuit data are shown in Table III. Each circuit is thermally limited to 100 MW, and for stability purposes the loading should not exceed that shown in Fig. 3. For a further discussion of loading limitations see [4]. The per-unit R and X values are from [3, p. 220].

The last column contains the nonelectrical quantity, the guide number, which will be used by the flow-estimation method to guide the power onto the most direct circuits from generators to load buses. Their use will be illustrated once the overload-path characteristics are introduced.

Overload paths initially provide routes for overloads, when they occur, and later locations for new circuits. The guide numbers for these paths are selected 1) to indicate the relative

TABLE III
CIRCUIT CHARACTERISTICS

Terminals	Length (miles)	R (pu)	X (pu)	Capacity (MW)	Guide Number
1–2	40	0.10	0.40	100	40
1–4	60	0.15	0.60	80	60
1–5	20	0.05	0.20	100	20
2–3	20	0.05	0.20	100	20
2–4	40	0.10	0.40	100	40
3–5	20	0.05	0.20	100	20

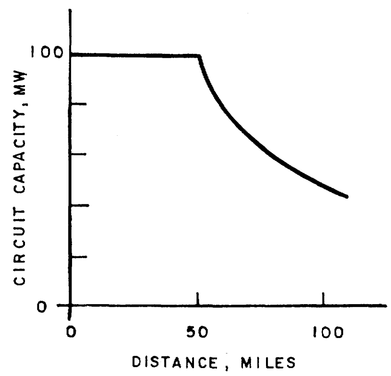

Fig. 3. Circuit capacity as a function of line length.

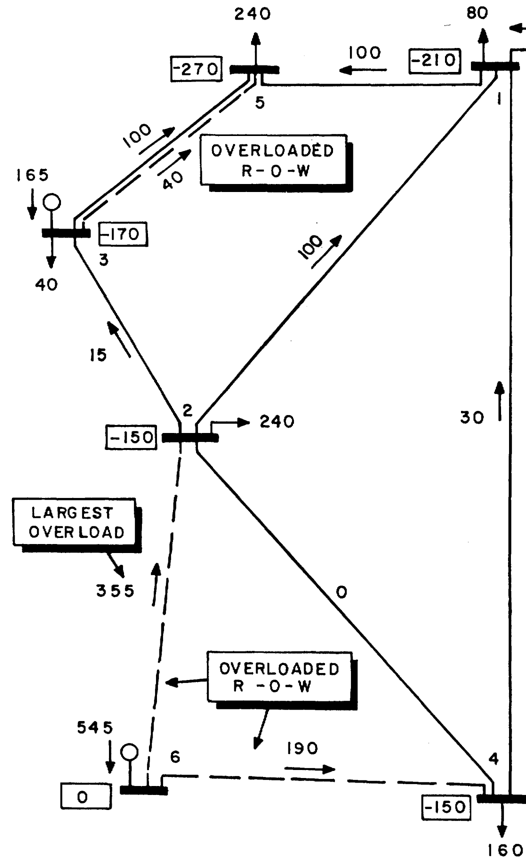

Fig. 4. First flow estimate with no additions.

costs of new circuits, and 2) to be large enough to penalize overloading until all circuit capacity has been used. Experience has shown that guide numbers 3 to 5 times larger than those for a parallel circuit produce the desired results. For this example five times the mileage is used as the path guide number (Table IV).

Table IV is a listing of all possible connections. Along with the mileages and guide numbers for the overload paths Table IV presents the characteristics to be assumed for new circuits should they be needed. The values are all related to mileage in the same manner as used for the data in Table III. Any number of these new circuits may be added. This assumption is not critical, and modifications may be used in larger studies. Also the assumption that all connections are possible is not required. In larger systems a limited number of overload paths may be considered.

The guide numbers need not be strictly mileages. The procedure is flexible enough to include the effects of various voltage levels and construction costs. Mileages are used in the following example to simplify the presentation. The data are now complete for a linear flow estimation.

Linear Flow Estimation

Linear flow estimation is a new network analysis technique capable of studying any bus arrangement and determining where circuits are needed for an adequate transmission network. This technique replaces the electrical network problem with a linear programming problem. For a discussion of linear programming and its application to linear networks see [5, ch. 17]. Because the linear network model differs from the electrical network model in some respects, a comparison of the two is presented in Table V.

As noted in Table V, the linear model has two types of links between buses. Circuits can carry power flow but only up to a given capacity limit. Circuits have low guide numbers and are effective power carriers. In contrast to circuits are the overload paths. These have high guide numbers and resist the flow of power. However, they are assumed to exist between all buses and have unlimited capacity.

The first flow estimate for the example system will aid in highlighting the network model comparison of Table V. Using the data of Tables III and IV and a linear programming solution method produces the flow estimate in Fig. 4. This flow pattern minimizes the loss function in Table V, the magnitude of the power times the link guide number summed for all links. In effect, the power moves over the most direct route from generator to load.

The dashed lines in Fig. 4 indicate the overload paths that have nonzero flows. The many other possible overload paths are not shown in Fig. 4 although they were considered in arriving at this flow pattern. The overload paths with flows become the possible rights-of-way for new circuit additions.

The bus quantities of guide potentials referred to in Table V are shown in the boxes next to the buses in Fig. 4. The method used to arrive at these values is presented in the Appendix.

The flows in Fig. 4 obey the network laws established for this linear model as listed in Table V. The sum of the power flows in and out of each bus balance, thus conserving flow. The circuit capacities are not exceeded. Between buses 3 and 5 the circuit carries its capacity of 100 MW and the overload path carries the additional 40-MW flow. Circuits 1–2 and 1–5 are also carrying their rated flows. The power flow on the 2–4 circuit is zero because the potential difference across the circuit is less than the guide number 40.

The circuits 1–2, 1–5, and 3–5 are in the direction of high-to-lower potential and are at their capacities because the potential

TABLE IV
OVERLOAD-PATH AND NEW-CIRCUIT DATA

Terminals	Overload Path Length (miles)	Overload Path Guide Number	New-Circuit Parameters R (pu)	X (pu)	Capacity (MW)	Guide Number
1–2	40	200	0.10	0.40	100	40
1–3	38	190	0.09	0.38	100	38
1–4	60	300	0.15	0.60	80	60
1–5	20	100	0.05	0.20	100	20
1–6	68	340	0.17	0.68	70	68
2–3	20	100	0.05	0.20	100	20
2–4	40	200	0.10	0.40	100	40
2–5	31	155	0.08	0.31	100	31
2–6	30	150	0.08	0.30	100	30
3–4	59	295	0.15	0.59	82	59
3–5	20	100	0.05	0.20	100	20
3–6	48	240	0.12	0.48	100	48
4–5	63	315	0.16	0.63	75	63
4–6	30	150	0.08	0.30	100	30
5–6	61	305	0.15	0.61	78	61

TABLE V
COMPARISON OF THE LINEAR LOSS FUNCTION NETWORK MODEL WITH AN ELECTRICAL NETWORK MODEL

	Linear Model	Electrical Model
Types of network links	Circuits Overload paths	Circuits
Circuit parameters	Low guide number (circuit length) Flow is constrained by capacity limit	Impedance Flow is unconstrained
Overload-path parameters	High guide number (five times length) Flow is unconstrained	
Flow quantity on links	Power	Current
Quantity at buses	Guide potential	Voltage
Loss function of links	|power| × (guide number)	(current)^2 × (impedance)
Network laws	Summation of power at buses equals zero Circuit capacities cannot be exceeded The power flow on a link is 1) zero if the potential difference is less than the guide number; 2) at capacity and in the direction from high potential to lower, when the potential difference is above the guide number; 3) from high to lower potential and determined by the network tree* when the potential difference equals the guide number. For links in the network tree the potential at the receiving end is less than the potential at the sending end by the guide number of the link.	Summation of current at buses equals zero Sum of (current) × (impedance) around each loop in network equals zero The current flow on a circuit is equal to the voltage difference across the circuit divided by the impedance and is in the direction from high to lower voltage.

* Network tree is a minimum set of links with no closed loops connecting all buses.

differences are greater than the guide numbers. For example the 3–5 circuit has a guide number of 20 and a bus potential difference across it of 100.

The two remaining circuits 2–3 and 1–4 and the overloaded paths form a tree connecting all of the buses and not containing any loops. The flows in a tree can be determined by computing back from the ends of the branches. For example, in Fig. 4 the circuit 1–4 is the only tree branch touching bus 1. Circuits 1–2 and 1–5 are both at capacity limits and are not branches in the tree. With the 1–4 flow undetermined, sum the flows to and from bus 1. It will be noted that in order to balance the flows at 1, circuit 1–4 must carry 30 MW into 1. Now the flow on 4–6 may be determined by repeating the above process around bus 4. Similarly, bus-6 conditions then determine the 2–6 flow, bus 2 determines the 2–3 flow, and bus 3 determines the 3–5 flow. As a check that the generation equals the load the bus-5 summation must be zero.

Fig. 4 illustrates a property of linear programming solutions that is very important for network estimation. Overload paths exist between all of the buses, but most of them have zero flow and need not be considered further by the circuit-selection logic.

Circuit Selection

The flow estimate of Fig. 4 indicates the rights-of-way on which circuits should be constructed to minimize new-circuit mileage. The solution in Fig. 4 indicates a need for 355 MW of circuit capacity between buses 2 and 6. Each new circuit on the 2–6 path is capable of 100 MW, and thus 3.55 circuits are required. Since part of a circuit cannot be built, an integer solution is required. For the reader interested in learning why integer programming is not used to solve this problem, [6] indicates that exact solution procedures for integer programming problems have not yet advanced to the state where they are practical for large problems. Therefore, a roundup procedure will be used.

The roundup rule to follow in solving for an integer number of circuits will be as follows. Select the overload path with the largest overload for the addition of one circuit. In the example of Fig. 4, the 2–6 path is selected because of the 355-MW overload. After the circuit addition a new flow estimate is made (Fig. 5). The only change is the reduction of the 2–6 overload to 255 MW. The foregoing illustrates one of the unique features of the linear network formulation. The guide number for two circuits in parallel is the same as that of one circuit. Only the capacity is changed; it becomes twice the single circuit amount. This leads one to identify the guide numbers more closely with the electrical quantity of voltage drop at full load, which is unchanged when paralleling a circuit with a duplicate, then with the impedance, which is halved when paralleling.

Because the guide numbers are unchanged, the only change with the addition of a parallel circuit whose capacity is less than the overload is to reduce the overload and leave the remainder of the flow pattern unchanged. Thus Fig. 4 flows are still correct even with the addition of a 2–6 circuit, with the exception of the 2–6 flow itself as shown in Fig. 5.

The 255-MW overload is the largest, and a second 2–6 circuit is added reducing the overload to 155 MW. Now the 190-MW overload on the 4–6 path of Fig. 4 is largest, and a circuit is added there reducing the overload to 90 MW with no other change.

The 155-MW overload on 2–6 is now the largest, and a circuit is added. Fig. 6 presents the flow estimate with these four new circuits in place. There is no change in the flow pattern from that shown in Fig. 4, except for the reductions in overloads.

Continuing to follow the largest overload selection rule results in the addition of a circuit on the 4–6 path. This addition causes the first change in the flow pattern (Fig. 7). The new 4–6 circuit is fully loaded as the minimization technique of linear programming seeks to use circuits and avoid overload paths. But there are still three overloads in the network, one between buses 2 and 6, one between buses 3 and 5, and a new overload between buses 1 and 5. At first glance it may appear that the new overload is not in the location to minimize the overload miles. But try to find a better path! Taking the 10-MW overload off of the 1–5 path would require using the paths 6–2 and 3–5, which in combined length are longer than 1–5. The next circuit addition is 2–6 because of the 45-MW overload.

In Fig. 8 six circuits have been added out of the new generating station at bus 6, and no further overloads remain there. However a 40-MW overload remains in 3–5 requiring the seventh new circuit addition. The final flow estimate in Fig. 9 indicates that the network contains no capacity shortages for this set of terminal conditions. The network in Fig. 9 is ready for testing with an ac load flow calculation.

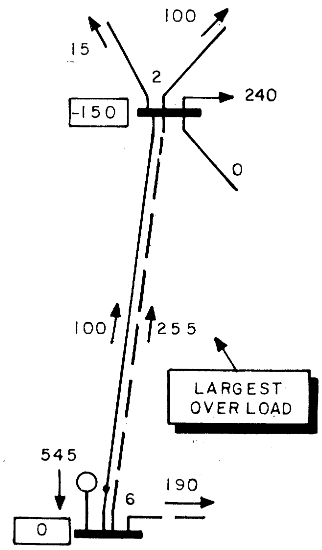

Fig. 5. Second flow estimate with one circuit addition.

AC Load Flow Test

The network in Fig. 9 has been developed using the flow-estimation technique and a simple circuit-selection rule. An ac load flow calculation confirms the fact that this network is capable of supporting the power movements. Table VI presents the bus data for the calculation.

The generation is increased in power output by two percent in anticipation of the power losses in the network. At bus 1 the generation became 51 MW resulting in a net real power schedule of $P = 51 - 80 = -29$ MW. The reactive power Q is unspecified and instead the voltage magnitude is set at 1.02 pu. As discussed in [3], the load flow solution solves for two dependent variables at each bus, i.e., reactive power and voltage angle in the case of bus 1.

At some load buses it may be assumed that the reactive power is supplied by capacitor banks up to the needs of the loads. At buses 2 and 5 the reactive power equals zero to simulate this condition. Bus 3 was treated like bus 1 with a higher voltage because it is a major generator. Bus 4 is a load bus with reactive power to be supplied by the transmission network. Bus 6 is the swing bus with the voltage magnitude and angle specified.

Table VII presents the line data for the ac load flow calculation. The first five entries are unchanged from Table III. The 2–6 line represents four parallel circuits by using one quarter of the resistance of 0.08 and the reactance of 0.30 listed in Table IV. The 3–5 and 4–6 lines represent double circuits by using one half of the values listed in Table IV.

The ac load flow test of the estimated network is shown in Fig. 10. The correlation between the estimated power flows (Fig. 9) and the ac results shows that linear flow estimation has successfully prepared a network for ac load flow testing. The power flows are in the directions indicated by the estimate and are generally of the same magnitude. There are no overloads in the network. There is a voltage problem at bus 4, but then this is only a preliminary design with which to begin developing a network using conventional methods.

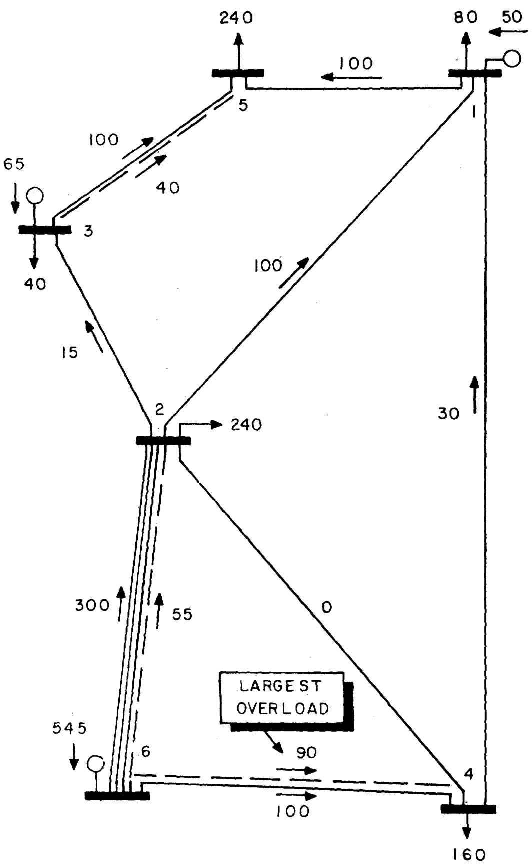

Fig. 6. Fifth flow estimate with four circuit additions.

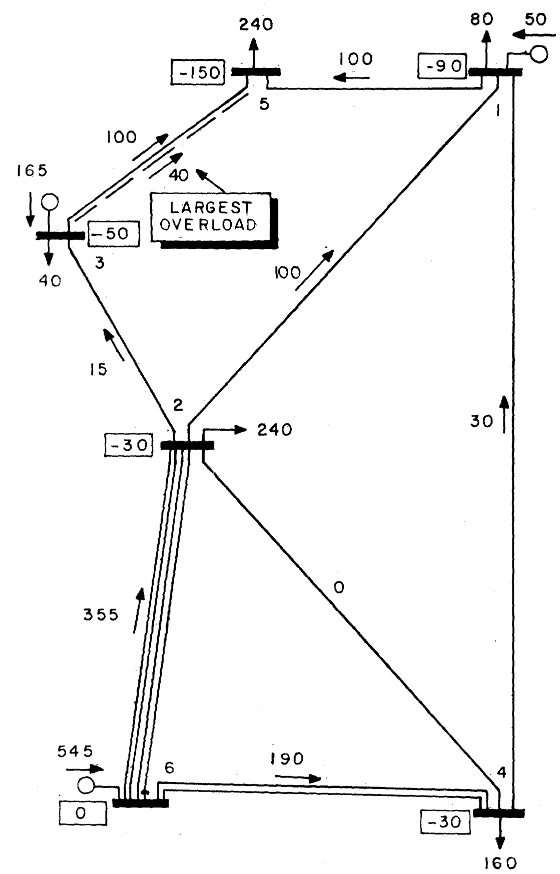

Fig. 8. Seventh flow estimate with six circuit additions.

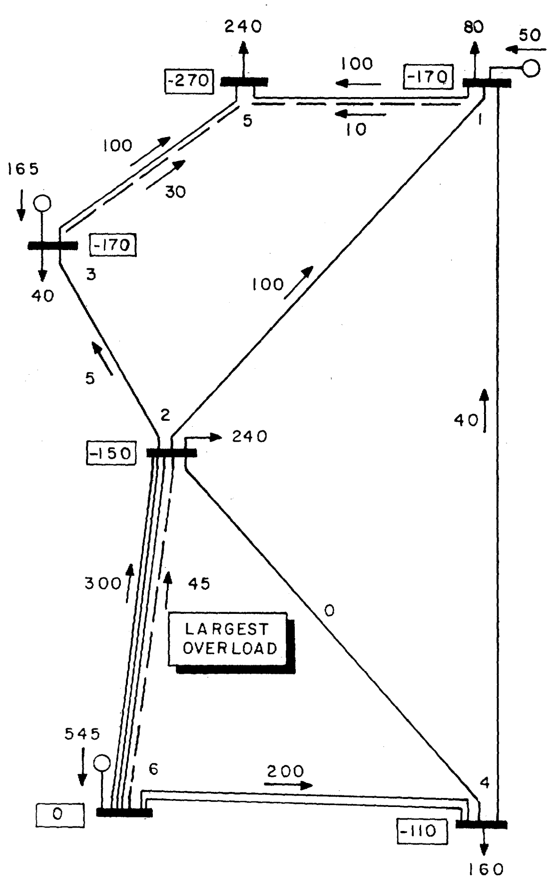

Fig. 7. Sixth flow estimate with five circuit additions.

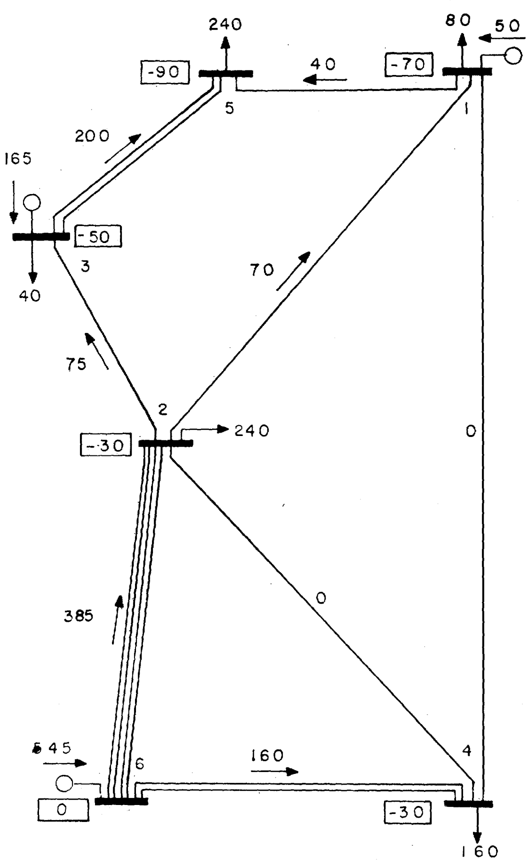

Fig. 9. Final flow estimate with seven circuit additions.

TABLE VI
BUS DATA FOR AC LOAD FLOW

Bus	P (MW)	Q (MW)	Voltage (pu)	Remarks
1	−29	*	1.02 ∠*	generator holds voltage
2	−240	0	* ∠*	load vars supplied locally
3	128.3	*	1.04 ∠*	generator holds voltage
4	−160	−40	* ∠*	load
5	−240	0	* ∠*	load vars supplied locally
6	*	*	1.04 ∠0	swing bus

* Quantities to be determined by ac load flow.

TABLE VII
LINE DATA FOR AC LOAD FLOW

Terminals	R (pu)	X (pu)
1–2	0.10	0.40
1–4	0.15	0.60
1–5	0.05	0.20
2–3	0.05	0.20
2–4	0.10	0.40
2–6	0.01875	0.075
3–5	0.025	0.10
4–6	0.0375	0.15

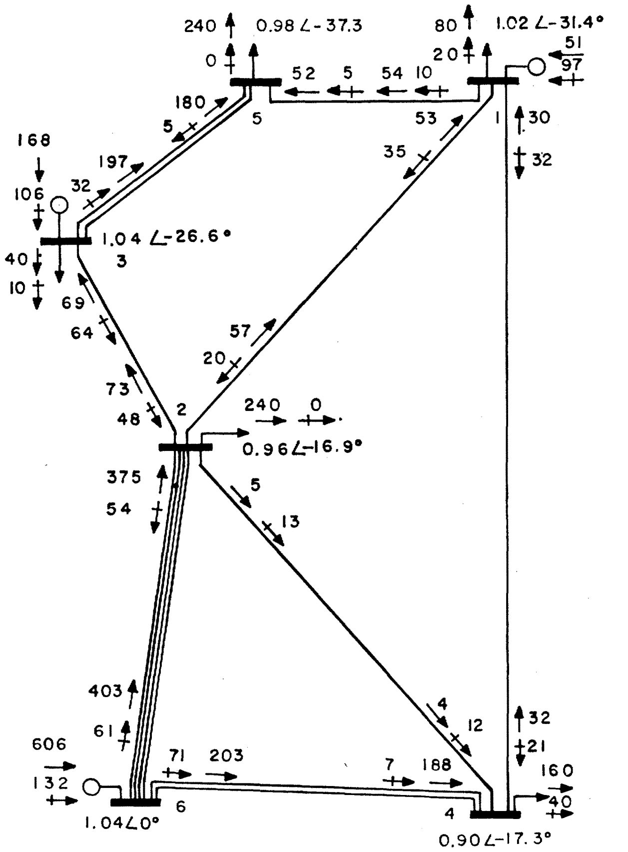

Fig. 10. Ac load flow test of estimated network.

The network of Fig. 10 appears to have little excess capability. If circuit outages or generation outages were tested on it, overload problems would develop. The further network additions may be determined using the network estimation procedure.

Two or More Voltage Levels

The example considered only one voltage level. Two or more voltage levels may be studied by specifying circuits of different capacities and different guide numbers depending on their voltage level and construction cost. Transformers are included as equivalent circuits with capacities and guide numbers. The guide number for a transformer may be computed by considering the phase angle across the transformer at rated conditions

$$\frac{\text{transformer guide number}}{\text{circuit guide number per mile}} = \frac{\theta_t}{\theta_c/\text{mi}} \tag{1}$$

where θ_t is the phase angle across the transformer at rated conditions and θ_c/mi is the phase angle per mile for a circuit at rated conditions. The guide numbers are thus an indication of electrical distance at rated conditions.

The phase angle may be approximated by the relation from [3]

$$\theta \approx \frac{PX}{E^2} \quad \text{rad}$$

where P is the power in MW, X is the reactance in ohms, and E is the voltage in kV. Thus with both the circuit and transformer at the same kV (1) becomes the ratio

$$\frac{\text{GN}_T}{\text{GN}_c/\text{mi}} = \frac{P_T X_T}{P_c X_c/\text{mi}} \tag{2}$$

where P_T is the transformer rated capacity, X_T is the transformer reactance, P_c is the circuit thermally rated capacity, and X_c/mi is the circuit reactance per mile. For example, a transformer rated at 300 MW and with a reactance of 4 ohms would be related to the guide number for a circuit of 100 MW and 0.8 Ω/mi by

$$\frac{(300)(4)}{(100)(0.8)} = 15.$$

Transformer guide numbers are very insensitive to transformer size since the product of rating and impedance tends to be a constant.

The network estimation technique has been used for multivoltage network planning with transformers included and in other cases with no transformers and a bus representing a load area with more than one voltage serving it. The particular formulation of the problem depends on the detail required in the answer.

Circuits of two different voltage levels have guide numbers per mile related by an expression similar to that for a transformer:

$$\frac{\text{high-voltage GN/mi}}{\text{low-voltage GN/mi}} = \frac{\theta_H}{\theta_L} \tag{3}$$

where θ_H is the phase angle per mile for a high-voltage circuit at rated conditions and θ_L is the phase angle per mile for a low-voltage circuit at rated conditions. The ratio in (3) tends to be unity and, therefore, mileage is a good stand-in for the guide number no matter what the voltage level.

The guide number per mile for overload paths in a multivoltage network may be five times a circuit guide number as in the previous example. The circuit selection rules become more complicated when the voltage class is a variable. Two selection procedures have been tried, one using slightly reduced guide

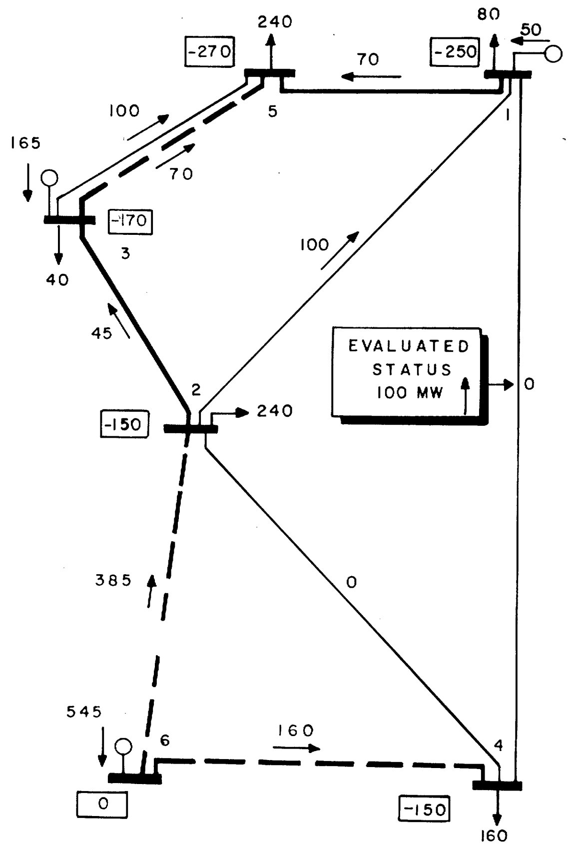

Fig. 11. Nonoptimum flow pattern.

numbers on overload paths for the preferred voltage, and the other using logic based on the voltages already present at the terminals and the magnitude and distance of the overload. A study of various voltage selection rules is beyond the scope of this paper.

Conclusions

The method of linear flow estimation has been shown to be an effective guide in the development of preliminary network designs for later study by conventional transmission analysis techniques. The method has the flexibility to study situations ranging from those where no network presently exists to those where the existing network includes more than one voltage level. The flow-estimation technique, when combined with a procedure for selecting circuit additions, produces a network estimation method for the synthesis of future transmission networks.

A set of computer programs for long-range transmission planning has been developed using linear flow estimation for preparing network designs and ac load flow calculations for modifying these designs. A number of large networks representing both utility companies and power pools have been designed using the linear flow-estimation method. The results were confirmed by ac load flow calculations and used in determining the transmission investment costs in generation–transmission expansion studies.

Appendix

Linear Programming

The linear programming method used in solving the linear network model minimizes a loss function defined as power times guide number summed over all network links. The minimum of this linear function is termed a program in its economic sense for it is a budget of resources. For example, each flow estimate (Figs. 4–9) is a program of flows to minimize the loss function. Linear programming problems are solved by computer programming, and much confusion about the meaning of the word programming has developed.

Linear Flow Estimation

Linear flow estimation uses the theory of linear programming to 1) determine if a flow pattern minimizes the loss function and, if not, 2) how to make one change at a time to move toward the minimum. The mathematical theory for the following steps is detailed in [5].

Step 1: Select links to form a tree connecting all buses without closing any loops. In Fig. 11 the tree branches are shown as heavy lines. The dashed lines are the overload paths in the tree.

Step 2: Define all of the flows on the nontree overload paths to be zero. Set the nontree circuit flows to either zero or to the circuit capacity. If a wrong choice is made at this step it will be discovered in step 5 and corrected in step 6. In Fig. 11, circuits 1–4 and 2–4 were chosen for zero flows. A flow at capacity on line 2–1 was chosen along with a flow at capacity on 3–5.

Step 3: Determine the flows in the tree branches by beginning at an end branch and working back down the tree. For example, bus 4 is at the end of a tree branch and the flow of 160 on 6–4 is necessary to supply the load. Now bus 6, requirements determine that 385 MW must flow to bus 2. Similarly bus 2 requirements determine the 45 MW flow to bus 3 and then the 70 MW overload to bus 5 is determined. Finally the 70 MW flow at 1–5 is determined by the bus 5 requirements. Bus 1 checks the balance of generation and load by the sum of the flows equaling 0.

Step 4: Compute bus-guide potentials by defining one bus at zero potential. Then compute the other bus potentials such that the following relation holds for all tree links:

$$\text{(sending-bus potential)} - \text{(receiving-bus potential)} = \text{(link guide number)}$$

For example, the guide potential for bus 2, shown in the box on Fig. 11, was computed by

$$0 - x = 150$$

or

$$x = -150$$

where 150 is the guide number for the overload path 2–6 in Table IV. The potential at bus 3 can now be computed and similarly on down the tree.

Step 5: Evaluate the flow settings of all nontree links using the network laws in Table V.

1) If the potential difference across any link is less than its guide number then the flow on that link should be zero. For example the overload path 1–3 in Table IV has a guide number of 190 and the potential difference between 1 and 3 in Fig. 11 is 80, so that the evaluated flow is zero and agrees with the flow set in step 1. All overload paths are correctly set in Fig. 11. Also the circuit 2–4 has a potential difference of zero across it indicating zero flow.

2) If the potential difference across any link is greater than the guide number, then the flow should be at capacity and in the

direction from high to lower potential. For example, buses 1 and 2 have a potential difference of 100 while the 1–2 circuit guide number is 20 indicating that flow is correctly set at rating from the −150 potential bus to the −250 potential bus. Circuit 1–4 has a potential difference of 100 and a guide number of 60 and is evaluated as having flow at capacity. Since this is not the case in Fig. 11, the flow pattern does not minimize the loss function and we proceed to step 6. When all evaluations agree with the flows, the flow estimate is complete.

Step 6: To correct the flow pattern, select one link where the evaluation does not agree with the flow, e.g., circuit 1–4 in the example. Change the flows on this link and in the branches of the tree which form a closed loop with the link. The change is in the direction evaluated and in magnitude to cause one of the changing links to 1) come to zero flow, or 2) come to a capacity limit.

For example, as circuit 1–4 begins to carry flow from 4 to 1 all of the tree branches in Fig. 11 change since they are all in the closed loop from bus 1 around to bus 4. The first link to reach a limit is 1–5, which goes to its capacity of 100 MW after a change of 30 MW. Therefore, the 1–5 circuit is removed from the tree, and 1–4 with a flow of 30 MW enters the tree. The resulting flow pattern is shown in Fig. 4 and is evaluated as being a minimum loss pattern.

This procedure allows a computer program to visualize where power flows are necessary, even before circuits are added. The real strength of this method lies in the fact that most of the overload paths never carry any flow and thus need not enter into a planning procedure.

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Discussion

C. A. Germain (Ebasco Services, Inc., New York, N. Y. 10006): The author shows a method of estimating what lines are needed to relieve overloads on a transmission system by using linear programming. The way the program goes about relieving overloads does not, as I see it, necessarily lead to the solution that adds the minimum number of new circuit miles. Consider the three buses in Fig. 12(a). Suppose we find that the power is flowing from C to B to A, and lines CB and BA are overloaded due to a heavy load on bus A. The program will first look to see which line has the greatest overload. Suppose this is line CB. A line will be built parallel to CB to relieve the overload (Fig. 12(b)). Then the program will build a line parallel to BA to relieve the overload on line BA (Fig. 12(c)). The overloaded path CA has not been considered. A line from C to A might be much shorter than one from C to B and B to A, but the

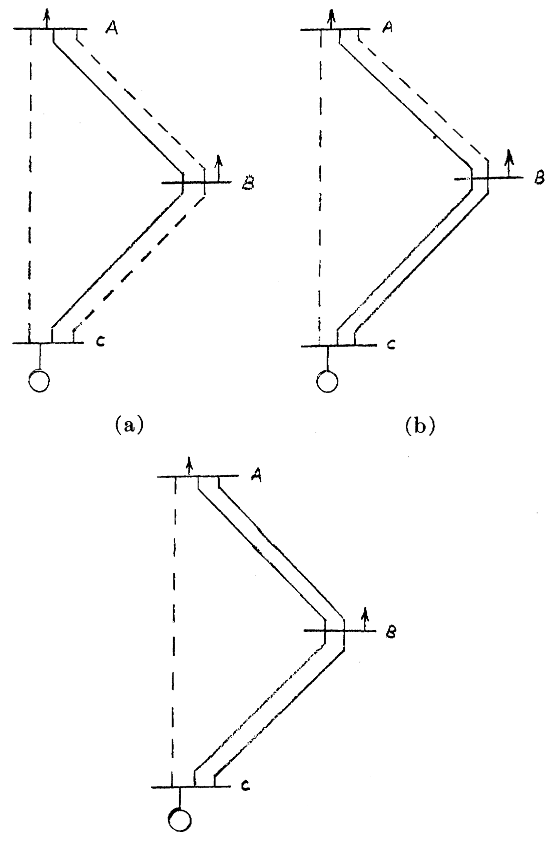

Fig. 12. Dashed lines—overload path, solid lines—existing or constructed lines.

program looks for the minimum mileage to relieve the overloaded line, and C to A is longer than C to B or B to A. The logic seemingly does not provide for this condition. Line CA would have to be added by the engineer when he analyzes the results of the program. Since the engineer would have to examine what the program built after each load level was run, there would be no reason to run the program for more than one successive load level at a time. A great deal of engineering time would not be saved.

For any one overload path, only one set of circuit characteristics are given. This limits the program to only one circuit type for any given right-of-way that the overload path specifies. The program seemingly does not provide for bundling or reconductoring old lines. Being limited to only being able to build new lines, the cost of relieving overloads can not be accurately reflected. If the cost depends only on the guide numbers, and all guide numbers are picked at, say, five times the mileage as in the example given, the author is implying that cost of all lines is a linear function of mileage. This could lead to erroneous results.

It would be interesting to see an example of how the author handles a system with more than one voltage level. Guide numbers would have to be assigned so that at the optimum stage of development, higher voltage buses would be established and higher voltage lines built. The program must be able to handle this type of problem if it is going to be used to study any actual transmission system.

R. H. Iveson (New York State Electric and Gas Corporation, Binghamton, N. Y. 13902): The author is to be commended for developing what is proving to be a valuable planning tool.

Several questions and possible refinements have come to mind. The author's comments on these would be appreciated.

1) Has a figure of merit or guideline been determined for the condition of heavy system throughflows at which the assumption of linearity may no longer be valid?

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2) Some question arises in the determination of circuit capability. Fig. 3 is essentially a surge impedance loading curve and, as such, represents an idealized condition. For relatively short line distances, high circuit capabilities will be input to the program. This condition assumes either the buses are closely coupled by the surrounding system or are associated with large blocks of generation. Suppose, for example, three line sections, each 75 miles long, are assumed in series and assigned equal ratings. Unless the line terminals are closely coupled through other paths, this can look like a 225-mile line with a considerably lower circuit capacity. Is not engineering judgment required to decide whether the line ratings should be biased for this effect?

3) The author does not mention the ability to analyze line contingencies, but our understanding is that the program will automatically analyze and design for single-line contingencies. The ability to incorporate multiline contingency design (for example, two contingencies) would be of great use when more stringent reliability criteria must be met.

4) In the Section *Two or More Voltage Levels* there is a brief discussion of the use of voltage selection rules and guide numbers. It would appear this may be critical to the performance of the program. It would also appear that some sort of heuristic reasoning is involved in these procedures. Although, as the author states, it is beyond the scope of this paper, it would seem important to prepare a discussion of this material in the near future.

C. C. Hancock (Southern Services, Inc., Birmingham, Ala. 35202): The author has presented a practical approach to the development of a transmission network to fit future generation expansion plans. The planning engineer does not need to start the transmission study for a distant year with a network capable of supporting a conventional ac load flow. He is thus free to consider any network design resulting from the linear flow estimation program. The method outlined not only adds the circuits required, but also eliminates from consideration those paths on which circuits are not required. This allows the load flow testing to start with a network that is relatively uncluttered. These features become very important in long-range studies.

In the development of a transmission network for a future year, it will be well to run several designs to allow for differences in engineering judgment when selecting a design for further study.

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Atif S. Debs (Systems Control, Inc., Palo Alto, Calif. 94306): This paper contains a technique for transmission network expansion in order to meet demand. The advantages of this technique as I see them are 1) computational speed and simplicity, and 2) the use of linear programming for optimization purposes.

The disadvantages are as follows.

1) No mathematical link is established between the true electrical laws and the linear model used by the author. An experimental link is, however, suggested. It seems that heuristic engineering judgment has been used extensively in order to arrive at the linear model.

2) The loss function chosen is not necessarily related to the dollar cost of transmission-line expansion. I think that a more meaningful criterion should be related, in some sense, to two major quantities:

a) cost of line additions
b) cost of power losses.

3) A major requirement in system planning is the reliability of the resulting network. Although the author mentions, near the end of the paper, that reliability considerations are possible in his algorithm, he attaches very little weight to this important aspect.

In conclusion, it is noted that this suggested technique for power system expansion offers some progress over traditional methods in its employment of some form of optimization in the planning process. It is questioned, however, if the linear planning model corresponds to the actual situation.

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In addition, reliability considerations should be stressed further.

The paper is well written and organized. It definitely ventures into a controversial and difficult task.

Len L. Garver: I appreciate the valuable questions and comments presented by the discussers and thank them for their efforts in preparing them.

Overloads Need Not Parallel Circuits: The paper presents a method to mathematically determine the flows and overloads in a fashion similar to an engineering estimate of the flows on a simplified map. Thus, as Mr. Germain points out, an engineer would see the direct route from C to A even though a load flow calculation would overload C–B–A. The linear programming solution would also place any overload between C and A directly on the path C–A even though no circuits exist there. Thus, instead of only paralleling existing circuits, overloads and then new circuit additions may appear on any bus pair. It was for this important reason, overloads where no circuits exist, that the dc load flow method of network analysis was set aside and this new concept of flow estimation developed. Mr. Germain's example would thus result in a design with a circuit added from C to A.

Overloads Guided by Economic Miles: The guide numbers described in the paper are meant to control the flow estimation so that economical construction possibilities will be sought out by overloads. If, as Mr. Germain suggests, some bus pairs would have lower costs per mile than others, these can be accommodated by setting the guide number lower than the mileage value.

More Then One Type of Construction: Various types of construction may be possible between two buses. Each of the possibilities are given a separate overload path and guide number. There is no limit to the number of parallel overload paths. Each path may have a circuit limit indicating that once line looping, reconductoring, second circuit on an existing tower line, or whatever has been used, the opportunity no longer exists.

Computer Program: The method described in the paper was developed to be implemented on a digital computer. The first computer programs using linear flow estimation were used in two transmission expansion studies in which Mr. Iveson and Mr. Hancock participated. Each study used the same programs, one for horizon-year studies to set a planning goal and the second for year-by-year planning. However, the applications were quite different. One application involves a power pool with three voltage levels to plan and two more lower voltage levels to approximate, while the second involves one company with two voltage levels to plan and a third to represent.

Line Capabilities: One of the most important inputs to this method is each circuit's capability. As Mr. Iveson indicates, this is a difficult quantity to exactly specify. The circuit ratings are set based on utility experience and are checked by developing a network using flow estimation and testing the network using ac load flow calculations. If the network has too large a phase-angle spread or the losses appear too high then all ratings could be adjusted. Rating changes have not been necessary when nominal ratings are used for normal test conditions. Increases of 25–50 percent over these ratings are used during generation contingency or circuit contingency tests.

Test Cases: Preparing flow estimates for many test cases before selecting any circuit addition is especially important. In the current program up to 30 normal and contingency generation-load tests are considered simultaneously. Circuit-selection rules are based on overload amounts and location in all of these tests.

As Mr. Iveson mentions, the program can automatically test all single-circuit outages. There is no theoretical reason why the computer could not take all multiple-circuit outages. More pressing program requirements have thus far postponed its development.

Many Designs: Mr. Hancock mentioned the advisability of developing several designs and then using engineering judgment. Besides networks developed for various input conditions such as generation locations, voltage levels to plan, and test cases, the program automatically develops several plans for one set of input conditions. This unique feature provides several near-optimum network plans for the engineer's consideration. These plans for the same conditions plus those for several varying conditions give the system planner insight into the future network requirements.

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