Día 1

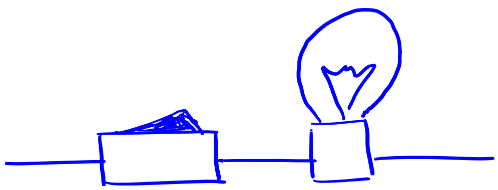
1. Los átomos de la computación

Objetivo: Conocer cómo se tiene la información en un computador.

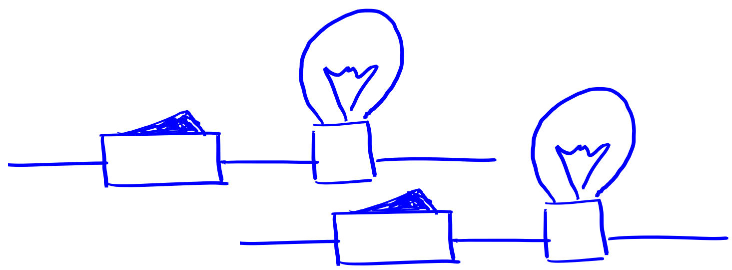
Átomos $\longrightarrow$ Bit : Unidad básica de información

0 1

Forma física del bit ?



encendido 1
apagado 0



apagado - apagado 00
" - encendido 01
encendido - apagado 10
encendido - encendido 11

Los interruptores no aguantan'!

transistores

Qué información almacenamos?

Números?　　　Letras?　　　Emojis?

$$9213 = \underline{9}\cdot 10^3 + \underline{2}\cdot 10^2 + \underline{1}\cdot 10^1 + \underline{3}\cdot 10^0$$

Base 2

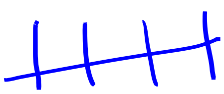
$$13 = \underline{1}\cdot 2^3 + \underline{1}\cdot 2^2 + \underline{0}\cdot 2^1 + \underline{1}\cdot 2^0$$

1 1 0 1

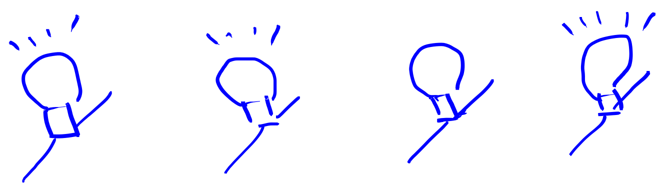
Cuántos bits? 

$$\begin{array}{rrrrr} 13 & \lfloor 2 & & & \\ 1 & 6 & \lfloor 2 & & \\ & 0 & 3 & \lfloor 2 & \\ & & 1 & 1 & \lfloor 2 \\ & & & 1 & 0 \end{array}$$

1101_2



$$\begin{aligned} 13 &= 2\cdot 6 + 1 \\ &= 2^1\cdot 6 + 1\cdot 2^0 \\ &= 2^1\cdot(2^1\cdot 3 + 0) + 1\cdot 2^0 \\ &= 2^2\cdot 3 + 0\cdot 2^1 + 1\cdot 2^0 \\ &= 2^2\cdot(1\cdot 2 + 1) + 0\cdot 2^1 + 1\cdot 2^0 \\ &= 1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0 \end{aligned}$$



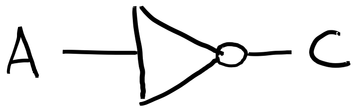
Preguntas! Z

Computador Clásico → Computador Cuántico

Bit　　　　　　　　Qubit

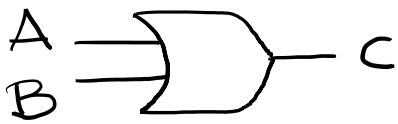
Diagramas → Operadores lógicos

NOT



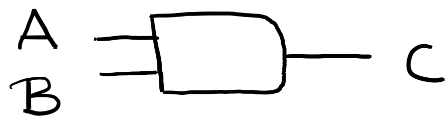
A	C
0	1
1	0

OR



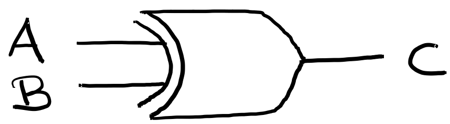
A	B	C
0	0	0
0	1	1
1	0	1
1	1	1

AND

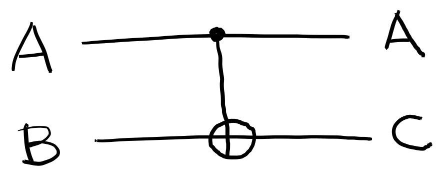


A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

En computación cuántica



XOR



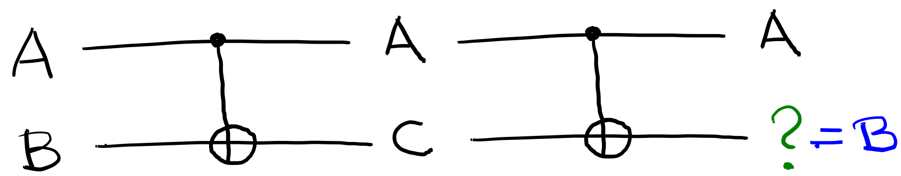
CNOT

A	B	C
0	0	0
0	1	1
1	0	1
1	1	0

→

A	B	A	C
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

Pregunta!

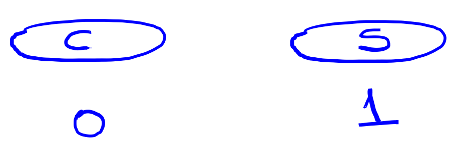


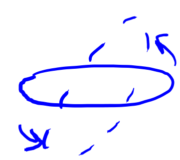
2. Representación de Qubits

Objetivo: Escribir un qubit de forma matemática

Bit

Qubit





c + s

0 + 1

También hay 0 y 1!

2.1. Álgebra lineal

2.1.1 Números Complejos

cartesiana

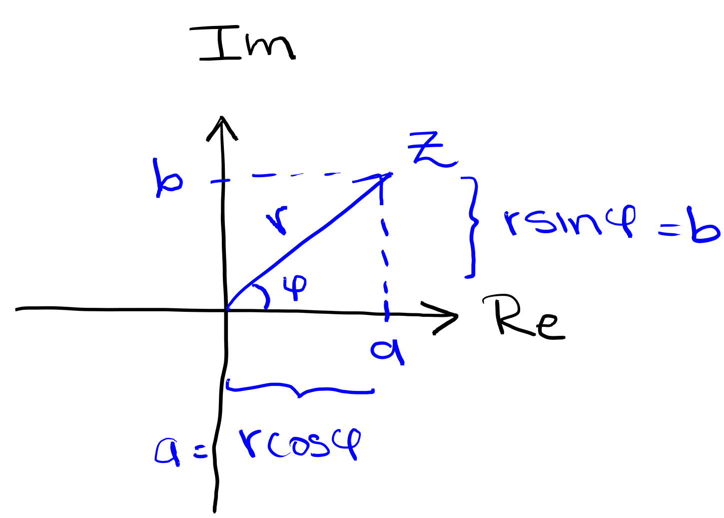
$z = a + bi \qquad i = \sqrt{-1}$

Polar

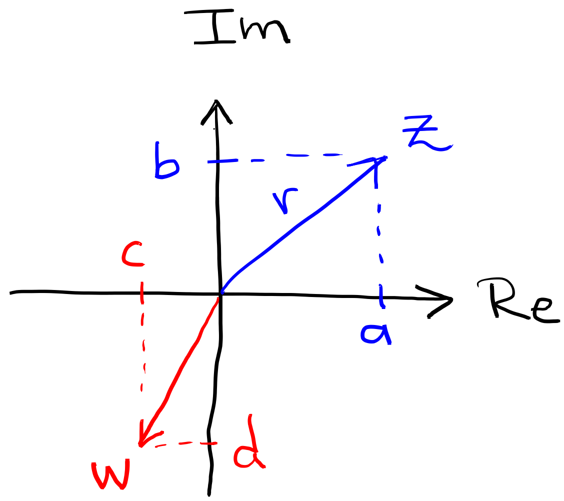
$z = r\cos\varphi + r\sin\varphi\, i$

$= r\underbrace{(\cos\varphi + i\sin\varphi)}_{e^{i\varphi}}$ Euler

$= re^{i\varphi}$



- Magnitud $|z| = r = \sqrt{a^2+b^2}$



- Suma $w = c + di$

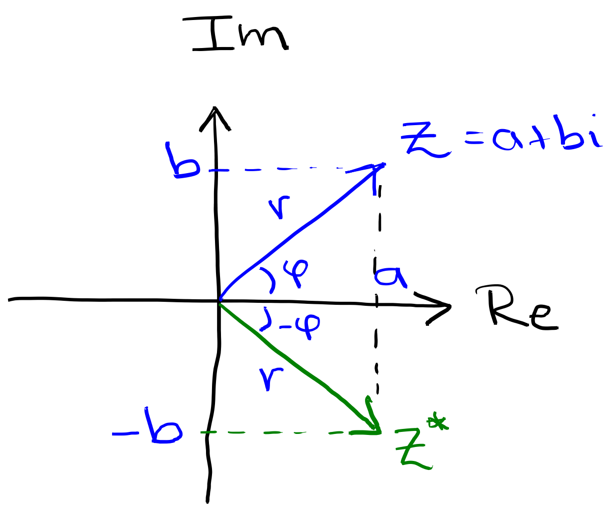
$z + w = a + bi + c + di$
$\quad = (a+c) + (b+d)i$

$i = \sqrt{-1}$
$i^2 = -1$

- Multiplicación

$z \cdot w = (a+bi)(c+di) = ac - bd + (ad+bc)i$

- División



- Conjugación $i \to -i$

$z^* = \bar{z} = a - bi$

$z = re^{i\varphi}$

$z^* = re^{-i\varphi}$

Otras propiedades

- $(z+w)^* = z^* + w^*$
- $(z^*)^* = z$
- $(zw)^* = z^* w^*$
- $z z^* = |z|^2$

2.1.2 Ahora sí álgebra lineal

$\{$ Los estados cuánticos se pueden representar como vectores. $\}$

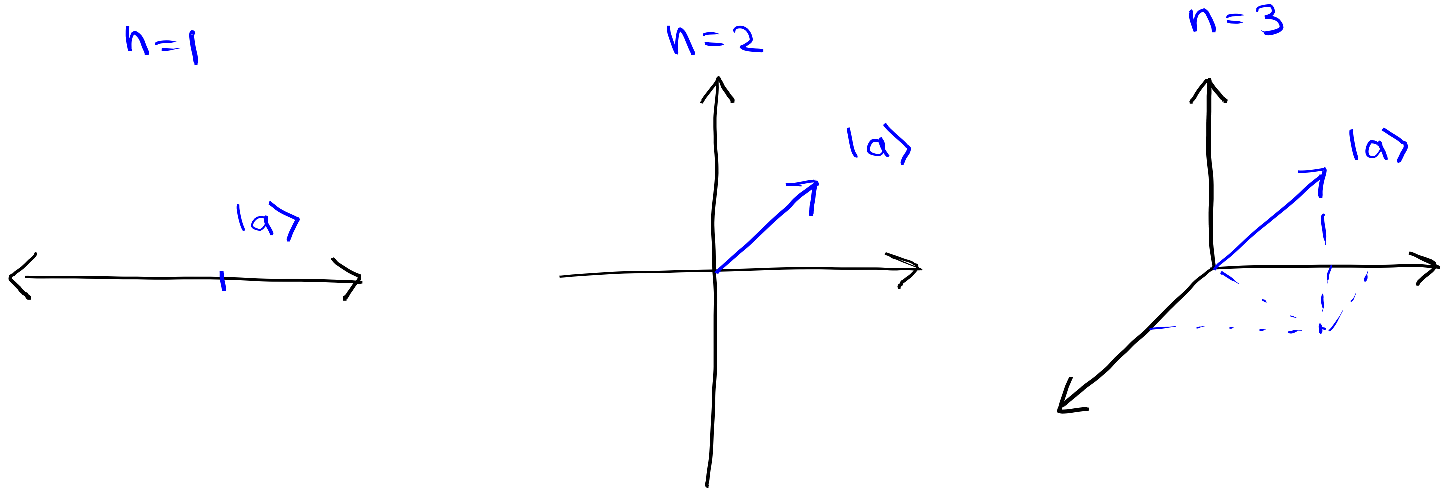
$\{$ Las compuertas cuánticas como matrices $\}$

2.1.2.1 Vectores + Notación Dirac

$$\vec{a} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \longrightarrow \text{"Ket"} \quad |a\rangle = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$$

n : dimensión

$$\vec{b} = \begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix} \longrightarrow \text{"Bra"} \quad \langle b| = \begin{bmatrix} b_1 & \cdots & b_n \end{bmatrix}$$



- Suma $\quad |a\rangle + |b\rangle = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_n + b_n \end{bmatrix}$

$$\langle c| + \langle d| = \begin{bmatrix} c_1 + d_1 & \cdots & c_n + d_n \end{bmatrix}$$

- Multiplicación por escalar

$$k|a\rangle = \begin{bmatrix} ka_1 \\ \vdots \\ ka_n \end{bmatrix} = |ka\rangle \qquad \langle b|k = \begin{bmatrix} kb_1 & \cdots & kb_n \end{bmatrix}$$

- Conjugado transpuesto

$$|a\rangle = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \longrightarrow |a\rangle^{*T} = \begin{bmatrix} a_1^* & a_2^* & \cdots & a_n^* \end{bmatrix}$$

$$= \langle a|$$

- Magnitud

$$|z|^2 = z \cdot z^*$$

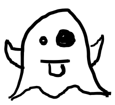
$$\| |a\rangle \|^2 = |a_1|^2 + |a_2|^2 + \cdots + |a_n|^2$$

$$= a_1 a_1^* + \cdots + a_n a_n^*$$

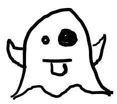
$$= \sum_{i=1}^{n} |a_i|^2$$

2.1.2.2 Espacios Vectoriales

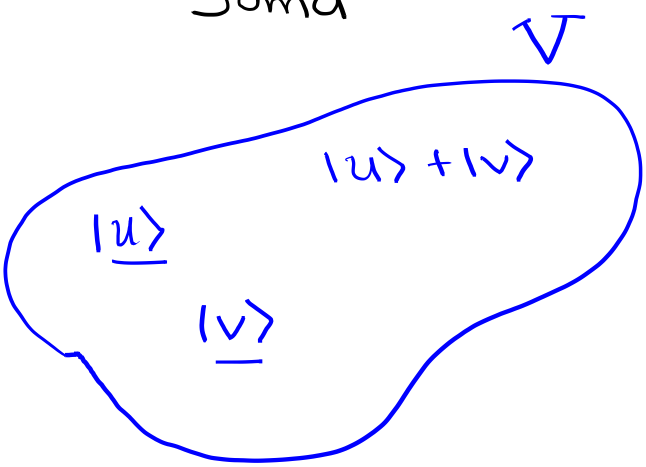
$$\underbrace{\text{Conjunto de vectores } \{|u\rangle, \dots, |v\rangle\} + \text{Escalares } (\mathbb{C}) + \text{Adición \& Multiplicación por escalar}}_{\text{Axiomas}}$$



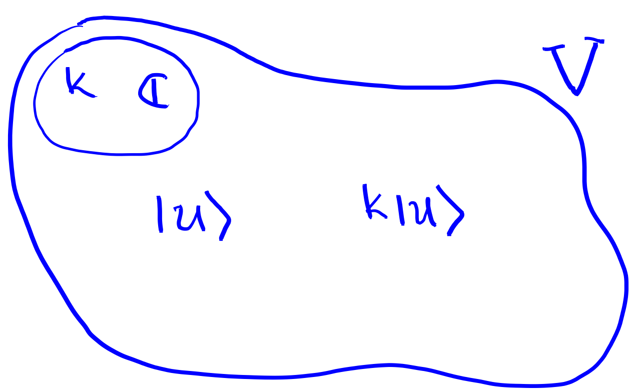
Axiomas

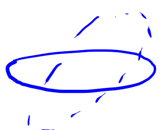


Suma



Producto por escalar $\mathbb{C}$





$k|u\rangle, |v\rangle \in V$

$k|u\rangle + |v\rangle \in V$

$|cara\rangle$

$|sello\rangle$

$|cara\rangle + |sello\rangle$

IMPORTANCIA

2.1.2.3 Producto Interno

$$\vec{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \qquad \in \mathbb{C}^{n=\text{dim}}$$

$$\vec{u}\cdot\vec{v} = \underbrace{\begin{array}{ccc} v_1 & \cdots & v_n \\ \hline u_1^* & \cdots & u_n^* \end{array}}_{\langle u|} \underbrace{\begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}}_{|v\rangle}$$

$$= u_1^* v_1 + u_2^* v_2 + \cdots + u_n^* v_n$$

$$= \sum_{i=1}^{n} u_i^* v_i = \langle u| \, |v\rangle$$

$$\langle u|u\rangle = \vec{u}\cdot\vec{u} = \|\vec{u}\|^2 = \||u\rangle\|^2$$

braket

Un vector es unitario si

$$\||u\rangle\| = 1 \qquad \langle u|u\rangle = 1$$

Dos vectores son ortogonales si

$$\vec{u}\cdot\vec{v} = 0 \qquad \langle u|v\rangle = 0$$

2.1.2.4 Base Ortonormal

$\underbrace{\{0\}}_{a}^{\psi}\underbrace{\}}_{b}^{\psi}$ $\underbrace{\{1\}}_{a}^{\psi}\underbrace{\}}_{b}^{\psi}$

$a, b \in \mathbb{C}$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} = a\begin{bmatrix} 1 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\left\{ \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{|0\rangle}, \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{|1\rangle} \right\} \rightarrow \text{Base Computacional}$$

Es ortonormal?

$\langle 0 | 1 \rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0$ $\qquad \langle 1 | 1 \rangle = 1$

$\langle 1 | 0 \rangle = 0$

$\langle 0 | 0 \rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1$

En general, $\qquad \langle u_i | u_i \rangle = 1 \quad \langle u_1 | u_2 \rangle = 0$

$$|v\rangle = \sum_{i=1}^{n} c_i |u_i\rangle = c_1 |u_1\rangle + \cdots + c_n |u_n\rangle$$

Preguntas!

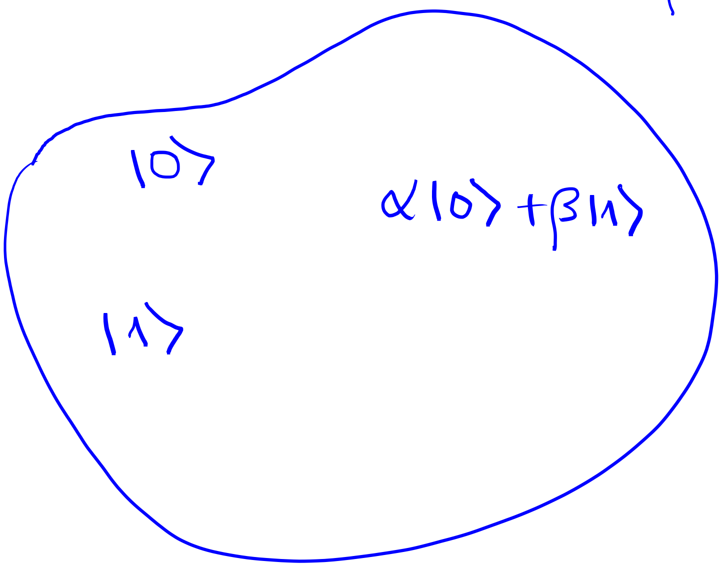
$$\left\{ \underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{|+\rangle}, \underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}}_{|-\rangle} \right\}$$ Base? Ortonormal?

2.2 Qubits como estados cuánticos

Un estado cuántico tiene la forma

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$\alpha, \beta \in \mathbb{C}$



$\mathcal{H}$: Estados cuánticos

$\hookrightarrow$ Espacio de Hilbert.

$|\text{moneda}\rangle = \alpha|\text{cara}\rangle + \beta|\text{sello}\rangle$

α, β : Amplitudes

Qubit : $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

α, β

$|\alpha|^2$: Probabilidad de encontrar el qubit en $|0\rangle$

$|\beta|^2$: Probabilidad de encontrar el qubit en $|1\rangle$

$$|\psi\rangle = \underline{\alpha}|0\rangle + \underline{\beta}|1\rangle \xrightarrow{\text{Medición}} |\psi\rangle = |0\rangle$$

con $P = |\alpha|^2$

$P_{cara} = \frac{1}{2}$ $\qquad P_{sello} = \frac{1}{2}$

$|cara\rangle$ $\qquad |sello\rangle$ $\qquad P_{total} = 1$

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$P_{|0\rangle} = |\alpha|^2$

$P_{|1\rangle} = |\beta|^2$

$$P_T = P_{|0\rangle} + P_{|1\rangle} = |\alpha|^2 + |\beta|^2 = 1$$

{ Las probabilidades deben sumar 1 }

Vector de estado!

$|\alpha|^2 + |\beta|^2 = 1$ $\qquad |\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$$\langle\underline{\psi|\psi}\rangle = [\alpha^* \ \beta^*]\begin{bmatrix}\alpha \\ \beta\end{bmatrix} = \alpha^*\alpha + \beta^*\beta = |\alpha|^2 + |\beta|^2 = 1$$

$||\psi\rangle|^2$

Preguntas! ¿Es $|u\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$

$|+i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$

un vector de estado?

$|u\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$ $\qquad \alpha = \frac{1}{\sqrt{2}}$ $\qquad |\alpha| = \frac{1}{\sqrt{2}}$ $\qquad |\alpha|^2 = \frac{1}{2}$

$\beta = \frac{i}{\sqrt{2}}$ $\qquad |\beta|^2 = \beta\cdot\beta^* = \left(\frac{i}{\sqrt{2}}\right)\left(-\frac{i}{\sqrt{2}}\right) = -\frac{1}{2}i^2 = \frac{1}{2}$

$|\alpha|^2 + |\beta|^2 = 1$