Día 3

1. Varios Qubits y entrelazamiento

Objetivo: Transformar sistemas de varios qubits y relacionarse con el entrelazamiento.

{ Uno de los poderes de la computación cuántica está en la interacción entre qubits }

Supongamos dos qubits $\begin{bmatrix} - \\ - \\ - \\ - \end{bmatrix}$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \qquad |\phi\rangle = \gamma|0\rangle + \delta|1\rangle = \begin{bmatrix} \gamma \\ \delta \end{bmatrix}$$

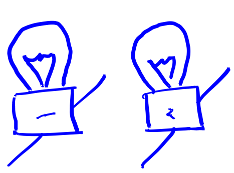
El estado combinado es $|\psi\rangle$ $|\phi\rangle$

$$\underline{|\psi\rangle \otimes |\phi\rangle} = (\alpha|0\rangle + \beta|1\rangle) \otimes (\gamma|0\rangle + \delta|1\rangle)$$

$$= \alpha\gamma\, \underline{|0\rangle \otimes |0\rangle} + \alpha\delta\, \underline{|0\rangle \otimes |1\rangle} + \beta\gamma\, \underline{|1\rangle \otimes |0\rangle}$$

$$+ \beta\delta\, \underline{|1\rangle \otimes |1\rangle}$$

0 0
0 1
1 0
1 1



$$= \underline{\alpha\gamma}\, |\underline{00}\rangle + \underline{\alpha\delta}\, |\underline{01}\rangle + \underline{\beta\gamma}\, |\underline{10}\rangle + \underline{\beta\delta}\, |\underline{11}\rangle$$

$$= \begin{bmatrix} \alpha\gamma \\ \alpha\delta \\ \beta\gamma \\ \beta\delta \end{bmatrix} = \alpha\gamma \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{|00\rangle} + \alpha\delta \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}}_{|01\rangle} + \cdots$$

1 qubit $\longrightarrow$ 2 estados

2 qubits $\longrightarrow$ 4 estados

3 qubits $\longrightarrow$ 8 estados

$\vdots$

n qubits $\longrightarrow$ 2^n estados

$|0\rangle \quad |1\rangle$

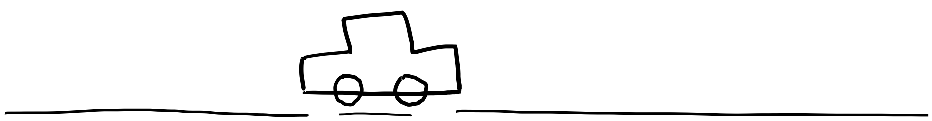
$|00\rangle \quad |01\rangle \quad |10\rangle \quad |11\rangle$

n qubits

$$\left.\begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix}\right\} 2^n$$

$n = 40 \qquad 2^{40} \sim 10^{12} \sim 1\,\text{Tb}$

Un vector de estado en un espacio de Hilbert de dimensión $\underline{2^n}$ representa un sistema de $\underline{n}$ qubits



0 0 0 1 0 0 0 0

$$|x\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow 2^3 \text{ componentes}$$

2^n

$\downarrow$

3 qubits

1.2 Producto de Kronecker $|\psi\rangle$ $|\phi\rangle$

$$|\psi\rangle \otimes |\phi\rangle = \begin{bmatrix}\alpha \\ \beta\end{bmatrix} \otimes \begin{bmatrix}\gamma \\ \delta\end{bmatrix}$$

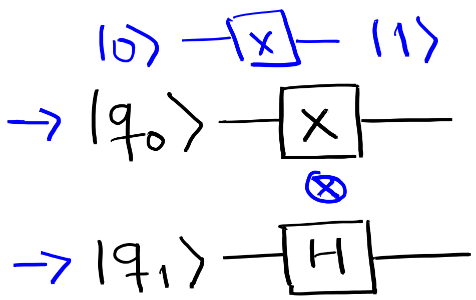
$$= \begin{bmatrix} \alpha \begin{bmatrix}\gamma \\ \delta\end{bmatrix} \\ \beta \begin{bmatrix}\gamma \\ \delta\end{bmatrix} \end{bmatrix} = \begin{bmatrix} \alpha \gamma \\ \alpha \delta \\ \beta \gamma \\ \beta \delta \end{bmatrix} = |\Phi\rangle$$

2 qubits

$$|00\rangle = |0\rangle \otimes |0\rangle \qquad |01\rangle = \begin{bmatrix}0\\1\\0\\0\end{bmatrix} \qquad |10\rangle = \begin{bmatrix}0\\0\\1\\0\end{bmatrix}$$

$$= \begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix} 1\begin{bmatrix}1\\0\end{bmatrix} \\ 0\begin{bmatrix}1\\0\end{bmatrix} \end{bmatrix} = \begin{bmatrix}1\\0\\0\\0\end{bmatrix} \qquad |11\rangle = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

1.3 Compuertas de un qubit en sistemas de varios qubits



Inicio

$$|q_0\rangle \otimes |q_1\rangle$$

Final

$$\underbrace{(X|q_0\rangle)}_{\rightarrow |q_{0f}\rangle} \otimes \underbrace{(H|q_1\rangle)}_{|q_{1f}\rangle}$$

$$\rightarrow C\,(|q_0\rangle \otimes |q_1\rangle)$$

$$C = X \otimes H$$

$$\underbrace{(X \otimes H)}\,(|q_0\rangle \otimes |q_1\rangle) = (X|q_0\rangle) \otimes (H|q_1\rangle)$$

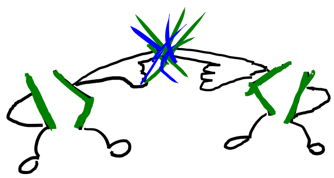
Matriz asociada

$$\overset{X}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}} \otimes \overset{H}{\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} & 1\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \\ 1\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} & 0\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \end{bmatrix}$$

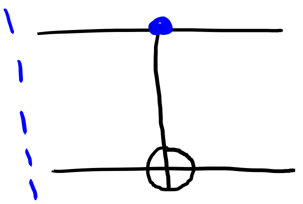
$$\rightarrow X \otimes H = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

1.4 Compuertas de varios qubits

Finalmente interacción!



CNOT Controlled NOT



$|q_0\rangle$ control

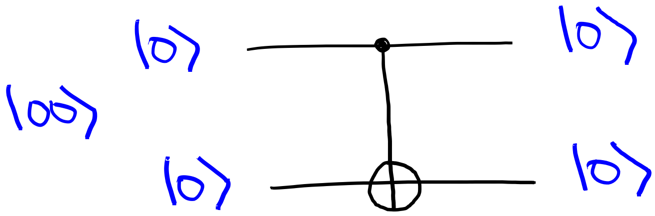
$|q_1\rangle$ Target

CNOT

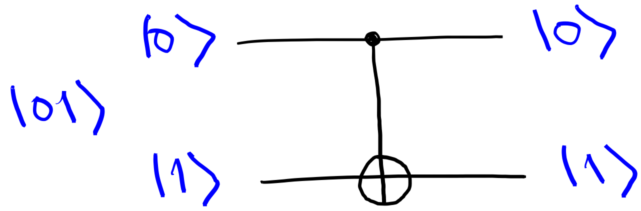
$|q_0\rangle \otimes |q_1\rangle$

$$\left.\begin{matrix} \text{control} \\ \text{Target} \end{matrix}\right\} CNOT(|q_0\rangle \otimes |q_1\rangle)$$

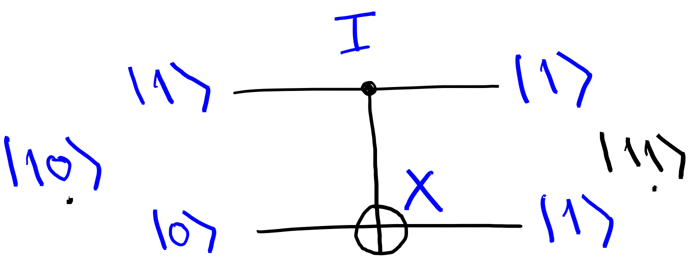
Cuando el control es $|1\rangle$, el target invierte su estado.



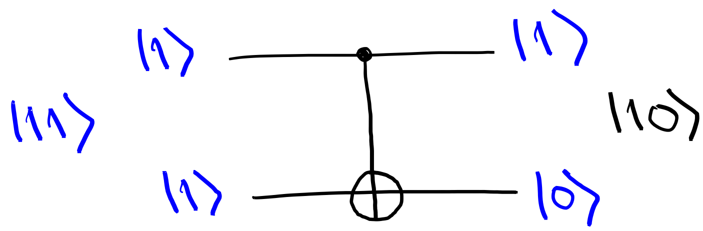
$\text{CNOT} = I \otimes I$



$I \otimes I$



$I \otimes X$



$I \otimes X$

Matricialmente,

$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

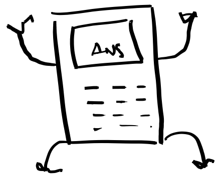
$$\text{CNOT} = \begin{bmatrix} I & 0 \\ 0 & X \end{bmatrix} \quad \underset{|00\rangle}{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}} \quad \underset{|01\rangle}{\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}} \quad \underset{|10\rangle}{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}} \quad \underset{|11\rangle}{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}$$

Tabla de verdad

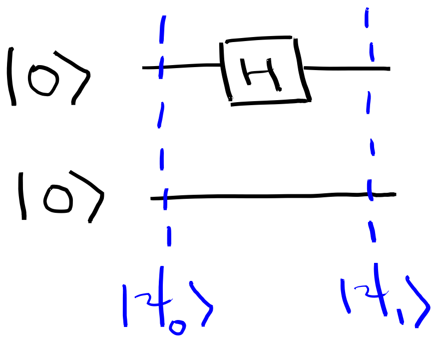
CNOT

$\|q_0\rangle$	$\|q_1\rangle$	$\underline{CX}\,\|q_0\rangle\|q_1\rangle$	
$\|0\rangle$	$\|0\rangle$	$\|00\rangle$	I
$\|0\rangle$	$\|1\rangle$	$\|01\rangle$	
$\|1\rangle$	$\|0\rangle$	$\|11\rangle$	X
$\|1\rangle$	$\|1\rangle$	$\|10\rangle$	

Ahora, calculemos!



$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

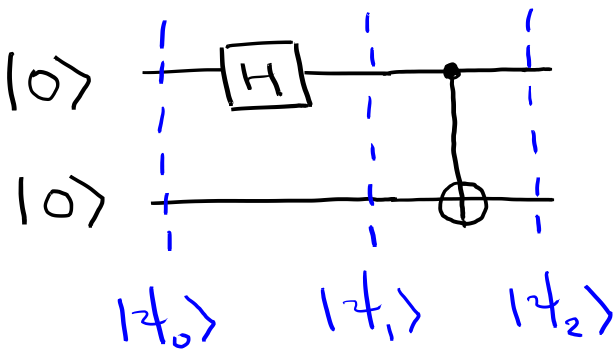


- $|\psi_0\rangle = |0\rangle \otimes |0\rangle = |00\rangle$
- $|\psi_1\rangle = H|0\rangle \otimes |0\rangle = (H \otimes I)|00\rangle$

$$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle$$

$$= \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |0\rangle)$$

$$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle$$



- $|\psi_2\rangle = \text{CNOT}\,\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$

$$= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

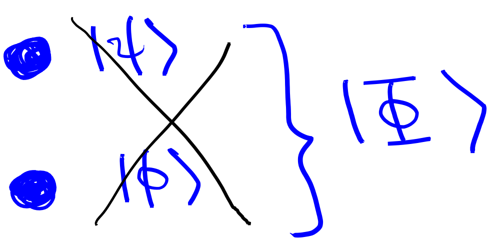
1.5 Estados entrelazados

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \neq \underbrace{|\psi\rangle}_{|q_0\rangle} \otimes \underbrace{|\phi\rangle}_{|q_1\rangle}$$

Interpretación

$$\alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle$$

$$\alpha\delta = 0 \qquad \beta\gamma = 0$$



EPR Can a quantum mechanical description of reality be considered complete?

Medición

$$\left.\begin{matrix}\bullet\\ \boxed{\bullet}\end{matrix}\right\} |\Phi\rangle = \frac{1}{\sqrt{2}}\boxed{|0}\boxed{0\rangle} + \frac{1}{\sqrt{2}}\boxed{|1}\boxed{1\rangle}$$

$$|0\rangle \longrightarrow |0\rangle$$

$$|1\rangle \longrightarrow |1\rangle$$

1.6 Estados de Bell

$$|\Phi^{00}\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$$

$$|\Phi^{01}\rangle = \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\0\end{bmatrix}$$

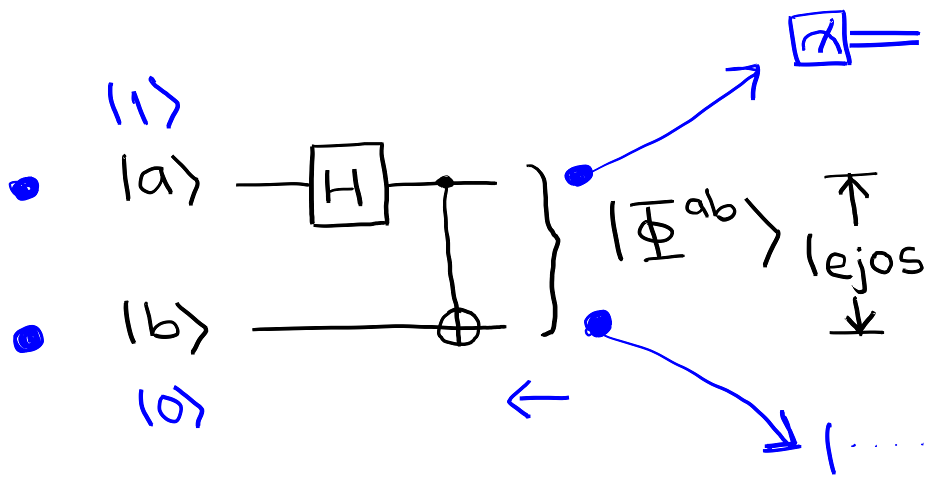
$$|\Phi^{10}\rangle = \frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}$$

$$|\Phi^{11}\rangle = \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\-1\\0\end{bmatrix}$$

$|\Phi^{ab}\rangle$

$a = 0, 1$

$b = 0, 1$



1.7 Medición de Bell

Invertimos el circuito anterior

$|\Phi^{00}\rangle \quad |\Phi^{01}\rangle \quad |\Phi^{10}\rangle \quad |\Phi^{11}\rangle$