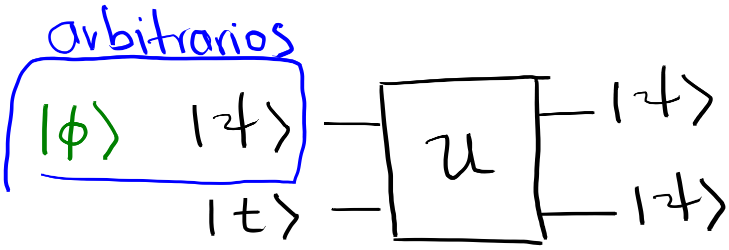
Día 4

1. Teorema de no-clonación

Objetivo: Entender por qué no es posible copiar estados arbitrarios

Afirmación: No existe $\mathcal{U}$ tal que



$$\underline{\mathcal{U}}\left(|\psi\rangle \otimes |t\rangle\right) = |\psi\rangle \otimes |\psi\rangle$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\phi\rangle = \gamma|0\rangle + \delta|1\rangle$$

Supongamos que podemos,

$$\mathcal{U}\underline{\left(|\psi\rangle \otimes |t\rangle\right)} = |\psi\rangle \otimes |\psi\rangle \qquad \mathcal{U}\left(|\phi\rangle \otimes |t\rangle\right) = |\phi\rangle \otimes |\phi\rangle$$

$$\underline{|\psi\rangle} = \underset{(2\times2)(2\times1)}{\mathcal{U}\,|\psi_0\rangle} \qquad \langle\underline{\psi}| = \left(\mathcal{U}|\psi_0\rangle\right)^\dagger = \underset{(1\times2)(2\times2)}{\langle\psi_0|\,\mathcal{U}^\dagger}$$

Tomando el producto interno entre ellos,

$$\left(\langle\underline{\psi}| \otimes \langle\bar{t}|\right)\underbrace{\mathcal{U}^\dagger\mathcal{U}}_{I}\left(|\underline{\phi}\rangle \otimes |\bar{t}\rangle\right) = \left(\langle\underline{\psi}| \otimes \langle\bar{\psi}|\right)\left(|\underline{\phi}\rangle \otimes |\bar{\phi}\rangle\right)$$

$$\langle t|t\rangle = \||t\rangle\|^2$$

$$\||t\rangle\| = 1$$

$$\langle\psi|\phi\rangle\cancel{\langle t|t\rangle}^{1} = \underline{\langle\psi|\phi\rangle}\,\underline{\langle\psi|\phi\rangle}$$

$$1 = 1^2$$

$$\langle\psi|\phi\rangle = \left(\langle\psi|\phi\rangle\right)^2$$

$$0 = 0^2$$

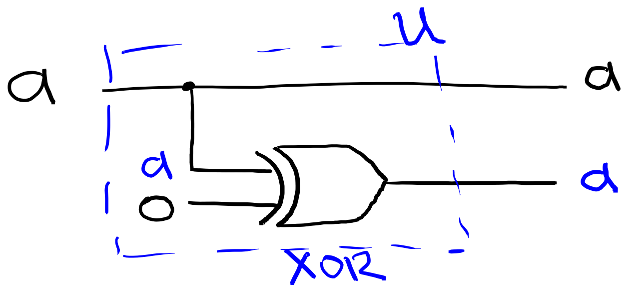
Sólo hay dos opciones $\rightarrow$ $\underline{\langle\psi|\phi\rangle = 0}$ Ortogonales

$\langle\psi|\phi\rangle = 1$ Mismo

$$\langle\psi|\psi\rangle = 1$$

$$\langle\phi|\phi\rangle = 1$$

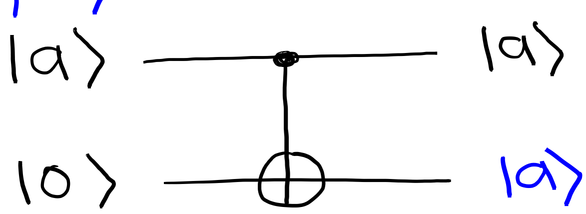
Clasicamente,



A	B	C
0	0	0
0	1	1
1	0	1
1	1	0

Cuánticamente,

$|0\rangle, |1\rangle$ CNOT

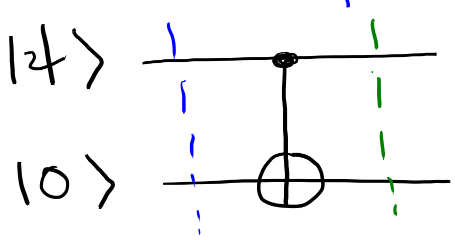


A	B	A	C
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

Ahora,

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



$$\left.\begin{array}{l} |\psi\rangle = \alpha|0\rangle + \beta|1\rangle \\ \\ \end{array}\right\} |\psi\rangle \otimes |\psi\rangle$$

$$|\psi\rangle \otimes |0\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes |0\rangle$$

$$= \alpha |0\rangle \otimes |0\rangle + \beta |1\rangle \otimes |0\rangle$$

$$= \alpha |00\rangle + \beta |10\rangle$$

$$\text{CNOT } |\psi\rangle \otimes |0\rangle = \alpha \text{ CNOT } |0\boxed{0}\rangle + \beta \text{ CNOT } |10\rangle$$

$$= \alpha|00\rangle + \beta|11\rangle$$

Z

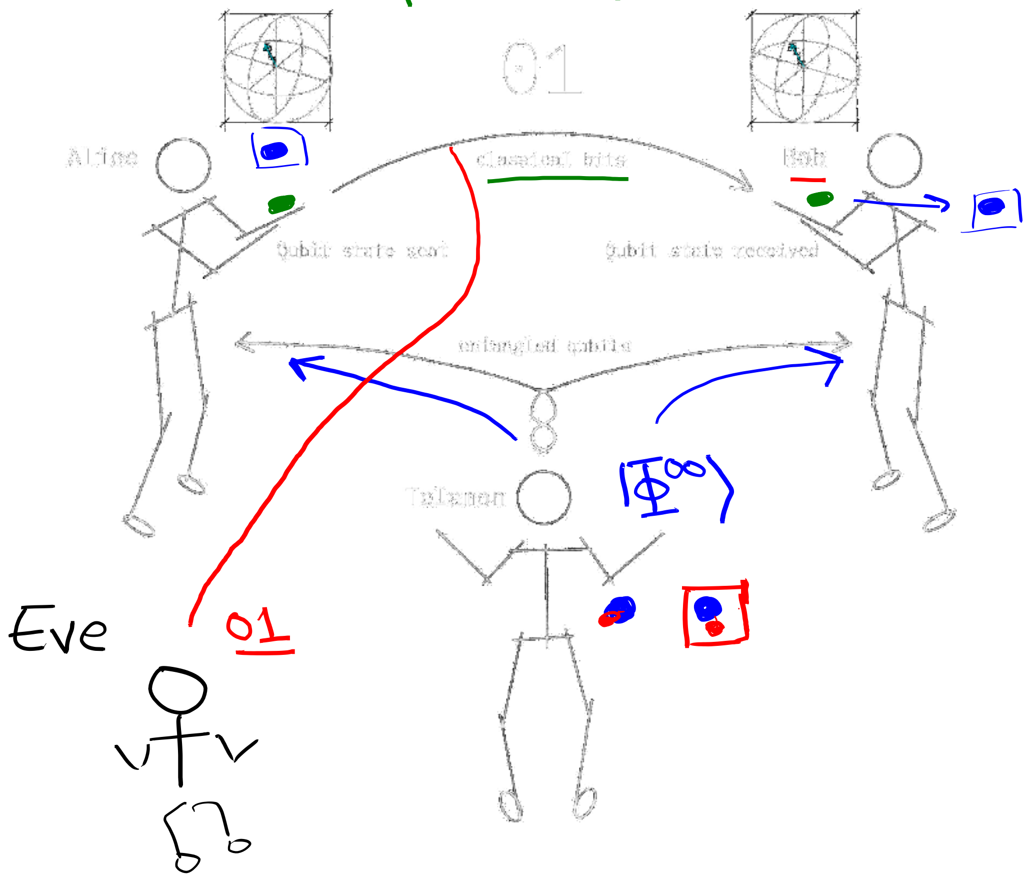
$$\underline{\alpha|00\rangle + \beta|11\rangle} \stackrel{?}{=} \underline{|\psi\rangle \otimes |\psi\rangle}$$

$$= (\alpha|0\rangle + \beta|1\rangle) \otimes (\alpha|0\rangle + \beta|1\rangle)$$

$$= \alpha^2 |00\rangle + \underline{\alpha\beta|01\rangle} + \alpha\beta|10\rangle + \beta^2|11\rangle$$

2. Protocolo de teletransportación cuántica

Objetivo: Teletransportar!



Recordemos los estados de Bell

$$\rightarrow |\Phi^{00}\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \qquad |\Phi^{01}\rangle = \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$$

$+$

$$\rightarrow |\Phi^{10}\rangle = \frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle \qquad |\Phi^{11}\rangle = \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle$$

Invirtiendo estas relaciones, $\{|00\rangle, \boxed{|01\rangle}, \boxed{|10\rangle}, |11\rangle\}$

$$|\Phi^{00}\rangle + |\Phi^{10}\rangle = \frac{2}{\sqrt{2}}|00\rangle = \sqrt{2}\,|00\rangle$$

$$\frac{2}{\sqrt{2}} = \sqrt{2}$$

$$\boxed{|00\rangle = \frac{1}{\sqrt{2}}|\Phi^{00}\rangle + \frac{1}{\sqrt{2}}|\Phi^{10}\rangle}$$

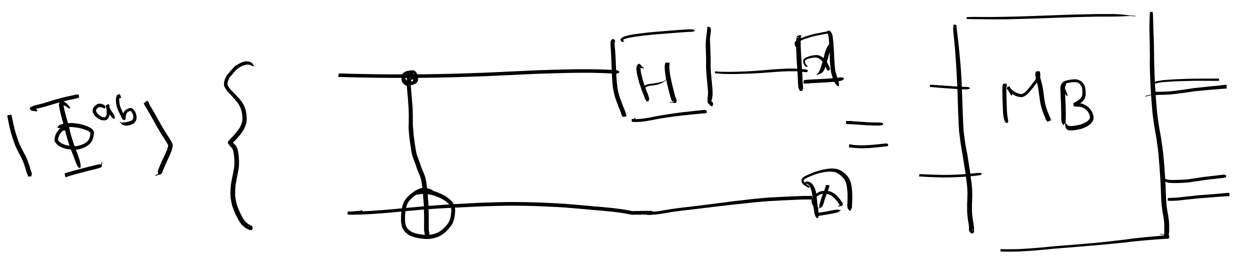
$$|\Phi^{01}\rangle + |\Phi^{11}\rangle = \frac{2}{\sqrt{2}}|01\rangle = \sqrt{2}\,|01\rangle$$

$$|00\rangle = \frac{1}{\sqrt{2}}|\Phi^{00}\rangle + \frac{1}{\sqrt{2}}|\Phi^{10}\rangle$$

$$|01\rangle = \frac{1}{\sqrt{2}}|\Phi^{01}\rangle + \frac{1}{\sqrt{2}}|\Phi^{11}\rangle$$

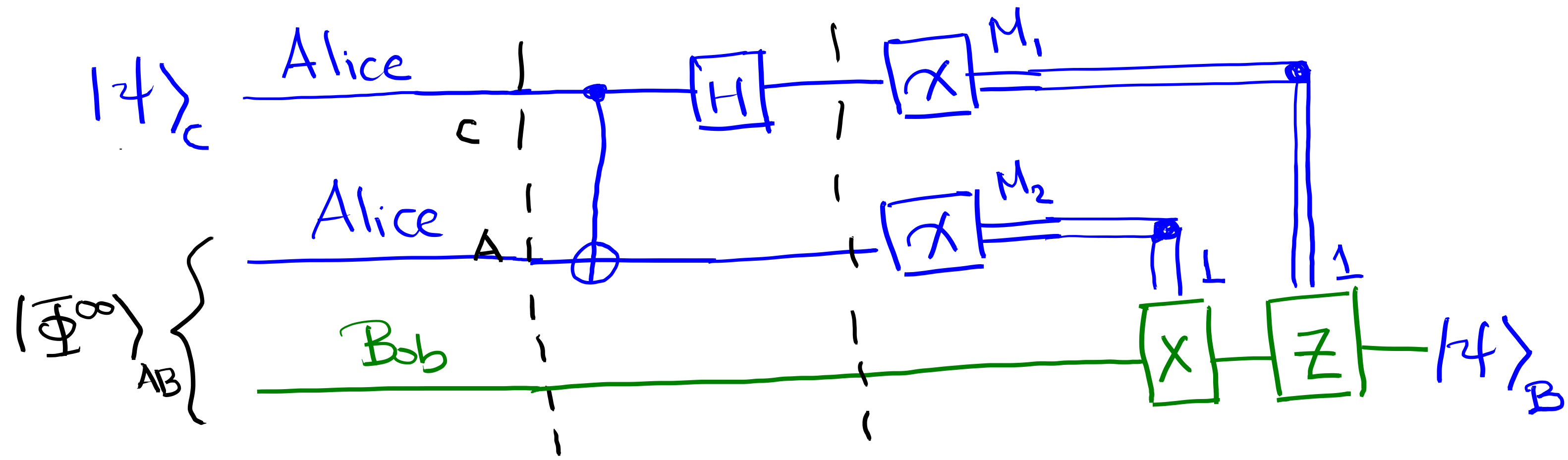
$$|10\rangle = \frac{1}{\sqrt{2}}|\Phi^{01}\rangle - \frac{1}{\sqrt{2}}|\Phi^{11}\rangle$$

$$|11\rangle = \frac{1}{\sqrt{2}}|\Phi^{00}\rangle - \frac{1}{\sqrt{2}}|\Phi^{10}\rangle$$



El circuito

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$



$|\Psi_0\rangle$ $|\Psi_1\rangle$

$$|\Phi^{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$|\Psi_0\rangle = |\psi\rangle_C \otimes |\Phi^{00}\rangle_{AB}$$

$$= \left(\alpha|0\rangle_C + \beta|1\rangle_C\right) \otimes \frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |11\rangle_{AB}\right)$$

$$= \frac{\alpha}{\sqrt{2}}|0\rangle_C|00\rangle_{AB} + \frac{\alpha}{\sqrt{2}}|0\rangle_C|11\rangle_{AB}$$

$$+ \frac{\beta}{\sqrt{2}}|1\rangle_C|00\rangle_{AB} + \frac{\beta}{\sqrt{2}}|1\rangle_C|11\rangle_{AB}$$

$$= \frac{\alpha}{\sqrt{2}}\boxed{|00\rangle_{CA}}|0\rangle_B + \frac{\alpha}{\sqrt{2}}\boxed{|01\rangle_{CA}}|1\rangle_B$$

$$+ \frac{\beta}{\sqrt{2}}\boxed{|10\rangle_{CA}}|0\rangle_B + \frac{\beta}{\sqrt{2}}\boxed{|11\rangle_{CA}}|1\rangle_B$$

$$|\Psi_0\rangle = \frac{\alpha}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(|\Phi^{00}\rangle_{CA} + |\Phi^{10}\rangle_{CA}\right)\right]|0\rangle_B$$

$$+ \frac{\alpha}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(|\Phi^{01}\rangle_{CA} + |\Phi^{11}\rangle_{CA}\right)\right]|1\rangle_B$$

$$+ \frac{\beta}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(|\Phi^{01}\rangle_{CA} - |\Phi^{11}\rangle_{CA}\right)\right]|0\rangle_B$$

$$+ \frac{\beta}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(|\Phi^{00}\rangle_{CA} - |\Phi^{10}\rangle_{CA}\right)\right]|1\rangle_B$$

$\frac{1}{2}$

$$= \frac{1}{2}|\Phi^{00}\rangle_{CA}\left(\underline{\alpha|0\rangle_B + \beta|1\rangle_B}\right) \leftarrow |\psi\rangle$$

$P = \frac{1}{4}$ $\boxed{00}$

$$+ \frac{1}{2}|\Phi^{01}\rangle_{CA}\left(\alpha|1\rangle_B + \beta|0\rangle_B\right) \quad X|\psi\rangle$$

$P = \frac{1}{4}$ $\boxed{01}$ X

$$+ \frac{1}{2}|\Phi^{10}\rangle_{CA}\left(\alpha|0\rangle_B - \beta|1\rangle_B\right) \quad Z|\psi\rangle$$

$P = \frac{1}{4}$ $\boxed{10}$ Z

$$+ \frac{1}{2}|\Phi^{11}\rangle_{CA}\left(\alpha|1\rangle_B - \beta|0\rangle_B\right) \quad XZ|\psi\rangle$$

$P = \frac{1}{4}$ $\boxed{11}$ ZX

$P_T = 1$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

M_1	M_2	Bob	Comp.	Estado final
0	0	$\alpha\|0\rangle + \beta\|1\rangle$	I	$\|\psi\rangle$
0	1	$\alpha\|1\rangle + \beta\|0\rangle$	X	$\|\psi\rangle$
1	0	$\alpha\|0\rangle - \beta\|1\rangle$	Z	$\|\psi\rangle$
1	1	$\alpha\|1\rangle - \beta\|0\rangle$	ZX	$\|\psi\rangle$

¿Se puede transmitir información más rápido que la velocidad de la luz usando el protocolo de teletransportación?