

# CSSS 554: Assignment 3

Department of Biostatistics @ University of Washington

Alejandro Hernandez

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Let  $Y_i$  and  $E_i$ ,  $i = 1, \dots, n$ , denote the observed and expected counts in region  $i$ ,  $i = 1, \dots, n$ . Then consider the model

$$Y_i | \theta_i \sim \text{Poisson}(E_i \theta_i)$$

1.

- (a) Provide a map of the observed counts  $Y_i$ .
- (b) Provide a map of the expected counts  $E_i$ .
- (c) Provide a map of the SMRs, defined as

$$\text{SMR}_i = \hat{\theta}_i = \frac{Y_i}{E_i}$$

for  $i = 1, \dots, n$ . Comment on the variability of the SMRs.

Plot the SMRs versus the estimated standard errors, which are given by  $\sqrt{\hat{\theta}_i / E_i}$ .

2. In this question we will smooth the SMRs using the disease mapping Poisson-Lognormal model:

$$Y_i | \beta_i, \epsilon_i \sim_{ind} \text{Poisson}(E_i e^{\beta_0} e^{\epsilon_i}) \quad \epsilon_i | \sigma_i \sim N(0, \sigma_i^2)$$

for  $i = 1, \dots, n$ .

- (a) Using the `inla` function in **R** fit this model using the default priors for  $\beta_0$  and  $\sigma_e$ . Report the posterior medians and 95% intervals for  $\beta_0$  and  $\sigma_e$ .
- (b) Extract the posterior medians of the relative risk (RR) estimates and provide a map of these.
- (c) Plot these posterior RR estimates against the SMRs, and comment.
- (d) Plot the posterior standard deviations of the RRs against the standard errors of the SMRs and comment.

3. In this question we will smooth the SMRs using the disease mapping Poisson-LognormalSpatial model:

...

- (a) Using the `inla` function in R fit this model using the `bym2` model, with the default prior for  $\beta_0$  and the following prior specification for the spatial and non-spatial random effects (note that you must be in the directory that contains the `VR.graph` file):

```
f(Region, model="bym2", graph="VR.graph", scale.model=T, constr=T, hyper=list(phi=list(prior="pc",
param=c(0.5, 0.5), initial=1), prec=list(prior="pc.prec", param=c(0.3,0.01), initial=5)))
```

These choices correspond to the prior belief that there is a 1% chance that the total residual standard deviation is greater than 0.3, and a 50% chance that the proportion of the variance that is spatial is bigger than 0.5.

Report both the posterior medians and 95% intervals for  $\beta_0$ , the total variance of the random effects, and the proportion of the total variance attributed to the spatial random effect.

- (b) Extract the relative risk estimates and provide a map of these. Compare these estimates with the SMRs and with those obtained from the Poisson-Lognormal model (i.e., the model with IID random effects only) that you fit in Question 2.

4. Bonus Question: Suppose that instead of having available the counts and expected numbers we have access to the relative risks and their standard errors. Take the data as

and fit the model

Fit this model using `inla` and plot the estimated relative risks from this model against the estimates from the Poisson-Lognormal model, and comment.

**End of report. Code appendix begins on the next page.**

## Code Appendix

```
# Clear environment
rm(list=ls())

# Setup options
knitr::opts_chunk$set(echo=FALSE, warning=FALSE, message=FALSE, results='hide')
options(knitr.kable.NA = '-', digits = 2)
labs = knitr::all_labels()
labs = labs[!labs %in% c("setup", "allcode")]
# Load relevant packages

# Load data
load("../data/HW3data.Rdata")
exp_counts <- Exp.mv3
obs_counts <- Obs.mv3

# Handle missing data
```

End of document.