Small area estimation

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In this vignette, we illustrate the use of the `smoothArea` and `smoothUnit` functions for generic small area estimation of outcomes other than under-five mortality rates.

Installation - Development Version

To download this version of the SUMMER package with `smoothArea` and `smoothUnit`, use the code below.

```
# run the below line if devtools is not already installed
install.packages("devtools")
devtools::install_github("richardli/SUMMER")
```

Load packages

First, we load the necessary packages and the necessary data. The required package `INLA` is not in a standard repository, so we install if it is not available. The `survey` package will be used to generate direct estimates, while `dplyr` and `tidyr` will be used to facilitate data manipulation.

```
library(sae)
if (!isTRUE(requireNamespace("INLA", quietly = TRUE))) {
  install.packages("INLA", repos=c(getOption("repos"),
                  INLA="https://inla.r-inla-download.org/R/stable"), dep=TRUE)
}
library(SUMMER)
library(survey)
library(dplyr)
library(tidyr)
```

Area level models

Income data

First, we compare the results from a Bayesian analysis using SUMMER with those obtained using the `sae` package using examples presented in the `sae` vignette. Molina and Marhuenda (2015) generate an artificial dataset on income and other related variables to illustrate the use of area level models in the `sae` package. In their simulated example, the objective is to estimate prevalence of poverty in Spanish counties.

```
data("incomedata")
data("sizeprov")
data("sizeprovedu")
povertyline <- 0.6*median(incomedata$income) # 6557.143
incomedata$poor <- as.integer(incomedata$income < povertyline)
```

The `incomedata` data frame contains information on `17199` observations of individuals in `52` Spanish provinces. Income values and sampling weights are provided for each individual along with covariate information including age group and education level. Molina and Marhuenda (2015) define the poverty line `z`, and calculate an indicator variable with value 1 if the corresponding income value is below the poverty line and 0 otherwise.

Direct estimation with `sae`: The `sae::direct` function computes the Horvitz-Thompson (direct) estimator of domain means, given by,

$$\widehat{\bar{Y}}_i^{ht} = \frac{1}{N_i} \sum_{k \in S_i} w_k y_k$$

where N_i is the population size of domain i, S_i is the set of sampled observations in domain i, w_k is the sampling weight for unit k, and y_k is the observation for unit k, for all $k \in S_i$. The `sae::direct` function also estimates standard deviation and coefficient of variation for each domain.

```r
Popn <- sizeprov[, c("provlab", "Nd")]
sae.DIR <- sae::direct(y = incomedata$poor, dom = incomedata$provlab,
    sweight = incomedata$weight, domsize = Popn)
knitr::kable(head(sae.DIR), digits = 3)
```

Domain	SampSize	Direct	SD	CV
Alava	96	0.255	0.048	19
Albacete	173	0.141	0.030	22
Alicante	539	0.205	0.022	11
Almeria	198	0.265	0.041	15
Avila	58	0.055	0.026	46
Badajoz	494	0.210	0.023	11

Direct estimation with `survey`: We can similarly use the `survey::svyby` function to compute the Horvitz-Thompson estimates:

```r
incomedata$pop <- sum(sizeprov$Nd[match(incomedata$provlab, sizeprov$provlab)])
design <- survey::svydesign(ids = ~1, weights = ~weight, data = incomedata,
    fpc = ~pop)

# estimate area totals
svy.DIR <- survey::svyby(~poor, ~provlab, design, svytotal)

# calculate corresponding area mean estimates
svy.DIR$prov_pop <- sizeprov$Nd[match(svy.DIR$provlab, sizeprov$provlab)]
svy.DIR$poor_mean <- svy.DIR$poor/svy.DIR$prov_pop
svy.DIR$poor_mean_se <- svy.DIR$se/svy.DIR$prov_pop
knitr::kable(head(svy.DIR) %>%
    select(provlab, poor_mean, poor_mean_se), digits = 3)
```

	provlab	poor_mean	poor_mean_se
Alava	Alava	0.255	0.048
Albacete	Albacete	0.141	0.030
Alicante	Alicante	0.205	0.022
Almeria	Almeria	0.265	0.041
Avila	Avila	0.055	0.026

	provlab	poor_mean	poor_mean_se
Badajoz	Badajoz	0.210	0.023

Fay-Herriot model: The Fay-Herriot model is a basic area level model used to obtain small area estimators Fay and Herriot (1979). The Fay-Herriot model is often specified as a two-stage model; the first stage is the sampling model and the second stage is the linking model.

First stage:

Let $\widehat{\theta}_i^{ht}$ be a direct estimator of θ_i:

$$\widehat{\theta}_i^{ht} = \theta_i + \epsilon_i; \qquad \epsilon_i \sim_{ind} N(0, V_i), \qquad i = 1, \dots, m,$$

where V_i is the **known sampling variance** of the direct estimator $\widehat{\theta}_i^{ht}$.

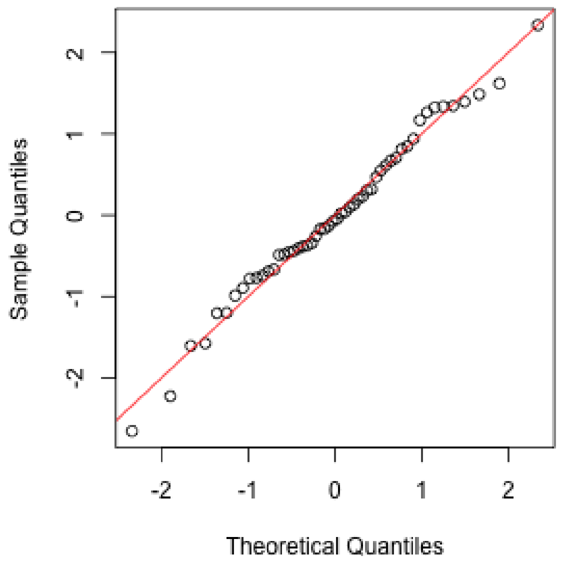
Second stage:

$$\theta_i = \mathbf{x}_i^T \boldsymbol{\beta} + \delta_i, \qquad \delta_i \sim_{ind} N(0, \sigma_\delta^2) \qquad i = 1, \dots, m,$$

where σ_δ^2 (between-area residual variance) is estimated. In this basic Fay-Herriot model, the area-specific random effects δ_i are assumed to be independent between areas.

Implementing Fay-Herriot model with sae: Below, we provide a quantile-quantile plot comparing the direct estimates to a Gaussian distribution. Here the observed quantiles align well with those from a Gaussian distribution, which lends some support to the basic IID model.

```
par(pty="s")
muD <- mean(sae.DIR$Direct)
sdD <- sd(sae.DIR$Direct)
qqnorm((sae.DIR$Direct-muD)/sdD,main="")
abline(0,1,col="red")
```



The `sae::mseFH` function calculates the empirical best linear unbiased predictors (EBLUP) for all domain means as well as their estimated MSEs.

```r
sae.FH <- sae::mseFH(sae.DIR$Direct ~ 1, sae.DIR$SD^2)
```

Implementing Fay-Herriot model in `SUMMER`: The `smoothArea` function compute direct estimates and then produces smoothed estimates using a Bayesian Fay-Herriot model. The main arguments of interest are:

- `formula`: describing the response variable and any area-level covariates
- `domain` a one-sided formula with the variable containing domain labels on the right. The domain labels variable should be contained in the dataset used to generate the design.
- `design`: a `survey.design` object containing survey data and specifying the survey design.

In addition, other commonly used optional arguments include:

- `Amat`: Optional adjacency matrix if a spatial smoothing model is desired.
- `responseType`: either `"gaussian"` or `"binary"` depending on the type of response variable. If `"binary"` is specified, a logit transform will be applied to the direct estimates before smoothing.
- `direct.est`: If desired, the direct estimates may be specified and smoothing will be applied directly to the direct estimates.
- `X.area`: Areal covariates data frame.
- `domain.size`: Domain size data frame used for computing direct estimates if domain sizes are known.

Other optional arguments can be specified to change the priors and are described further in the documentation.

We obtain direct estimates and smoothed estimates below, fitting the Fay-Herriot model to the untransformed direct estimates.

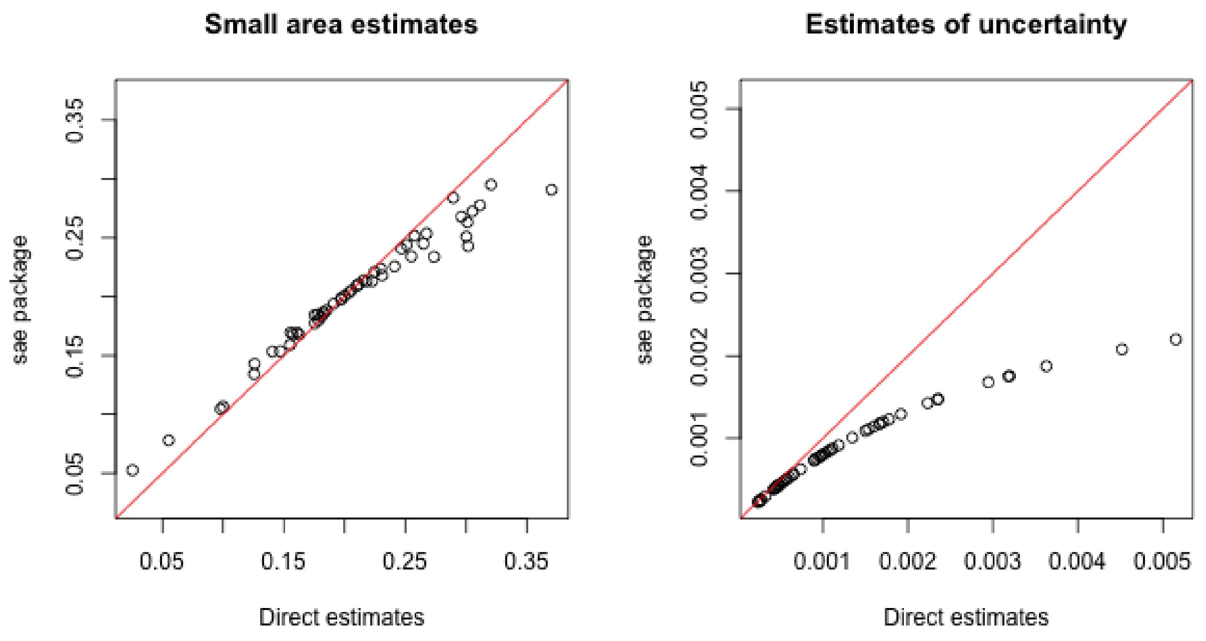
```r
# fit the model
domain.size <- sizeprov[, c("provlab", "Nd")]
colnames(domain.size)[2] <- "size"

summer.FH <- smoothArea(formula = poor ~ 1, domain = ~provlab,
    design = design, domain.size = domain.size)
```

The fitted parameters from `sae` (obtained via likelihood-based methods) and estimated parameter posterior distribution from `SUMMER` (obtained from Bayesian methods, implemented via `INLA`) are in reasonable agreement. The estimated intercept β_0 from `sae` is `0.202` ; the posterior median of β_0 from `SUMMER` is `0.2`. In the absence of strong priors, fixed effects are usually in close agreement, with the posterior being symmetric. The estimated precision $1/\sigma_\delta^2$ from `sae` is `281.35` , while the posterior median of $1/\sigma_\delta^2$ from `SUMMER` is `273.25`. The differences are larger here, but the posterior for the variance is skewed, and we would expect the posterior median to be smaller than the REML estimate. The area estimates and measures of uncertainty are in close agreement, however.

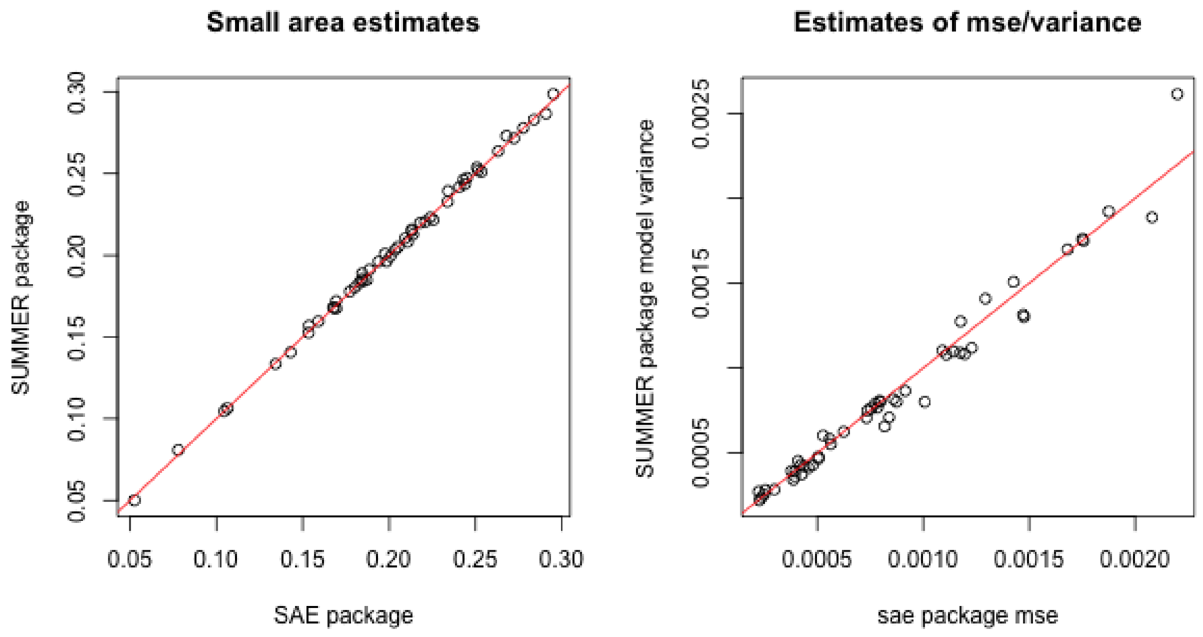
We first illustrate the shrinkage of the EBLUP estimates, and the reduced uncertainty:

```r
par(mfrow = c(1, 2))
plot(sae.DIR$Direct, sae.FH$est$eblup,
     xlab = "Direct estimates",ylab = "sae package",
     xlim=c(min(sae.DIR$Direct, sae.FH$est$eblup),max(sae.DIR$Direct, sae.FH$est$eblup)),
     ylim=c(min(sae.DIR$Direct, sae.FH$est$eblup),max(sae.DIR$Direct, sae.FH$est$eblup)),
     main = "Small area estimates")
abline(0,1,col="red")
plot(sae.DIR$SD^2, sae.FH$mse,
     xlab = "Direct estimates",ylab = "sae package",
     xlim=c(min(sae.DIR$SD^2, sae.FH$mse),max(sae.DIR$SD^2, sae.FH$mse)),
     ylim=c(min(sae.DIR$SD^2, sae.FH$mse),max(sae.DIR$SD^2, sae.FH$mse)),
     main = "Estimates of uncertainty")
abline(0,1,col="red")
```



Now compare EBLUP and HB, using posterior variance from `SUMMER` and estimated MSE from `sae` to measure uncertainty:

```
par(mfrow = c(1, 2))
plot(sae.FH$est$eblup, summer.FH$s.dir.iid.est$median,
     xlab = "SAE package",ylab = "SUMMER package",
     main = "Small area estimates")
abline(0,1,col="red")
plot(sae.FH$mse,
     summer.FH$s.dir.iid.est$var,
     xlab = "sae package mse",
     ylab = "SUMMER package model variance",
     main = "Estimates of mse/variance")
abline(0,1,col="red")
```



Milk expenditures:

Molina and Marhuenda (2015) also use a real dataset on milk expenditures Arora et al. (1997) to illustrate the implementation of area level models in the `sae` package. The `milk` dataset contains information on direct estimators for area level expenditures on milk based on data from the dairy survey component of the Consumer Expenditure Survey conducted by the U.S. Census Bureau. Weights are based on inverse sampling probabilities and adjusted for non-response/post-stratification.

```
data(milk)
milk$Var <- milk$SD^2
knitr::kable(head(milk))
```

SmallArea	ni	yi	SD	CV	MajorArea	Var
1	191	1.10	0.16	0.15	1	0.03
2	633	1.07	0.08	0.07	1	0.01
3	597	1.10	0.08	0.07	1	0.01
4	221	0.63	0.11	0.17	1	0.01
5	195	0.75	0.12	0.16	1	0.01
6	191	0.98	0.14	0.14	1	0.02

Here, `SmallArea` denotes areas of inferential interest, `yi` gives the direct estimates, `ni` the sample sizes, `SD` and `CV` give the estimated standard deviation and coefficient of variation, and `MajorArea` corresponds to major areas (US regions defined by You et al 2006).

Results using sae: Fit F-H model with area-level covariates

```
attach(milk)
sae.FH.milk <- mseFH(yi ~ as.factor(MajorArea), SD^2)
names(sae.FH.milk)
```

```
## [1] "est" "mse"
```

```
names(sae.FH.milk$est)
```

```
## [1] "eblup" "fit"
```

```
names(sae.FH.milk$est$fit)
```

```
## [1] "method"      "convergence" "iterations"  "estcoef"
## [5] "refvar"      "goodness"
```

Results are compared to `SUMMER` below:

Results using `SUMMER`: Since the `milk` dataset only provides area level direct estimates and not unit level data, we use the `direct.est` argument as input.

```
# Major Area fixed effects
Xmat <- milk[, c("SmallArea", "MajorArea")]
Xmat$MajorArea <- as.factor(Xmat$MajorArea)

# format direct estimates for SUMMER
milk.dir <- milk[, c("SmallArea", "yi", "Var")]
# Fit the model with Major Area intercepts
summer.FH.milk<- smoothArea(formula = yi~MajorArea,
                            direct.est = milk.dir,
                            X.area = Xmat,
                            domain = ~SmallArea)
```

Again note that the posterior median for σ_u^2 is larger than the REML estimate.

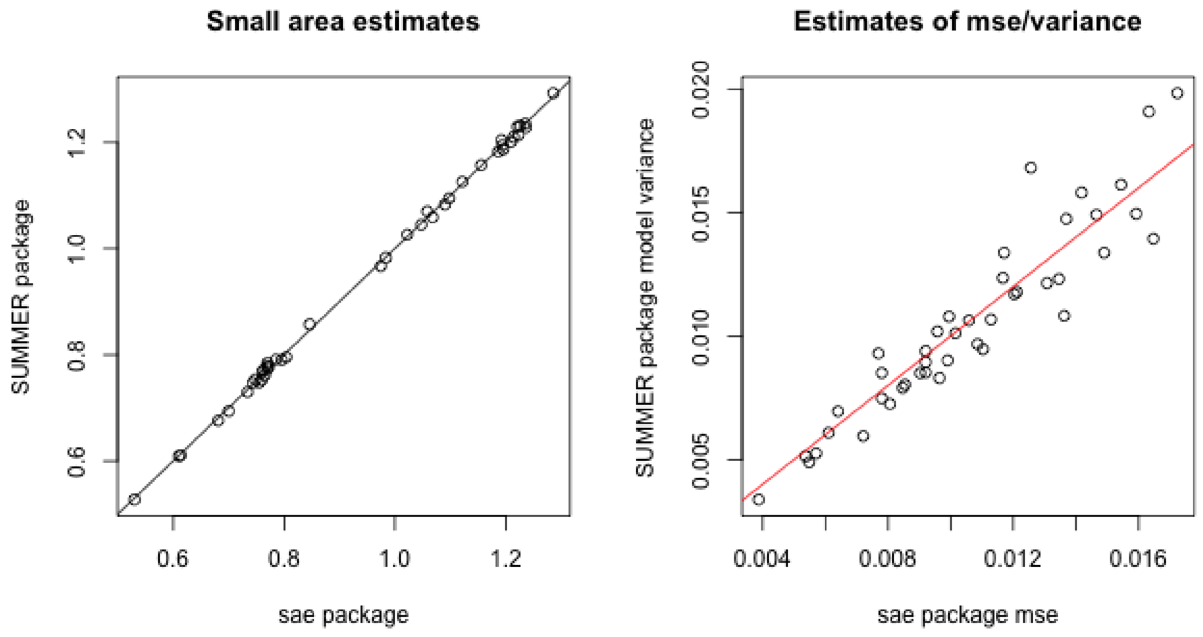
```
# random effects precision
1 / sae.FH.milk$est$fit$refvar
## [1] 54
summer.FH.milk$s.dir.iid.fit$summary.hyperpar[,c(4)]
## [1] 54

# estimated fixed effects
sae.FH.milk$est$fit$estcoef
##                          beta std.error tvalue  pvalue
## X(Intercept)             0.97     0.069   14.0 2.8e-44
## Xas.factor(MajorArea)2  0.13     0.103    1.3 2.0e-01
## Xas.factor(MajorArea)3  0.23     0.092    2.5 1.4e-02
## Xas.factor(MajorArea)4 -0.24     0.082   -3.0 3.1e-03
summer.FH.milk$s.dir.iid.fit$summary.fixed[,c(1:5)]
##              mean    sd 0.025quant 0.5quant 0.97quant
## (Intercept)  0.97 0.071      0.829     0.97     1.103
## MajorArea2   0.13 0.106     -0.073     0.13     0.335
## MajorArea3   0.23 0.094      0.041     0.23     0.405
## MajorArea4  -0.24 0.084     -0.406    -0.24    -0.083
```

Again, we observe good agreement.

```
par(mfrow = c(1, 2))
plot(sae.FH.milk$est$eblup[as.numeric(summer.FH.milk$s.dir.iid.est$domain)],
     summer.FH.milk$s.dir.iid.est$median,
     xlab = "sae package",
     ylab = "SUMMER package",
     main = "Small area estimates")
abline(0,1,color="red")
```

```
plot(sae.FH.milk$mse[as.numeric(summer.FH.milk$s.dir.iid.est$domain)],
     summer.FH.milk$s.dir.iid.est$var,
     xlab = "sae package mse",
     ylab = "SUMMER package model variance",
     main = "Estimates of mse/variance")
abline(0,1,col="red")
```



Spatial Fay-Herriot

The `sae` package also provides tools for implementing a spatial version of the Fay-Herriot model which assumes that the vector of area specific effects follows a first order simultaneous autoregressive, or SAR(1), process:

$$\mathbf{u} = \rho_1 \mathbf{W}\mathbf{u} + \boldsymbol{\epsilon}, \quad \boldsymbol{\epsilon} \sim N(\mathbf{0}_m, \sigma_I^2 \mathbf{I}_m),$$

where $\mathbf{I}_m$ is the identity matrix for the m areas and $\mathbf{0}_m$ is a vector of zeroes of size m. Additionally, $\rho_1 \in (-1, 1)$ is an autoregression parameter and $\mathbf{W}$ is an adjacency matrix (with rows standardized to sum to 1).

The `sae::mseSFH` function estimates the unknown variance parameters, the resulting EBLUP small area estimators, and then uses bootstrap methods to estimate the MSE of the estimators.

To illustrate the use of this function, Molina and Marhuenda (2015) consider a simulated dataset concerning grape production surface area for 274 Italian municipalities. Below we load the relevant objects from the `sae` package. The `grapes` dataset containes direct estimators of the mean surface area in hectares for grape production in each municipality (`grapehect`), the sampling variance of these direct estimators (`var`), and relevant covariates including number of working dats and overall agrarian surface area. The `grapesprox` object contains the relevant adjacency matrix representing the municipalities' neighborhood structure.

```
data("grapes")
data("grapesprox")
```

```r
sae.FH.grapes <- sae::mseSFH(grapehect ~ area + workdays - 1, var, grapesprox, data = grapes)

results <- data.frame(DIR = grapes$grapehect,
                      eblup.SFH = sae.FH.grapes$est$eblup,
                      mse = sae.FH.grapes$mse)
# reorder results for comparison later
results$area_name <- paste0('area_', rownames(results))
```

Results using sae

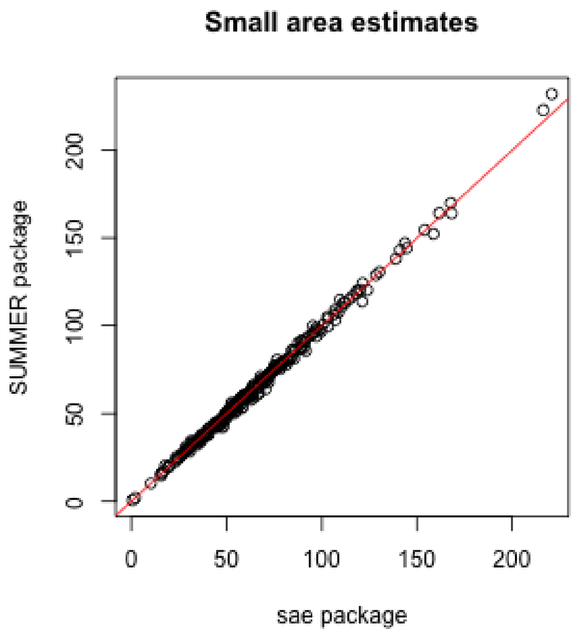
Results using `SUMMER` The `smoothSurvey` function also allows the use of a model with spatially correlated area effects, but the default implementation assumes a BYM2 model for $\mathbf{u}$ rather than a simultaneous autoregressive model as in the SFH model implemented in `sae`.

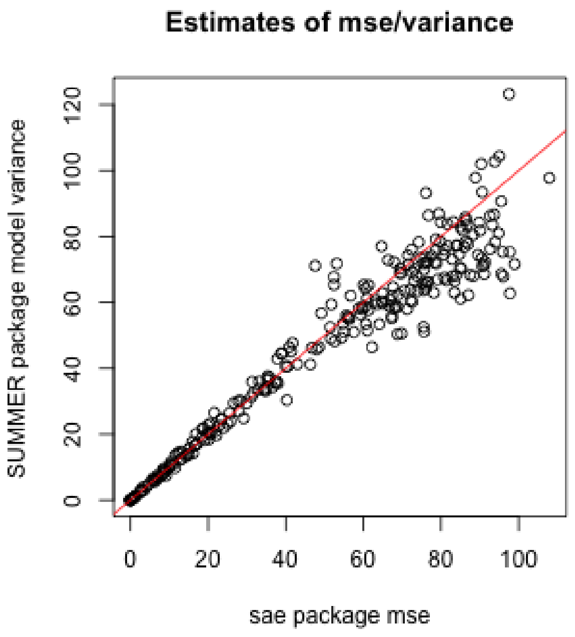
```r
# create area_name as SUMMER requires rownames of A_mat to match area variable
grapes$area_name <- paste0('area_', rownames(grapes))
Amat_grapes <- as.matrix(grapesprox)
rownames(Amat_grapes) <- colnames(Amat_grapes) <- grapes$area_name
X_grapes <- grapes[,c('area_name', 'area', 'workdays')]

# format direct estimates for SUMMER
grapes.dir <- grapes[, c(5, 1, 4)]
# scale direct estimates for use with INLA
grapes.dir$grapehect <- grapes.dir$grapehect / 10
grapes.dir$var <- grapes.dir$var/ 100
summer.FH.grapes<- smoothArea(formula = grapehect~area + workdays,
                              direct.est = grapes.dir, X.area = X_grapes,
                              domain = ~area_name, Amat = Amat_grapes)
```

Despite the differing models, we again observe good agreement with the estimates, though less so with the estimates of uncertainty, which is interesting.

```r
par(mfrow = c(1, 2))
#???
plot(results$eblup.SFH,
     summer.FH.grapes$s.dir.sp.est$median * 10,
     xlab = "sae package",
     ylab = "SUMMER package",
     main = "Small area estimates")
abline(0,1, col = 'red')
plot(results$mse,
     summer.FH.grapes$s.dir.sp.est$var * 100,
     xlab = "sae package mse",
     ylab = "SUMMER package model variance",
     main = "Estimates of mse/variance")
abline(0,1, col = 'red')
```





Unit Level Models

The nested error model, introduced by @battese_error-components_1988, uses auxiliary data at the unit level.

Nested error model:

$$y_{ij} = \mathbf{x}_{ij}^T \boldsymbol{\beta} + \delta_i + \epsilon_{ij}, \quad \delta_i \sim_{ind} N(0, \sigma_\delta^2), \quad \epsilon_{ij} \sim_{ind} N(0, \sigma_\epsilon^2)$$

Here u_d are area random effects and ϵ_{ij} are unit level errors.

This model assumes there is **no sample selection bias**.

The `sae` package conducts estimation of domain means by first estimating variance parameters σ_δ^2 and σ_ϵ^2. Next, given known variance parameters, domain means θ_d are predicted by calculating the EBLUPs.

The area fitted values are:

$$\widehat{y}_i^{\text{EBLUP}} = f_i \overline{y}_{Si} + (\overline{X}_i - f_i \overline{x}_{Si})\widehat{\beta} + (1 - f_i)\widehat{\delta}_i,$$

where

- $f_i = n_i / N_i$ is the domain sampling fraction.
- $\overline{y}_{Si}$ is the mean response in the sampled units.
- $\overline{x}_{Si}$ is the mean of the covariates in the sampled units.
- $\overline{X}_i$ is the mean of the covariates in the population.
- $\widehat{\delta}_i$ is the estimated random effect.

Corn and Soy Production

The `cornsoybean` and `cornsoybeanmeans` datasets contain info on corn and soy beans production in 12 Iowa counties Battese et al. (1988). The objective here is use satellite imagery of the number of pixels assigned to corn and soy to estimate the hectares grown of corn.

- `SampSegments`: sample size.
- `PopnSegments`: population size.
- `MeanCornPixPerSeg`: county mean of the number of corn pixels (satellite imagery).
- `MeanSoyBeansPixPerSeg` county mean of the number of soy beans (satellite imagery) pixels.

The variables `MeanCornPixPerSeg` and `MeanSoyBeansPixPerSeg` provide the known county means of the auxiliary variables.

We load the sample data:

```
data("cornsoybean")
head(cornsoybean,n=10)
```

```
##    County CornHec SoyBeansHec CornPix SoyBeansPix
## 1       1     166         8.1     374          55
## 2       2      96       106.0     209         218
## 3       3      76       103.6     253         250
## 4       4     185         6.5     432          96
## 5       4     116        63.8     367         178
## 6       5     162        43.5     361         137
## 7       5     152        71.4     288         206
## 8       5     162        42.5     369         165
## 9       6      93       105.3     206         218
## 10      6     150        76.5     316         221
```

Next, we load the population auxiliary information:

```
data("cornsoybeanmeans")
Xmean <-
  data.frame(cornsoybeanmeans[, c("CountyIndex",
                                  "MeanCornPixPerSeg",
                                  "MeanSoyBeansPixPerSeg")])
Popn <-
  data.frame(cornsoybeanmeans[, c("CountyIndex",
                                  "PopnSegments")])
head(Xmean)
```

```
##   CountyIndex MeanCornPixPerSeg MeanSoyBeansPixPerSeg
## 1           1               295                   190
## 2           2               300                   197
## 3           3               290                   205
## 4           4               291                   220
## 5           5               318                   188
## 6           6               257                   247
```

The `sae::pbmseBHF` function obtains EBLUPs under the nested error model and then uses a parametric bootstrap approach to estimate MSEs.

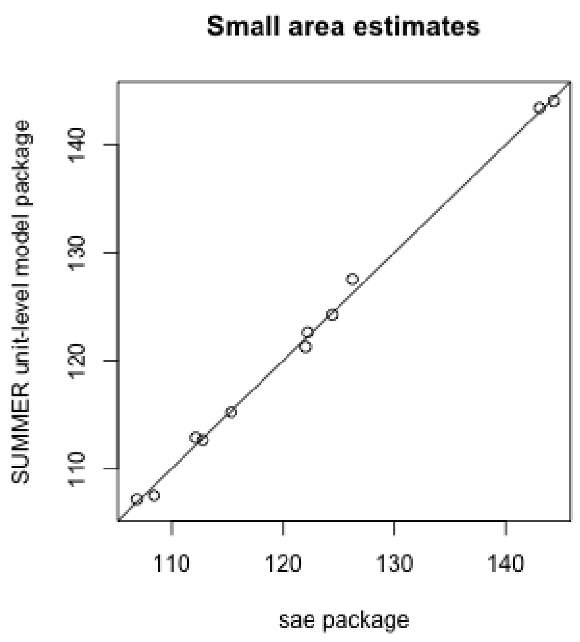
```
cornsoybean <- cornsoybean[-33, ] # remove outlier
sae.bhf <-
  pbmseBHF(CornHec ~ CornPix + SoyBeansPix,
           dom = County, meanxpop = Xmean,
           popnsize = Popn, B = 200,
           data = cornsoybean)
```

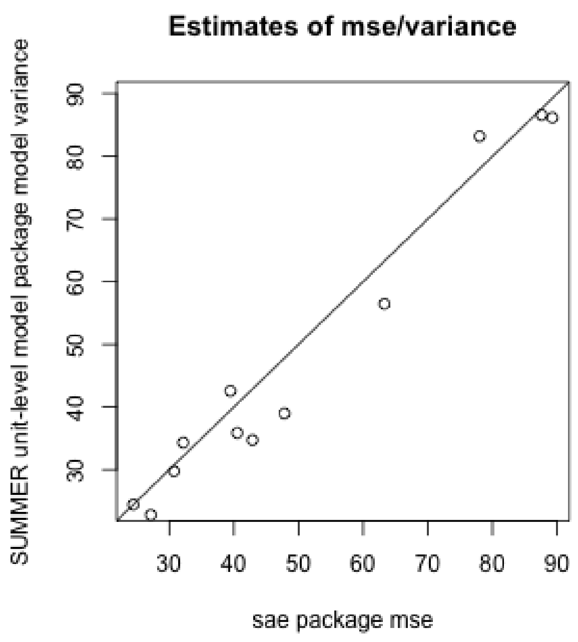
`SUMMER::smoothSurvey` provides the ability to fit unit level models with unit level covariates for Gaussian response variables. Below we use the `is.unit` argument to specify a unit level model and then provide the column names of the unit level covariates in `X.unit`. Finally, the `X` argument provides the area level means of each covariate for use when generating predictions. Note that in order to align the SUMMER estimates with those from the sae package, we specify a relatively flat prior on the variance of the area-specific random effect (pc.u = 100, pc.alpha = 0.01 specifies a penalized complexity prior such that $P(\sigma_u > 100) = 0.01$ where σ_u is the standard deviation of the area-specific random effects).

```
cornsoybean$id <- 1:dim(cornsoybean)[1]
Xsummer <- Xmean
colnames(Xsummer) = c("County", "CornPix", "SoyBeansPix")
des0 <- svydesign(ids = ~1, data = cornsoybean)
summer.bhf.unit <- smoothUnit(formula = CornHec ~ CornPix + SoyBeansPix,
                              family = "gaussian",
                              domain = ~County,
                              design = des0, X.pop = Xsummer,
                              pc.u = 1000, pc.alpha = 0.01, CI = 0.95)
```

Below, we plot comparisons of the sae and SUMMER results.

```
par(mfrow = c(1, 2))
range1 <- range(c(sae.bhf$est$eblup$eblup,summer.bhf.unit$median))
plot(sae.bhf$est$eblup$eblup,summer.bhf.unit$model.est$median,
     xlab = "sae package",
     ylab = "SUMMER unit-level model package",
     main = "Small area estimates",
     xlim=range1,ylim=range1)
abline(0,1)
range2 <- range(c(sae.bhf$mse$mse, summer.bhf.unit$var))
plot(sae.bhf$mse$mse, summer.bhf.unit$model.est$var,
     xlab = "sae package mse",
     ylab = "SUMMER unit-level model package model variance",
     main = "Estimates of mse/variance",
     xlim=range2,ylim=range2)
abline(0,1)
```





BRFSS Data: Another Area-Level Example

Below, we provide an example comparing spatial models from `sae` and `SUMMER` using data from the Behavioral Risk Factor Surveillance System (BRFSS).

```
library(ggplot2)
library(patchwork)
data(BRFSS)
data(KingCounty)
BRFSS <- subset(BRFSS, !is.na(BRFSS$diab2))
BRFSS <- subset(BRFSS, !is.na(BRFSS$hracode))
mat <- getAmat(KingCounty, KingCounty$HRA2010v2_)
design <- svydesign(ids = ~1, weights = ~rwt_llcp,
                    strata = ~strata, data = BRFSS)
direct <- svyby(~diab2, ~hracode, design, svymean)
```

Results using `sae` Below, we use `sae` to smooth the logit-transformed direct estimates.

```
direct$var <- direct$se ^ 2
direct$logit.diab2 <- SUMMER::logit(direct$diab2)
direct$logit.var <- direct$var / (direct$diab2 ^ 2 * (1 - direct$diab2) ^ 2)
SFH.brfss <- sae::mseSFH(logit.diab2 ~ 1, logit.var, mat, data = direct)

results <- data.frame(region = direct$hracode,
                      eblup.SFH = SUMMER::expit(SFH.brfss$est$eblup),
                      mse = SFH.brfss$mse)
```

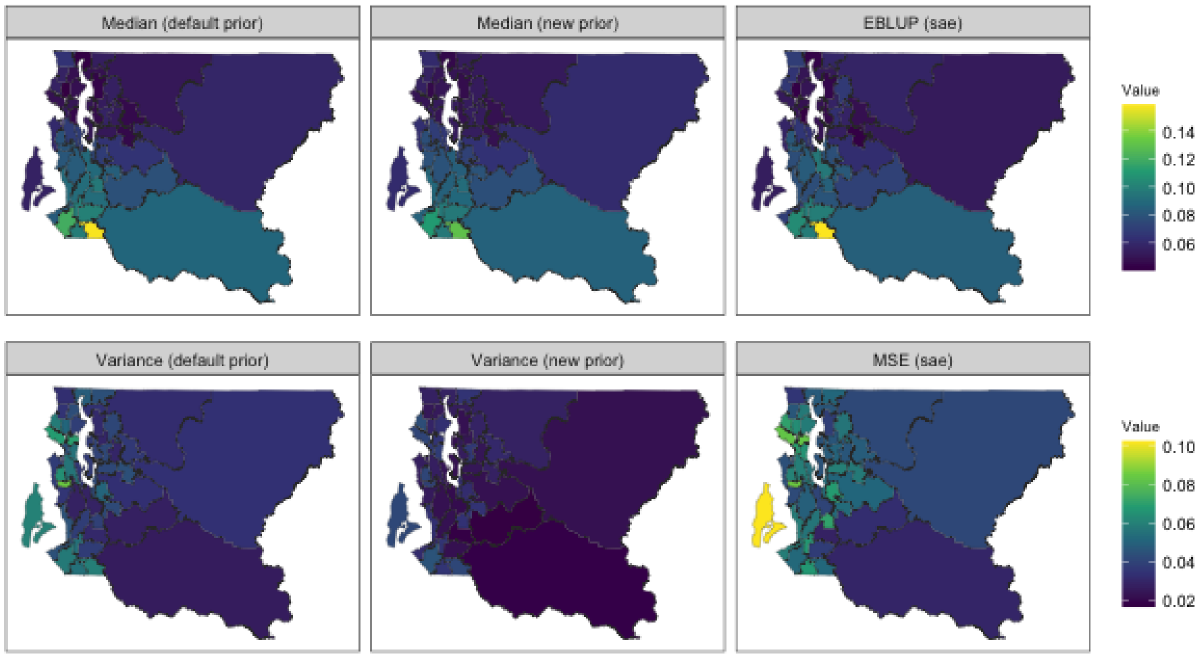
Results using `SUMMER` Below, we fit two versions of the spatial area levelmodel in `SUMMER`. If we change `pc.u` and `pc.alpha` from the default value $u = 1, \alpha = 0.01$ to $u = 0.1, \alpha = 0.01$, we assign more prior mass on smaller variance of the random effects, inducing more smoothing.

```
summer.brfss <- smoothArea(diab2~1, domain= ~hracode,
                           design = design,
                           responseType = "binary",
                           Amat = mat, CI = 0.95)
summer.brfss.alt <- smoothArea(diab2~1, domain= ~hracode,
                           design = design,
                           responseType = "binary",
                           Amat = mat, CI = 0.95,
                           pc.u = 0.1, pc.alpha = 0.01)
```

Finally, we use `SUMMER::mapPlot` to compare median estimates and uncertainty estimates obtained via `sae` and `SUMMER`.

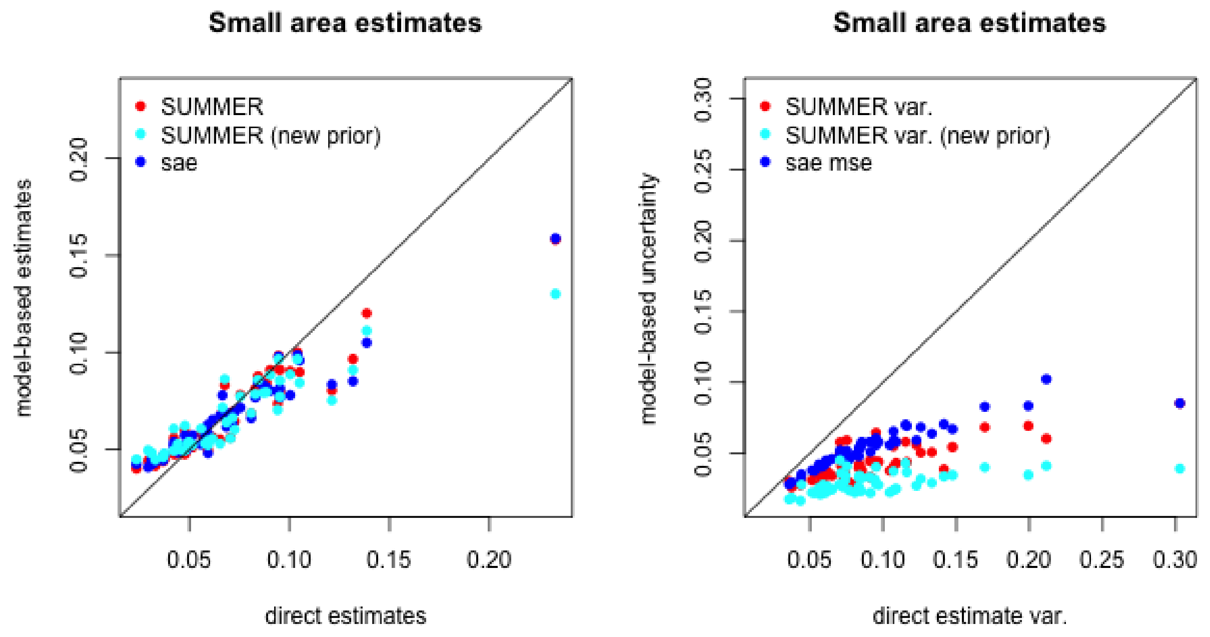
```
toplot <-  summer.brfss$s.dir.sp.est
toplot$logit.var <- toplot$var /
  (summer.brfss$s.dir.sp.est$median ^ 2 *
     (1 - summer.brfss$s.dir.sp.est$median) ^ 2)
toplot$median.alt <-  summer.brfss.alt$s.dir.sp.est$median
toplot$logit.var.alt <-  summer.brfss.alt$s.dir.sp.est$var /
  (summer.brfss.alt$s.dir.sp.est$median ^ 2 *
     (1 - summer.brfss.alt$s.dir.sp.est$median) ^ 2)
toplot$median.sae <- results$eblup.SFH
toplot$mse.sae <- results$mse
variables <- c("median", "median.alt",  "median.sae",
               "logit.var", "logit.var.alt", "mse.sae")
```

```r
names <- c("Median (default prior)", "Median (new prior)",  "EBLUP (sae)",
           "Variance (default prior)", "Variance (new prior)", "MSE (sae)")
g1 <- mapPlot(data = toplot, geo = KingCounty,
              variables=variables[1:3],
              labels = names[1:3], by.data = "domain",
              by.geo = "HRA2010v2_", size = 0.1)
g2 <- mapPlot(data = toplot, geo = KingCounty,
              variables=variables[4:6], labels = names[4:6],
              by.data = "domain", by.geo = "HRA2010v2_", size = 0.1)
g1 / g2
```



```r
par(mfrow = c(1, 2))
range1 <- range(c(direct$diab2,toplot$median.alt))
plot(direct$diab2,toplot$median,
     xlab = "direct estimates",
     ylab = "model-based estimates",
     main = "Small area estimates", col = 'red', pch = 16,
     xlim=range1,ylim=range1)
points(direct$diab2,toplot$median.sae,  col = 'blue', pch = 16)
points(direct$diab2,toplot$median.alt,  col = 'cyan', pch = 16)
legend('topleft', pch = 16, col = c('red', 'cyan', 'blue'),
       legend = c("SUMMER",'SUMMER (new prior)', "sae"),bty="n")
abline(0,1)
range2 <- range(c(direct$logit.var,toplot$mse.sae,toplot$logit.var.alt))
plot(direct$logit.var,toplot$logit.var,
     xlab = "direct estimate var.",
     ylab = "model-based uncertainty",
     main = "Small area estimates", col = 'red', pch = 16,
     xlim=range2,ylim=range2)
points(direct$logit.var,toplot$mse.sae,  col = 'blue', pch = 16)
points(direct$logit.var,toplot$logit.var.alt,  col = 'cyan', pch = 16)
```

```
legend('topleft', pch = 16, col = c('red', 'cyan','blue'),
       legend = c("SUMMER var.", 'SUMMER var. (new prior)',"sae mse"),bty="n")
abline(0,1)
```


Small area estimates
SUMMER
SUMMER (new prior)
sae
model-based estimates
direct estimates
Small area estimates
SUMMER var.
SUMMER var. (new prior)
sae mse
model-based uncertainty
direct estimate var.