# Loss Leading as an Exploitative Practice

*(Chen and Rey 2012)*

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This document describes the steps to generate Figure 1.

## Analytical Solutions

### Baseline

In the baseline, the firm L has monopoly power over A and B. This is derived in the main text:

$$\pi^m_{AB} = \frac{w^2_{AB}}{4}$$

### Case (i): $w_{AB} < w'_B$

In this scenario, as discussed in the main text, firm L exploits its monopoly power over A. Since $w'_B > w_A$, a subset of consumers who purchase B from firm S will also purchase A from L. If firm L applies zero margin, $r_A = 0$, then the share of consumers is $\min\{w'_B/2, w_A\}$. Let us consider the two cases:

- If $w_A < w'_B/2$, then demand is $w_A - r_A$. Profits are:

$$\pi^m_A := \max_{r_A} r_A(w_A - r_A)$$

- If $w_A \geq w'_B/2$, then demand at zero margin is $w'_B/2$. The optimal margin is at least $w_A - w'_B/2$, because it does not reduce demand until this level. Profits are:

$$\pi^m_A := r_A(w_A - [r_A - (w_A - w'_B/2)]) = r_A(w_A - r_A)$$

From the FOC, $r^m_A = w_A/2$, so that $\pi^m_A = w^2_A/4$.

### Case (ii): $2w'_B \leq w_{AB}$

As derived in the main text:

$$\pi^*_{AB} = \pi^m_{AB} + \frac{(c_B - c'_B)^2}{4} > \pi^m_{AB}$$

### Case (iii): $w'_B \leq w_{AB} < 2w'_B$

As derived in the main text:

$$\tilde{\pi}^*_{AB} = (w_{AB} - w'_B)w'_B + \frac{(c_B - c'_B)^2}{4}$$

Since profits are increasing in $w_{AB}$, conceptually consider that $u_A$ increases *ceteris paribus*, we know that there is a level $w_{AB}$ such that profits match the baseline level $\pi^m_{AB}$. This occurs when $w_{AB} = w'_B + w_B$, graphically, at the point where the two lines cross. To derive this midpoint, consider that $w_{AB} = w'_B + \tau, \tau \in [0, w'_B)$. Then, equating both profits:

$$\begin{aligned}
\pi^m_{AB} = \tilde{\pi}^*_{AB} \Leftrightarrow \frac{w^2_{AB}}{4} &= (w_{AB} - w'_B)w'_B + \frac{(w'_B - w_B)^2}{4} \\
\Leftrightarrow \frac{(w'_B + \tau)^2}{4} &= \tau w'_B + \frac{(w'_B - w_B)^2}{4} \\
\Leftrightarrow \frac{\tau^2}{4} + \frac{2\tau w'_B}{4} &= \tau w'_B + \frac{w^2_B}{4} - \frac{2w'_B w_B}{4} \\
\Leftrightarrow \frac{\tau^2}{4} &= \frac{2\tau w'_B}{4} + \frac{w^2_B}{4} - \frac{2w'_B w_B}{4} \\
\Leftrightarrow \tau^2 - 2w'_B\tau + w_B(2w_B - w'_B) &= 0
\end{aligned}$$

The roots of this quadratic equation are:

$$\tau = w'_B \pm (w'_B - w_B) = \begin{cases} 2w'_B - w_B > w'_B \\ w_B \end{cases}$$

Only the smaller (second) root is admissible in this third case, so that $\tilde{\pi}^*_{AB} = \pi^m_{AB}$ whenever $w_{AB} = w'_B + w_B$.

## Simulation

Parameters:

```
cA <- 0
cB.p <- 0
uA.0 <- 1
uA.1 <- 5 # st. uA.1>2*uB
uB <- 2 # st. uA.0 < uB < uA.1
min.wB <- .5 # 0<wB<uB
```

Vectors:

```
cB.seq <- seq(from=cB.p,to=uB-min.wB,by=.02)
uA.seq <- seq(from=uA.0,to=uA.1,by=.02)
cB <- c(rep(uB-min.wB,length(uA.seq)),rev(cB.seq))
uA <- c(uA.seq,rep(uA.1,length(cB.seq)))

wA <- uA - cA
wB.p <- uB - cB.p
wB <- uB - cB
wAB <- wA + wB
```

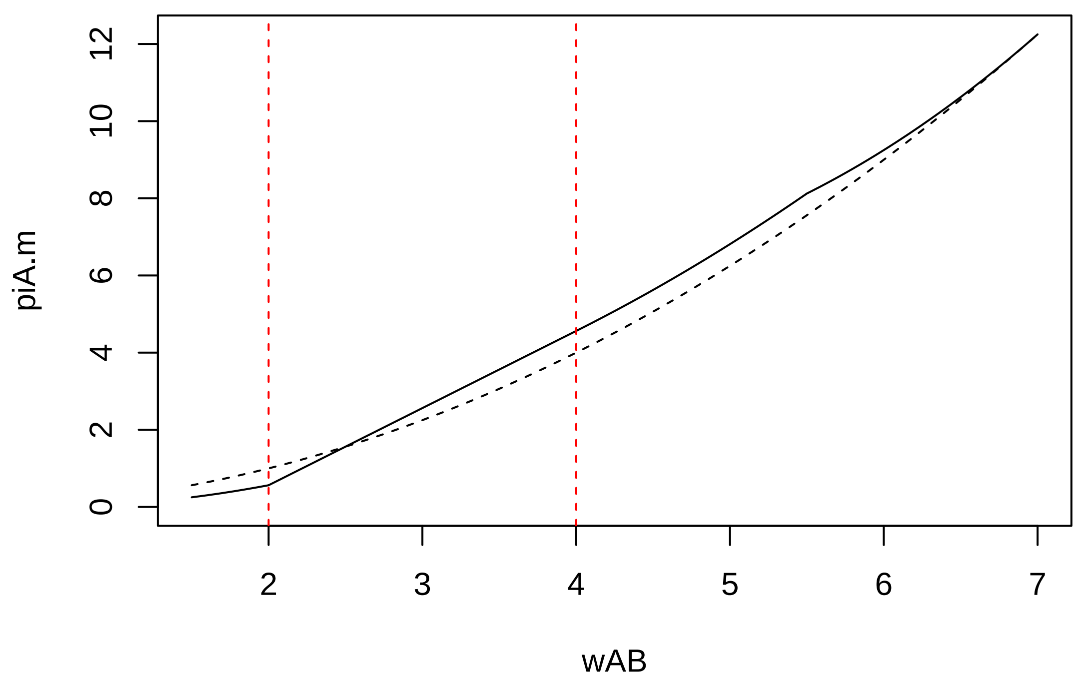
Profit functions:

```
piA.m <- ifelse(wAB<=wB.p,(wA^2)/4,NA)
piAB.m <- (wAB^2)/4
piAB.star <- ifelse(wAB>=2*wB.p, piAB.m + (cB-cB.p)^2/4,NA)
piAB.tilde <- ifelse(wAB>=wB.p & wAB<=2*wB.p, (wAB-wB.p)*wB.p + (cB-cB.p)^2/4,NA)
```

Raw plot:

```
ymax <- max(piAB.star,na.rm=T)

# case 1
plot(wAB,piA.m,xlim=c(min(wAB),max(wAB)),ylim=c(0,ymax),ty="l")
# case 2
lines(wAB,piAB.star)
# case 3
lines(wAB,piAB.tilde)
# baseline
lines(wAB,piAB.m,lty=2)
# thresholds
abline(v=wB.p,lty=2,col="red")
abline(v=2*wB.p,lty=2,col="red")
```




Adjusted plot:

```r
plot(wAB,piA.m,xlim=c(min(wAB),4.5),ylim=c(0,6),ty="l",ylab="profits",
     #xaxt="n",yaxt="n"
     )
# case 2
lines(wAB,piAB.star)
# case 3
lines(wAB,piAB.tilde)
# baseline
lines(wAB,piAB.m,lty=3)
# cases
abline(v=wB.p,lty=3,col="red")
abline(v=2*wB.p,lty=3,col="red")
# legend
legend("top", legend="baseline", lty=3,cex=.7)
text(1.75, 3.5, " Case (i)",cex=.7)
text(1.75, 3, "wAB<wB'",cex=.7)
text(4.3, 3.5, " Case (ii)",cex=.7)
text(4.3, 3, "wB'<wAB<2wB'",cex=.7)
text(2.5, 3.5, " Case (iii)",cex=.7)
text(2.5, 3, "2wB<2wAB'",cex=.7)
```