TSP v3

July 4, 2021

1 The Travelling Salesman Problem (TSP)

1.1 Librerías

[115]:
```
import os
import sys
import math
from matplotlib import pyplot as plt
import numpy as np

import random
```

1.2 Leer archivo

[113]:
```
def leer_fichero(file_path):

    # Lista para guardar las posiciones
    positions = []
    if not os.path.isfile(file_path):
        print("ERROR - no se ha encontrado el fichero")
        sys.exit()

    with open(file_path, "r") as file:
        for line in file:
            if line is not None:
                if line[0].isdigit():

                    line = line.strip()
                    line_elements = line.split()
                    positions.append((float(line_elements[1]),␣
↪float(line_elements[2])))

    # Extrae el fichero
    file_name = os.path.basename(file_path)

    return positions, file_name
```

1.3 Calcular la matriz de pesos

[3]:
```
def get_distances(positions:list):
    distances = np.zeros((len(positions),len(positions)))
    npuntos = len(positions)
    for i in range(npuntos):
        for j in range(npuntos):
            if positions[j][0]==positions[i][0] and positions[j][1] ==␣
↪positions[i][1]:
                aux = np.inf
            else:
                aux = math.sqrt(math.
↪pow((positions[j][0]-positions[i][0]),2)+math.
↪pow((positions[j][1]-positions[i][1]),2))
            distances[i][j] = aux
    return distances
```

1.4 Pintar resultado

[85]:
```
def pintar(positions,path):
    for position in positions:
        plt.plot(position[0],position[1], marker="o", color="red")
    x = []
    y = []
    for i in path:
        position = positions[i]
        x.append(position[0])
        y.append(position[1])

    plt.plot(x,y,linestyle='solid',color='green')
    plt.show()
```

2 Ant colony optimization algorithm

[5]:
```
class Grafo(object):
    def __init__(self, distances: list, num_hormigas, alpha, beta,Q):
        self.distances = distances
        self.npuntos = len(distances)
        self.visitados = []
        self.permitidos = [i for i in range(0,self.npuntos)]
        self.alpha = alpha
        self.beta = beta
        self.Q = Q

        # Inicializamos el grafo con un nodo aleatorio
```

```
        inicial = random.randint(0,self.npuntos-1)
        # Añadimos el nodo inicial a la lista de visitados
        self.visitados.append(inicial)
        # Ponemos el puntero del nodo actual en el nodo inicial
        self.actual = inicial
        # Eliminamos el nodo inicial de la lista de permitidos
        self.permitidos.remove(inicial)
        # Inicializamos la distancia total a 0
        self.distancia_total = 0

        # Calculamos el camino escogiendo aquel camino que sea el más corto
        while len(self.permitidos) != 0:
            minimo = np.inf
            siguiente = -1
            for i in self.permitidos:
                if self.distances[self.actual][i]<minimo:
                    minimo = self.distances[self.actual][i]
                    siguiente = i
            self.permitidos.remove(siguiente)
            self.visitados.append(siguiente)
            self.distancia_total += minimo
            self.actual = siguiente

        # Añadimos de nuevo el nodo inicial para cerrar el camino
        self.visitados.append(inicial)
        self.distancia_total += self.distances[self.actual][inicial]

        # Inicializamos las feromonas como (num_hormigas*Q)/(npuntos *␣
↪distancia_camino_minimo)
        self.feromonas = [[(num_hormigas*self.Q)/(self.npuntos*self.
↪distancia_total) for i in range(self.npuntos)] for j in range(self.npuntos)]
```

[6]:
```
class Colonia(object):
    def __init__(self,num_hormigas,iteraciones,rho):
        self.num_hormigas = num_hormigas
        self.iteraciones = iteraciones
        self.rho = rho

    # Actualiza las feromonas
    #  xy = (1- ) xy + Σ xy
    def actualiza_feromonas(self, grafo:Grafo, hormigas: list):
        for i, fila in enumerate(grafo.feromonas):
            for j, columna in enumerate(fila):
                grafo.feromonas[i][j] *= 1-self.rho
                for hormiga in hormigas:
                    grafo.feromonas[i][j] += hormiga.feromonas[i][j]
```

```
    def resolver(self,grafo:Grafo):
        minimo = np.inf
        solucion = []
        for i in range(self.iteraciones):
            hormigas = [Hormiga(self,grafo) for id_hormiga in range(self.
↪num_hormigas)]
            for hormiga in hormigas:
                for nodo in range(grafo.npuntos - 1):
                    hormiga.selecciona_nodo()

                hormiga.distancia_total += grafo.distances[hormiga.
↪visitados[-1]][hormiga.visitados[0]]
                hormiga.visitados.append(hormiga.visitados[0])
                if hormiga.distancia_total < minimo:
                    minimo = hormiga.distancia_total
                    solucion = hormiga.visitados

                # Calcula las delta_feromonas (Δ xy) para el cálculo que␣
↪realizaremos a continuación
                hormiga.delta_feromonas()

            # Una vez que la hormiga llega a su destino, se debe actualizar las␣
↪feromonas de cada camino
            # respecto al desgaste o evaporación  y si pasó la hormiga por el␣
↪camino dejando feromonas.
            self.actualiza_feromonas(grafo,hormigas)

        return solucion, minimo
```

[110]:
```
class Hormiga(object):
    def __init__(self, colonia: Colonia, grafo: Grafo):
        #self.colonia = colonia # colonia
        self.grafo = grafo
        self.distancia_total = 0.0
        self.visitados = []
        self.feromonas = []
        self.permitidos = [i for i in range(grafo.npuntos)]

        # Situamos a la hormiga en un nodo aleatorio del grafo
        inicio = random.randint(0, grafo.npuntos - 1)
        # Añadimos el nodo a la lista de visitados por la hormiga
        self.visitados.append(inicio)
        # Situamos el puntero del nodo actual en el nodo de inicio
        self.actual = inicio
        # Eliminamo el nodo inicial de la lista de permitidos
        self.permitidos.remove(inicio)
```

```
   # Selecciona un nodo
   # Para calcular el posible camino a tomar de una hormiga, se utiliza la␣
↪Ecuación:
   # Pxy = ( (e)*N(e))/(Σ (e)*N(e)), donde  (e) son las feromonas depositadas␣
↪por ese
   # camino (A a B), N(e) es la visibilidad del camino dado por N(e) = 1/
↪peso_del_camino,
   # y el denominador de esta ecuación siendo la suma del cálculo anterior␣
↪( (e)*N(e)) de
   # todos los posibles que tiene esa hormiga
   def selecciona_nodo(self):
       N = 1/self.grafo.distances
       numerador = []
       denominador = 0
       for i in self.permitidos:
           aux = (self.grafo.feromonas[self.actual][i]**self.grafo.
↪alpha)*(N[self.actual][i]**self.grafo.beta)
           denominador += aux
           numerador.append(aux)
       P = numerador/denominador
       peso_minimo = -1
       nodo_minimo = -1
       for i,p in enumerate(P):
           if p>peso_minimo:
               peso_minimo = p
               nodo_minimo = self.permitidos[i]

       self.permitidos.remove(nodo_minimo)
       self.visitados.append(nodo_minimo)
       self.distancia_total += self.grafo.distances[self.actual][nodo_minimo]
       self.actual = nodo_minimo

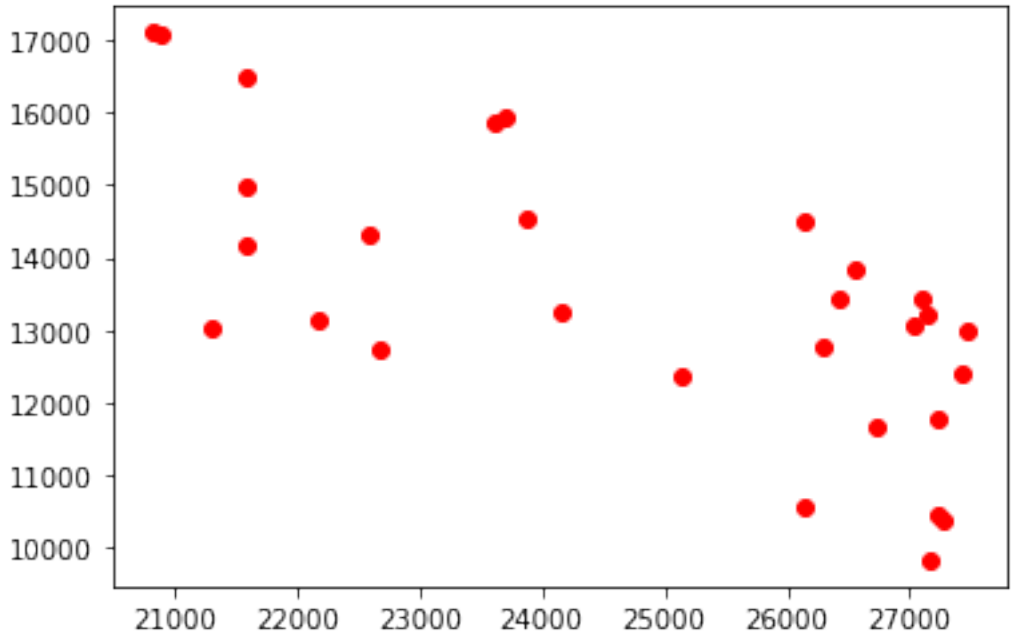
   # Calcula las Δ xy como Q/Lk si se usó xy ó 0 en caso contrario
   def delta_feromonas(self):
       self.feromonas = [[0 for j in range(self.grafo.npuntos)] for i in␣
↪range(self.grafo.npuntos)]
       for i in range(0,len(self.visitados)-1):
           nodo = self.visitados[i]
           siguiente = self.visitados[i+1]
           # Q es el parámetro de aprendizaje y Lk (distancia total) es el␣
↪costo del camino por hormiga
           self.feromonas[nodo][siguiente] = self.grafo.Q/self.distancia_total
```

3 Ejecución

3.1 Leemos el archivo y lo mostramos

[8]:
```
positions, fname = leer_fichero("./wi29.tsp")
```

[9]:
```
for position in positions:
    plt.plot(position[0],position[1], marker="o", color="red")
plt.show()
```



3.2 Calculamos la matriz de distancias:

[10]:
```
distances = get_distances(positions)
```

3.3 Definimos los Parámetros:

[11]:
```
# Parámetros
num_hormigas = [5, 10, 20]
iteraciones = 625
rho = [0.15, 0.25, 0.5]
q = [0.3, 0.6, 0.9]
alpha = [0.05, 0.1, 0.35]
beta = [0.5, 1, 2]
```

3.4 Buscamos el camino más corto

[12]:
```
def buscar(d,m,r,q,a,b):

    colonia = Colonia(m,iteraciones,r)
    grafo = Grafo(d,m,a,b,Q)
    camino,longitud = colonia.resolver(grafo)
    return camino,longitud
```

3.4.1 Debido al warning RunTime, se ha optado por ejecutar el programa poco a poco para diferentes parámetros

[59]:
```
caminos1 = []
longitudes1 = []
for m in num_hormigas:
    for r in rho:
        a = alpha[0]
        Q = q[0]
        b = beta[0]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos1.append(camino)
        longitudes1.append(longitud)
```

[14]:
```
caminos2 = []
longitudes2 = []
for m in num_hormigas:
    for r in rho:
        a = alpha[0]
        Q = q[0]
        b = beta[1]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos2.append(camino)
        longitudes2.append(longitud)
```

[15]:
```
caminos3 = []
longitudes3 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[0]
        Q = q[0]
        b = beta[2]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos3.append(camino)
```

```
        longitudes3.append(longitud)
```

[16]:
```
caminos4 = []
longitudes4 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[0]
        Q = q[1]
        b = beta[0]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos4.append(camino)
        longitudes4.append(longitud)
```

[17]:
```
caminos5 = []
longitudes5 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[0]
        Q = q[1]
        b = beta[1]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos5.append(camino)
        longitudes5.append(longitud)
```

[18]:
```
caminos6 = []
longitudes6 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[0]
        Q = q[1]
        b = beta[2]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos6.append(camino)
        longitudes6.append(longitud)
```

[19]:
```
caminos7 = []
longitudes7 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[0]
```

```
        Q = q[2]
        b = beta[0]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos7.append(camino)
        longitudes7.append(longitud)
```

[20]:
```
caminos8 = []
longitudes8 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[0]
        Q = q[2]
        b = beta[1]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos8.append(camino)
        longitudes8.append(longitud)
```

[21]:
```
caminos9 = []
longitudes9 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[0]
        Q = q[0]
        b = beta[0]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos9.append(camino)
        longitudes9.append(longitud)
```

[22]:
```
caminos10 = []
longitudes10 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[0]
        Q = q[2]
        b = beta[2]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos10.append(camino)
        longitudes10.append(longitud)
```

[23]:
```
caminos11 = []
longitudes11 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[1]
        Q = q[0]
        b = beta[0]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos11.append(camino)
        longitudes11.append(longitud)
```

[24]:
```
caminos12 = []
longitudes12 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[1]
        Q = q[0]
        b = beta[1]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos12.append(camino)
        longitudes12.append(longitud)
```

[25]:
```
caminos13 = []
longitudes13 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[1]
        Q = q[0]
        b = beta[2]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos13.append(camino)
        longitudes13.append(longitud)
```

[26]:
```
caminos14 = []
longitudes14 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[1]
        Q = q[1]
        b = beta[0]
```

```
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos14.append(camino)
        longitudes14.append(longitud)
```

[27]:
```
caminos15 = []
longitudes15 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[1]
        Q = q[1]
        b = beta[1]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos15.append(camino)
        longitudes15.append(longitud)
```

[28]:
```
caminos16 = []
longitudes16 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[1]
        Q = q[1]
        b = beta[2]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos16.append(camino)
        longitudes16.append(longitud)
```

[29]:
```
caminos17 = []
longitudes17 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[1]
        Q = q[2]
        b = beta[0]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos17.append(camino)
        longitudes17.append(longitud)
```

[30]:
```
caminos18 = []
longitudes18 = []
```

```
for m in num_hormigas:
    for r in rho:
        a = alpha[1]
        Q = q[2]
        b = beta[1]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos18.append(camino)
        longitudes18.append(longitud)
```

[31]:
```
caminos19 = []
longitudes19 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[1]
        Q = q[2]
        b = beta[2]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos19.append(camino)
        longitudes19.append(longitud)
```

[32]:
```
caminos20 = []
longitudes20 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[2]
        Q = q[0]
        b = beta[0]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos20.append(camino)
        longitudes20.append(longitud)
```

[33]:
```
caminos21 = []
longitudes21 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[2]
        Q = q[0]
        b = beta[1]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos21.append(camino)
```

```
        longitudes21.append(longitud)
```

[34]:
```
caminos22 = []
longitudes22 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[2]
        Q = q[0]
        b = beta[2]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos22.append(camino)
        longitudes22.append(longitud)
```

[35]:
```
caminos23 = []
longitudes23 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[2]
        Q = q[1]
        b = beta[0]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos23.append(camino)
        longitudes23.append(longitud)
```

[36]:
```
caminos24 = []
longitudes24 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[2]
        Q = q[1]
        b = beta[1]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos24.append(camino)
        longitudes24.append(longitud)
```

[37]:
```
caminos25 = []
longitudes25 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[2]
```

```
        Q = q[1]
        b = beta[2]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos25.append(camino)
        longitudes25.append(longitud)
```

[38]:
```
caminos26 = []
longitudes26 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[2]
        Q = q[2]
        b = beta[1]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos26.append(camino)
        longitudes26.append(longitud)
```

[39]:
```
caminos27 = []
longitudes27 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[2]
        Q = q[2]
        b = beta[1]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos27.append(camino)
        longitudes27.append(longitud)
```

[40]:
```
caminos28 = []
longitudes28 = []

for m in num_hormigas:
    for r in rho:
        a = alpha[2]
        Q = q[2]
        b = beta[2]
        d = distances.copy()
        camino,longitud = buscar(d,m,r,Q,a,b)
        caminos28.append(camino)
        longitudes28.append(longitud)
```

3.4.2 Juntamos todos los caminos y longitudes en una sola lista

[69]:
```
caminos = caminos1[:]
longitudes = longitudes1[:]
caminos.extend(caminos2)
caminos.extend(caminos3)
caminos.extend(caminos4)
caminos.extend(caminos5)
caminos.extend(caminos6)
caminos.extend(caminos7)
caminos.extend(caminos8)
caminos.extend(caminos9)
caminos.extend(caminos10)
caminos.extend(caminos11)
caminos.extend(caminos12)
caminos.extend(caminos13)
caminos.extend(caminos14)
caminos.extend(caminos15)
caminos.extend(caminos16)
caminos.extend(caminos17)
caminos.extend(caminos18)
caminos.extend(caminos19)
caminos.extend(caminos20)
caminos.extend(caminos21)
caminos.extend(caminos22)
caminos.extend(caminos23)
caminos.extend(caminos24)
caminos.extend(caminos25)
caminos.extend(caminos26)
caminos.extend(caminos27)
caminos.extend(caminos28)
longitudes.extend(longitudes2)
longitudes.extend(longitudes3)
longitudes.extend(longitudes4)
longitudes.extend(longitudes5)
longitudes.extend(longitudes6)
longitudes.extend(longitudes7)
longitudes.extend(longitudes8)
longitudes.extend(longitudes9)
longitudes.extend(longitudes10)
longitudes.extend(longitudes11)
longitudes.extend(longitudes12)
longitudes.extend(longitudes13)
longitudes.extend(longitudes14)
longitudes.extend(longitudes15)
longitudes.extend(longitudes16)
longitudes.extend(longitudes17)
```

```
longitudes.extend(longitudes18)
longitudes.extend(longitudes19)
longitudes.extend(longitudes20)
longitudes.extend(longitudes21)
longitudes.extend(longitudes22)
longitudes.extend(longitudes23)
longitudes.extend(longitudes24)
longitudes.extend(longitudes25)
longitudes.extend(longitudes26)
longitudes.extend(longitudes27)
longitudes.extend(longitudes28)
print(caminos)
print(longitudes)
```

```
[[0, 1, 5, 4, 3, 7, 6, 8, 2, 12, 13, 16, 17, 18, 21, 22, 20, 28, 27, 25, 19, 15,
24, 26, 23, 14, 11, 9, 10, 0], [7, 3, 4, 5, 1, 0, 9, 10, 11, 12, 13, 16, 17, 18,
14, 21, 22, 20, 28, 27, 25, 19, 15, 24, 26, 23, 8, 6, 2, 7], [7, 3, 4, 5, 1, 0,
10, 9, 11, 12, 13, 16, 17, 18, 14, 21, 22, 20, 28, 27, 25, 19, 15, 24, 26, 23,
8, 6, 2, 7], [4, 3, 7, 6, 8, 2, 12, 13, 16, 17, 18, 21, 22, 20, 28, 27, 25, 19,
15, 24, 26, 23, 14, 11, 9, 10, 5, 1, 0, 4], [10, 9, 11, 7, 3, 4, 5, 1, 0, 2, 6,
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3, 4, 5, 1, 0, 9, 10, 11, 12, 13, 16, 17, 18, 14, 21, 22, 20, 28, 27, 25, 19,
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[88]:
```
i_min = []
for i,l in enumerate(longitudes):
    if len(i_min)<6:
        i_min.append(i)
    else:
        maximo = longitudes[i_min[0]]
        pos = 0
        for j in range(1,6):
            if longitudes[i_min[j]]>maximo:
                maximo = longitudes[i_min[j]]
                pos = j


        if l<maximo:
            i_min[pos]= i
```

3.4.3 Obtenemos las posiciones de los caminos más cortos y los pintamos

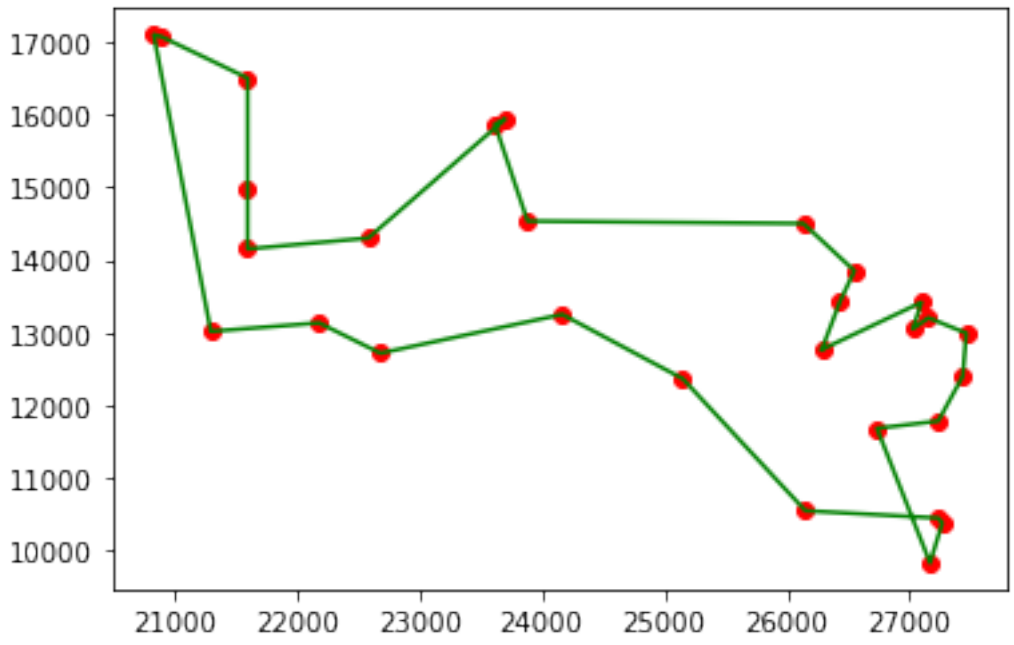
[89]:
```
i_min
```

[89]: [37, 55, 113, 211, 200, 229]

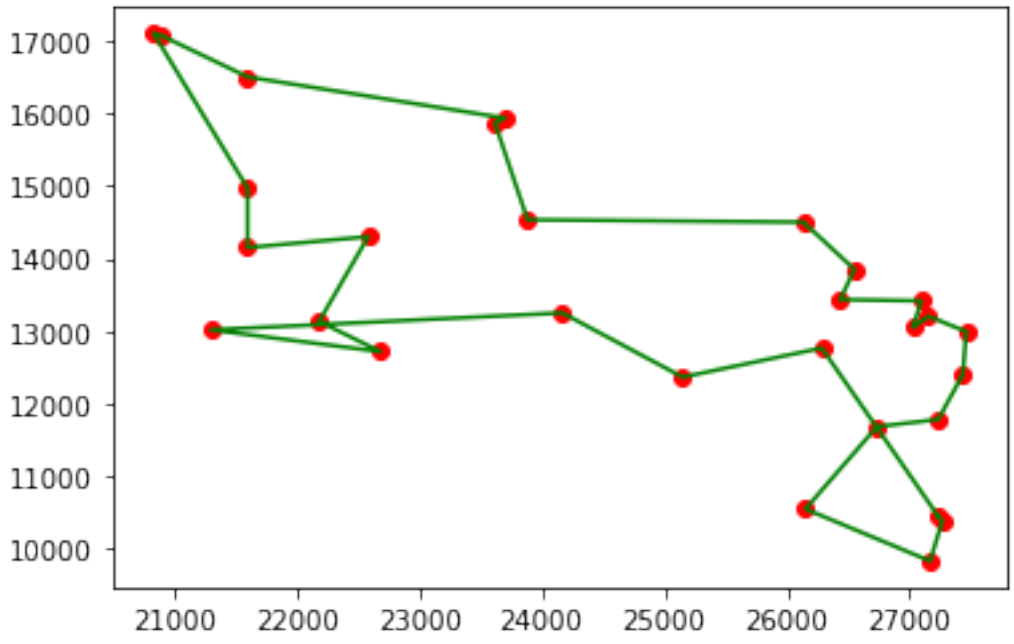
[104]:
```
infos = []
for m in num_hormigas:
    for r in rho:
        for a in alpha:
            for Q in q:
                for b in beta:
                    info =[m,r,a,Q,b]
                    infos.append(info)
```

[105]:
```
for i in i_min:
    print(longitudes[i])
    print(infos[i])
    pintar(positions,caminos[i])
```

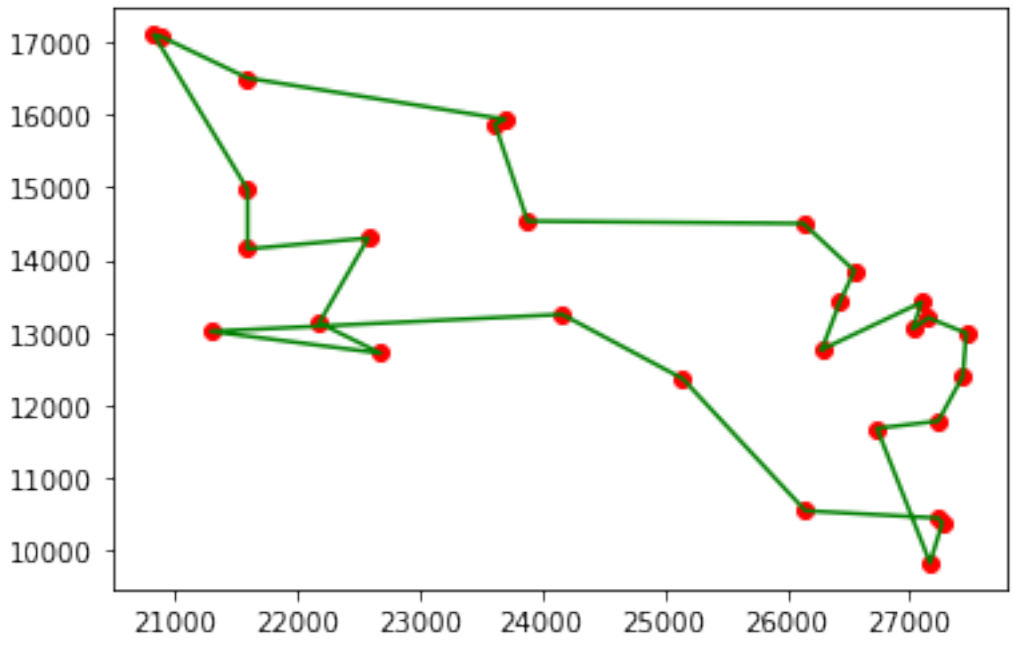
```
29881.981999188767
[5, 0.25, 0.1, 0.3, 1]
```



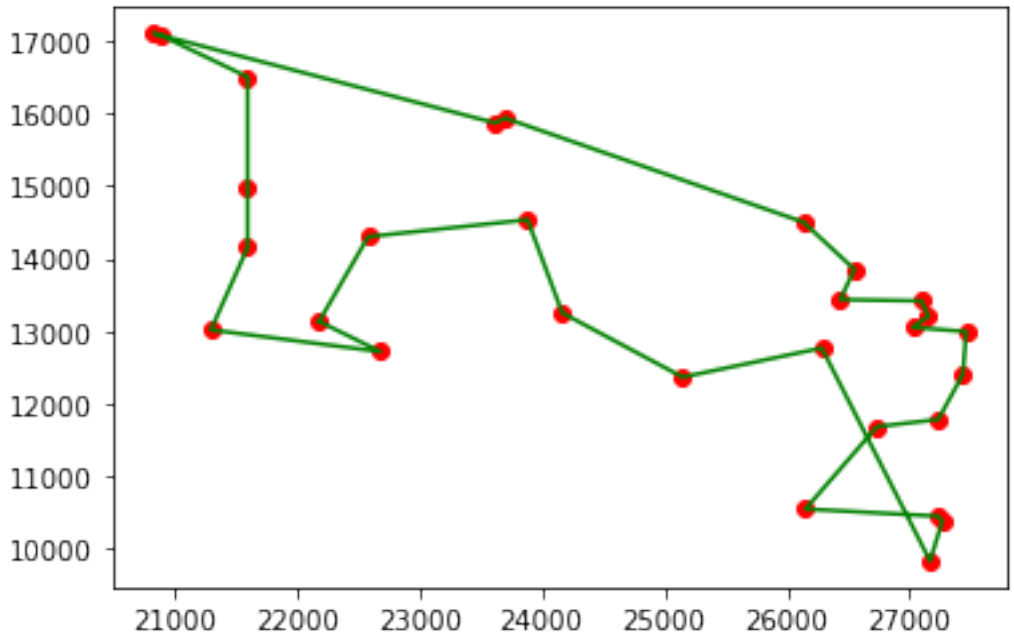
29863.25398663174
[5, 0.5, 0.05, 0.3, 1]



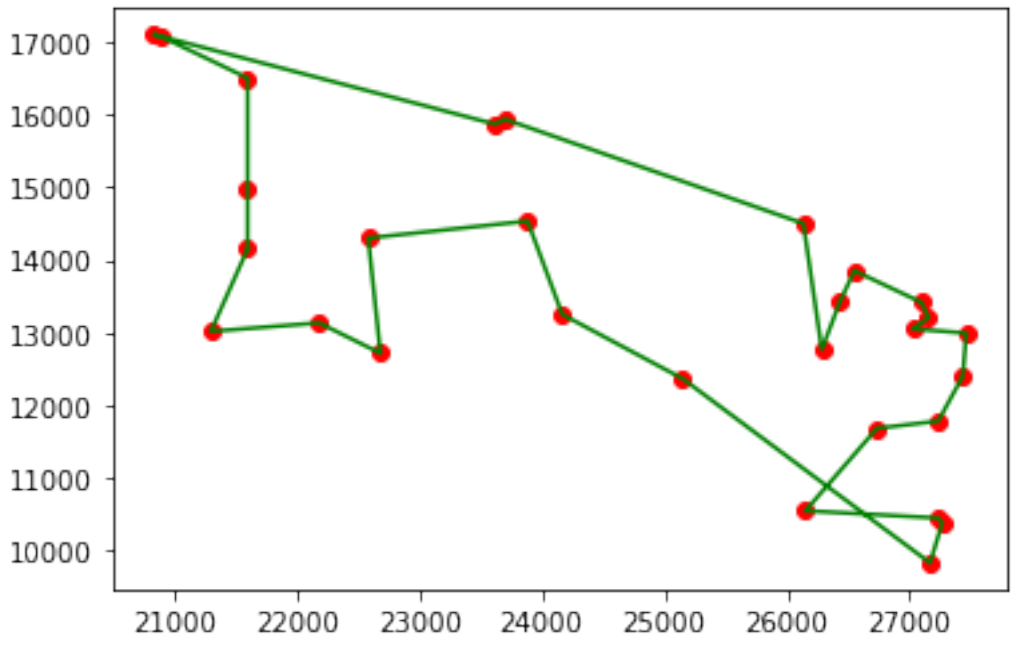
29759.858615278194
[10, 0.25, 0.05, 0.6, 2]



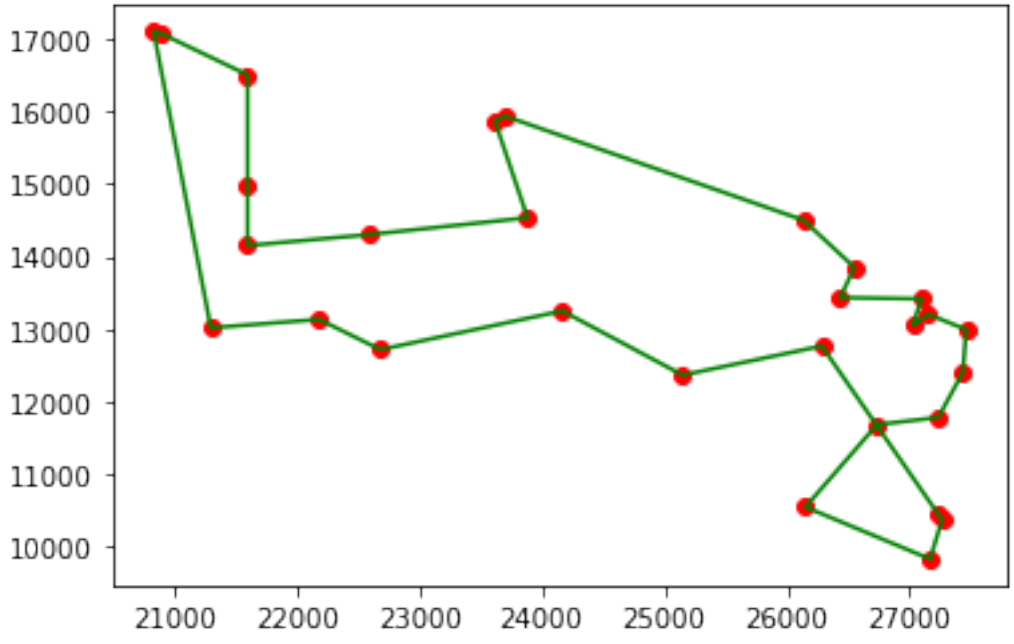
29469.080386194644
[20, 0.25, 0.35, 0.6, 1]



29954.985818037243
[20, 0.25, 0.1, 0.3, 2]




```
29901.441524353922
[20, 0.5, 0.1, 0.6, 1]
```




```
[ ]:
```