Math 105A   **Student's Name (Print):** _____

Fall 2017

Midterm Exam   **Student's ID:** _____

Fri Nov 3 2017

12.00pm   **Discussion Section Code:** _____

Print your name and student ID on the top of this page.

This exam contains 6 pages (including this cover page) and 5 problems. You may *not* use your books, notes, or any calculator in this exam. Do not write in the grading table below.

The following rules apply to the answers you provide in this exam:

- **Organize your work**, in a neat and coherent way.

- **Unsupported answers will not receive full credit**. Calculation or verbal explanation is expected.

- **If you need more space, use the back of the pages**; clearly indicate when you have done this.

- **Box your final answer** for full credit.

Question	Points	Score
1	10	
2	10	
3	5	
4	10	
5	20	
Total:	55	

1. (a) (5 points) Consider the sequence $\{p_n\}$ defined by

$$p_n = \sum_{k=1}^{n} \frac{1}{k}.$$

This sequence diverges (to see this intuitively note that $p_n \approx \int_1^\infty dx/x$ for large n). Show that, nevertheless,

$$\lim_{n\to\infty} (p_n - p_{n-1}) = 0.$$

Less: Be mindful of this when testing for convergence in your iterative algorithms!

(b) (5 points) Show that $p_n = 10^{-2^n}$ converges quadratically.

2. (10 points) Find the unique fixed point of $g(x) = 1 + \sqrt{x}$.

3. (5 points) Find all values of α for which the following system has no solutions. Explain your answer.

$$
\begin{aligned}
2x - y + 3z &= 5 \\
4y - 4z &= -4 \\
(5 + \alpha)z &= 8 + \alpha
\end{aligned}
$$

4. (10 points) Use Gaussian Elimination to compute the determinant of

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}.$$

5. (20 points) Use partial pivoting to compute a PLU factorization of

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}.$$

(You may use this page to complete your solution to Q5.)