

Lab 06 Gauss Jordan.

Ex Construct inverse of

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 0 \\ -1 & 1 & 2 \end{bmatrix}$$

using Gauss-Jordan method (w/ partial pivot).

SLⁿ

STEP 1:

$$\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ -1 & 1 & 2 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{l} \rightarrow \\ \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 2 & 1 & 1 & 0 & 1 \end{array} \right. \end{array}$$

$$\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}$$

Gaussian
Elimination
w/ partial
pivoting

STEP 2: Scale last row by a_{nn} :

$$R_n \leftarrow R_n / a_{nn}.$$

Next, eliminate last column.

$$\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 & R_1 \leftarrow R_1 - (-1)R_3 \\ 0 & 2 & 0 & 2 & -1 & 1 & R_2 \leftarrow R_2 - (1)R_3 \\ 0 & 0 & 1 & -1 & 1 & 0 & \end{array}$$

General formula:

n^{th} column: $R_j \leftarrow R_j - a_{jn} R_n.$

STEP 3: Scale 2nd row by a_{22} :

$$R_2 \leftarrow R_2 / a_{22}$$

$$\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}$$

Eliminate 2nd col:

$$\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & \frac{3}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array} \quad R_1 \leftarrow R_1 - (1)R_2$$

Since the matrix on the left is the identity matrix, the matrix on the right is A^{-1} .

CHK:

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 0 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} -1 & \frac{3}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & \frac{1}{2} \\ -1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ as req'd.}$$