Quiz 2

Choose 2 of the following 3 problems:

(1) Given a function $g : [a, b] \to [a, b]$ such that $|g'(x)| \le K < 1$, on (a, b), there exists a unique fixed point in the interval $[a, b]$. The fixed point iteration scheme $x_n = g(x_{n-1})$ converges to this fixed point. Show that in general, this convergence is linear. (hint, taylor expand about the fixed point).

(2) Let $g(x) = 2^{-x}$. g has unique fixed point in $[1/3, 1]$, and $|g'(x)| \le 3/4$. Using the error formula given in class, what is the maximum amount of iterations needed before the iterates converge to within 10^{-3}, assuming $x_0 = 1$. (Note, don't find the exact value, just solve the equation for n)

(3) Let $f(x) = x^3 - 1$, and let $x_0 = 2$. Find the first two iterates x_1, x_2 of Newton's method applied to f. (Do not simplify x_2).