Lab 3: Closed Loop Response of Control Systems

Thomas Ledford (tll160130)

Alejandro Pasillas (axp171430)

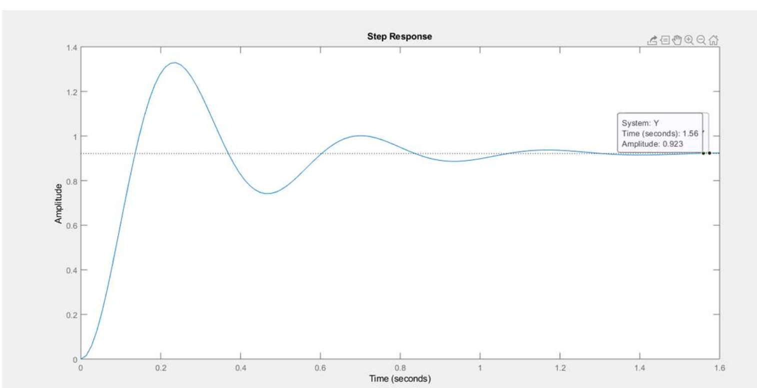
1. **Tracking**

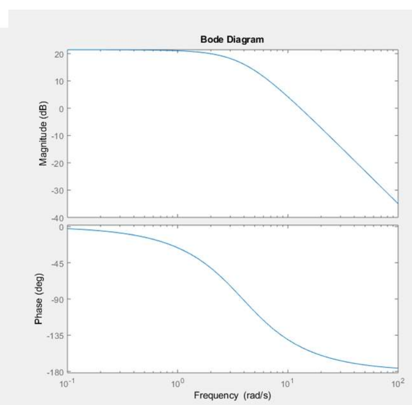
(1)

```
r =

  -3.4890 + 1.7175i
  -3.4890 - 1.7175i
```

Steady state error

```
ans =

    0.0784
```

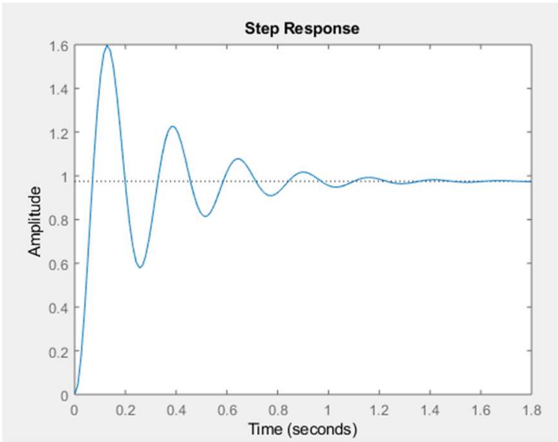


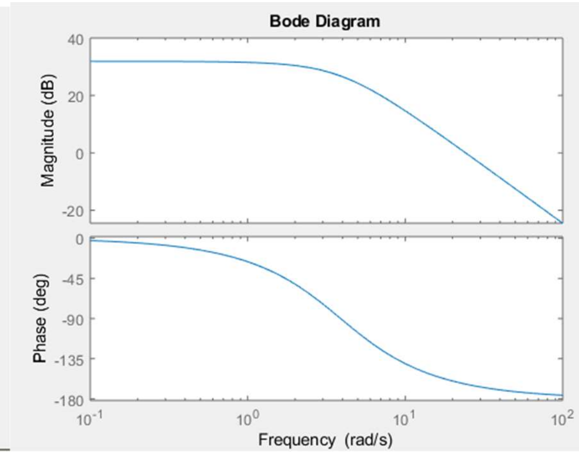


(2)

Steady state error

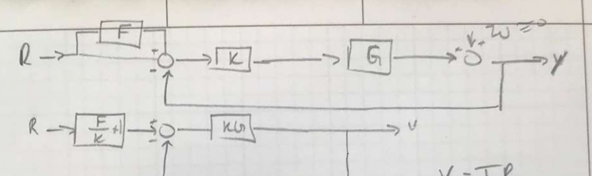
```
ans =

    0.0249
```

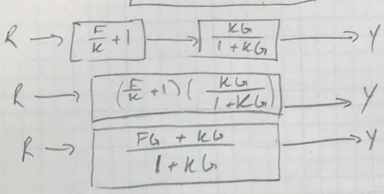




The transient response between K= 10 and K = 3 is represented on the overshoot and the steady state error which is much smaller for K =10 meaning as our graph approaches infinity the error difference is much closer to 1 than K=3

(3)





$Y = TR$

$T = \frac{FG+KG}{1+KG}$

Solve for $T=1$

$\frac{FG+KG}{1+KG} = 1$

$F = \frac{1+KG-KG}{G}$

$\boxed{F = \frac{1}{G}}$

$E = R - Y = R - TR = R(1-T)$

$E = R\left(1 - \frac{FG+KG}{1+KG}\right)$

F as constant; $(K=3)$

$e_{ss} = \lim_{s\to 0} s\cdot\left(\frac{1}{s}\right)\left(1 - \frac{FG+KG}{1+KG}\right) = 0$

$e_{ss} = \lim_{s\to 0} \left(1 - \frac{59.292F + 3(59.292)}{(s^2+6.798s+15.123) + 3(59.292)}\right) = 0$

$e_{ss} = 1 - \frac{59.292F + 3(59.292)}{15.123 + 3(59.292)} = 0$

$e_{ss} = 15.123 + \cancel{3(59.292)} - 59.292F - \cancel{3(59.292)} = 0$

$e_{ss} = F = 0.2551$

Generalized: $e_{ss} = \lim_{s\to 0}\left(s\cdot\frac{1}{s}\right)\left(1 - \frac{F_{constant}G(0) + K(0)G(0)}{1+K(0)G(0)}\right) = 0$

$\to e_{ss} = \lim_{s\to 0}\left(1 - \frac{F_c G(0) + K(0)G(0)}{1+K(0)G(0)}\right) = 0$

$1 - \frac{F_c G(0) + K(0)G(0)}{1+K(0)G(0)} = 0$

$F_c G(0) + K(0)G(0) = 1 + K(0)G(0)$

F=

$\boxed{F = \frac{1}{G(0)}}$

From the picture, you can see that the value of “F” for perfect steady-state tracking error would be 0.2551.

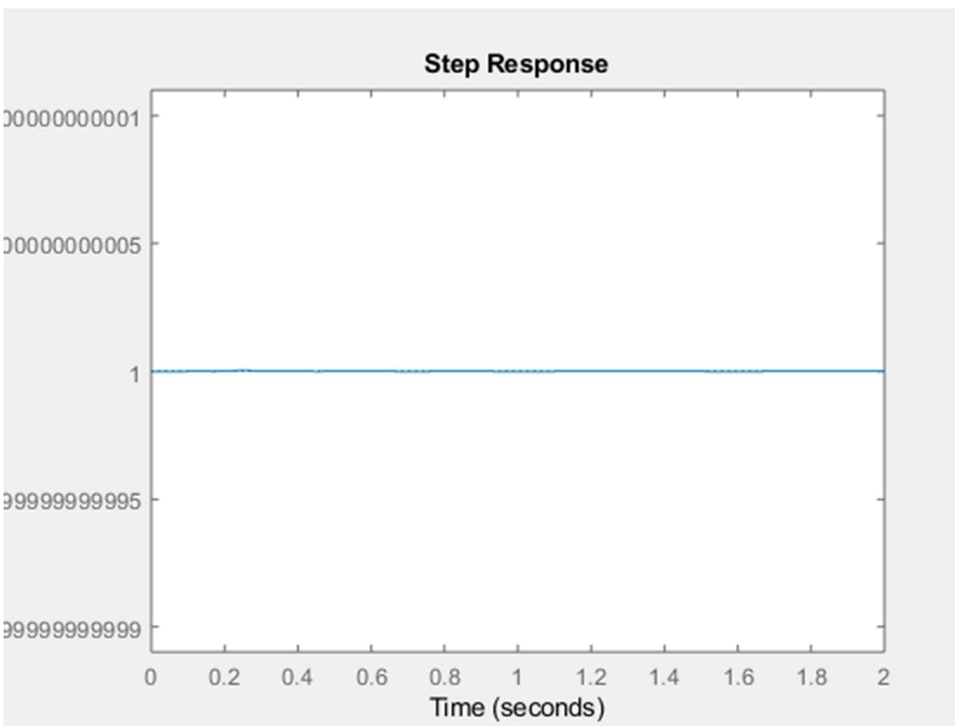
Description:

Similarly to in part 3, we can find what happens to our system when s is 0 (G=G(0)). With this G(0), which is more likely for us to know because we can understand our starting point a lot easier than our end point, we can use this information later in our studies to better monitor the steady-state tracking error of our system.

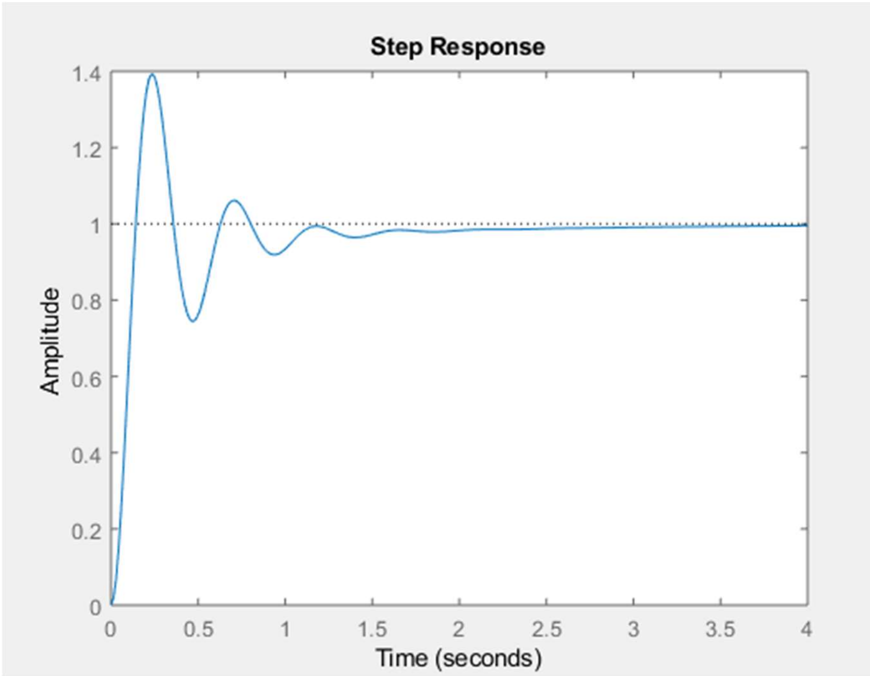
```
ans =

  -1.7793e-16
```

Plot:

Here you can clearly see that Y=R meaning that you have 0 steady-state tracking error.

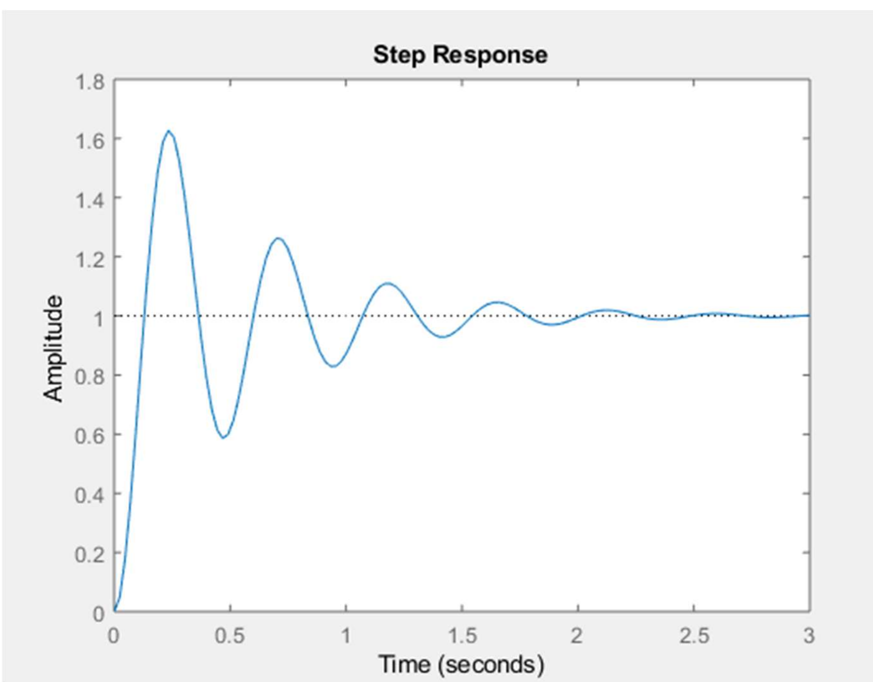


(4)



Step response which shows the steady-state tracking error when Kp = 3 and KI=2.

(5)



Step response showing the steady-state tracking error shown when Kp =3 and KI = 10.

Looking at the feedforward function, you implement a transfer function “F” to make your steady-state tracking error zero. You do this by finding “F” as (1/G(0)) and then using FVT to demonstrate that this feedforward function does actually make your steady-state error go to zero (Shown in the picture under “Generalized”.

For the PI controllers, instead of implementing a transfer function “F” we are now changing our controller to get our zero-tracking error. This would directly impact our “Y” value which is equal to ((G*K)/(1+G*K)). In the case of 4), we are using an integral controller to slowly integrate our error from our original 7.84% to 0 over about a 4 second period (Kp = 3, KI = 2). To contrast this with part 5), the higher kI value allows use to more quickly adjust for this error so that instead of slowly integrating the error to zero, the overshoot goes higher and oscillates to zero quicker.

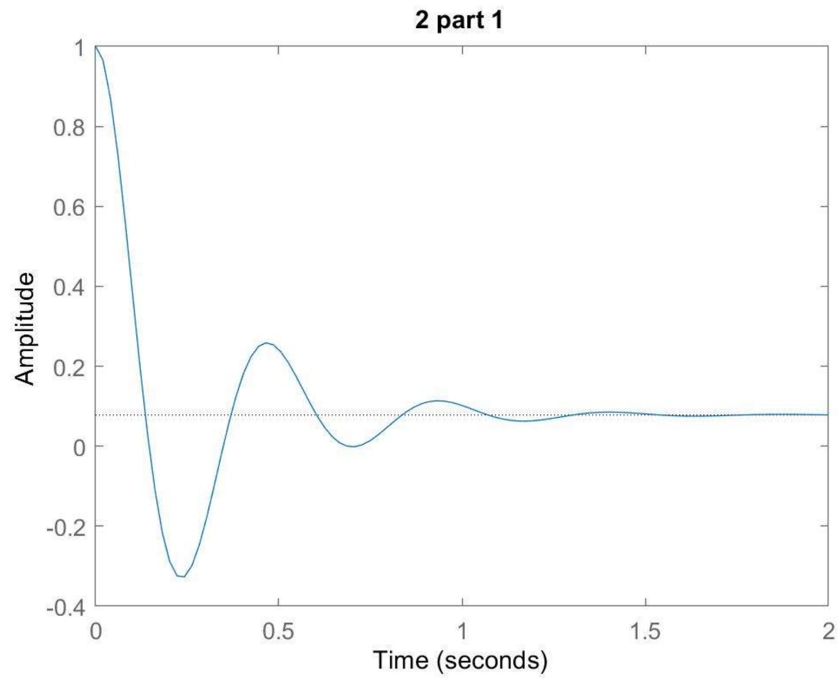
Table 1. Pros and Cons of Feedforward Vs. PI Controllers

	Pros	Cons
Feedforward	• Easier to find • Works well with original block diagrams	• Not very robust • Requires more changes to your original block diagrams
Lower Integral Controller	• Changing only the controller • Less oscillation. • Lower overshoot and settling time. • Quite robust	• Takes longer to get zero steady state tracking error.
High Integral Controller	• No slow integration to zero steady-state tracking error. • Extremely robust	• More oscillation • Higher overshoot and settling time.

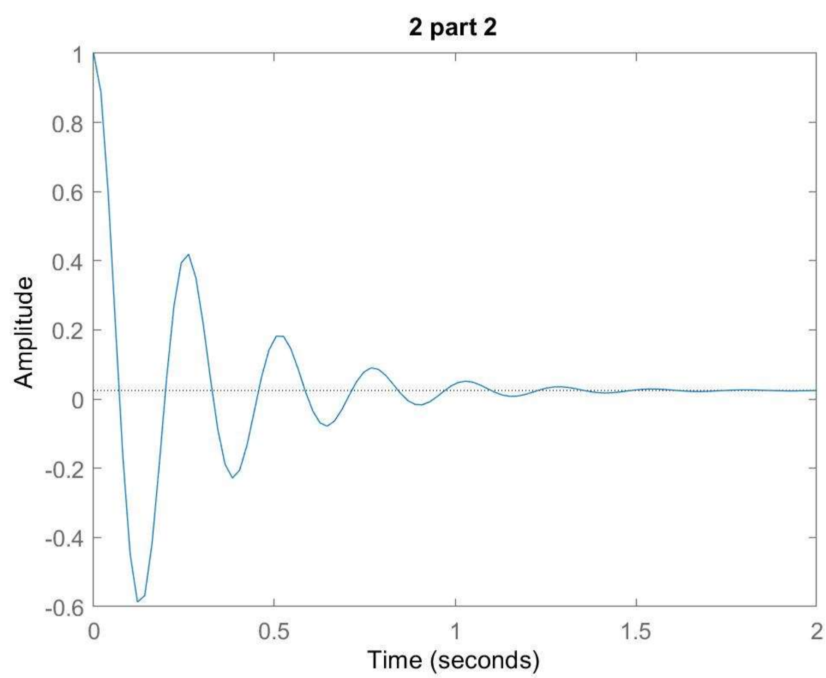
2. Disturbance Rejection

```
ess_21 =

     0


ess_22 =

     0


ess_23 =

     0


ess_24 =

     0


ess_25 =

     0
```

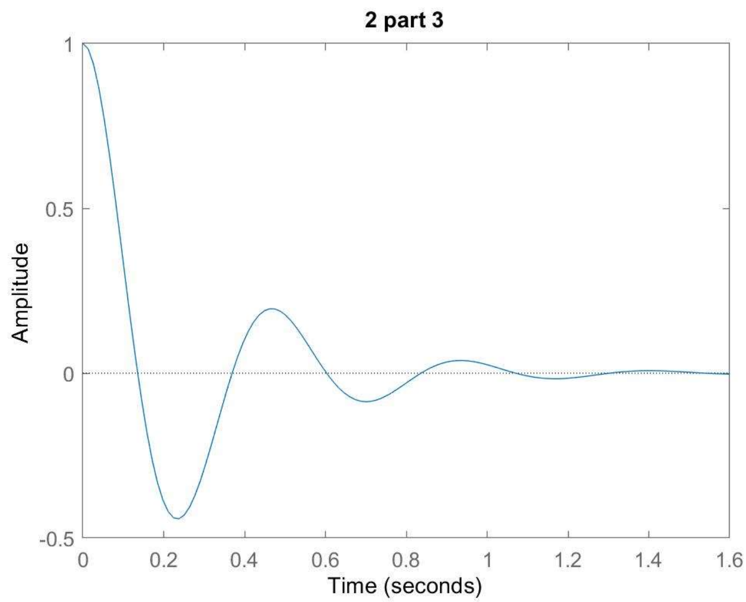
1)


2 part 1
Amplitude
Time (seconds)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
0.5
1
1.5
2

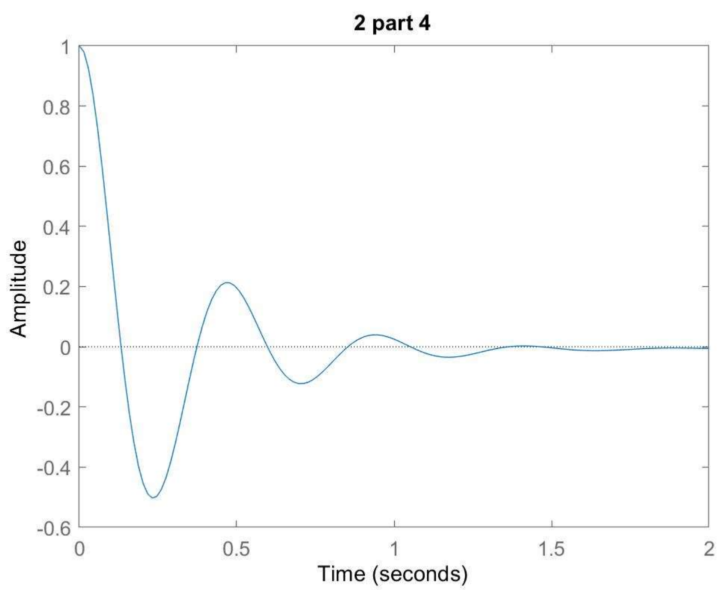
2)


2 part 2
Amplitude
Time (seconds)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
0.5
1
1.5
2

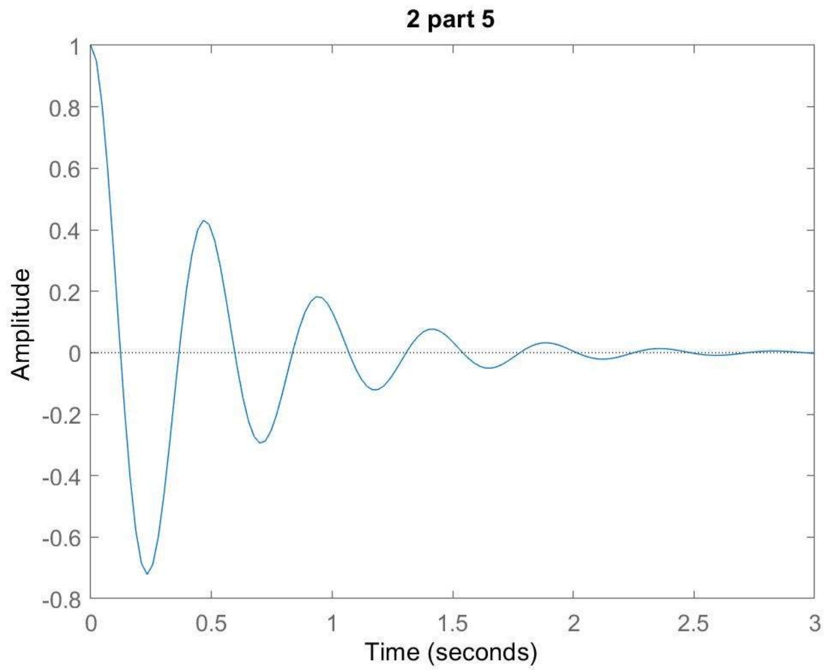
3)


2 part 3
1
0.5
0
-0.5
Amplitude
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
Time (seconds)

4)


2 part 4
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
Amplitude
0
0.5
1
1.5
2
Time (seconds)

5)

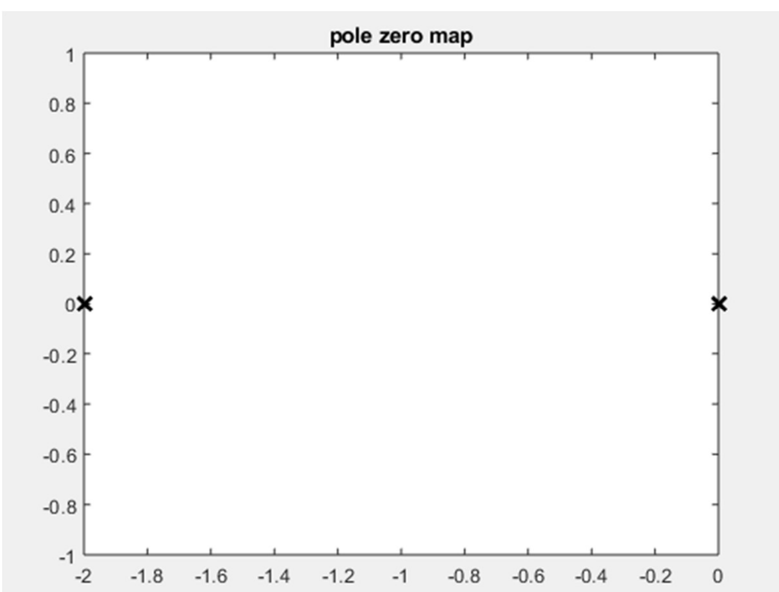


Proper value for the feedforward gain should be F = 1/G(0) = 3.92 in this case.

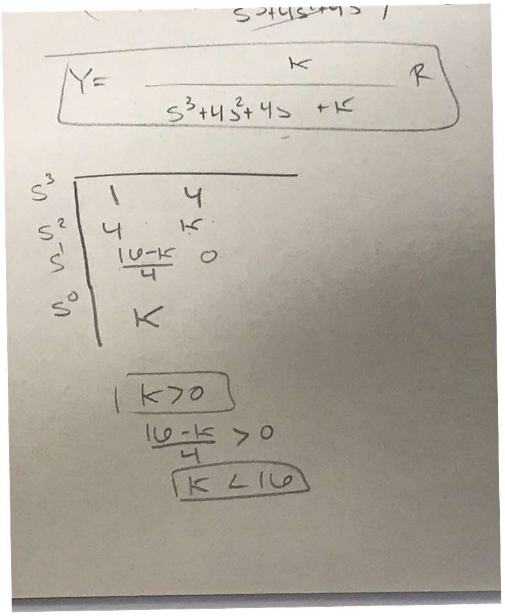
3. Stability through Dynamic Pole-Zero Maps

3.1 System 1

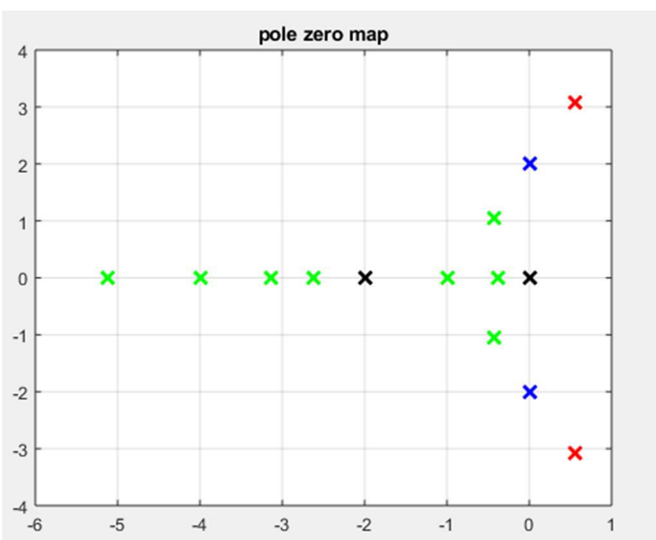
(1)



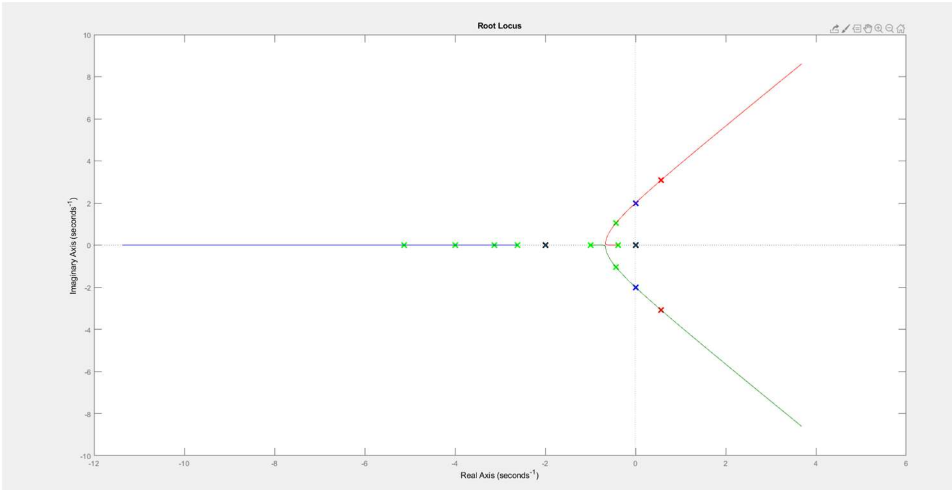
(2)



(3)



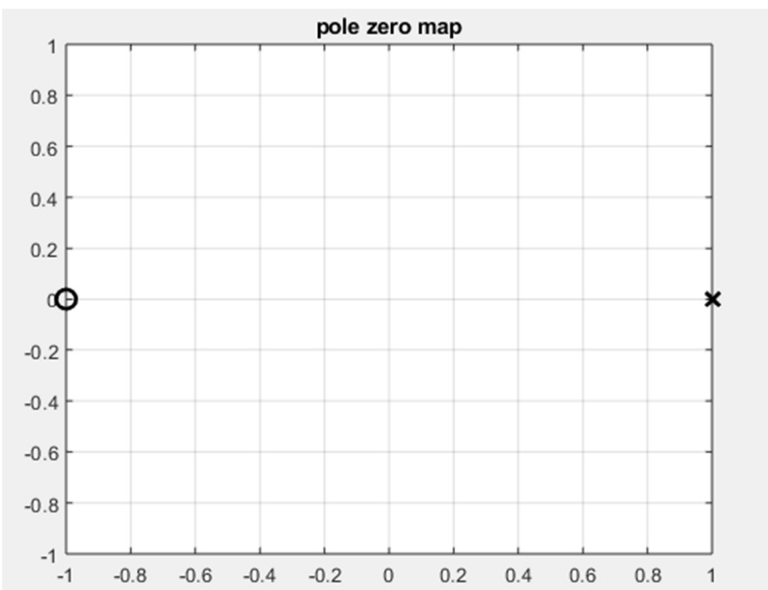
From our plot we can see that we only have stability between 0 and –4, but this stability will fall off as you become more negative or become more positive. This is shown in the Routh array through k > 0 && k < 16. Therefore, two differ based on the square of the k value putting it between 0 and 16.



We used rlocusplot to plot this picture however our code doesn't relay this. In our code we trued making the function however we kept getting an error.

3.2 System 2

(1)



As you change this value, you will be changing your zeros only. This would not impact your stability at all. Therefore, this situation is useful because you are just ensuring that your controller is keeping your zeros in order.

(2)

$$\frac{s^2\ k(s+1)}{(s-1)^2+k(s-1)}$$

$$s^2-2s+1+ks-k$$

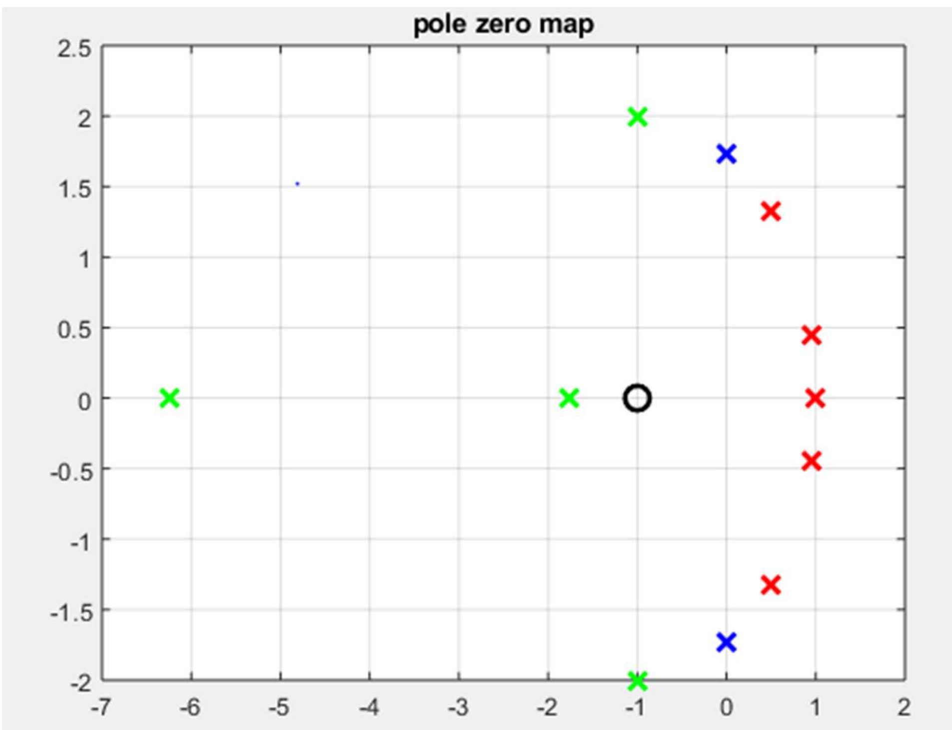
$$s^2+(-2+k)s+(k+1)$$

1	k+1
-2+k	0
k+1	

$-2+k>0 \quad k>2$

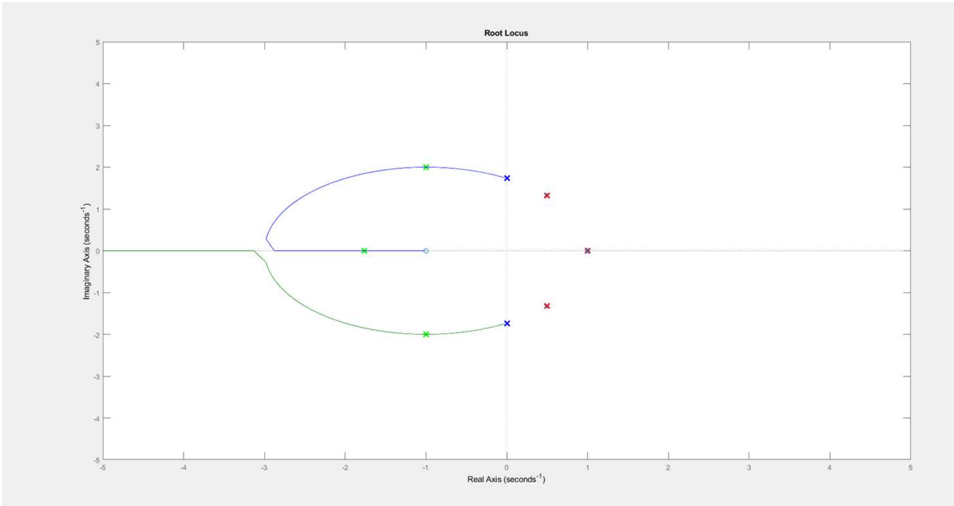
$k+1>0 \quad k>-1$

(3)

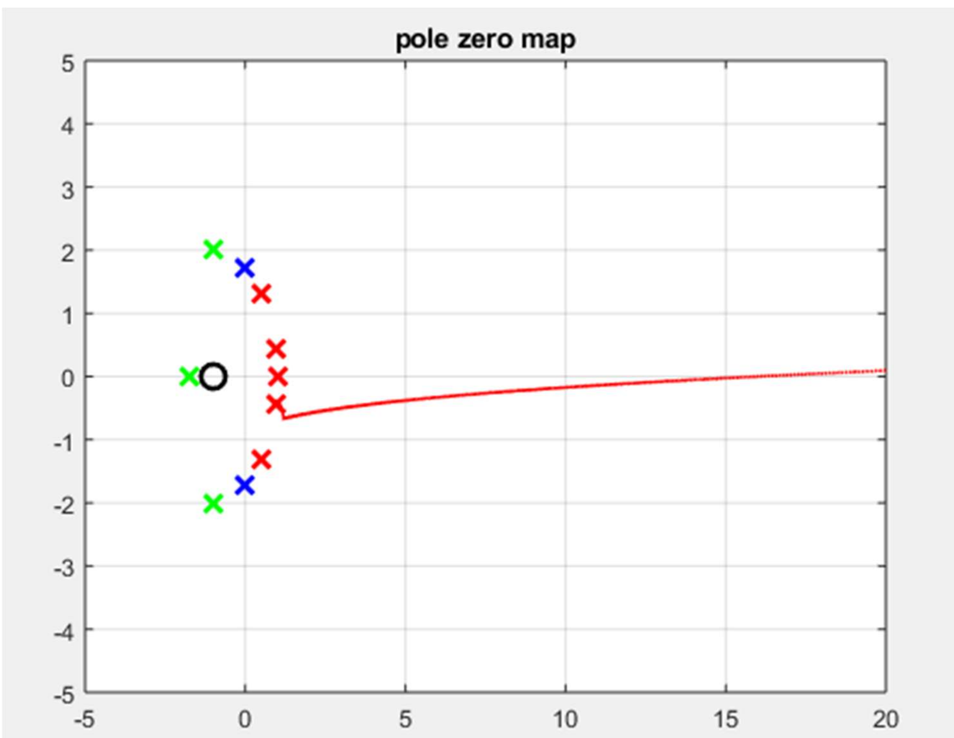


In this problem, we have the stability in the ranges of k>2. This is presented through our work on the Routh array. Therefore, we would go from unstable at 0 to 2 and then become stable from 2 to infinity.

(4)



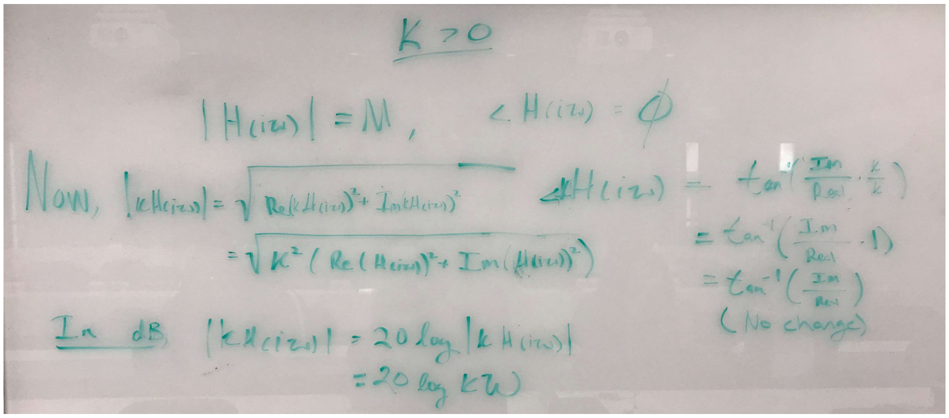
We used rlocusplot to plot this picture however our code doesn't relay this. In our code we trued making the function however we kept getting an error.



This is what our function gives us if we actually tried to plot the rootlocus as a line. This needed to be commented out due to the error created through the attempt to plot.

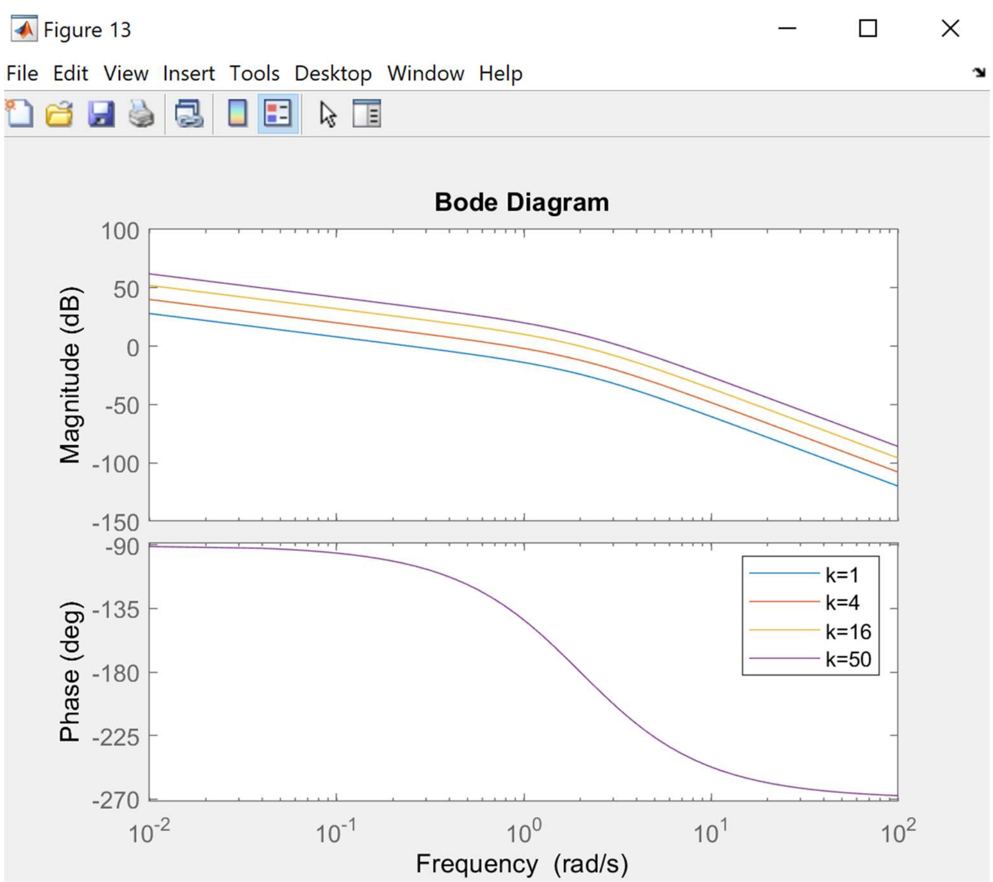
4. Bode Plots and Stability

What happens when you multiply your transfer function H by a positive constant “k”?

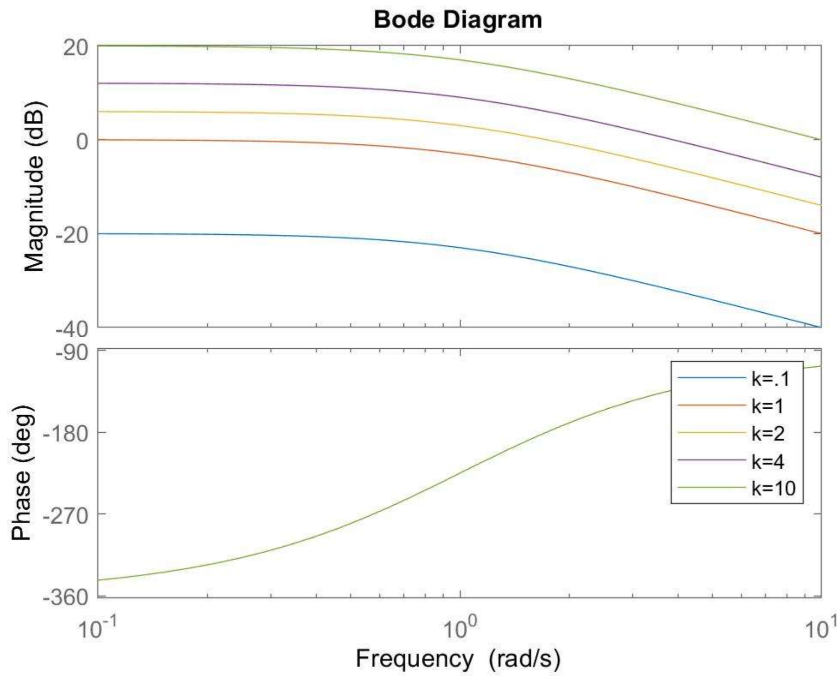


From my writing, you can see that the magnitude for the bode would be impacted proportionally by the multiplication of this positive multiplication. This is also the case as it pertains to the dB values for the bode as well. However, from my math you can tell this is not the same for the phase angle. I will talk more about this later.

Bode Plot for 3.1:



Bode Plot for 3.2:



What did we observe?

From both of our bode plots, you can see the cases I presented before unfolding. For instance, when observing the magnitude of the bode plot you can see that the k values determine the dB values at which the bode plot is formed. Meaning that the higher your k value, the higher your plot for the magnitude. Also, the inverse the true. Decreasing the value of your k, between 0 and 1, would shift your magnitude down. Finally, the phase of the bode diagrams, from the math I presented in the first part of this section, would not be impacted by any value of positive k.

What is the connection between the bode plot and stability (pay attention to the location of the dB crossing in magnitude and the –180 crossing in the phase)?

Table for 3.1 Frequency values as they correspond to k values

K values	Frequency Values w (rad/s)
1	~0.25
4	~0.8
16 (End of Stability from Routh)	~2
50	~3.5

Looking at 3.1, we can see that the bode plot shows us that the stability is connected to the frequency value of w = 2 rad/s. Having a larger w, or a higher k value, would then start to make your system unstable. Also, from the phase that w = 2 rad/s would also be the crossing would be 180 degrees.

Table for 3.2 Frequency values as they correspond to k values

K values	Frequency Values w (rad/s)
0.1	Under zero
1	~0.01
2	~1.8
4	~4
10	~9

Looking at 3.2, we can see that the bode plot shows us that the stability is connected to the frequency value of w = 1.8 rad/s. Having a larger w, or a higher k value, would then start to make your system unstable.

Finally, when we look at the rearranging of this equation we found that the magnitude would be equal to zero and the phase angle would –180 degrees when your frequency, w, is 1 rad/s. Below is the proof of this idea:

$kH(i\omega) = -1$, $|kH(i\omega)| = 1$

$\Rightarrow |kH(i\omega)|_{dB} = 0$

Also, $\angle kH(i\omega) = \angle H(i\omega) = -180^\circ$

When, $\omega = 1$ rad/s