Physics 777: Introduction to Astrophysics

Problem Set 1

Due: Monday February 17, 2020

1 The Solar System

1. In a circular restricted three-body problem, where the third body has negligible mass compared to the other two, there are five stationary solutions called the Lagrange points.
   (a) Describe and derive all five Lagrange points for the Earth-Sun system. Ignore the small eccentricity of the Earth's orbit and assume that it is circular. Calculate the positions numerically and express the results in appropriate units.
   (b) There are no closed-form exact solutions, but analytical approximations can be found by Taylor expansion. Derive analytical solutions and calculate the corresponding numerical values.
2. The Hill sphere around an astronomical body is defined as the region within which it dominates the gravity of its satellites. The L1 and L2 points of a planet lie on the surface of its Hill sphere.
   (a) For each planet in the Solar System, calculate the Hill radius and compare it to the separation between the planet and the nearest neighboring planet. What does this tell you about how planets are located relative to one another?
   (b) If the planet has a moon, compare the Hill radius to the semi-major axis length of the most distant moon. What does this tell you about how satellites are located relative to their parent planet?
3. Compare the insolation temperature for each of the planets in the Solar System to the observed values.

2 Astronomical Observations

1. Optical broadband photometry is one way to 'sample' the SED of astronomical objects which is pretty "cheap". As we have discussed in class, stars can be approximated as blackbodies.
   (a) Analytically, show that for sufficiently high temperatures, optical broadband photometry with standard visible filters cannot distinguish between different effective temperature stars.
   (b) Numerically calculate (and plot) the the relation between $(g - r)$ color and the effective temperature, $T_{\rm eff}$, of a blackbody using the standard SDSS filters. Tabulated response curves of the SDSS $ugriz$ filters are included with this assignment.
2. The light rays coming from an object do not, in general, travel parallel to the optical axis of a lens or mirror system. In figure 1 there is an arrow representing some object on interest, located a distance p from the center of a simple converging lens of focal length f, such that $p > f$. Assume that the arrow is perpendicular to the optical axis of the system with the tail of the arrow located on the axis. To locate the image, draw two light rays coming from the tip of the arrow:
   - One ray should follow a path parallel to the optical axis until it strikes the lens. It then bends toward the focal point of the side of the lens opposite the object.
   - A second ray should pass directly through the center of the lens un-deflected. (This assumes that the lens is sufficiently thin.)

   The intersection of the two rays is the location of the tip of the image arrow. All other rays coming from the tip of the object that pass through the lens will also pass through the image tip. The tail of the image is located on the optical axis, a distance q from the center of the lens. The image should also be oriented perpendicular to the optical axis.

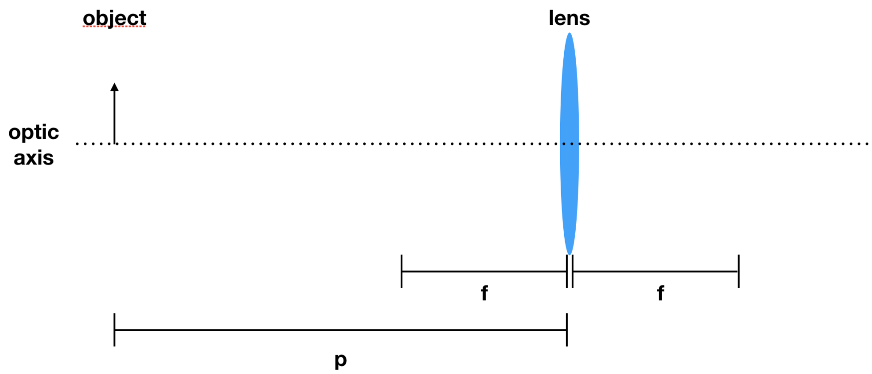

Figure 1: configuration for imaging an object a distance p away with a lens with a focal length f.

(a) Using similar triangles, prove the relation:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

(b) Show that if the distance of the object is much larger than the focal length of the lens ($p >> f$), then the image is effectively located on the focal plane. This is essentially always the situation for astronomical observations.

The analysis of a diverging lens or a mirror (either converging or diverging) is similar and leads to the same relation between object distance, image distance, and focal length.

3. Demonstrate the infeasibility of resolving any extra-solar star with visible light observations.

4. Prove that surface brightness is independent of the distance between an observer and an extended source (ignoring cosmological effects).

3 Exoplanets

1. In class we examined some of physical equations which relate observable quantities to planetary (e.g m_p, r_p, a_p, and i) and stellar parameters (e.g. m_* and r_*) for several exoplanet detection methods: astrometry, radial velocity, transit, and microlensing methods. In order to get a quantitative feel for the observables, look up or derive the equations and normalize the mass, distance, periods, etc., by appropriate astrophysical units. For example, stellar masses m_* should be given in solar units ($m_*/M_\odot$). Use appropriate units, and show all calculations.

   (a) Astrometry Method
      i. angular shift α
   (b) Radial Velocity Method
      i. semi-amplitude K
   (c) Transit Method
      i. flux decrement $\Delta F/F$
      ii. transit duration Δt
      iii. transit probability P
   (d) Microlensing Method

i. Angular Einstein radius θ_E

2. In a planet-star system, each of the bodies follows an elliptical orbit with the barycenter at one of the foci. In the radial velocity method, we can derive expressions for the line-of-sight position, z, and the radial velocity, v_r.

   (a) Derive or look up the radial velocity equation, $v_r(\theta)$.

   (b) For elliptical orbits, give a physical explanation for why the radial velocity curve $v_r()$ is not symmetrical about the $v_r = 0$ line over one orbital period.

   (c) Show that the time integral of v_r over one orbital period is exactly zero since there is no net motion.

3. Kepler-10b is the first confirmed terrestrial exoplanet, detected by the Kepler satellite in 2011. The parent star has mass Ms = 0.895 M, radius Rs = 1.05 R, effective temperature $T_{\text{eff}} = 5627$ K, and is located at a distance d = 173 pc from the Earth. The planet is in a circular orbit with a period P = 0.837 days. Transiting measurements yield a fractional flux decrement $\Delta F/F = 1.53 \times 10^4$ and a transit duration $\Delta t = 1.81$ hours. Radial velocity measurements give a semi-amplitude K = 3.32 m/s.

   (a) What is the density of Kepler-10b?

   (b) What angular accuracy is required to observe this system through astrometry?

   (c) What photometric accuracy is required to observe this system through direct imaging?