33-777

Today

I Scattering Processes
II Opacity
III Spectral Line Formation

The radiative transfer function describes how a ray of radiation is attenuated as it travels through a gas.

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

Today, we will focus on the physical sources of the absorption coefficient, the concept of opacity, and the effect on stellar spectra.

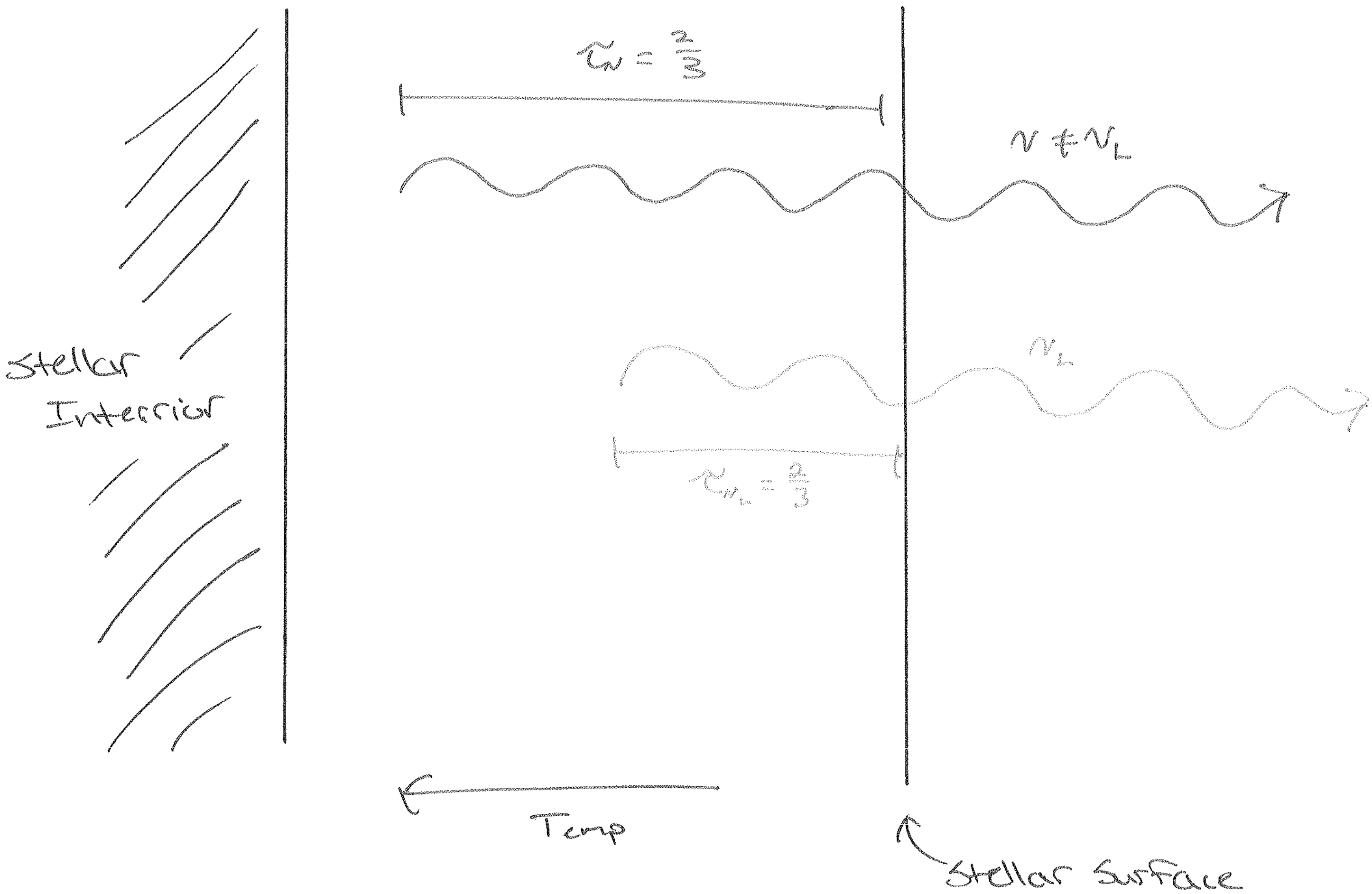
At this point, we have examined methods to solve for the source function, S_ν, and the intensity of radiation, I_ν, in stellar atmospheres. We saw that for a grey atmosphere (where the absorption coefficient, α_ν is independent of frequency) we can get a rough idea for the global properties of the stellar atmosphere. For example, we expect the temperature to increase with the depth below the surface

$$T^4 \simeq \frac{3}{4} T_{eff}^4 \left(\tau + \frac{2}{3}\right)$$

From this we showed that we "see" down to $\tau = \frac{2}{3}$. For a grey atmosphere, this is true for all frequencies, i.e. an optical depth of $\tau = \frac{2}{3}$ corresponds to the same physical depth below the stellar surface.

However, in a real stellar atmosphere, the absorption coefficient is a function of ν.

Consider a toy stellar atmosphere where the absorption coefficient is constant except over a narrow frequency range centered on ν_L.



$$\alpha(\nu) = \begin{cases} \alpha_L & \text{if } \nu = \nu_L \\ \alpha_c & \text{else} \end{cases}$$

$+$

$$\alpha_L > \alpha_c$$

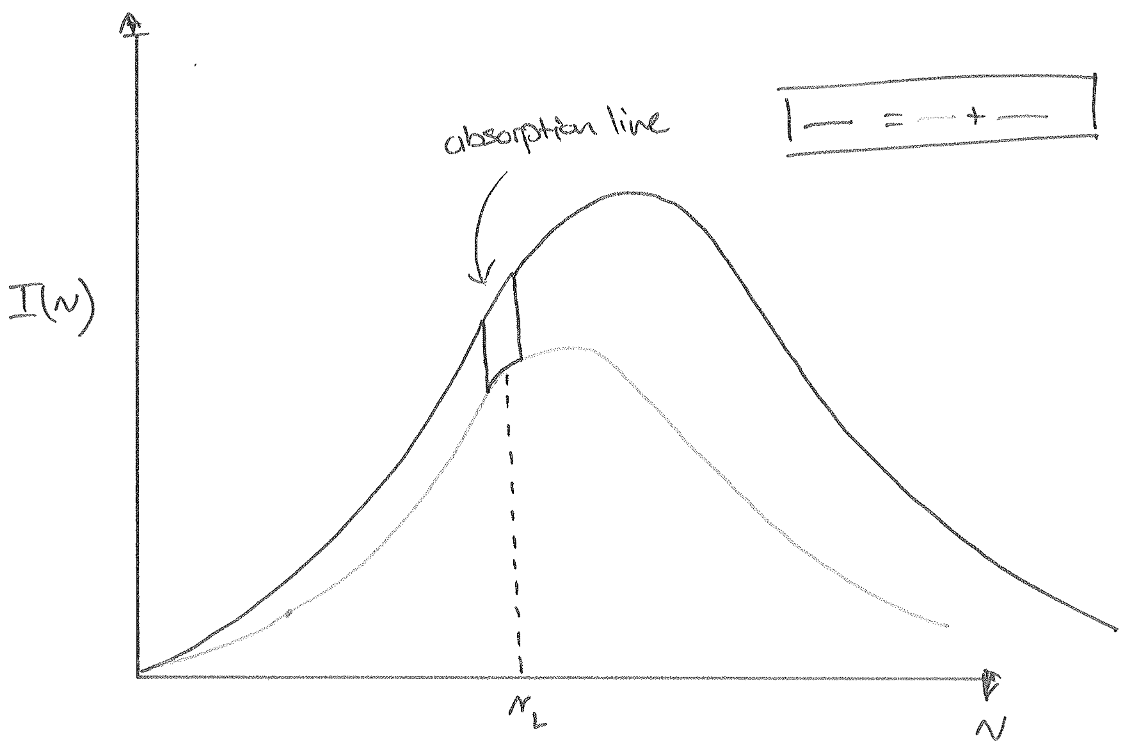
We saw that under certain circumstances in stellar atmospheres,

$$\langle I \rangle \simeq S_\nu = B_\nu(T)$$

Because the optical depth associated with the frequency ν_L corresponds to a shallower depth

$$\Rightarrow \quad T_L < T_c$$

At the location of an absorption line, we see blackbody emission with a lower effective temperature.



To understand stellar spectra in more detail it becomes necessary to talk about the interaction between photons and matter in more detail.

First, recal that for a collection of particles, we can define the <u>mean free path</u>, ℓ, as the average distance between collisions.

$$\ell = \frac{1}{n\sigma}$$

n: number density

σ: collisional cross section

In the classical picture, the cross section is a geometric quantity where

$$\sigma \equiv \pi(2r)^2$$

r: radius of particle

However, for quantum mechanical particles we will talk about effective cross sections which describe the "likelihood" of a photon being scattered or absorbed by a target particle, typically an electron or atom.

We can relate the effective cross section, σ_ν, to the absorption coefficient, α_ν

$$\sigma_\nu n = \alpha_\nu$$

Finally, we can define the <u>opacity</u> as the mass absorption coefficient, κ_ν,

mass density

$$\kappa_\nu \rho = \alpha_\nu$$

These quantities all define how effectively the intensity is attenuated along a ray path.

$$d\tau_\nu = \sigma_\nu n\, ds = \kappa_\nu \rho\, ds = \alpha_\nu ds$$

The interactions that can attenuate the intensity of a ray can be split into two categories:

1. scattering
2. absorption

Let's first deal with scattering processes, which we will define as any process where,

$$\text{photon} + \text{matter} \longrightarrow \text{photon} + \text{matter}$$

Note that we can ignore photon - photon scattering as this is extremely rare for any astrophysical process we will consider.

Scattering processes can be split into:

- elastic ("coherent")
- inelastic ("incoherent")

Photon - Matter elastic scattering comes in three forms:

- Thomson Scattering

$$\gamma + e \rightarrow \gamma + e$$

- Resonant Scattering

$$\gamma + X \rightarrow X^{*} \rightarrow \gamma + X$$

- Rayleigh Scattering

$$\gamma + X \rightarrow \gamma + X$$

Key

γ - photon

e - free electron

X - atom/ion

X^{*} - excited atom/ion

Inelastic scattering comes in two forms

- Compton Scattering

$$\gamma + e \rightarrow \gamma' + e'$$

- Fluorescence

$$\gamma + X \rightarrow X^{**} \rightarrow \gamma' + X^{*} \rightarrow \gamma' + \gamma'' + X$$

Here γ', e' indicate a photon/electron with a different energy.

- Thomson Scattering

In thomson scattering, an electromagnetic wave encounters a free electron. When this occurs, the electron will sympathetically oscillate with the electric field of the EM wave.

The Thomson cross section is given by

$$\sigma_T = \frac{8\pi}{3} r_e^2 = \frac{8\pi e^4}{3 m_e^2 c^4} \simeq 6.65 \times 10^{-25} \, cm^2$$

This process is most relavent when

$$E_\gamma < m_e c^2 = 0.511 \, MeV$$

Note that for low enough energy photons this scattering process is independent of frequency.

- Compton Scattering

This is the high energy relative of Thomson scattering.

This is an inelastic scattering process where a photon transfers energy/momentum to a free charged particle. This is only important when the photon has energy comparable to the charged particles rest mass energy. For electrons this means

$$E_\gamma \gtrsim m_e c^2 \simeq 0.511 \text{ MeV}$$

This corresponds to X-rays and gamma rays.

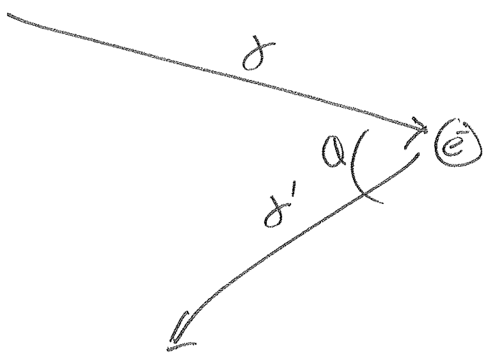
The cross section for compton scattering is given by the Klein-Nishina cross section, $\sigma_{KN}(\nu)$, which is a function of frequency. The general effect is that at low frequencies/energies ($E_\gamma < 0.511$ MeV) σ_{KN} approaches the thomson cross section. At higher frequencies, $\sigma_{KN} < \sigma_T$.

At high energies $\sigma_{KN}(\nu) \propto \dfrac{1}{\nu}$.

The outgoing photon has an energy

$$E'_\gamma = \frac{E_\gamma}{1 + \frac{E_\gamma}{m_e c^2}(1-\cos\theta)}$$

Here, θ is the angle between the incoming and outgoing photon path.



This can also be written in terms of wavelength

$$\lambda' - \lambda = \lambda_c(1-\cos\theta)$$

where λ_c is the compton wavelength

$$\lambda_c = \frac{h}{m_e c} \simeq 2.43 \times 10^{-10} \text{ cm}$$

This is not so relavent for our discussion of stellar atmospheres, but there is also another related process called <u>inverse Compton scattering</u>. Here electrons can transfer some of their kinetic energy to photons, increasing ν.

For (inverse) compton scattering the average change in photon energy is given by

$$\left\langle \frac{\Delta E_\gamma}{E_\gamma} \right\rangle = \frac{4 k_b T_e - h\nu}{m_e c^2}$$

Here T_e is the temperature of the electron gas,

$$T_e = \frac{m_e \langle v^2 \rangle}{3 k_b}$$

If,

$$4 k_b T_e > h\nu \Rightarrow \text{photons gain energy!}$$

The other type of process is <u>absorption</u>. The absorption of a photon by matter can have three effects,

1. heating of the medium
2. acceleration of the medium
3. change of state of the medium

- Heating

This process occurs by the excitation of particles in the gas followed by collisional de-excitation before spontaneous de-excitation occurs.

- Acceleration

Recal that photons carry momentum,

$$p = \frac{h}{\lambda}$$

If a photon is absorbed by matter, this momentum must be transfered to the medium.

The inverse compton effect is important for explaing various astrophysical phenomina like

- AGN
- Sunyaev - Zel'dovic (SZ) effect

We may look at these phenomina later in the course.

- Resonant scattering

This type of scattering is also known as "bound-bound scattering" and "line scattering". This occurs when photons scatter off electrons bound to an atom/ion.

With a quantum mechanical viewpoint, there are three regimes for scattering between a photon and a bound electron.

For an atom/ion with an electron transition with a corresponding energy change ΔE_{ij}, with an associated photon frequency ν_{ij}

① $\nu \gg \nu_{ij}$

$\sigma = \sigma_T$ (Thomson scattering)

② $\nu \simeq \nu_{ij}$

$$\sigma = \frac{\pi e^2}{m_e c} f_{ij} \phi_\alpha(\nu)$$

(Resonant scattering)

③ $\nu \ll \nu_{ij}$

$$\sigma = \sigma_T f_{ij} \left(\frac{\nu}{\nu_{ij}}\right)^4$$

(Rayleigh scattering)

For the high-energy photon case, the electron acts as it would if it were free. This is simply Thomson scattering.

In the low-energy photon case, the system behaves like a forced, damped, harmonic oscillator with natural frequency ν_{ij}.

This type of scattering is not so important for stellar atmospheres, but it is important for other astrophysical phenomina, e.g. this is the reason the sky appears blue, and the sun appears redder as it sets. The key is that Rayleigh scattering is more effective for bluer light.

Finally, when the incident photon has a frequency near a line frequency ν_{ij}, the scattering cross section may increase dramatically,

$$\sigma_{line} \gg \sigma_T$$

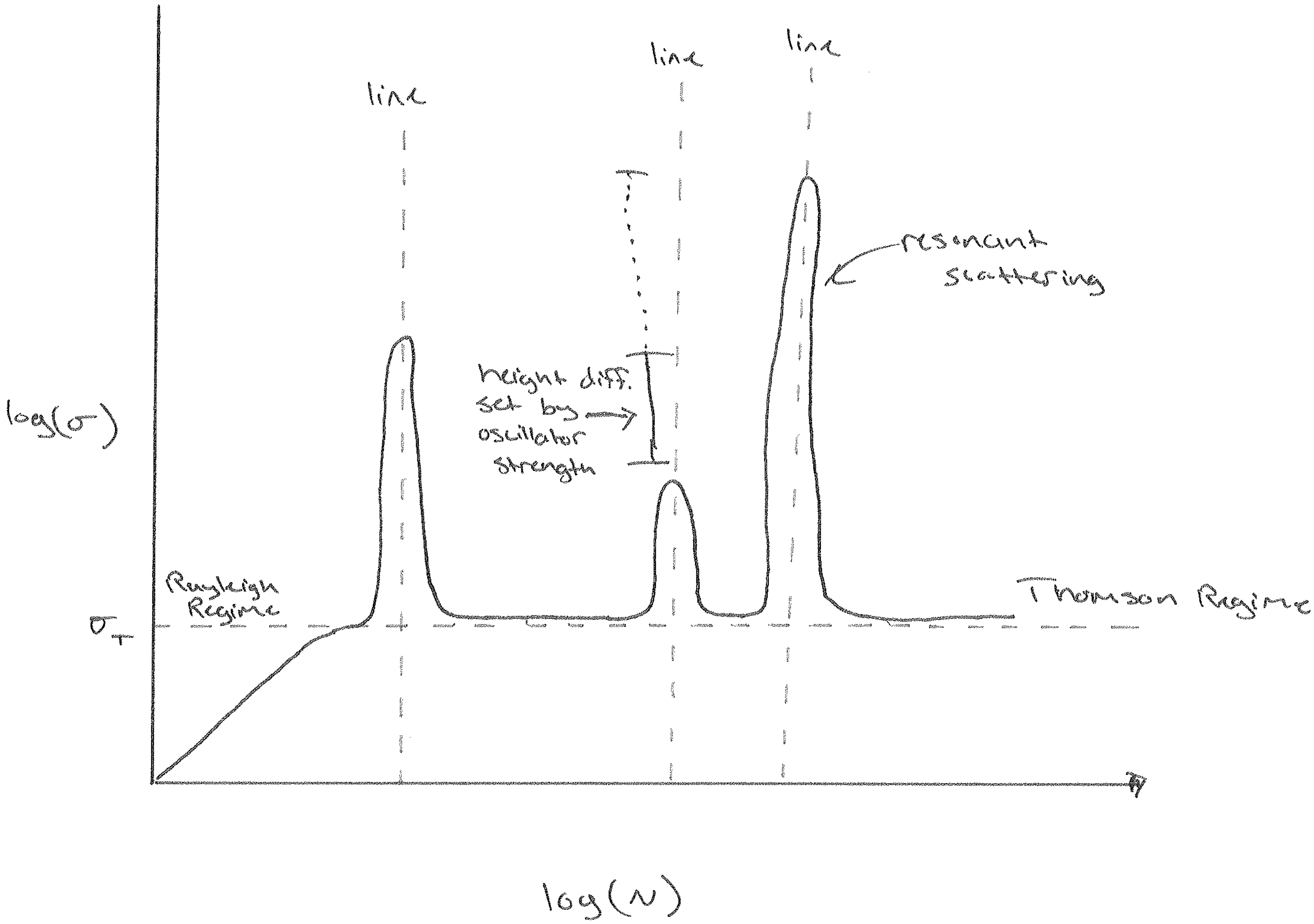
The constant f_{ij} is a dimensionless constant which expresses the strength of the $i \to j$ quantum transition. This is known as the <u>oscillator strength</u>. f_{ij} is approximatly proportional to the probability on incident photon of frequency ν_{ij} results in the atom making the $i \to j$ transition.

$\phi_L(\nu)$ is the <u>Lorentz profile</u> which accounts for "natural line broadening". This accounts for the slight mismatch in incoming and outgoing photon enery as required by the <u>Heisenberg Uncertainty Principle</u>,

$$\Delta E \Delta t \geq \hbar/2$$

time it takes for electron to spontaneously de-excite.

Given this overview of how photons scatter off bound electrons, we can sketch the scattering cross section of an atom with at least one bound electron.



One important thing to note about resonant scattering is that if the atom undergoes collisional excitment or de-excitation, the photon is "lost" and we have an absorption process.

– Fluorescence

This last scattering process is closely related to resonant scattering. Here an incident photon must excite an electron by at least two energy states. The electron may then cascade down any number of intermediate states. As a result, the incident photon's energy is changed into one ore more photons of differing energy.

- Change of state

Finally, an absorbed photon's energy may go into changing the state of the matter. This may be,

* ionization
* sublimation
* dissociation

An important example in astrophysics is the photo-ionization of hydrogen. The cross section depends on the principle quantum state n via,

$$\sigma = (1.31 \times 10^{-19}) \left(\frac{1}{n}\right)^5 \left(\frac{\lambda}{500\ \text{nm}}\right)^3 \ m^2$$

units

Photo-ionization is an important source of opacity in stellar atmospheres. Hydrogen in the first excited state can be ionized by a photon with energy $E_\gamma \geq 3.40$ eV.

This corresponds to a wavelength of 364.7 nm. The Sun shows an abrupt drop in its continious spectrum at this wavelength. This feature is called the Balmer jump.

For less energetic photons, the primary source of opacity is the H^- ion. This is a hydrogen atom with a loosely bound second electron. The binding energy of this second electron is 0.754 eV, corresponding to 1640 nm wavelength photon.

In detail, the opacity in a stellar atmosphere has contributions from many sources,

Photoionization

$$\kappa_\nu = \kappa_{\nu,bb} + \kappa_{\nu,bf} + \kappa_{\nu,ff}$$

resonant scattering

Thomson + Compton scattering

For some purposes, we are not concerned with the detailed, frequency dependent, opacity function κ_ν. Insted it is useful to consider the "total" opacity of the material. For this purpose, it is common to use the Rosseland mean opacity, $\bar{\kappa}$

$$\frac{1}{\bar{\kappa}} = \frac{\int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial B_\nu(T)}{\partial T} \, d\nu}{\int_0^\infty \frac{\partial B_\nu(T)}{\partial T} \, d\nu}$$

This is simply the weighted average of the opacity over the planck frequency spectrum.

Entire careers are spent calculating opacities for different compositions, temperatures, and densities, so we will not go over this in too much detail.

Here are a few rule's of thumb for the scaling of $\overline{\kappa}$ given here without any derivation.

- photo-ionization + Compton scattering

$$\overline{\kappa} \propto \frac{\rho}{T^{3.5}} \leftarrow \text{mass density}$$

This type of opacity scaling is known as Kramers opacity law

- Thomson scattering

$$\kappa = \text{const.}$$

- H^- opacity

$$\kappa \propto \sqrt{\rho}\, T^9$$

One interesting application of this is to place an upper-limit on the luminosity of a star.

Consider an outer layer of a star with thickness l and opicity $\bar{\kappa}$ such that

$$\tau = \kappa \rho l = 1$$

If this is the case, most photons will be absorbed that pass through this layer. In this case, the photons apply a force to the layer,

luminosity of star

$$F_{rad} = \frac{L_*}{c}$$

We can set this equal to the gravitational force holding onto the layer

$$F_{grav} = \frac{G M_* M_{layer}}{r^2}$$

If we note that the mass of the layer is given by,

$$M_{layer} = 4\pi r^2 l \rho = \frac{4\pi r^2}{\bar{\kappa}}$$

By setting $F_{rad} = F_{grav}$ and solving for luminosity, we arrive at the Eddington Luminosity,

$$L_{Edd} = \frac{4\pi G M_* c}{\bar{\kappa}}$$

Stars that have a luminosity $L_* > L_{edd}$ would blow themselves apart.

– Spectral Line Profiles

The shape of the absorption lines in stellar spectra are influenced by three phenomena

- natural broadening

This is a consequence of the Heisenberg uncertainty principle

$$\Delta E \, \Delta t \geq \frac{\hbar}{2}$$

This corresponds to a line "width"

$$\Delta \lambda \simeq \frac{\lambda^2}{\pi c} \frac{1}{\Delta t}$$

where Δt is the lifetime of the excited atomic state. Typical values for Δt are $\sim 10^{-8}$ s. The detailed shape of this broadening effect is given by the <u>Lorentz profile</u>.

- Thermal Doppler Broadening

This effect is caused by the thermal motions of particles in a gas. The wavelengths of the absorbed or emited photons will be doppler shifted such that

$$\frac{\Delta\lambda}{\lambda} = \frac{\pm |V_r|}{c}$$

Recal that the most probable velocity in a gas in T.E. is,

$$V = \sqrt{\frac{2KT}{m}}$$

The line width for this phenomena is

$$\Delta\lambda = \frac{2\lambda}{c}\sqrt{\frac{2KT\ln(2)}{m}}$$

- Pressure Broadening

Through close encounters with charged particles, the quantum states can be perturbed. The net result of this is similar to natural broadening.

$$\Delta \lambda = \frac{\lambda^2}{c} \frac{n\sigma}{\pi} \sqrt{\frac{2kT}{m}}$$

The combination of all these effects produces the total line profile, called the Voigt Profile.