performance_attribution_solution

April 24, 2023

1 Performance Attribution

In this exercise, we're going to use artificial data and a PCA risk model to show you the nuts and bolts of performance attribution. We use artificial data here so that you can focus on the calculations without having to worry about extra packages and data details. Let's get started!

```
In [1]: import pandas as pd
        import matplotlib.pyplot as plt
        import numpy as np
        import scipy.stats as stats
        import plotly as py
        import plotly.graph_objs as go
        import helper
        from scipy.stats import zscore
        import statsmodels.api as sm


        py.offline.init_notebook_mode(connected=True)
        %matplotlib inline
        plt.style.use('ggplot')
```

```
/opt/conda/lib/python3.6/site-packages/statsmodels/compat/pandas.py:56: FutureWarning:

The pandas.core.datetools module is deprecated and will be removed in a future version. Please u
```

1.0.1 Load the data

These will be our example price data.

```
In [2]: prices = pd.read_csv('stock_prices_attribution.csv', parse_dates=['date'], index_col=0)
```

```
In [3]: prices.head()
```

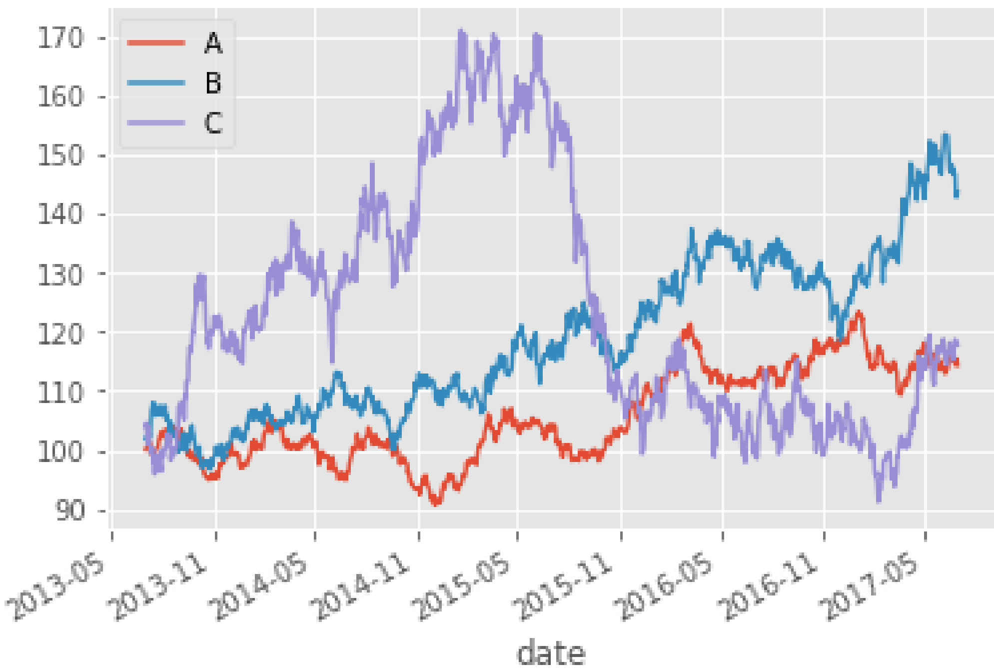
```
Out[3]:                        A           B           C
        date
        2013-07-02  100.000000  102.000000  104.000000
```

```
2013-07-03  100.350611  102.209243  104.589219
2013-07-05  100.724889  101.640083  100.913246
2013-07-08  100.742305  102.861043  103.910356
2013-07-09  101.360592  102.182024  101.238888
```

```
In [4]: rets = prices.pct_change()[1:].fillna(0)
```

```
In [5]: prices.plot()
```

```
Out[5]: <matplotlib.axes._subplots.AxesSubplot at 0x7f5e6ce88c88>
```



This will be our example alpha factor.

```
In [6]: # 1-yr momentum alpha

        def log_returns(series):
            return np.log(series[-1])-np.log(series[0])

        alpha = prices.rolling(window=252).apply(log_returns, raw=True).rank(axis='columns').app
```

```
In [7]: # Take most recent set of values
        alpha_vector = alpha.iloc[-1]
        print(alpha_vector)
```

```
A   -1.224745
B    0.000000
C    1.224745
Name: 2017-06-30 00:00:00, dtype: float64
```

1.0.2 Risk model

Let's build a PCA risk model.

In [8]:
```
m = rets.mean()
rets = rets - m
```

In [9]:
```
from sklearn.decomposition import PCA

class RiskModelPCA():

    ANN_FACTOR = 252

    def __init__(self, num_factors):
        self._num_factors = num_factors
        self.num_stocks_ = None
        self.factor_betas_ = None
        self.factor_returns_ = None
        self.common_returns_ = None
        self.residuals_ = None
        self.factor_cov_matrix_ = None
        self.idio_var_matrix_ = None
        self.explained_variance_ratio_ = None

    def fit(self, returns):
        self.num_stocks_ = len(returns.columns)
        mod = PCA(n_components=self._num_factors, svd_solver='full')
        mod.fit(returns)

        self.factor_betas_ = pd.DataFrame(
            data=mod.components_.T,
            index=returns.columns
        )

        self.factor_returns_ = pd.DataFrame(
            data=mod.transform(returns),
            index=returns.index
        )

        self.explained_variance_ratio_ = mod.explained_variance_ratio_

        self.common_returns_ = pd.DataFrame(
            data=np.dot(self.factor_returns_, self.factor_betas_.T),
            index=returns.index
        )
        self.common_returns_.columns = returns.columns

        self.residuals_ = (returns - self.common_returns_)
```

```
        self.factor_cov_matrix_ = np.diag(
            self.factor_returns_.var(axis=0, ddof=1)*RiskModelPCA.ANN_FACTOR
        )

        self.idio_var_matrix_ = pd.DataFrame(
            data=np.diag(np.var(self.residuals_))*RiskModelPCA.ANN_FACTOR,
            index=returns.columns
        )

        self.idio_var_vector_ = pd.DataFrame(
            data=np.diag(self.idio_var_matrix_.values),
            index=returns.columns
        )

        self.idio_var_matrix_.columns = index=returns.columns

    def get_factor_exposures(self, weights):
        B = self.factor_betas_.loc[weights.index]
        return B.T.dot(weights)
```

```
In [10]: rm = RiskModelPCA(2) # create an instance of the class with 2 factors
         rm.fit(rets.iloc[:-2,:]) # fit the model on all the data up to 2 days before the last d
```

The part that we're going to use from the risk model is the *risk model exposures*. Can you remember how to extract them from the fitted risk model?

```
In [11]: # TODO: Write the code to extract the "B" matrix from the fitted risk model.
         B = rm.factor_betas_
```

Let's make a plot to remember what we have so far.

```
In [12]: PC_scaler = 0.04 # The PC vectors have length 1, but this is larger than the scale of o

         # Trace for PC 0
         pc0 = np.vstack((np.full(3, 0), rm.factor_betas_[0].values)).T*PC_scaler

         hover_text4 = helper.generate_hover_text(pc0[0], pc0[1], pc0[2], 'Return of Stock A', '

         trace4 = go.Scatter3d(
             x=pc0[0],
             y=pc0[1],
             z=pc0[2],
             mode='lines+markers',
             marker=dict(
                 size=4,
                 color='#45B39D',
                 opacity=0.9,
                 symbol="diamond"
             ),
```

```
    line=dict(
        color='#45B39D',
        width=3
    ),
    name = 'PC 0',
    text = hover_text4.flatten(),
    hoverinfo = 'text'

)

# Trace for PC 1
pc1 = np.vstack((np.full(3, 0), rm.factor_betas_[1].values)).T*PC_scaler

hover_text5 = helper.generate_hover_text(pc1[0], pc1[1], pc1[2], 'Return of Stock A', '

trace5 = go.Scatter3d(
    x=pc1[0],
    y=pc1[1],
    z=pc1[2],
    mode='lines+markers',
    marker=dict(
        size=4,
        color='#FFC300',
        opacity=0.9,
        symbol="diamond"
    ),
    line=dict(
        color='#FFC300',
        width=3
    ),
    name = 'PC 1',
    text = hover_text5.flatten(),
    hoverinfo = 'text'

)

# Trace for data
hover_text6 = helper.generate_hover_text(rets['A'].iloc[:-2].values, rets['B'].iloc[:-2

trace6 = go.Scatter3d(
    x=rets['A'].values,
    y=rets['B'].values,
    z=rets['C'].values,
    mode='markers',
    marker=dict(
        size=4,
        color='#7FB3D5',
        opacity=0.3,
```

```
    ),
    name = 'daily returns',
    text = hover_text6.flatten(),
    hoverinfo = 'text'
)

data = [trace4, trace5, trace6]

layout = helper.create_standard_layout('Returns Data with Factor (PC) Directions', 'Ret
fig = go.Figure(data=data, layout=layout)
py.offline.iplot(fig)
```

1.0.3 Factor Returns

We're going to calculate the factor returns in a slightly different way than we did back when we studied risk models with PCA. We want to estimate the factor returns of our entire model, including both risk and alpha factors, so we're going to fit a least squares regression model to the risk factors and alpha factor we've come up with, in order to estimate the factor returns. We're going to run the regression for a single time period only—the last one our dataset covers.

```
In [13]: # create an r vector for a single time period (the last time period)
         r = rets.iloc[-1,:]
```

```
In [14]: estu = B.rename({0:'risk_factor_zero', 1:'risk_factor_one'}, axis='columns')
         estu['alpha'] = alpha_vector
         estu['return'] = r
         estu
```

```
Out[14]:    risk_factor_zero  risk_factor_one     alpha    return
         A          0.009675         0.018538 -1.224745 -0.007382
         B          0.019312         0.999638  0.000000  0.004861
         C          0.999767        -0.019489  1.224745 -0.002491
```

```
In [15]: # TODO: use statsmodels.OLS to fit an ordinary linear regression model to `return`, as
         # `risk_factor_one`, and `alpha`
         model = sm.OLS(estu['return'], estu[['risk_factor_zero', 'risk_factor_one','alpha']])
         results = model.fit()
```

Let's take a look at the results.

```
In [16]: factor_returns = results.params
         factor_returns
```

```
Out[16]: risk_factor_zero   -0.009776
         risk_factor_one     0.005051
         alpha               0.006027
         dtype: float64
```

The idiosyncratic term comes from the model's residuals.

```
In [17]: s = results.resid
         s
```

```
Out[17]: A    2.602085e-18
         B   -1.994932e-17
         C    4.336809e-19
         dtype: float64
```

1.0.4 Exposure Vector

Let's do our performance attribution for a single time period—the last one our dataset covers. Now let's assume that we've run our optimization and come up with a set of holdings we wish to take on Stocks A, B and C. Let's say we want to go \$500k short on Stock B and \$500k long on Stock C.

```
In [18]: h = pd.Series({'A': 0, 'B': -500000, 'C': 500000})
         h
```

```
Out[18]: A         0
         B   -500000
         C    500000
         dtype: int64
```

Now, let's add our alpha factor to the B matrix, so that B contains *both alpha and risk factors*, as explained in the lecture.

```
In [19]: B = B.rename({0:'risk_factor_zero', 1:'risk_factor_one'}, axis='columns')
         B['alpha'] = alpha_vector
         B
```

```
Out[19]:    risk_factor_zero  risk_factor_one     alpha
         A          0.009675         0.018538 -1.224745
         B          0.019312         0.999638  0.000000
         C          0.999767        -0.019489  1.224745
```

Now let's calculate the exposure vector: $\mathbf{B}^T\mathbf{h}$.

```
In [20]: # TODO: Calculate the exposure vector.
         E = B.T.dot(h)
         E
```

```
Out[20]: risk_factor_zero    490227.482019
         risk_factor_one    -509563.426113
         alpha               612372.435696
         dtype: float64
```

1.0.5 The profit calculation

The total portfolio pnl is $h^T r$.

```
In [21]: # TODO: Calculate the total pnl.
         total_pnl = h.T.dot(r)
         print('Total pnl: ', total_pnl)
```

```
Total return:  -3676.03547723
```

1.0.6 Performance Attribution

Now let's calculate the factor contributions.

```
In [22]: # TODO: Calculate the contribution to pnl due to alpha
         alpha_contribution = E['alpha']*factor_returns['alpha']
         alpha_contribution
```

```
Out[22]: 3690.6507934360025
```

```
In [23]: # TODO: Calculate the contribution to pnl due to factor zero
         factor_zero_contribution = E['risk_factor_zero']*factor_returns['risk_factor_zero']
         factor_zero_contribution
```

```
Out[23]: -4792.6745247506024
```

```
In [24]: # TODO: Calculate the contribution to pnl due to factor one
         factor_one_contribution = E['risk_factor_one']*factor_returns['risk_factor_one']
         factor_one_contribution
```

```
Out[24]: -2574.0117459142366
```

```
In [25]: total_factor_pnl =  alpha_contribution + factor_zero_contribution + factor_one_contribu
```

Now let's calculate the idiosyncratic pnl.

```
In [26]: # TODO: Calculate the idiosyncratic pnl.
         print('Idiosyncratic pnl: ', h.T.dot(s))
```

```
Idiosyncratic pnl:  1.01915004214e-11
```

```
In [ ]:
```

```
In [ ]:
```