overfitting_exercise_solution

April 24, 2023

0.1 Overfitting Exercise

In this exercise, we'll build a model that, as you'll see, dramatically overfits the training data. This will allow you to see what overfitting can "look like" in practice.

```
In [1]: import os
        import pandas as pd
        import numpy as np
        import math
        import matplotlib.pyplot as plt
```

For this exercise, we'll use gradient boosted trees. In order to implement this model, we'll use the XGBoost package.

```
In [2]: ! pip install xgboost
```

```
Requirement already satisfied: xgboost in /Users/ehamel/anaconda3/lib/python3.6/site-packages
Requirement already satisfied: scipy in /Users/ehamel/anaconda3/lib/python3.6/site-packages (fro
Requirement already satisfied: numpy in /Users/ehamel/anaconda3/lib/python3.6/site-packages (fro
You are using pip version 9.0.1, however version 19.0.3 is available.You should consider upgradi
```

```
In [3]: import xgboost as xgb
```

Here, we define a few helper functions.

```
In [4]: # number of rows in a dataframe
        def nrow(df):
            return(len(df.index))

        # number of columns in a dataframe
        def ncol(df):
            return(len(df.columns))

        # flatten nested lists/arrays
        flatten = lambda l: [item for sublist in l for item in sublist]

        # combine multiple arrays into a single list
        def c(*args):
            return(flatten([item for item in args]))
```

In this exercise, we're going to try to predict the returns of the S&P 500 ETF. This may be a futile endeavor, since many experts consider the S&P 500 to be essentially unpredictable, but it will serve well for the purpose of this exercise. The following cell loads the data.

```
In [5]: df = pd.read_csv("SPYZ.csv")
```

As you can see, the data file has four columns, `Date`, `Close`, `Volume` and `Return`.

```
In [6]: df.head()
```

```
Out[6]:          Date     Close    Volume    Return
        0  1999-12-31  146.8750   3172700  0.001598
        1  2000-01-03  145.4375   8164300 -0.009787
        2  2000-01-04  139.7500   8089800 -0.039106
        3  2000-01-05  140.0000  12177900  0.001789
        4  2000-01-06  137.7500   6227200 -0.016071
```

```
In [7]: n = nrow(df)
```

Next, we'll form our predictors/features. In the cells below, we create four types of features. We also use a parameter, `K`, to set the number of each type of feature to build. With a `K` of 25, 100 features will be created. This should already seem like a lot of features, and alert you to the potential that the model will be overfit.

```
In [8]: predictors = []

        # we'll create a new DataFrame to hold the data that we'll use to train the model
        # we'll create it from the `Return` column in the original DataFrame, but rename that co
        model_df = pd.DataFrame(data = df['Return']).rename(columns = {"Return" : "y"})

        # IMPORTANT: this sets how many of each of the following four predictors to create
        K = 25
```

Now, you write the code to create the four types of predictors.

```
In [9]: for L in range(1,K+1):
            # this predictor is just the return L days ago, where L goes from 1 to K
            # these predictors will be named `R1`, `R2`, etc.
            pR = "".join(["R",str(L)])
            predictors.append(pR)
            for i in range(K+1,n):
                # TODO: fill in the code to assign the return from L days before to the ith row
                model_df.loc[i, pR] = df.loc[i-L,'Return']

            # this predictor is the return L days ago, squared, where L goes from 1 to K
            # these predictors will be named `Rsq1`, `Rsq2`, etc.
            pR2 = "".join(["Rsq",str(L)])
            predictors.append(pR2)
            for i in range(K+1,n):
```

```
        # TODO: fill in the code to assign the squared return from L days before to the
        # in `model_df`
        model_df.loc[i, pR2] = (df.loc[i-L,'Return']) ** 2

    # this predictor is the log volume L days ago, where L goes from 1 to K
    # these predictors will be named `V1`, `V2`, etc.
    pV = "".join(["V",str(L)])
    predictors.append(pV)
    for i in range(K+1,n):
        # TODO: fill in the code to assign the log of the volume from L days before to t
        # in `model_df`
        # Add 1 to the volume before taking the log
        model_df.loc[i, pV] = math.log(1.0 + df.loc[i-L,'Volume'])

    # this predictor is the product of the return and the log volume from L days ago, wh
    # these predictors will be named `RV1`, `RV2`, etc.
    pRV = "".join(["RV",str(L)])
    predictors.append(pRV)
    for i in range(K+1,n):
        # TODO: fill in the code to assign the product of the return and the log volume
        # ith row of this predictor in `model_df`
        model_df.loc[i, pRV] = model_df.loc[i, pR] * model_df.loc[i, pV]
```

Let's take a look at the predictors we've created.

```
In [10]: model_df.iloc[100:105,:]
```

```
Out[10]:             y        R1      Rsq1         V1       RV1        R2      Rsq2  \
         100  0.016304 -0.014726  0.000217  15.892349 -0.234024 -0.007529  0.000057
         101 -0.017157  0.016304  0.000266  16.221058  0.264474 -0.014726  0.000217
         102  0.001133 -0.017157  0.000294  15.929221 -0.273290  0.016304  0.000266
         103  0.034194  0.001133  0.000001  15.387039  0.017437 -0.017157  0.000294
         104  0.000657  0.034194  0.001169  15.494960  0.529838  0.001133  0.000001

                     V2       RV2        R3    ...          V23      RV23       R24  \
         100  16.198698 -0.121956 -0.018688    ...    15.959991  0.076664 -0.009302
         101  15.892349 -0.234024 -0.007529    ...    16.372203 -0.177882  0.004804
         102  16.221058  0.264474 -0.014726    ...    16.461827  0.683503 -0.010865
         103  15.929221 -0.273290  0.016304    ...    15.858172 -0.178954  0.041520
         104  15.387039  0.017437 -0.017157    ...    16.562480 -0.054770 -0.011285

                 Rsq24        V24      RV24       R25     Rsq25        V25      RV25
         100  0.000087  15.695540 -0.145995  0.026421  0.000698  16.209371  0.428273
         101  0.000023  15.959991  0.076664 -0.009302  0.000087  15.695540 -0.145995
         102  0.000118  16.372203 -0.177882  0.004804  0.000023  15.959991  0.076664
         103  0.001724  16.461827  0.683503 -0.010865  0.000118  16.372203 -0.177882
         104  0.000127  15.858172 -0.178954  0.041520  0.001724  16.461827  0.683503

         [5 rows x 101 columns]
```

Next, we create a DataFrame that holds the recent volatility of the ETF's returns, as measured by the standard deviation of a sliding window of the past 20 days' returns.

```
In [11]: vol_df = pd.DataFrame(data = df[['Return']])

         for i in range(K+1,n):
             # TODO: create the code to assign the standard deviation of the return from the tim
             # 20 days before day i, up to the day before day i, to the ith row of `vol_df`
             vol_df.loc[i, 'vol'] = np.std(vol_df.loc[(i-20):(i-1),'Return'])
```

Let's take a quick look at the result.

```
In [12]: vol_df.iloc[100:105,:]

Out[12]:        Return       vol
         100  0.016304  0.013069
         101 -0.017157  0.013615
         102  0.001133  0.014007
         103  0.034194  0.014008
         104  0.000657  0.015792
```

Now that we have our data, we can start thinking about training a model.

```
In [13]: # for training, we'll use all the data except for the first K days, for which the predi
         model = model_df.iloc[K:n,:]
```

In the cell below, first split the data into train and test sets, and then split off the targets from the predictors.

```
In [14]: # Split data into train and test sets
         train_size = 2.0/3.0
         breakpoint = round(nrow(model) * train_size)

         # TODO: fill in the code to split off the chunk of data up to the breakpoint as the tra
         # assign the rest as the test set.
         training_data = model.iloc[1:breakpoint,:]
         test_data = model.loc[breakpoint : nrow(model),]

         # TODO: Split training data and test data into targets (Y) and predictors (X), for the
         X_train = training_data.iloc[:,1:ncol(training_data)]
         Y_train = training_data.iloc[:,0]
         X_test = test_data.iloc[:,1:ncol(training_data)]
         Y_test = test_data.iloc[:,0]
```

Great, now that we have our data, let's train the model.

```
In [15]: # DMatrix is a internal data structure that used by XGBoost which is optimized for both
         # and training speed.
         dtrain = xgb.DMatrix(X_train, Y_train)
```

```
# Train the XGBoost model
param = { 'max_depth':20, 'silent':1 }
num_round = 20
xgModel = xgb.train(param, dtrain, num_round)
```

Now let's predict the returns for the S&P 500 ETF in both the train and test periods. If the model is successful, what should the train and test accuracies look like? What would be a key sign that the model has overfit the training data?

Todo: Before you run the next cell, write down what you expect to see if the model is overfit.

An overfit model will have low error on the training set, but high error on the testing set.

```
In [16]: # Make the predictions on the test data
         preds_train = xgModel.predict(xgb.DMatrix(X_train))
         preds_test = xgModel.predict(xgb.DMatrix(X_test))
```

Let's quickly look at the mean squared error of the predictions on the training and testing sets.

```
In [17]: # TODO: Calculate the mean squared error on the training set
         msetrain = sum((preds_train-Y_train)**2)/len(preds_train)
         msetrain

Out[17]: 1.6237099209080407e-06

In [18]: # TODO: Calculate the mean squared error on the test set
         msetest = sum((preds_test-Y_test)**2)/len(preds_test)
         msetest

Out[18]: 7.7118554982160437e-05
```

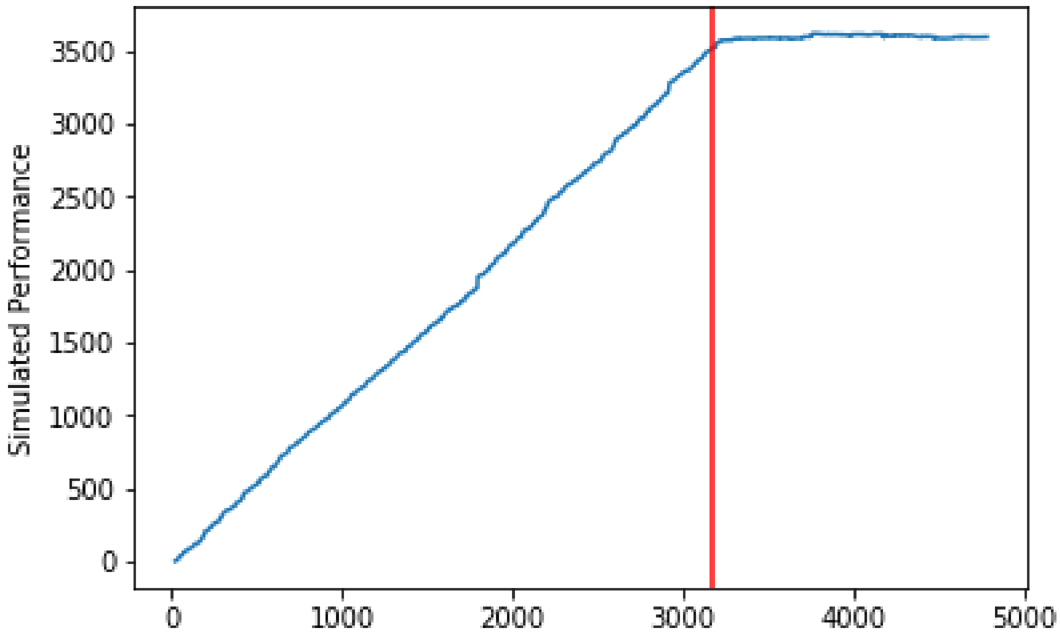
Looks like the mean squared error on the test set is an order of magnitude greater than on the training set. Not a good sign. Now let's do some quick calculations to gauge how this would translate into performance.

```
In [19]: # combine prediction arrays into a single list
         predictions = c(preds_train, preds_test)
         responses = c(Y_train, Y_test)

         # as a holding size, we'll take predicted return divided by return variance
         # this is mean-variance optimization with a single asset
         vols = vol_df.loc[K:n,'vol']
         position_size = predictions / vols ** 2

         # TODO: Calculate pnl. Pnl in each time period is holding * realized return.
         performance = position_size * responses

         # plot simulated performance
         plt.plot(np.cumsum(performance))
         plt.ylabel('Simulated Performance')
         plt.axvline(x=breakpoint, c = 'r')
         plt.show()
```



Our simulated returns accumulate throughout the training period, but they are absolutely flat in the testing period. The model has no predictive power whatsoever in the out-of-sample period.

Can you think of a few reasons our simulation of performance is unrealistic?

```
In [20]: # TODO: Answer the above question.
```

1. We left out any accounting of trading costs. If we had included trading costs, the performance in the out-of-sample period would be downward!

2. We didn't account for any time for trading. It's most conservative to assume that we would make trades on the day following our calculation of position size to take, and realize returns the day after that, such that there's a two-day delay between holding size calculation and realized return.

```
In [ ]:
```