Diff. Equations

Any partial diff equation has the form,

$$A U_{xx} + 2B U_{xy} + C U_{yy} + D U_x + E U_y + F = 0$$

where, A, B, C, D, E, F are constants,

and, $U_{xy} = \frac{\partial}{\partial x}\frac{\partial}{\partial y} U$, same for the others,

This form is analogous to,

$$Ax^2 + 2Bxy + Cy^2 + Dx + Ey + F = 0$$

When,

$B^2 = AC$, we have a parabola

Say, $y = Ax^2 + bx + C$

One example would be the heat equation

$$U_t = \beta U_{xx} \rightarrow \beta \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

The last equation defines a parabola by setting

$$(Ax + Cy)^2 = u^2$$

If $B^2 - AC > 0$; Defines a Hiperbolic Equation,

One example,

$y^2 - c^2 x^2 = 0$; its a hyperbola,

which is an example of the wave equation,

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

If $B^2 - AC < 0$, defines an eliptic equation, Example, $x^2 + y^2 = 0$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \nabla^2 u = 0 \leftarrow \text{Laplace equation}$$

Example 2,

$$u_{xx} + u_{yy} = f(x, y)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y) \leftarrow \text{Poisson equation}$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \nabla^2 V = -\frac{\rho}{\varepsilon_0} \leftarrow \text{Gauss Law,}$$

$$\vec{\nabla} \cdot (-\vec{\nabla} V) = -\frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \leftarrow \text{Gauss Law,}$$