In [ ]:

```
import pandas as pd
import numpy as np
import numpy.linalg as la
from sklearn.preprocessing import StandardScaler
import statsmodels.formula.api as smf
import statsmodels.api as sm
import matplotlib.pyplot as plt
from scipy.stats import t
```

Parcial 1: Estadística en Analítica

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Punto 1

In [ ]:

```
df = [
    [-2, 0],
    [-1, 0],
    [0, 1],
    [1, 1],
    [2, 3]
]
df = pd.DataFrame(df, columns="X Y".split())
df
```

Out[ ]:

	X	Y
0	-2	0
1	-1	0
2	0	1
3	1	1
4	2	3

In [ ]:

```
lm = smf.ols('Y~X', df)
lm_fit = lm.fit()
print(lm_fit.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      Y   R-squared:                       0.817
Model:                            OLS   Adj. R-squared:                  0.756
Method:                 Least Squares   F-statistic:                     13.36
Date:                Sat, 07 May 2022   Prob (F-statistic):             0.0354
Time:                        12:04:32   Log-Likelihood:                -3.3094
```

```
No. Observations:                   5   AIC:                             10.62
Df Residuals:                       3   BIC:                             9.838
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      1.0000      0.271      3.693      0.034       0.138       1.862
X              0.7000      0.191      3.656      0.035       0.091       1.309
==============================================================================
Omnibus:                          nan   Durbin-Watson:                   2.509
Prob(Omnibus):                    nan   Jarque-Bera (JB):                0.396
Skew:                          -0.174   Prob(JB):                        0.821
Kurtosis:                       1.667   Cond. No.                         1.41
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifi
ed.
c:\Users\lenovo\anaconda3\lib\site-packages\statsmodels\stats\stattools.py:74: ValueWarn
ing: omni_normtest is not valid with less than 8 observations; 5 samples were given.
  warn("omni_normtest is not valid with less than 8 observations; %i "
```

Con statsmodels, se obtiene el modelo de regresión lineal:

$$Y = 1 + 0.7X$$

Los mismos coeficientes se pueden obtener de manera matricial, resolviendo la ecuación

$$\beta = \left(X^T X\right)^{-1} X^T Y$$

In [ ]:

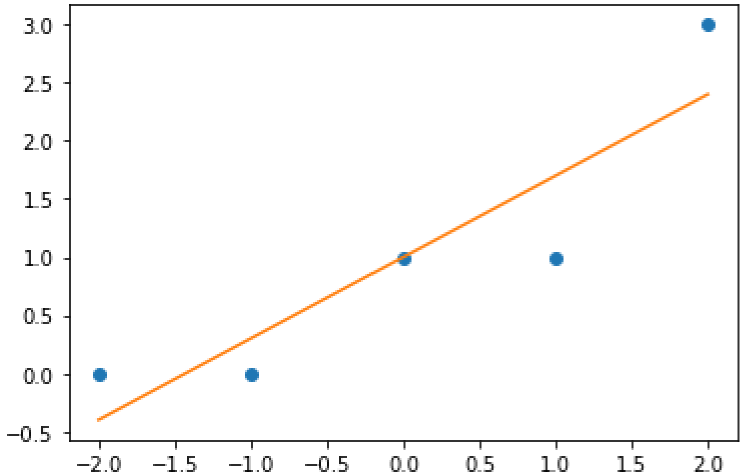
```
Y = df.Y
df['Ones'] = 1
X=df[['Ones','X']]
betas = la.inv(X.T @ X) @ (X.T @ Y)
betas
```

Out[ ]:

```
array([1. , 0.7])
```

In [ ]:

```
xx= np.linspace(df.X.min(), df.X.max())
yy = 1 + 0.7*xx
fig = plt.plot(df.X, df.Y,'o',xx,yy)
```



Cuando $X = 1$, el valor esperado de Y es

$$Y = 1 + 0.7(1) = 1.7$$

In [ ]:

```
SSE = Y.T @ Y - betas.T @ X.T @ Y
n=len(Y)
S2=SSE/(n-2)
S2
```

Out[ ]:

```
0.366666666666666
```

In [ ]:

```
P = la.inv(X.T @ X)

C_00 = P[0,0]
C_11 = P[1,1]

var_b0 = C_00 * S2
var_b1 = C_11 * S2
```

Para $X = 2$, se tiene $a^T = [1,\ 2]$. El intervalo de confianza de Y viene dado por:

$$a^T \hat{\beta} \pm t_{\alpha/2} S \sqrt{1 + a^T (X^T X)^{-1} a}$$

In [ ]:

```
# El 1 del primer valor del vector corresponde al intercepto
a = np.array([1, 2])
P = la.inv(X.T @ X)
var_y = S2 * (1 + a.T @ P @ a)
var_y
```

Out[ ]:

```
0.5866666666666656
```

In [ ]:

```
# El 1 del primer valor del vector corresponde al intercepto
lower_bound_y = a.T @ betas - t.ppf(0.975, df = len(Y) - len(betas)) * np.sqrt(var_y)
upper_bound_y = a.T @ betas + t.ppf(0.975, df = len(Y) - len(betas)) * np.sqrt(var_y)
lower_bound_y, upper_bound_y
```

Out[ ]:

```
(-0.03756828932652212, 4.837568289326523)
```

El intervalo de confianza para la predicción de Y con $X = 2$ es:

$$[-0.037,\ 4.837]$$

Punto 2

Carga de datos

In [ ]:
```
df = pd.read_csv('../datos/paises.csv')
df.tail()
```

Out[ ]:

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11
91	2.4	23	33	65382	73116	111	382	52	1.2	2186	5.7
92	2.2	41	49	17634	12270	11	414	26	1.5	101	0.3
93	4.2	100	29	4044	2159	12	335	8	0.0	206	0.7
94	2.6	109	45	3605	7785	8	186	43	1.1	149	0.3
95	2.8	55	44	5933	7334	14	136	23	0.7	438	1.8

Se calcula primero la matriz de covarianzas y se obtiene su inversa

In [ ]:
```
S = df.cov()
S_inv = la.inv(S)
```

In [ ]:
```
R2 = 1 - 1/(np.diag(S)*np.diag(S_inv))
print(df.columns)
R2.round(2)
```

```
Index(['X1', 'X2', 'X3', 'X4', 'X5', 'X6', 'X7', 'X8', 'X9', 'X10', 'X11'], dtype='objec
t')
```

Out[ ]:
```
array([0.68, 0.68, 0.64, 0.67, 0.63, 0.82, 0.3 , 0.19, 0.22, 0.87, 0.81])
```

Calculando el los coeficientes de correlación múltiples y obteniendo $R^2 = 1 - \frac{1}{S^{jj}S_{jj}}$, se puede determinar que la variable que mejor se puede explicar en términos de las otras es X_{10}

PCA

Primero, los datos deben estar estandarizados para se tenga media 0 y desviación 1

Se obitene las autovectores y autovalores de la matriz de covarianza

In [ ]:
```
df_stand = StandardScaler().fit_transform(df)
print(df_stand.mean(axis=0))
print(df_stand.std(axis=0))
```

```
[-1.85037171e-16  3.70074342e-17  3.70074342e-17 -2.77555756e-17
 -3.23815049e-17  0.00000000e+00 -9.25185854e-18  7.86407976e-17
```

```
  0.00000000e+00  2.77555756e-17  5.55111512e-17]
[1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
```

Ahora se calcula la matriz de covarianza de los datos estandarizados

In [ ]:

```
S = np.cov(df_stand.T)
eig_values, eig_vectors = la.eig(S)
print(eig_values.round(2))
print(eig_vectors.round(2))
```

```
[4.07 1.95 1.38 0.07 0.15 0.17 0.33 0.58 0.87 0.8  0.74]
[[ 0.31 -0.35 -0.07 -0.1   0.35  0.52  0.09 -0.16  0.44  0.33 -0.18]
 [ 0.39  0.04 -0.18  0.17 -0.39 -0.29  0.32 -0.64  0.13 -0.08 -0.09]
 [-0.12  0.58  0.17 -0.17  0.43  0.24 -0.05 -0.53 -0.06 -0.19  0.17]
 [-0.3   0.18 -0.53 -0.15  0.35 -0.37  0.45  0.15  0.26  0.14  0.05]
 [-0.26  0.17 -0.61  0.12 -0.34  0.31 -0.5  -0.11  0.17  0.08  0.03]
 [-0.45  0.03  0.15  0.45 -0.21  0.45  0.57  0.   -0.05  0.05  0.02]
 [-0.09 -0.32 -0.37 -0.07  0.21  0.08  0.06 -0.24 -0.74 -0.03 -0.31]
 [-0.01  0.46  0.16 -0.01 -0.09 -0.07 -0.04  0.08 -0.04  0.42 -0.75]
 [ 0.24  0.15 -0.03  0.07 -0.03 -0.02  0.02 -0.01 -0.34  0.73  0.51]
 [-0.42 -0.23  0.21 -0.68 -0.36 -0.04  0.05 -0.27  0.07  0.23  0.06]
 [-0.37 -0.29  0.21  0.47  0.29 -0.38 -0.31 -0.34  0.15  0.24 -0.03]]
```

In [ ]:

```
print(eig_values[0]/eig_values.sum().round(2))
print(eig_values[1]/eig_values.sum())
```

```
0.36621392092378485
0.17545381348370237
```

Se seleccionan los dos primeros autovalores que son los que tienen mayor valor: $\lambda_1 = 4.07$ y $\lambda_2 = 1.95$

In [ ]:

```
new_df = []
for index, row in enumerate(df_stand):
    new_df.append([np.dot(eig_vectors[:,0], row), np.dot(eig_vectors[:,1], row)])
new_df = pd.DataFrame(np.array(new_df), columns="Z1 Z2".split())
new_df.tail()
```

Out[ ]:

	Z1	Z2
91	0.165907	0.114074
92	1.366391	1.072347
93	2.599022	-1.350077
94	2.340780	1.230173
95	1.505448	0.447563

In [ ]:

```
new_df.iloc[13]
```

Out[ ]:

```
Z1   -2.465597
Z2    1.024444
Name: 13, dtype: float64
```

Para el registro $x^{(13)}$, las componentes en el plano de Z_1 y Z_2 son $(-2.46,\ 1.02)$

Para saber cual es la variable que más aporta en el componente principal Z_2, se debe observar el autovector asociado

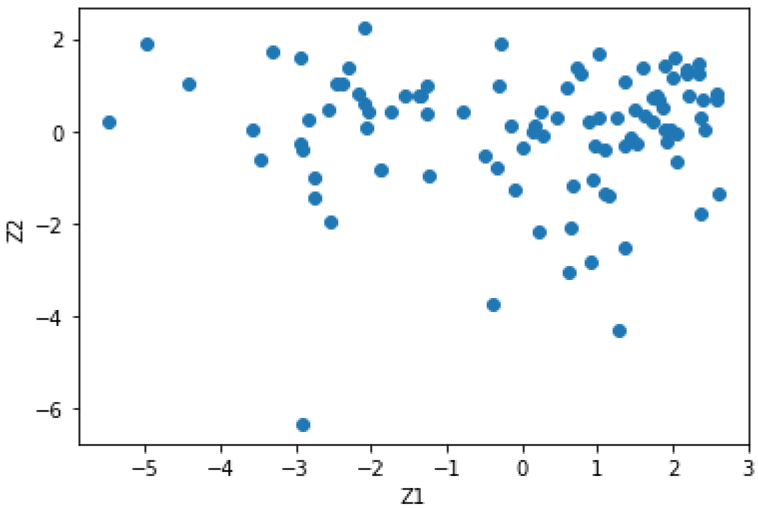
In [ ]:
```
print(df.columns)
print(eig_vectors[1].round(2))
```

```
Index(['X1', 'X2', 'X3', 'X4', 'X5', 'X6', 'X7', 'X8', 'X9', 'X10', 'X11'], dtype='objec
t')
[ 0.39  0.04 -0.18  0.17 -0.39 -0.29  0.32 -0.64  0.13 -0.08 -0.09]
```

La variable que más aporta al componente principal Z_2 es X_8, con un coeficiente de -0.64

In [ ]:
```
plt.plot(new_df.Z1, new_df.Z2, 'o')
plt.xlabel("Z1")
plt.ylabel("Z2")
```

Out[ ]:
```
Text(0, 0.5, 'Z2')
```



A partir de la gráfica, se puede observar un outlier alrededor de $(-3, 3)$

Punto 3

In [ ]:
```
df = [
    [-2, 0, 0],
    [-1, 0, 1],
    [0, 1, 1],
    [1, 1, 0],
    [2, 3, 1]
]
df = pd.DataFrame(df, columns="X1 X2 Y".split())
df
```

Out[ ]:

	X1	X2	Y
0	-2	0	0
1	-1	0	1
2	0	1	1

	X1	X2	Y
3	1	1	0
4	2	3	1

La función de pérdida de un modelo de regresión logística viene dada por:

$$L = \frac{1}{m}\sum_{i}^{m}\left[-Y^{(i)}\cdot\ln\left[g(X^{(i)}\cdot\beta)\right]-(1-Y^{(i)})\left(1-\ln\left[g(X^{(i)}\cdot\beta)\right]\right)\right]$$

donde

$$g(X\cdot\beta)=\frac{1}{1+e^{-(X\cdot\beta)}}=\frac{1}{1+e^{-(\beta_0+\beta_1 X)}}$$

Ajustemos los datos a un modelo de regresión logística y determinemos la predicción para el punto $(0,\ 2)$

In [ ]:

```
X = df.drop('Y', axis=1)
y = df.Y
lr = sm.Logit(y, X)
lr_fit = lr.fit()
```

```
Optimization terminated successfully.
         Current function value: 0.601406
         Iterations 6
```

In [ ]:

```
print(lr_fit.summary())
```

```
                           Logit Regression Results
==============================================================================
Dep. Variable:                      Y   No. Observations:                    5
Model:                          Logit   Df Residuals:                        3
Method:                           MLE   Df Model:                            1
Date:                Sat, 07 May 2022   Pseudo R-squ.:                  0.1064
Time:                        12:04:37   Log-Likelihood:                -3.0070
converged:                       True   LL-Null:                       -3.3651
Covariance Type:            nonrobust   LLR p-value:                    0.3974
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
X1             0.0406      0.855      0.047      0.962      -1.635       1.716
X2             0.6684      1.012      0.660      0.509      -1.315       2.652
==============================================================================
```

In [ ]:

```
a = np.array([0, 2])
betas = lr_fit.params
1 / (1 + np.exp(-(a @ betas)))
```

Out[ ]:

```
0.7919551575395981
```

Como el valor es mayor a 0.5, se dice que la clase predicha para el punto $(0,\ 2)$ es 1