Parcial 2: Estadística en Ciencia de Datos

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EAFIT 2022

In [ ]:
```
import pandas as pd
import numpy as np
import numpy.linalg as la
from sklearn.cluster import KMeans
from sklearn.preprocessing import StandardScaler, MinMaxScaler
from scipy.spatial.distance import euclidean
import matplotlib.pyplot as plt
```

Punto 1

In [ ]:
```
df1 = pd.read_excel('./../datos/Paises2.xlsx', index_col='Pais')
df1
```

Out[ ]:

Pais	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11
Albania	1.0	30	41	2199	3903	12	94	53	0.0	341	1.2
Angola	3.0	124	46	4422	955	6	57	19	0.7	89	0.5
ArabiaSaudi	4.3	21	13	133540	91019	96	497	1	0.0	4566	13.1
Argelia	2.5	34	24	44609	19883	42	180	2	0.8	906	3.0
Argentina	1.3	22	31	278431	65962	160	1043	22	0.1	1504	3.5
Australia	1.4	6	43	337909	167155	510	933	19	0.0	5341	15.3
Austria	0.6	6	41	216547	53259	465	304	47	-0.4	3301	7.2
Bangladesh	2.0	79	42	28599	9891	2	220	6	4.1	64	0.2
Bélgica	0.3	8	40	250710	72236	457	917	20	-0.3	5120	10.1
Benin	3.0	95	48	2034	6	5	26	45	1.3	20	0.1
Bielrorrusia	0.4	13	49	21356	31397	190	295	31	-0.4	2392	9.9
Bolivia	2.3	69	37	5905	2824	35	201	45	1.2	373	1.0
Brasil	1.6	44	35	579787	260682	75	246	66	0.6	718	1.4
Bulgaria	-0.6	15	48	11225	381333	335	1544	33	-0.2	2438	6.4
Camerún	2.9	56	38	8615	2740	4	38	44	0.6	103	0.2
Canadá	1.3	6	45	573695	554227	590	1602	49	-1.1	7854	14.4
Colombia	1.8	26	37	70263	43354	100	174	52	0.7	622	1.8
Congo	3.1	90	43	1784	435	8	20	58	0.2	331	1.6
Corea del Norte	1.8	26	45	12870	38000	47	687	74	0.0	1129	11.2
Corea del Sur	0.9	10	40	435137	164993	415	632	66	0.1	2982	6.6

In [ ]:
```
rows, cols = df1.shape
```

Estandarizar los datos

In [ ]:
```
X_stand = StandardScaler().fit_transform(df1)
```

In [ ]:
```
X_stand[0]
```

Out[ ]:
```
array([-0.64955357, -0.26451146,  0.20159678, -0.78056703, -0.66004874,
       -0.84002104, -0.81992424,  0.7226464 , -0.39223227, -0.77136178,
       -0.83385733])
```

Cálculo de matriz de distancias con norma usual sobre los datos estandarizados

In [ ]:
```
distance_matrix = np.zeros((rows,rows))
for i in range(0, distance_matrix.shape[0]):
    for j in range(0, distance_matrix.shape[1]):
        distance_matrix[i][j] = euclidean(X_stand[i],X_stand[j])
D = pd.DataFrame(distance_matrix, index=df1.index, columns=df1.index)
D
```

Out[ ]:

Pais	Albania	Angola	ArabiaSaudi	Argelia	Argentina	Australia	Austria	Bangladesh
Pais								
Albania	0.000000	3.752513	6.030931	3.527386	3.309341	5.489904	3.312303	4.901438
Angola	3.752513	0.000000	6.268867	3.896872	4.757918	6.577615	5.535951	3.789780
ArabiaSaudi	6.030931	6.268867	0.000000	3.567421	4.469589	5.190911	5.661563	6.851553
Argelia	3.527386	3.896872	3.567421	0.000000	2.929080	5.435038	4.557730	4.185382
Argentina	3.309341	4.757918	4.469589	2.929080	0.000000	3.824587	3.120367	5.213299
Australia	5.489904	6.577615	5.190911	5.435038	3.824587	0.000000	2.946859	6.994499
Austria	3.312303	5.535951	5.661563	4.557730	3.120367	2.946859	0.000000	6.326376
Bangladesh	4.901438	3.789780	6.851553	4.185382	5.213299	6.994499	6.326376	0.000000
Bélgica	4.607470	6.164894	5.326790	4.858081	3.012241	1.709158	2.105569	6.626921
Benin	3.039147	1.621430	6.493453	4.064277	4.669275	6.472718	5.112193	3.544790
Bielrorrusia	2.741305	4.808254	5.857646	4.326536	3.431028	3.584179	2.238666	5.745261
Bolivia	2.101209	2.448090	5.433299	2.822211	3.430620	5.755907	4.217477	3.468933
Brasil	3.783482	5.138529	6.172855	4.726401	3.584200	5.330778	4.082009	5.771028
Bulgaria	4.961830	6.593824	7.164312	6.044946	4.054756	4.051269	4.014800	6.968986
Camerún	2.011646	2.508673	5.330601	2.799295	3.610613	5.920155	4.296617	4.051590
Canadá	7.949236	9.102662	7.584069	8.363009	6.276066	3.947964	5.577810	9.793485
Colombia	1.288551	3.680783	5.293859	2.919532	2.817597	5.023523	3.104766	4.371223

Pais	Albania	Angola	ArabiaSaudi	Argelia	Argentina	Australia	Austria	Bangladesh
Pais								
Congo	2.577407	2.188151	6.007697	3.958317	4.390937	6.238834	4.662920	4.680734
Corea del Norte	2.709219	4.802033	5.859170	4.738049	3.757117	4.596372	3.367440	5.908987
Corea del Sur	3.904275	5.974873	6.008146	5.163171	3.171085	3.196468	1.905280	6.531700

Cálculo de los autovectores y autovalores de la matriz de covarianzas de la matriz de distancias estandarizada

In [ ]:
```
D_stand = StandardScaler().fit_transform(distance_matrix)
covariance = np.cov(D_stand)
eig_values, eig_vectors = la.eig(covariance)
eig_values.round(2)
```

Out[ ]:
```
array([ 9.15,  2.17,  1.42,  1.02,  0.64,  0.46,  0.28,  0.21,  0.19,
        0.14,  0.12,  0.09,  0.05,  0.05, -0.  ,  0.03,  0.03,  0.01,
        0.02,  0.02])
```

In [ ]:
```
eig_vectors[0:2][:].round(2)
```

Out[ ]:
```
array([[-0.12,  0.25,  0.14,  0.07,  0.07,  0.14,  0.  ,  0.36, -0.08,
         0.08, -0.11,  0.27,  0.52,  0.24,  0.22,  0.14, -0.35, -0.02,
        -0.29, -0.21],
       [-0.28, -0.08, -0.27,  0.1 , -0.01, -0.14,  0.25, -0.33, -0.12,
         0.09,  0.26, -0.06,  0.46,  0.19,  0.22, -0.11,  0.39, -0.02,
        -0.16,  0.24]])
```

Varianza explicada

In [ ]:
```
eig_values[0]/eig_values.sum(), eig_values[1]/eig_values.sum()
```

Out[ ]:
```
(0.5680757872686832, 0.13487130419294746)
```

In [ ]:
```
eig_values[0]/eig_values.sum() + eig_values[1]/eig_values.sum()
```

Out[ ]:
```
0.7029470914616307
```

Cálculo de las dos primeras coordenadas principales

In [ ]:
```
new_df = []
for index, row in enumerate(D_stand):
    new_df.append([np.dot(eig_vectors[:,0], row), np.dot(eig_vectors[:,1], row),])
new_df = pd.DataFrame(np.array(new_df), columns="Z1 Z2".split(), index=df1.index)
new_df
```

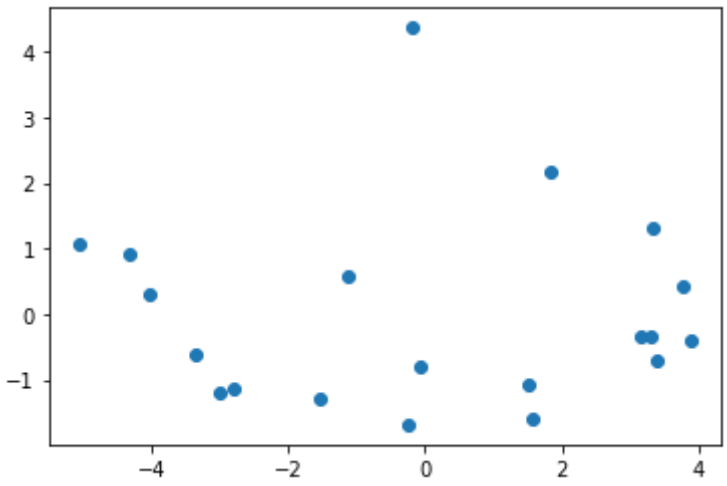
Out[ ]:

	Z1	Z2
Pais		

Pais	Z1	Z2
Albania	1.557898	-1.605406
Angola	3.752294	0.435125
ArabiaSaudi	-0.180428	4.366517
Argelia	1.822185	2.180006
Argentina	-1.114068	0.583447
Australia	-4.329564	0.922385
Austria	-2.794215	-1.128194
Bangladesh	3.335146	1.306430
Bélgica	-4.025695	0.299815
Benin	3.869947	-0.408090
Bielrorrusia	-1.538936	-1.290714
Bolivia	3.154868	-0.336673
Brasil	-0.071033	-0.808418
Bulgaria	-3.345540	-0.600146
Camerún	3.294522	-0.339461
Canadá	-5.045254	1.078929
Colombia	1.522746	-1.066007
Congo	3.379260	-0.709721
Corea del Norte	-0.235631	-1.682122
Corea del Sur	-3.008504	-1.197703

Se puede identificar un outlier alrededor de $(0,\ 4)$, que corresponde al pais de Arabia Saudita

In [ ]:
```
plt.plot(new_df.Z1, new_df.Z2, 'o')
plt.show()
```



El país más cercano a Colombia es Albania, según la matriz de distancias:

In [ ]:
```
distance_matrix = np.zeros((rows,rows))
for i in range(0, distance_matrix.shape[0]):
    for j in range(0, distance_matrix.shape[1]):
        distance_matrix[i][j] = euclidean(new_df.to_numpy()[i],new_df.to_numpy()[j])
D = pd.DataFrame(distance_matrix, index=df1.index, columns=df1.index)
```

In [ ]:
```
D.Colombia.sort_values()[0:2]
```

Out[ ]:
```
Pais
Colombia    0.000000
Albania     0.540543
Name: Colombia, dtype: float64
```

Punto 2

Se tiene la matriz de distancia

$$\begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$$

Como la matriz tiene dimensión 2×2, la matriz de datos debe tener solo 2 registros. El número de columnas puede ser cualquiera, con tal de que la distancia entre los registros sea 3 ya que la matriz de distancia es simétrica. Se debe cumplir entonces

$$||X^{(1)} - X^{(2)}||_2 = 3$$

Con esto en cuenta, una matriz en particular que cumple esto es

$$\begin{bmatrix} 0 & 0 \\ 3 & 0 \end{bmatrix}$$

In [ ]:
```
X = np.array([
    [0, 0],
    [3, 0]
])
distance_matrix = np.zeros((2,2))
for i in range(0, distance_matrix.shape[0]):
    for j in range(0, distance_matrix.shape[1]):
        distance_matrix[i][j] = euclidean(X[i],X[j])
distance_matrix
```

Out[ ]:
```
array([[0., 3.],
       [3., 0.]])
```

Punto 3

In [ ]:
```
df3 = pd.read_csv('./../datos/Clientes_D1.csv').drop('IDCliente', axis=1)
df3
```

Out[ ]:

	Sexo	Edad	IngresoDiario (miles)	PuntajeGastos(1-100)
0	Female	47	120	16
1	Male	21	15	81
2	Female	20	16	6

	Sexo	Edad	IngresoDiario (miles)	PuntajeGastos(1-100)
3	Female	23	16	77
4	Female	31	17	40
...	...	...	...	...
185	Male	30	99	97
186	Female	54	101	24
187	Male	28	101	68
188	Female	41	103	17
189	Female	36	103	85

190 rows × 4 columns

Cambiar a variable binaria la variable categórica Sexo

In [ ]:
```
df3 = pd.get_dummies(df3, drop_first=True)
df3
```

Out[ ]:

	Edad	IngresoDiario (miles)	PuntajeGastos(1-100)	Sexo_Male
0	47	120	16	0
1	21	15	81	1
2	20	16	6	0
3	23	16	77	0
4	31	17	40	0
...	...	...	...	...
185	30	99	97	1
186	54	101	24	0
187	28	101	68	1
188	41	103	17	0
189	36	103	85	0

190 rows × 4 columns

In [ ]:
```
X = MinMaxScaler().fit_transform(df3)
```

In [ ]:
```
kmeans = KMeans(n_clusters=2, random_state=42)
kmeans.fit(X)
```

Out[ ]:
```
KMeans(n_clusters=2, random_state=42)
```

In [ ]:
```
df3['Label'] = kmeans.labels_
df3
```

Out[ ]:

	Edad	IngresoDiario (miles)	PuntajeGastos(1-100)	Sexo_Male	Label
0	47	120	16	0	1
1	21	15	81	1	0
2	20	16	6	0	1
3	23	16	77	0	1
4	31	17	40	0	1
...	...	...	...	...	...
185	30	99	97	1	0
186	54	101	24	0	1
187	28	101	68	1	0
188	41	103	17	0	1
189	36	103	85	0	1

190 rows × 5 columns

La persona del primer registro pertence al cluster 1

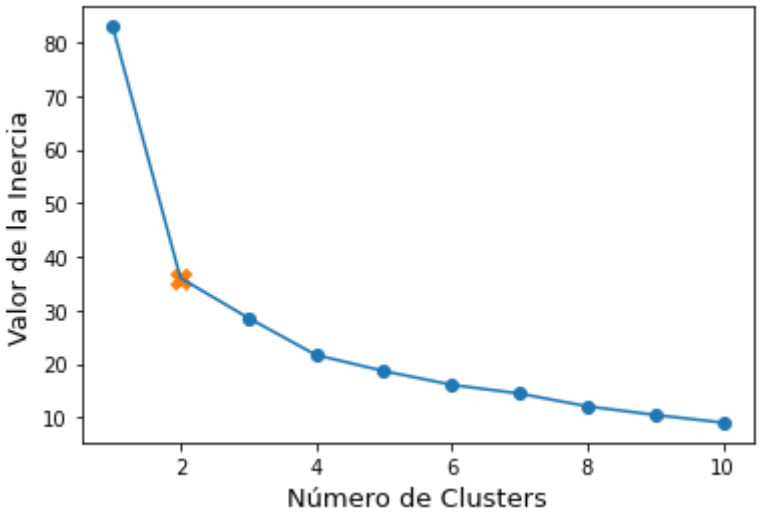
Número k de grupos óptimo

In [ ]:

```
inertia_list = []
for num_clusters in range(1, 11):
    kmeans = KMeans(n_clusters=num_clusters, random_state=42)
    kmeans.fit(X)
    inertia_list.append(kmeans.inertia_)

plt.plot(range(1,11),inertia_list)
plt.scatter(range(1,11),inertia_list)
plt.scatter(2, inertia_list[1], marker='X', s=100)
plt.xlabel("Número de Clusters", size=13)
plt.ylabel("Valor de la Inercia", size=13)
plt.show()
```

```
c:\Users\lenovo\anaconda3\lib\site-packages\sklearn\cluster\_kmeans.py:881: UserWarn
ing: KMeans is known to have a memory leak on Windows with MKL, when there are less
chunks than available threads. You can avoid it by setting the environment variable
OMP_NUM_THREADS=1.
  warnings.warn(
```



Por el método del codo, un valor adecuado para el número de grupos es 2

In [ ]:
```
kmeans = KMeans(n_clusters=2, random_state=42)
kmeans.fit(df3)
```

Out[ ]:
```
KMeans(n_clusters=2, random_state=42)
```

In [ ]:
```
df3['Label'] = kmeans.labels_
df3
```

Out[ ]:

	Edad	IngresoDiario (miles)	PuntajeGastos(1-100)	Sexo_Male	Label
0	47	120	16	0	0
1	21	15	81	1	1
2	20	16	6	0	0
3	23	16	77	0	1
4	31	17	40	0	0
...	...	...	...	...	...
185	30	99	97	1	1
186	54	101	24	0	0
187	28	101	68	1	1
188	41	103	17	0	0
189	36	103	85	0	1

190 rows × 5 columns

La persona del primer registro pertenece al cluster 0 cuando se definen 2 grupos

Punto 4

Demostrar que la esparanza condicional $E[Y|X]$ es la solución al problema de minimización

$$f(x) = \text{argmin}_c E((Y-c)^2|X=x)$$

Diferenciando f respecto c:

$$\frac{\partial f}{\partial c} = \frac{\partial}{\partial c}\left(E[(Y-c)^2|X=x]\right) \tag{1}$$
$$\frac{\partial f}{\partial c} = \frac{\partial}{\partial c}\left(E[Y^2|X=x] - 2cE[Y|X=x] + c^2\right) \tag{2}$$
$$\frac{\partial f}{\partial c} = -2E[Y|X=x] + 2c = 0 \tag{3}$$

Lo que entrega

$$c = E[Y|X=x]$$

Esta c minimiza f porque la segunda derivada es mayor a 0.