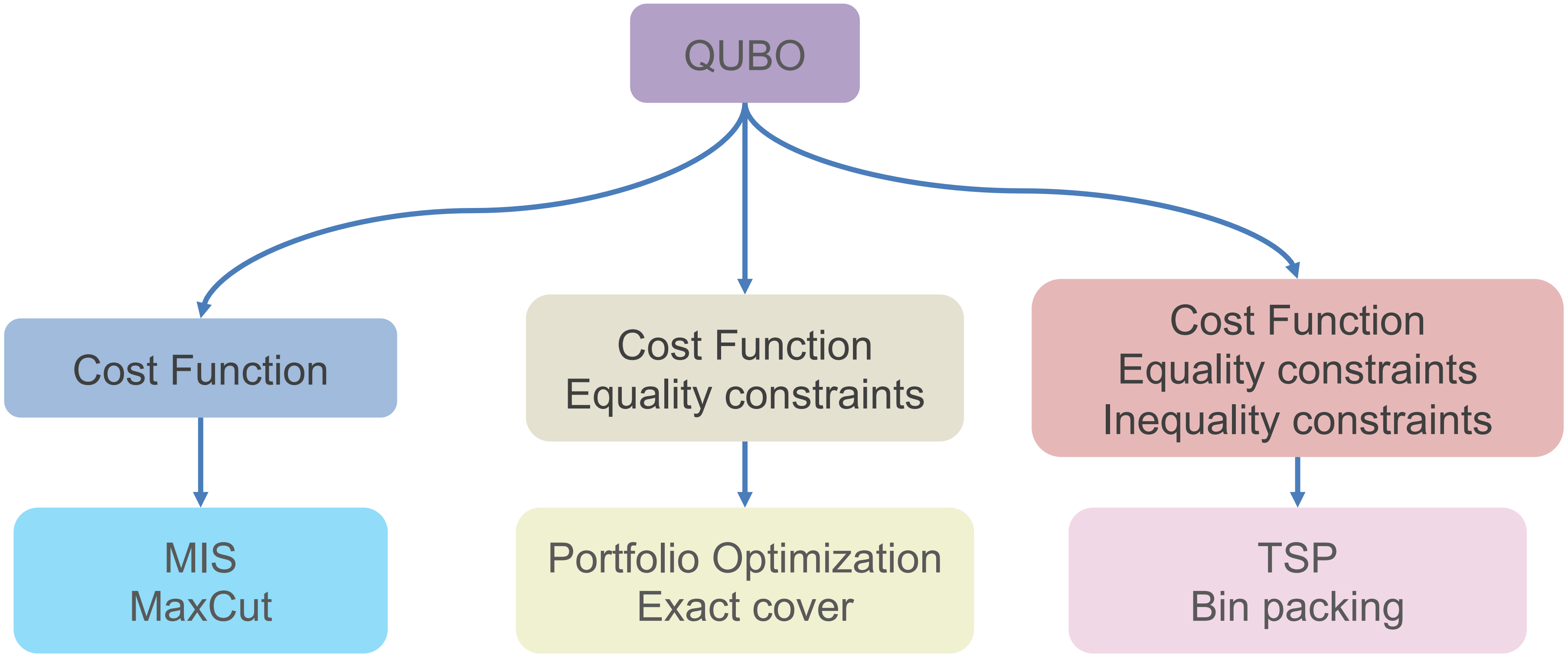
Solución del problema de optimización de portafolios de inversión usando computación cuántica

Docplex, QAOA, VQE, Problemas de optimización combinatoria

Octubre 18, 2022 I Alejandro Montanez-Barrera | FZJ - JSC

Outline

1. Combinatorial Optimization Problems



QUBO = Quadratic Unconstrained Binary Optimization
MIS = Maximum Independent Set
TSP = Traveling Salesman Problem

1.1 Portfolio Optimization

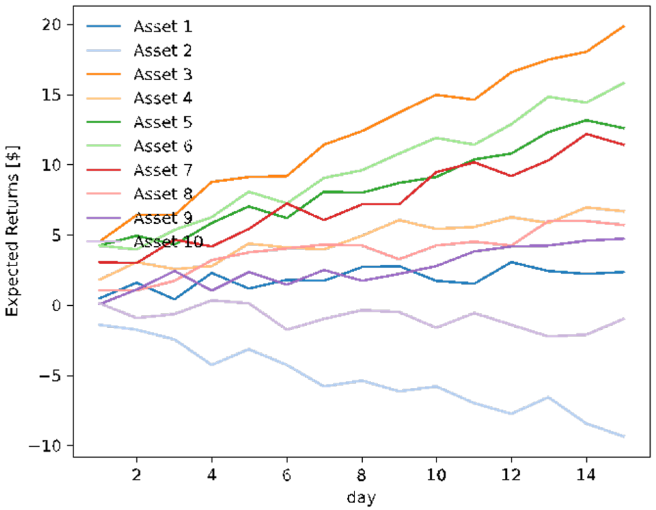
Objective Function

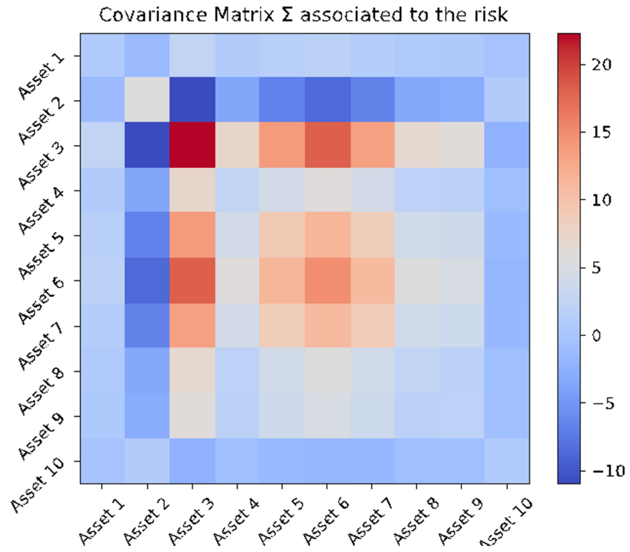
$$-\sum_{i=1}^{N} \mu_i x_i + q \sum_{i=1}^{N} \sum_{j=1}^{N} \Sigma_{ij} x_i x_j$$

Equality Constraints

$$\sum_{i=1}^{N} c_i x_i = B$$

This problem searches for the best set of assets to buy given a budget to maximize the profit while minimizing the risk of such an investment.





μ_i Return of the asset i $\quad$ x_i Binary var. if the asset i is bought

N Number of assets $\quad$ B Budget

c_i Cost of the asset i $\quad$ Σ_{ij} Covariance between i and j

2. Encoding Combinatorial Optimization Problems

Objective Function + penalization terms

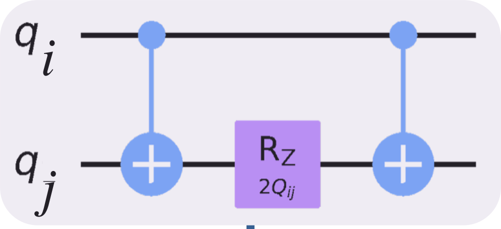
Ising Hamiltonian representation

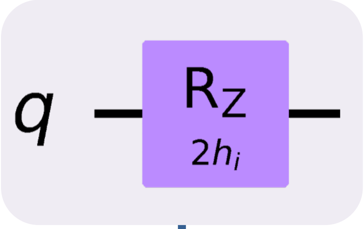
Circuit representation

$$\min_{x} \sum_{i=1}^{n} \sum_{j\neq i}^{n} c_{ij} x_i x_j + \sum_{i=1}^{n} h_i x_i + \lambda_0 \left(\sum_{i=1}^{n} q_i x_i - C \right)^2 + \lambda_1 \left(\sum_{i=1}^{n} l_i x_i + \sum_{k=1}^{N} 2^k s_k - B \right)^2 .$$

$$x_i \in \{0,1\} \rightarrow z_i \in \{-1,1\}$$

$$\hat{H}_C = \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \boxed{Q_{ij} z_i z_j} + \sum_{i}^{n} \boxed{h_i z_i}$$





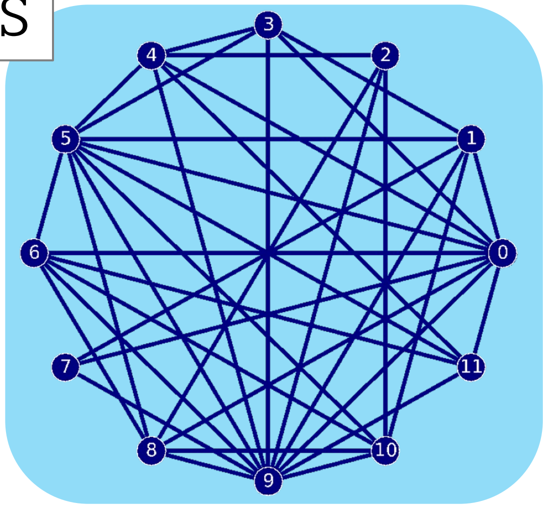
$$U_{\hat{H}_C} = e^{-i\gamma \hat{H}_C}$$

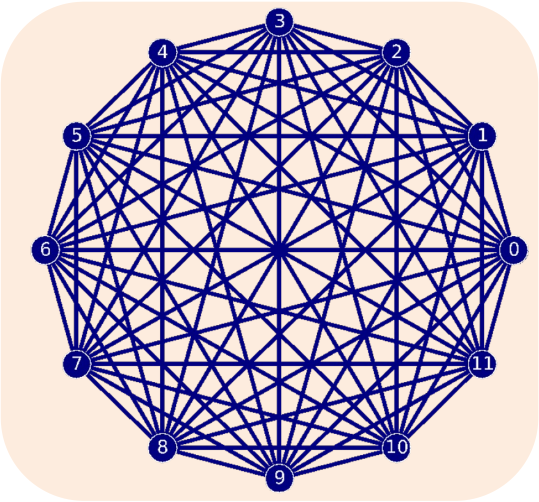
2.1 The Quadratic Unconstrained Binary Optimization (QUBO)

Objective Function | Equality Constraints | Inequality Constraints

$$\min_{x} \sum_{i=1}^{n}\sum_{j\neq i}^{n} c_{ij}x_ix_j + \sum_{i=1}^{n} h_ix_i + \lambda_0\left(\sum_{i=1}^{n} q_ix_i - C\right)^2 + \lambda_1\left(\sum_{i=1}^{n} l_ix_i + \sum_{k=1}^{N} 2^k s_k - B\right)^2 .$$

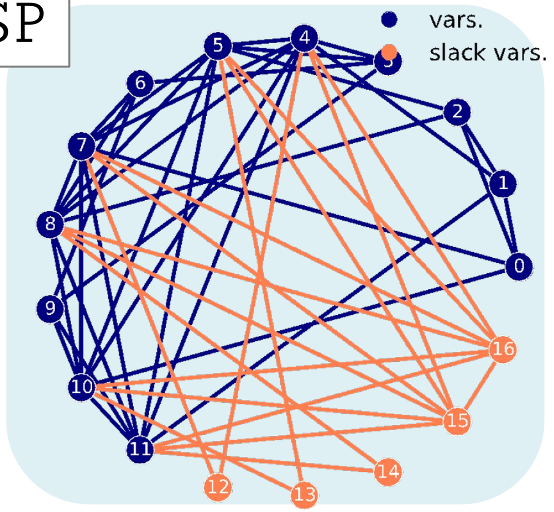
MIS





Portfolio Optimization

TSP



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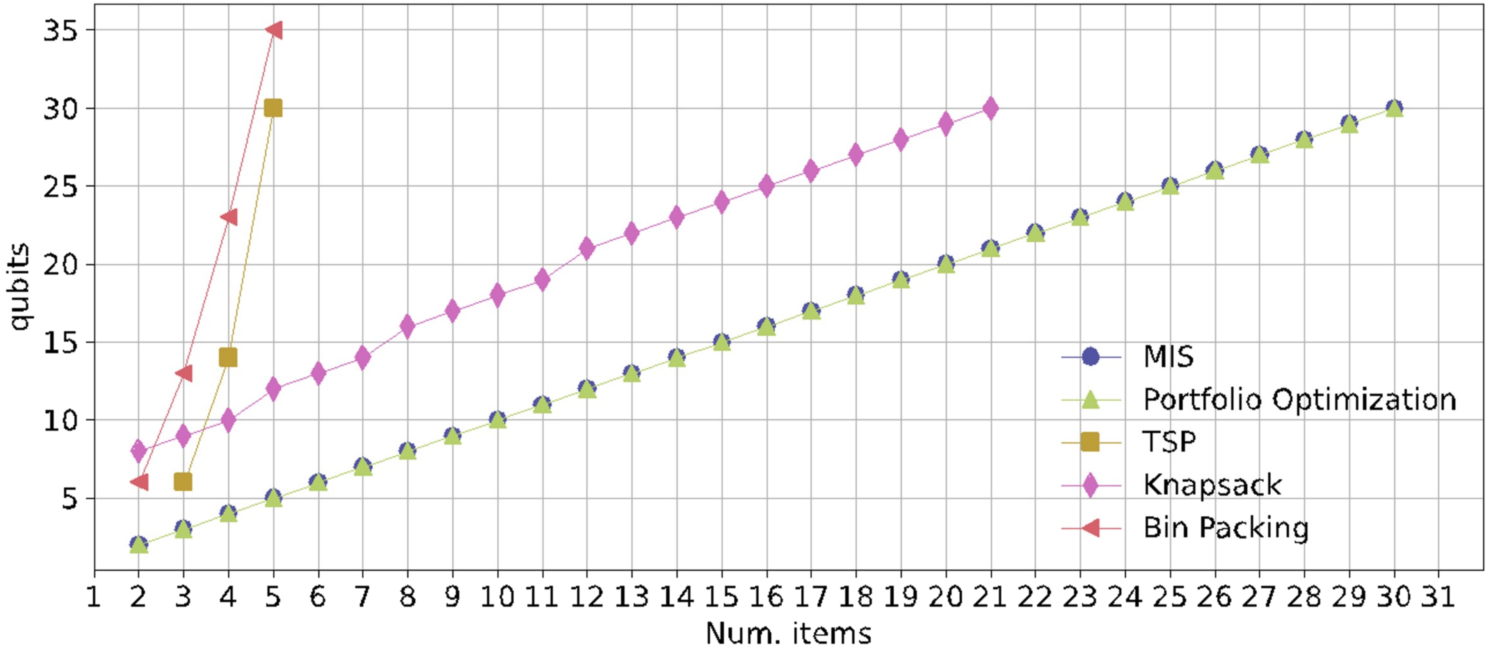
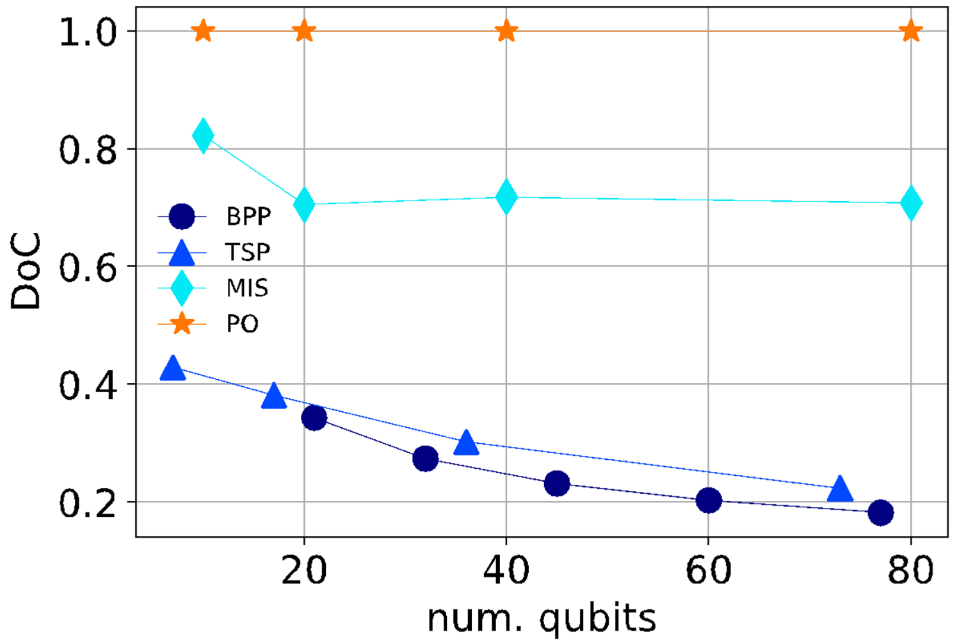
2.2 The number of qubits needed to encode different QUBO problems

Additional to the combinatorial optimization problems presented before, we include the [Knapsack](#) problem (KP) and the [Bin Packing](#) problem (BPP).





$$DoC = \frac{2N}{n(n-1)}$$

DoC = Density of Connections

n = num. qubits

N = Num. of connections QUBO

The Num. items mean nodes for MIS, assets for Portfolio Optimization, cities for TSP, items for KP, and items for BPP.

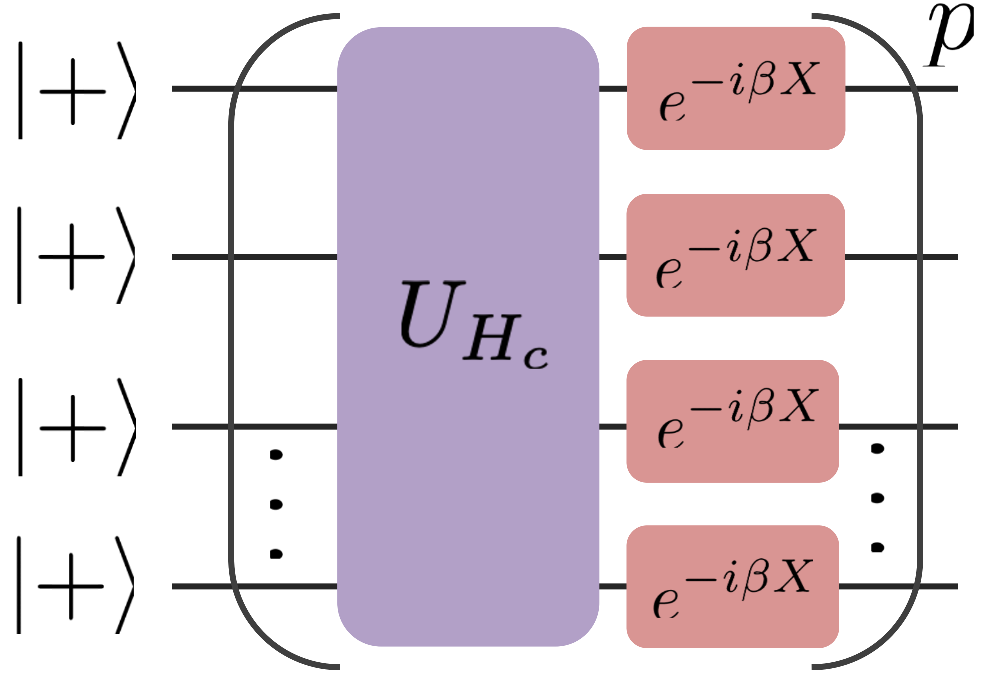
2.3 The Quantum Approximate Optimization Algorithm (QAOA)

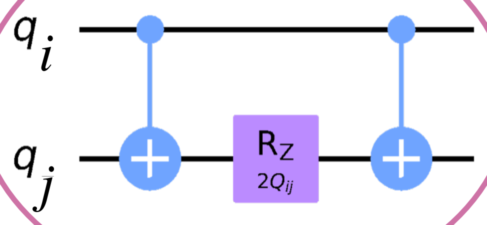
$$\min_x \sum_{i=1}^{n}\sum_{j\neq i}^{n} c_{ij}x_ix_j + \sum_{i=1}^{n} h_ix_i + \lambda_0\left(\sum_{i=1}^{n} q_ix_i - C\right)^2 + \lambda_1\left(\sum_{i=1}^{n} l_ix_i + \sum_{k=1}^{N} 2^k s_k - B\right)^2$$

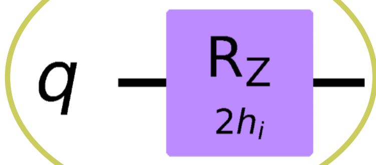
$$x_i \in \{0,1\} \to z_i \in \{-1,1\}$$

$$\hat{H}_C = \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Q_{ij}z_iz_j + \sum_{i}^{n} h_iz_i$$

$$U_{\hat{H}_C} = e^{-i\gamma \hat{H}_C} \qquad U_{\hat{H}_B} = e^{-i\beta \hat{B}}$$







p Repetitions

γ β Parameters to optimize

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Notebook:
https://github.com/alejomonbar/Qiskit_Fall_Fest_Mexico_2022