Solución del problema de optimización de portafolios de inversión usando computación cuántica

Docplex, QAOA, VQE, Problemas de optimización combinatoria

Octubre 18, 2022 I Alejandro Montanez-Barrera | FZJ - JSC

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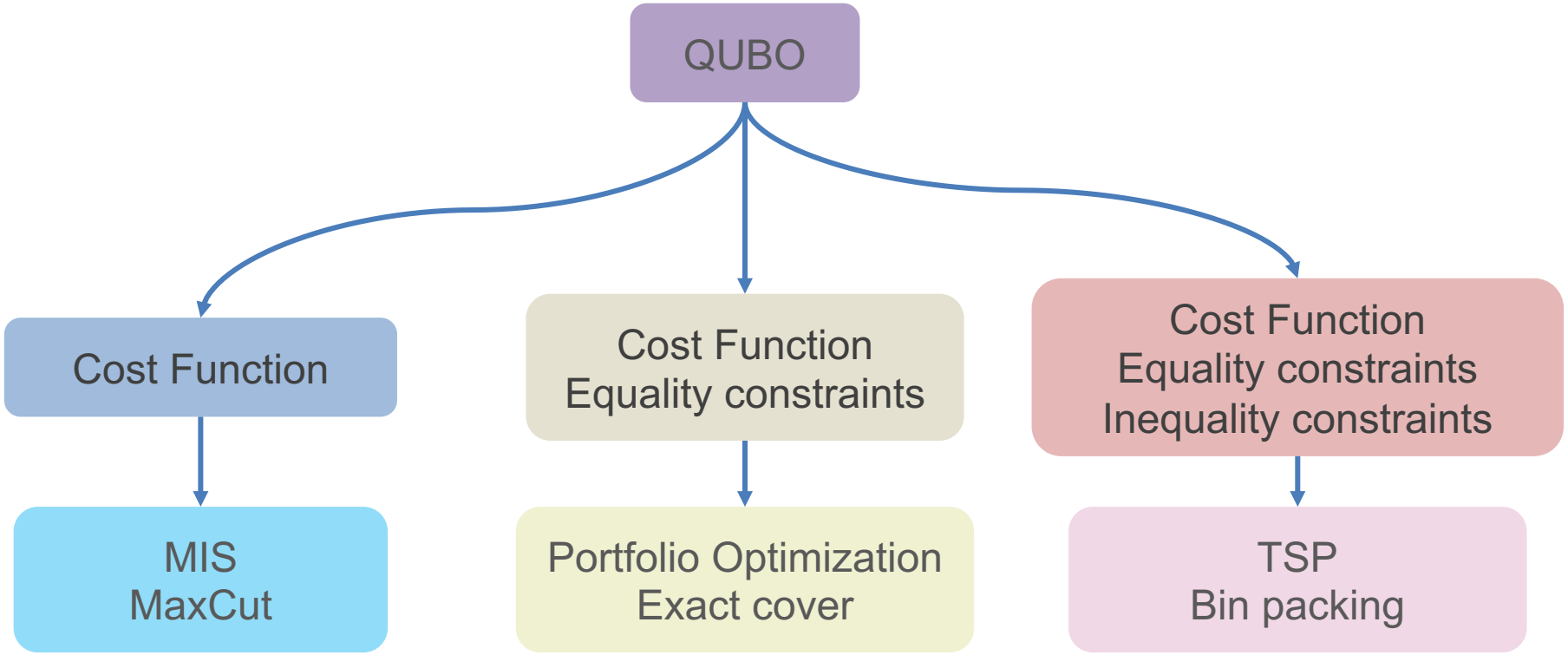
1. Combinatorial Optimization Problems



QUBO = Quadratic Unconstrained Binary Optimization
MIS = Maximum Independent Set
TSP = Traveling Salesman Problem

1.1 Portfolio Optimization

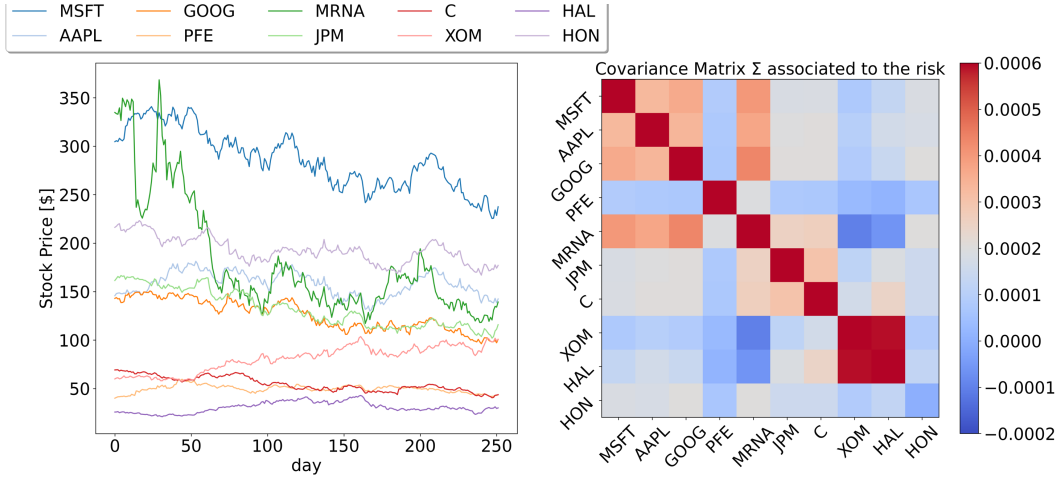
Objective Function

$$-\sum_{i=1}^{N} \mu_i x_i + q \sum_{i=1}^{N} \sum_{j=1}^{N} \Sigma_{ij} x_i x_j$$

Equality Constraints

$$\sum_{i=1}^{N} c_i x_i = B$$

This problem searches for the best set of assets to buy given a budget to maximize the profit while minimizing the risk of such an investment.



μ_i Return of the asset i

N Number of assets

c_i Cost of the asset i

x_i Binary var. if the asset i is bought

B Budget

Σ_{ij} Covariance between i and j

2. Encoding Combinatorial Optimization Problems

Objective Function + penalization terms

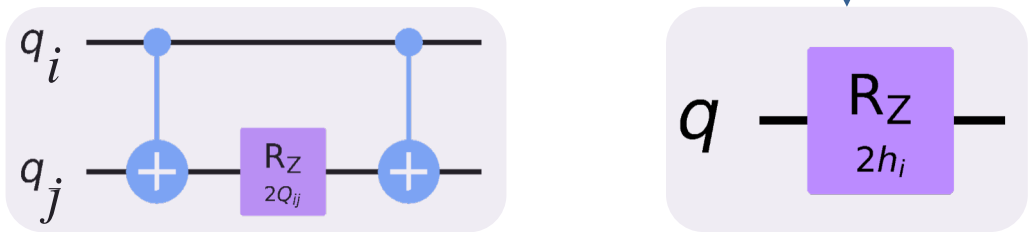
Ising Hamiltonian representation

Circuit representation

$$\min_x \sum_{i=1}^{n}\sum_{j\neq i}^{n} c_{ij}x_i x_j + \sum_{i=1}^{n} h_i x_i + \lambda_0 \left(\sum_{i=1}^{n} q_i x_i - C\right)^2 + \lambda_1 \left(\sum_{i=1}^{n} l_i x_i + \sum_{k=1}^{N} 2^k s_k - B\right)^2$$

$$x_i \in \{0,1\} \rightarrow z_i \in \{-1,1\}$$

$$\hat{H}_C = \sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n} Q_{ij} z_i z_j + \sum_{i}^{n} h_i z_i$$



$$U_{\hat{H}_C} = e^{-i\gamma \hat{H}_C}$$

2.1 The Quadratic Unconstrained Binary Optimization (QUBO)

Objective Function

$$\min_x \sum_{i=1}^{n} \sum_{j \neq i}^{n} c_{ij} x_i x_j + \sum_{i=1}^{n} h_i x_i$$

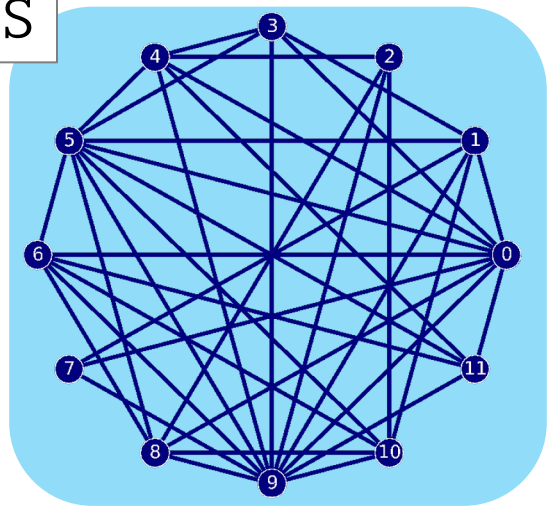
Equality Constraints

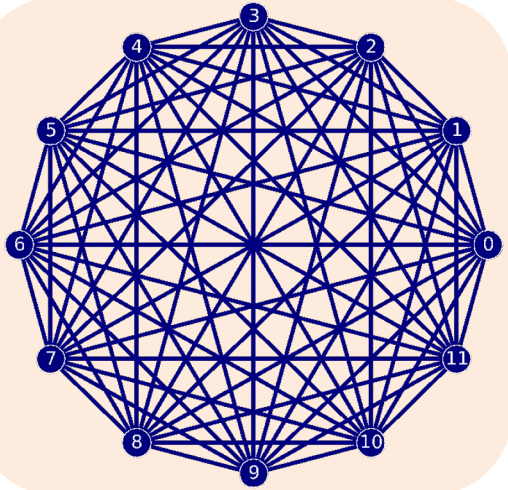
$$+ \lambda_0 \left(\sum_{i=1}^{n} q_i x_i - C \right)^2$$

Inequality Constraints

$$+ \lambda_1 \left(\sum_{i=1}^{n} l_i x_i + \sum_{k=1}^{N} 2^k s_k - B \right)^2 .$$

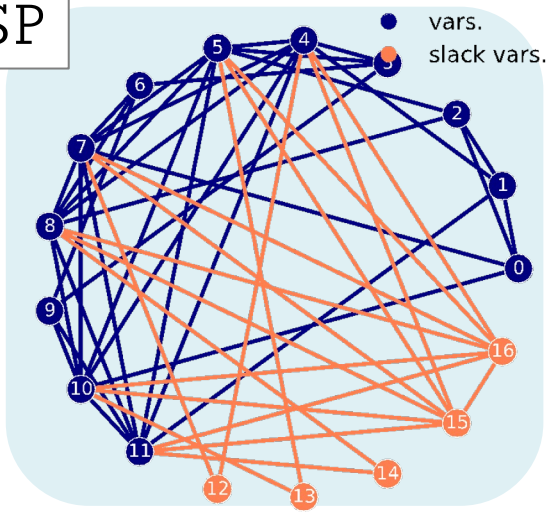
MIS





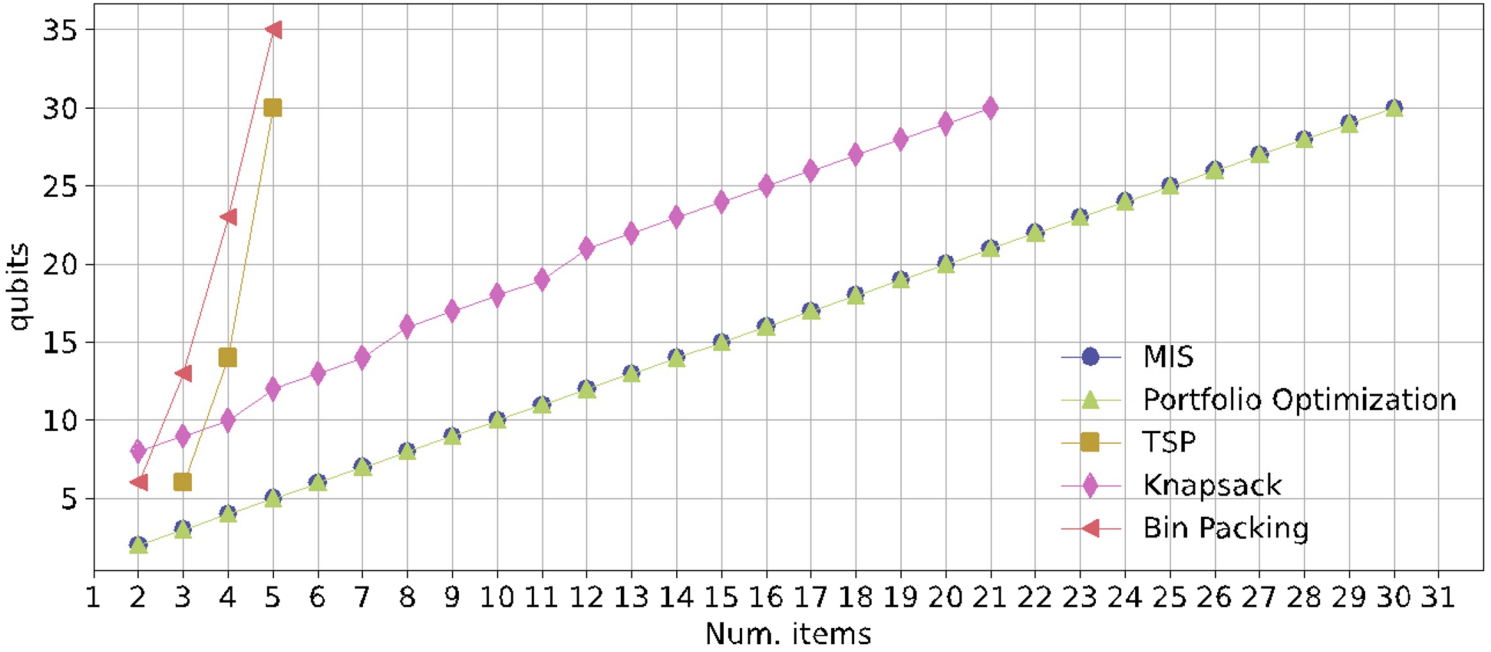
Portfolio Optimization

TSP

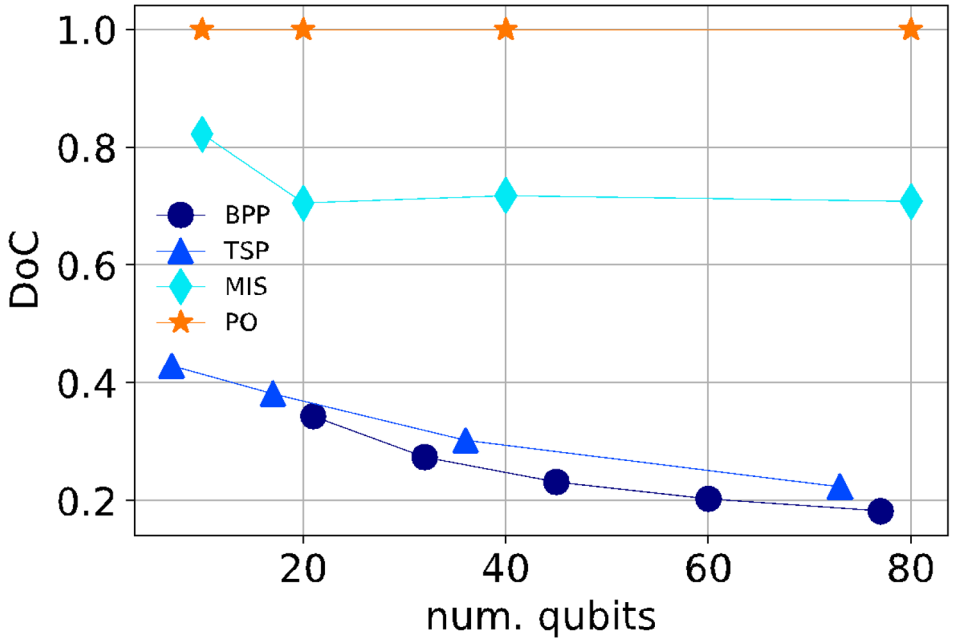


2.2 The number of qubits needed to encode different QUBO problems

Additional to the combinatorial optimization problems presented before, we include the Knapsack problem (KP) and the Bin Packing problem (BPP).



The Num. items mean nodes for MIS, assets for Portfolio Optimization, cities for TSP, items for KP, and items for BPP.



$$DoC = \frac{2N}{n(n-1)}$$

DoC = Density of Connections

n = num. qubits

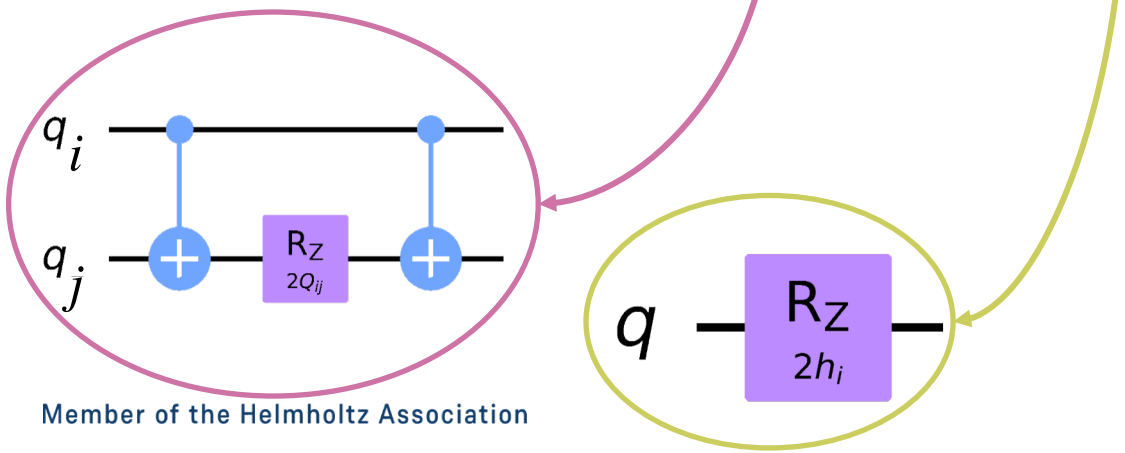
N = Num. of connections QUBO

2.3 The Quantum Approximate Optimization Algorithm (QAOA)

$$\min_x \sum_{i=1}^{n}\sum_{j\neq i}^{n} c_{ij}x_i x_j + \sum_{i=1}^{n} h_i x_i + \lambda_0 \left(\sum_{i=1}^{n} q_i x_i - C\right)^2 + \lambda_1 \left(\sum_{i=1}^{n} l_i x_i + \sum_{k=1}^{N} 2^k s_k - B\right)^2$$

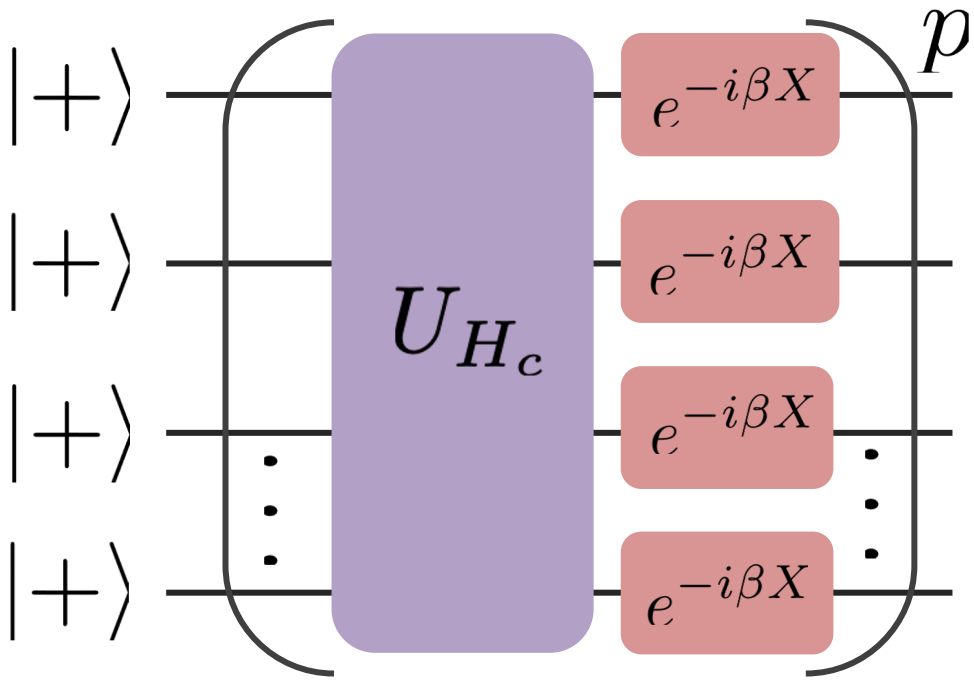
$$x_i \in \{0,1\} \rightarrow z_i \in \{-1,1\}$$

$$\hat{H}_C = \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Q_{ij} z_i z_j + \sum_{i}^{n} h_i z_i$$



$$U_{\hat{H}_C} = e^{-i\gamma \hat{H}_C}$$

$$U_{\hat{H}_B} = e^{-i\beta \hat{B}}$$



p Repetitions

γ β Parameters to optimize

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Notebook:
https://github.com/alejomonbar/Qiskit_Fall_Fest_Mexico_2022