Problem 2

a) Show that an $N \times N$ matrix M can be expressed as $M = VLV^{-1}$, where L is a diagonal matrix of eigenvalues and V is the matrix formed by columns of eigenvectors.

$$M v_i = \lambda_i v_i \leftarrow \text{by Definition}$$

$$MV = VL$$

$$M \underbrace{VV^{-1}}_{I} = VLV^{-1}$$

$$\Rightarrow M = VLV^{-1}$$

b) $M = VLV^{-1} \Rightarrow M^n = (VLV^{-1})^n$

$$= (VL\underbrace{V^{-1})(V}_{I}L\underbrace{V^{-1})(V}_{I}LV^{-1}) \ldots (VLV^{-1})$$

$$= VLL\ldots LV^{-1} = VL^nV^{-1} = M^n$$

Now since L is a diagonal matrix containing eigenvalues (λ_i) of M, eigenvalues of M^n are λ_i^n