Computational Physics HW 6 Alek Hutson

Problem 1

a) $\dot{P} = DP'' + CP$

Reflecting Boundary Conditions: $\left.\frac{\partial P}{\partial x}\right|_{x=\pm 1/2} = 0$

P can be solved using seperation of variables: $P(x,t) = X(x)\,T(t)$

$$\Rightarrow \frac{\cancel{X(x)}\,T'(t)}{\cancel{X(x)}\,T(t)} = \frac{D\,\cancel{T(t)}\,X''(x) + CX(x)\,\cancel{T(t)}}{X(x)\,\cancel{T(t)}}$$

$$\Rightarrow \frac{T'(t)}{T(t)} = \frac{DX''(x)}{X(x)} + C$$

$$\Rightarrow \frac{T'(t)}{DT(t)} = \frac{X''(x)}{X(x)} + \frac{C}{D} = \lambda$$

$$\Rightarrow T'(t) = D\lambda T(t) \Rightarrow \underline{T(t) = Ae^{D\lambda t}}$$

$$X''(x) = (\lambda - \tfrac{C}{D})\,X(x)$$

$$\Rightarrow \underline{X(x) = Me^{\sqrt{\lambda - C/D}\,x} + Ne^{-\sqrt{\lambda - C/D}\,x}}$$

$$X(x) = Me^{\sqrt{\lambda - c/D}\,x} + Ne^{-\sqrt{\lambda - c/D}\,x}$$

$$T(t) = Ae^{D\lambda t}$$

$$\rho(x,t) = T(t)X(x)$$

$$\rho = Ae^{D\lambda t}\left(Me^{\sqrt{\lambda - c/D}\,x} + Ne^{-\sqrt{\lambda - c/D}\,x}\right)$$

$$\frac{\partial \rho}{\partial x} = Ae^{D\lambda t}\sqrt{\lambda - c/D}\left(Me^{\sqrt{\lambda - c/D}\,x} - Ne^{-\sqrt{\lambda - c/D}\,x}\right)$$

$$\Rightarrow \cancel{Ae^{D\lambda t}\sqrt{\lambda - c/D}}\left(Me^{\sqrt{\lambda - c/D}\,L/2} - Ne^{-\sqrt{\lambda - c/D}\,L/2}\right) = 0$$

$$\cancel{Ae^{D\lambda t}}\sqrt{\lambda - c/D}\left(Me^{-\sqrt{\lambda - c/D}\,L/2} - Ne^{\sqrt{\lambda - c/D}\,L/2}\right) = 0$$

$$Me^{\sqrt{\lambda - c/D}\,L/2} = Ne^{-\sqrt{\lambda - c/D}\,L/2}$$

$$Me^{-\sqrt{\lambda - c/D}\,L/2} = Ne^{\sqrt{\lambda - c/2}\,L/2}$$

$$M\left(2\sinh\left(\sqrt{\lambda - c/D}\,L/2\right)\right) = N\left(2\sinh\left(-\sqrt{\lambda - c/D}\,L/2\right)\right)$$

$$\Rightarrow \sqrt{\lambda - c/D}\,\cancel{L/2} = -\sqrt{\lambda - c/D}\,\cancel{L/2}$$

$$\lambda = c/D$$

Plugging into $T(t)$: $T(t) = Ae^{\cancel{D}(c/\cancel{D})t} = Ae^{ct}$

For neutron emitters $c > 0$

$\therefore$ Ae^{ct} diverges as $t \to \infty$