Computational Physics HW 3 Alek Hutson

Problem 1

$$E_{tot} = \frac{mL^2}{2}\omega^2(t) + \frac{mgL}{2}\theta^2(t) - mgL$$

$$\frac{dE_{tot}}{dt} = \frac{mL^2}{2}\left[\frac{d}{dt}\omega^2(t)\right] + \frac{mgL}{2}\left[\frac{d}{dt}\theta^2(t)\right]$$

$$= \frac{mL^2}{\cancel{2}}\cancel{2}\omega(t)\frac{d\omega(t)}{dt} + \frac{mgL}{\cancel{2}}\cancel{2}\theta(t)\frac{d\theta(t)}{dt}$$

$$= mL^2\omega(t)\,\dot{\omega}(t) + mgL\,\theta\dot{\theta}(t)$$

Euler Method:
$$\theta_{n+1} = \theta_n + \tau\omega_n$$
$$\omega_{n+1} = \omega_n + \tau\alpha_n$$

$\frac{d\theta}{dt}$ $\quad \frac{\theta_{n+1}-\theta_n}{\Delta t}$

$$\frac{dE_{tot}}{dt} = mL^2\omega(t)\,\alpha(t) + mgL\,\theta(t)\,\omega(t)$$

$$\omega(t) = \frac{\theta_{n+1}-\theta_n}{\tau}, \quad \alpha(t) = \frac{\omega_{n+1}-\omega_n}{\tau}$$

$$\Rightarrow \frac{dE_{tot}}{dt} = mL^2\omega(t)\left(\frac{\omega_{n+1}-\omega_n}{\tau}\right) + mgL\left(\frac{\theta_{n+1}-\theta_n}{\tau}\right)$$

$$\frac{E_{n+1}-E_n}{\tau} = \left(\frac{mL^2\omega_n\omega_{n+1}}{\tau} + \frac{mgL\theta_n\theta_{n+1}}{\tau}\right) - \underbrace{\left(\frac{mL^2\omega_n^2}{\tau} + \frac{mgL\theta_n^2}{\tau}\right)}_{E_n}$$

$$\therefore \boxed{E_{n+1} = mL^2\omega_n\omega_{n+1} + mgL\,\theta_n\theta_{n+1} > 0}$$