$x = r\cos\theta$
$y = r\sin\theta$
$\dot{x} = \dot{r}\cos\theta - r\sin\theta\,\dot{\theta}$
$\dot{y} = \dot{r}\sin\theta + r\cos\theta\,\dot{\theta}$

Problem 1

$\dot{x} = ax + y - x(\underbrace{x^2+y^2}_{r^2})$

$\dot{r}\cos\theta - r\sin\theta\,\dot{\theta} = ar\cos\theta + r\sin\theta - r\cos\theta\, r^2$ — Pull out $\sin\theta$ (1)

$\dot{y} = -x + ay - y(x^2+y^2)$

$\dot{r}\sin\theta + r\cos\theta\,\dot{\theta} = ar\sin\theta - r\cos\theta - r\sin\theta\, r^2$ (2)

Pull out $\cos\theta$

$\Rightarrow \dot{r}\cot\theta - r\dot{\theta} = ar\cot\theta + r - r^3\cot\theta$

$\Rightarrow \dot{r}\tan\theta + r\dot{\theta} = ar\tan\theta - r - r^3\tan\theta$

$+$

$\dot{r}(\cot\theta + \tan\theta) = ar(\cot\theta + \tan\theta) - r^3(\cot\theta + \tan\theta)$

$\frac{\dot{r}(\cot\theta+\tan\theta)}{\cot\theta+\tan\theta} = \frac{ar(\cot\theta+\tan\theta) - r^3(\cot\theta+\tan\theta)}{\cot\theta+\tan\theta}$

$\Rightarrow \boxed{\dot{r} = ar - r^3}$

Equilibrium : $0 = ar - r^3$

$a - r^2 = 0 \Rightarrow a = r$

for $a > 0$, otherwise it approaches 0

$\dot{r} - r\tan\theta\,\dot{\theta} = ar + r\tan\theta - r^3$

$\dot{r} + r\cot\theta\,\dot{\theta} = ar - r\cot\theta - r^3$

$-$

$-r(\tan\theta + \cot\theta)\dot{\theta} = r(\tan\theta + \cot\theta)$

$\frac{-r(\tan\theta+\cot\theta)\dot{\theta}}{-r(\tan\theta+\cot\theta)} = \frac{r(\tan\theta+\cot\theta)}{-r(\tan\theta+\cot\theta)}$

$\Rightarrow \boxed{\dot{\theta} = -1}$