

Functions

Problem 2

$$f_1 \Rightarrow x^2 e^{-x^2} + y^2 - z = 0$$

$$f_2 \Rightarrow \frac{x^4}{1+x^2 y^2} - z = 0$$

$$f_3 \Rightarrow z - 1 = 0$$

$$\begin{bmatrix} x^2 e^{-x^2} + y^2 - z \\ \frac{x^4}{1+x^2 y^2} - z \\ z - 1 \end{bmatrix} = F$$

$$\frac{\partial f_1}{\partial x} = e^{-x^2} 2x + x^2 e^{-x^2} (-2x) = 2x e^{-x^2} (1 - x^2)$$

$$\frac{\partial f_1}{\partial y} = 2y$$

$$\frac{\partial f_1}{\partial z} = -1$$

$$\frac{\partial f_2}{\partial x} = \frac{4x^3}{1+x^2 y^2} + x^4 (-1) \frac{1}{(1+x^2 y^2)^2} (2y^2 x) = \frac{2x^3}{1+x^2 y^2} \left(2 - \frac{y^2 x^2}{1+y^2 x^2} \right)$$

$$\frac{\partial f_2}{\partial y} = 0 + x^4 (-1) \frac{1}{(1+x^2 y^2)^2} (2x^2 y) = \frac{-2x^6 y}{(1+x^2 y^2)^2}$$

$$\frac{\partial f_2}{\partial z} = -1$$

$$\frac{\partial f_3}{\partial x} = 0$$

$$\frac{\partial f_3}{\partial y} = 0$$

$$\frac{\partial f_3}{\partial z} = 1$$

Jacobian

$$J = \begin{bmatrix} 2x e^{-x^2} (1 - x^2) & 2y & -1 \\ \frac{2x^3}{1+x^2 y^2} \left(2 - \frac{y^2 x^2}{1+x^2 y^2} \right) & \frac{-2x^6 y}{(1+x^2 y^2)^2} & -1 \\ 0 & 0 & 1 \end{bmatrix}$$