

# Problem 3 a)

Aluk Hutton

$$\frac{\partial u(x,t)}{\partial t} + 6u(x,t) \frac{\partial u(x,t)}{\partial x} + \frac{\partial^3 u(x,t)}{\partial x^3} = 0$$

let  $u(x,t) = v(y)$  where  $y = x - ct$

$$\Rightarrow \frac{\partial v(y)}{\partial y} \left( \frac{\partial y}{\partial t} \right) + 6v(y) \left( \frac{\partial v(y)}{\partial y} \right) \left( \frac{\partial y}{\partial x} \right) + \frac{\partial^3 v(y)}{\partial y^3} \left( \frac{\partial y}{\partial x} \right)^3 = 0$$

$$\frac{\partial y}{\partial t} = \frac{\partial}{\partial t} (x - ct) = -c \quad ; \quad \frac{\partial y}{\partial x} = \frac{\partial}{\partial x} (x - ct) = 1$$

$$\Rightarrow -c \frac{\partial v(y)}{\partial y} + 6v(y) \frac{\partial v(y)}{\partial y} + \frac{\partial^3 v(y)}{\partial y^3} = 0$$

b) Let  $z = y\sqrt{c} \Rightarrow y = \frac{z}{\sqrt{c}}$

$$-c \left( \frac{\partial v(y)}{\partial z} \right) \left( \frac{\partial z}{\partial y} \right) + 6v(y) \left( \frac{\partial v(y)}{\partial z} \right) \left( \frac{\partial z}{\partial y} \right) + \frac{\partial^3 v(y)}{\partial z^3} \left( \frac{\partial z}{\partial y} \right)^3 = 0$$

$\frac{\partial z}{\partial y} = \sqrt{c}$  Now let  $v(y) = cw(z)$

$$\Rightarrow -c\sqrt{c} \frac{\partial (cw(z))}{\partial z} + 6(cw(z))\sqrt{c} \frac{\partial (cw(z))}{\partial z} + (\sqrt{c})^3 \frac{\partial^3 (cw(z))}{\partial z^3} = 0$$

$$\Rightarrow -c^{5/2} \frac{\partial w(z)}{\partial z} + 6c^{5/2} w(z) \frac{\partial w(z)}{\partial z} + c^{5/2} \frac{\partial^3 w(z)}{\partial z^3} = 0$$

Divide by  $c^{5/2}$

$$\Rightarrow -\frac{\partial w(z)}{\partial z} + 6w(z) \frac{\partial w(z)}{\partial z} + \frac{\partial^3 w(z)}{\partial z^3} = 0 \quad \checkmark$$