

$$\nabla^2 \phi = 0 \Rightarrow \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$\Rightarrow \frac{\phi_{j-1}^n - 2\phi_j^n + \phi_{j+1}^n}{\Delta x^2} + \frac{\phi_j^{n-1} - 2\phi_j^n + \phi_j^{n+1}}{\Delta y^2} = 0$$

$$\text{let } \Delta x = \Delta y$$

$$\Rightarrow \frac{1}{\Delta x^2} (\phi_{j-1}^n + \phi_{j+1}^n + \phi_j^{n-1} + \phi_j^{n+1} - 4\phi_j^n) = 0$$

$$(\phi_{j-1}^n + \phi_{j+1}^n + \phi_j^{n-1} + \phi_j^{n+1}) = 4\phi_j^n$$

$$\phi_j^n = \frac{1}{4} (\phi_{j-1}^n + \phi_{j+1}^n + \phi_j^{n-1} + \phi_j^{n+1})$$

Finite Difference Method

