$$P_t = DP_{xx} + P \Rightarrow \left(D\frac{\partial^2}{\partial x^2} + 1\right)\rho = P_t$$

$$\frac{\rho_j^{n+1} - \rho_j^n}{\Delta t} = \frac{D}{2\Delta x}\left(\rho_{j+1}^{n+1} - 2\rho_j^{n+1} + \rho_{j-1}^{n+1} + \rho_{j+1}^n - 2\rho_j^n + \rho_{j-1}^n\right) + \frac{1}{2}\left(\rho_j^{n+1} + \rho_j^n\right)$$

$$\Rightarrow \rho_j^{n+1} = \rho_j^n + \frac{D\Delta t}{2\Delta x}\left(\rho_{j+1}^{n+1} - 2\rho_j^{n+1} + \rho_{j-1}^{n+1} + \rho_{j+1}^n - 2\rho_j^n + \rho_{j-1}^n\right) + \frac{\Delta t}{2}\left(\rho_j^{n+1} + \rho_j^n\right)$$

$$\Rightarrow \rho_j^{n+1} - \frac{D\Delta t}{2\Delta x}\left(\rho_{j+1}^{n+1} - 2\rho_j^{n+1} + \rho_{j-1}^{n+1}\right) - \frac{\Delta t}{2}\rho_j^{n+1} = \rho_j^n + \frac{D\Delta t}{2\Delta x}\left(\rho_{j+1}^n - 2\rho_j^n + \rho_{j-1}^n\right) + \frac{\Delta t}{2}\rho_j^n$$

$$\cancel{\Rightarrow \frac{\rho_j^{n+1} - \rho_j^n}{\Delta t} = \frac{D}{\Delta x^2}\left(\rho_{j+1}^n - 2\rho_j^n + \rho_{j-1}^n\right) + \frac{1}{2}\left(\rho_j^n + \rho_j^n\right)}$$

$$\cancel{\rho^{n+1} - \rho_j^n = \frac{D\Delta t}{\Delta x^2}\left(\rho_{j+1}^n - 2\rho_j^n + \rho_{j-1}^n\right) + \frac{\Delta t}{2}\left(\rho^{n+1} + \rho^n\right)}$$

$$\cancel{\rho^{n+1}\left(1 - \frac{\Delta t}{2}\right) = \rho_j^n\left(1 + \frac{D\Delta t}{\Delta x^2}(\ ) + \frac{\Delta t}{2}\right)}$$

$$\Rightarrow \rho^{n+1}\left(\mathbb{1} - \frac{D\Delta t}{2\Delta x}A - \frac{\Delta t}{2}\mathbb{1}\right) = \rho^n\left(\mathbb{1} + \frac{D\Delta t}{2\Delta x}A + \frac{\Delta t}{2}\mathbb{1}\right)$$

$$\rho^{n+1} = \rho^n\left(\mathbb{1} + \frac{D\Delta t}{2\Delta x}A + \frac{\Delta t}{2}\mathbb{1}\right)\left(\mathbb{1} - \frac{D\Delta t}{2\Delta x}A - \frac{\Delta t}{2}\mathbb{1}\right)^{-1}$$