a - 30 oct

CAT-1 27 Oct
Unit-4,5 => Google forms

12-Oct

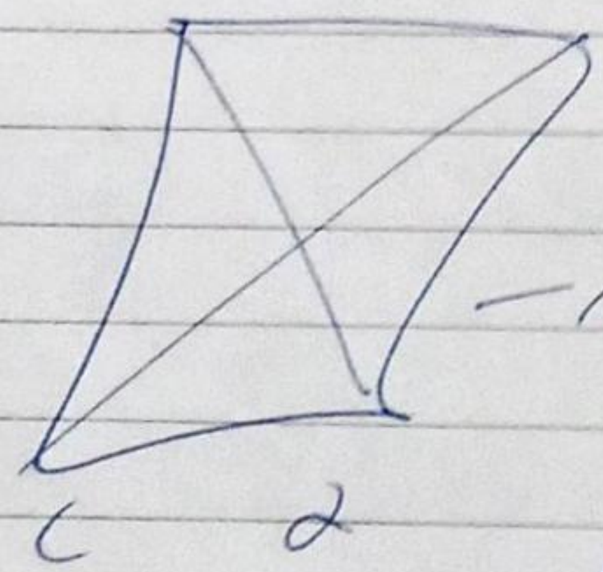
Unit-5

Planar Graph

eg. Map
↳ Map of India, or world
↳ Map Representation

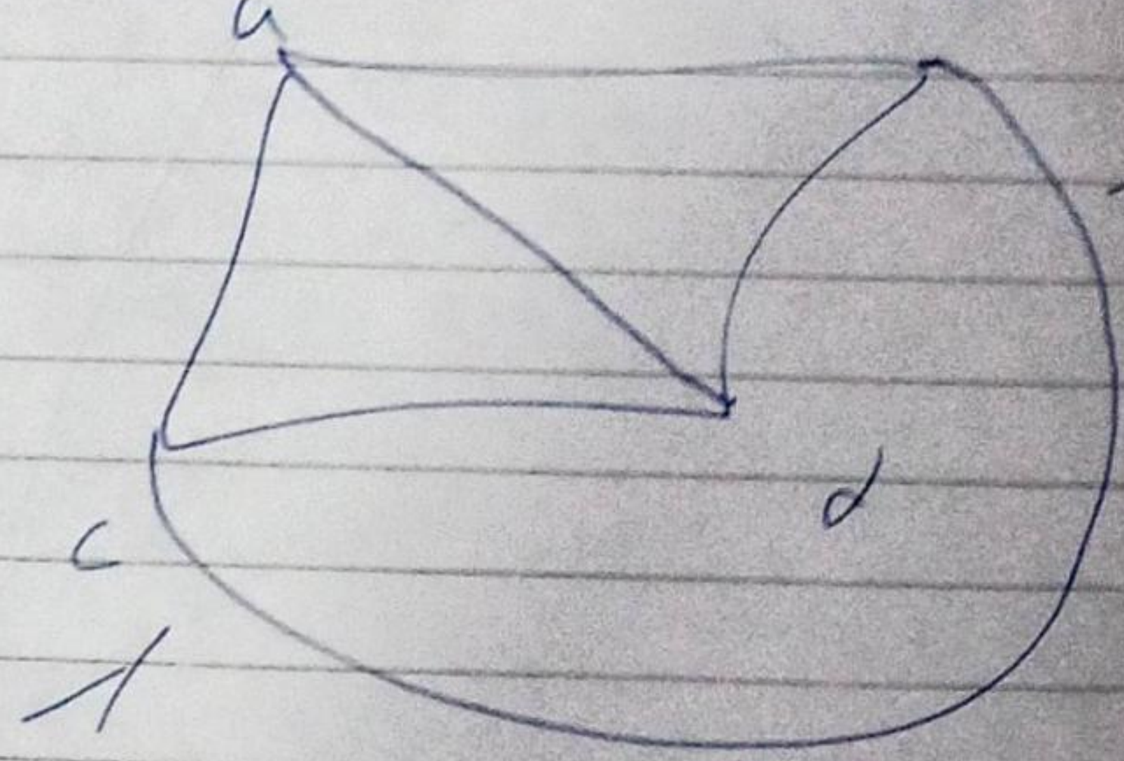
edges must not cross each other.

K_4 =>
complete graph



=> not planar

K_4

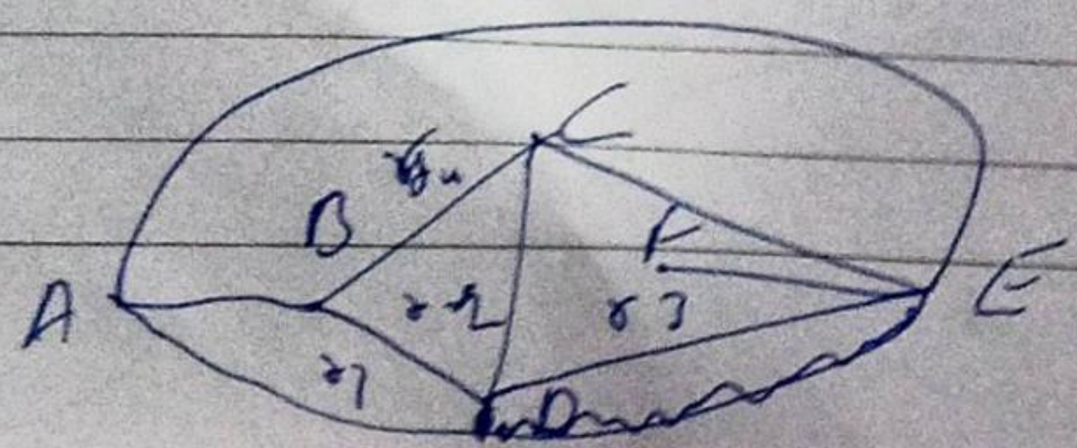


=> planar graph

K_4

Map Representation / Planar representation

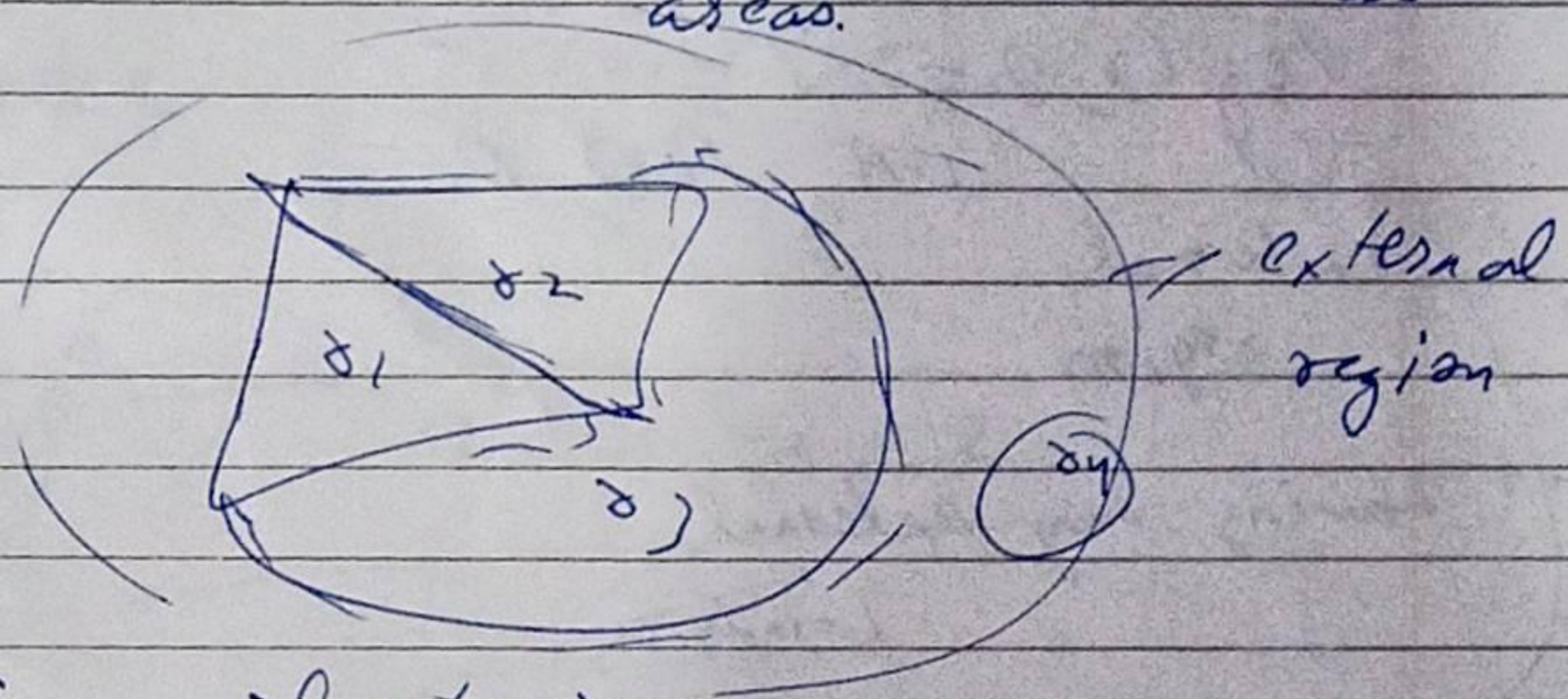
K_4



Sectors bounded by the edges = regions.

internal closed areas.

closed areas



r egions = closed areas

with internal regions there always exists an external region to the map

The sum of the deg of the regions is equal to twice the no. of edges

Deg of region = No. of edges bounding the closed area.

Map, region

AB, BD, DA

we have to include this as part of ~~the~~ f3

r1 deg (r1) = 3
deg (r2) = 3
deg (r3) = 5
L, ~~DE~~ CD, DE, EF, FE, EC

If start from f_1 we have to come back to f_1 only

$\overset{\frown}{\quad}$ ~~AB~~ BC, CE, EA

$\deg(f_4) = 4$

$\deg(f_3) = 3$

CA, AD, DE

external region

not having any external boundries

the edges with which it is adjacting

Sum of deg $= 3+3+5+4+3$
$= 18$

edges $= 9$

There is always one external to the map.

$\therefore$ deg $= 2 \times$ edges

the deg of external region $=$ edges from which it is adjacting

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Euler formula

$v - e + r = 2$

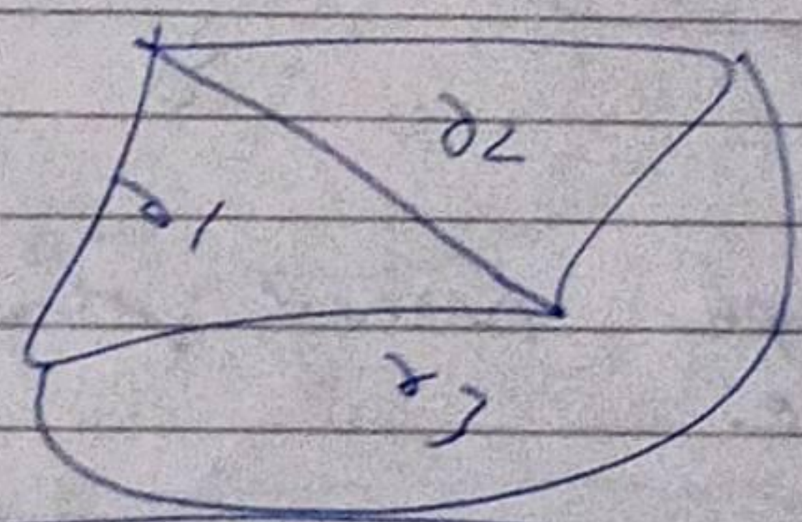
vertices, edges, region

no. of region

$6 - 9 + 5 = 2$

$2 = 2$

K_4



$4 - 6 + 4 = 2$

$2 = 2$

r_4

not no. of edges in any simple graph

n = No. of vertices

In K_4 ⇒ degree of every vertex $= 3$

$(n-1)$

sum of $\deg = n(n-1) = 2e$

handshaking theorem

no. of edges $= e = \frac{n(n-1)}{2}$

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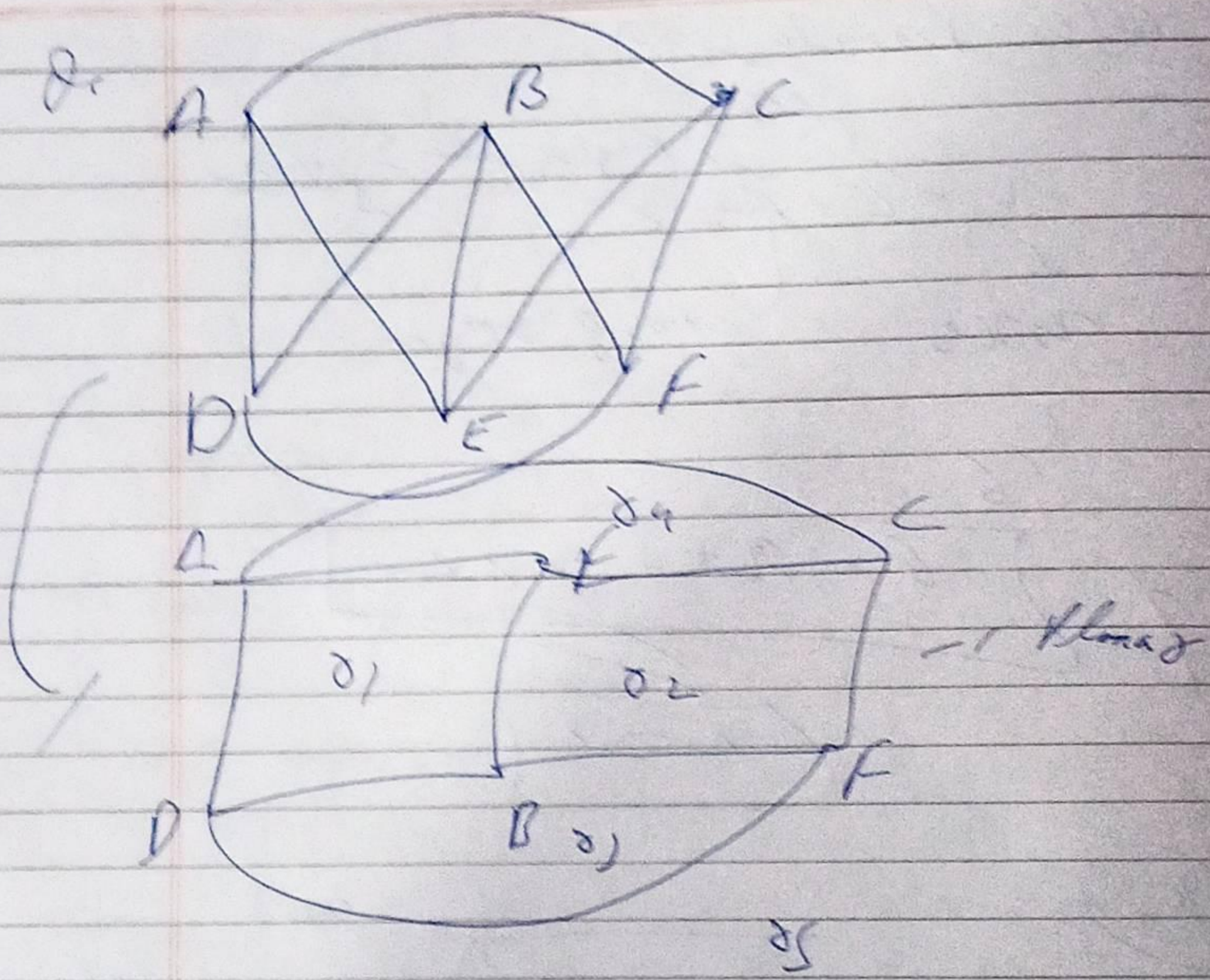

The graph drawn planar Euler formula holds true.

$$\boxed{\begin{aligned} 6 - 9 + 5 &= 2 \\ 2 &= 2 \end{aligned}}$$

$$\sum \deg(\gamma) = 18$$

sum of degree of regions

$$\underset{\gamma_1}{4} + \underset{\gamma_2}{4} + \underset{\gamma_3}{3} + \underset{\gamma_4}{3} + \underset{\gamma_5}{4}$$

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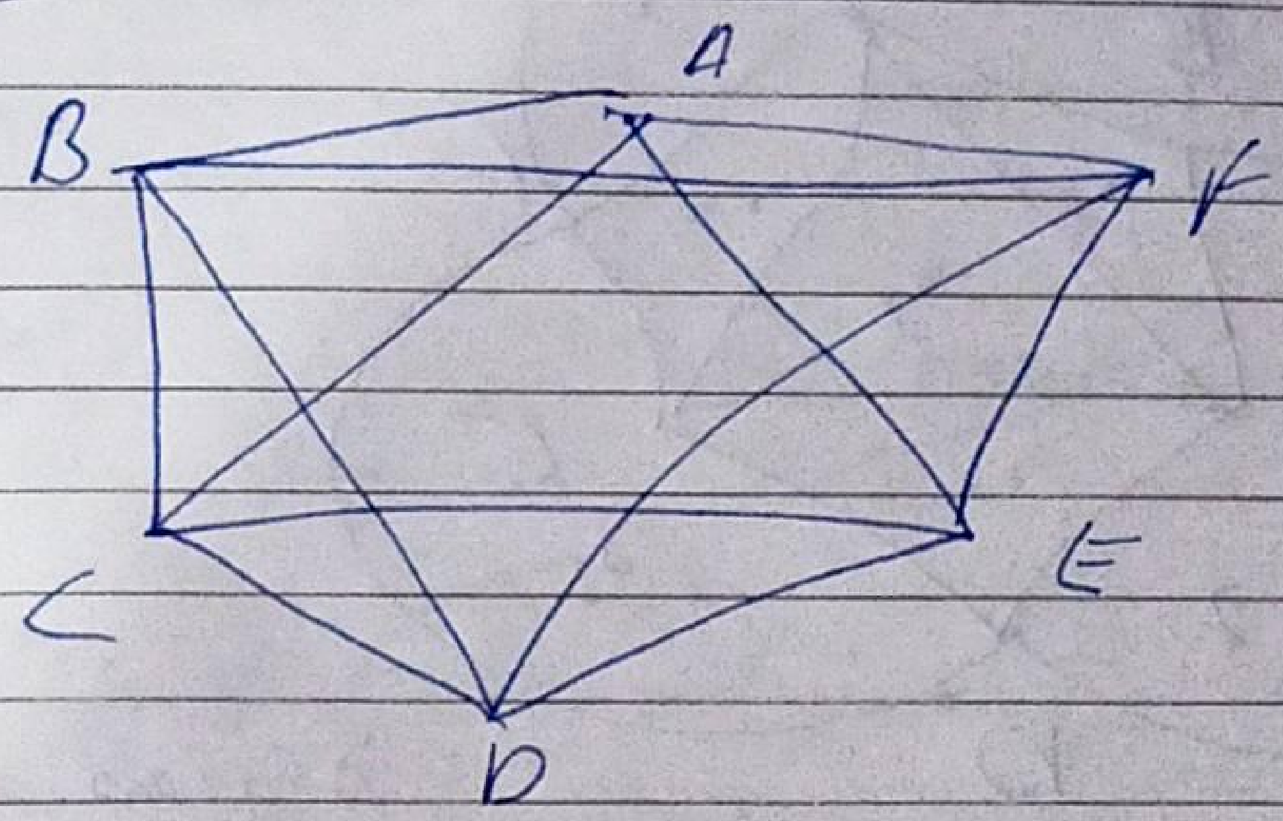
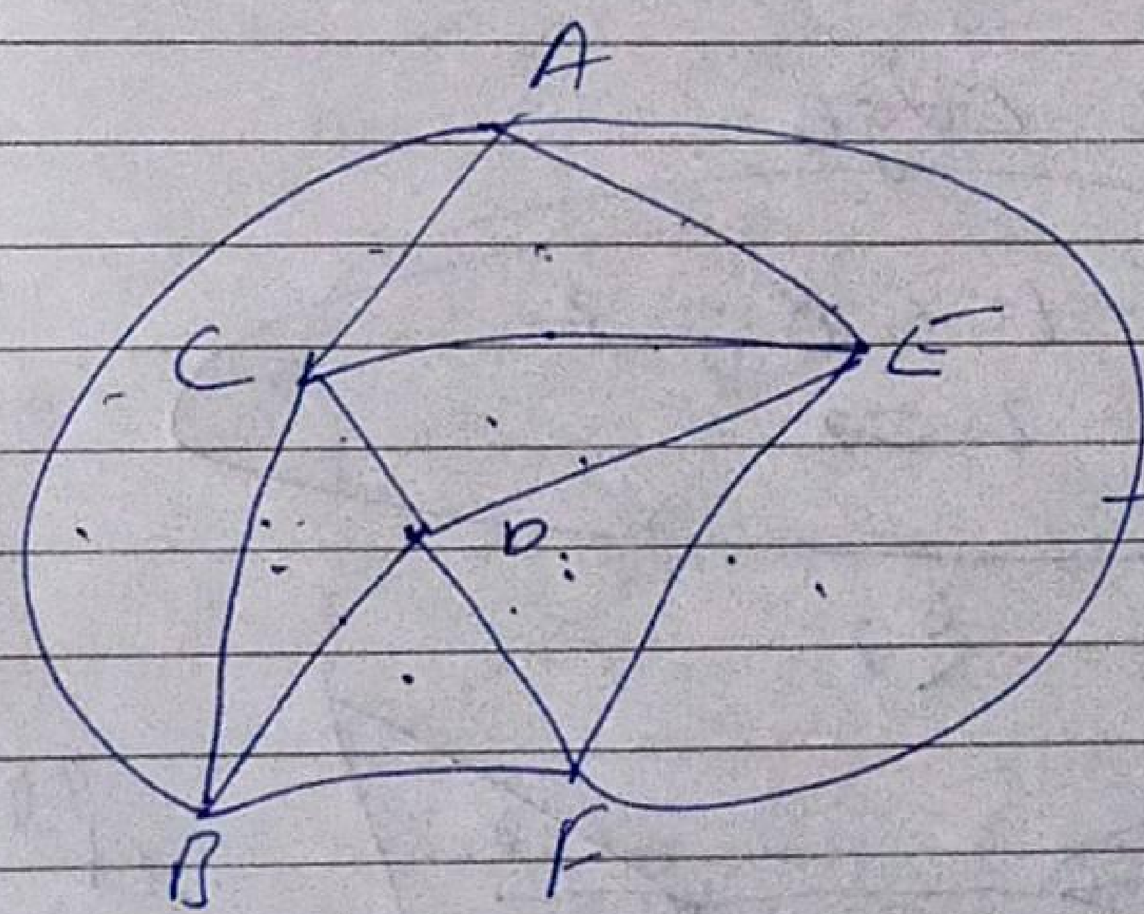
Page:

2: Is the graph planar



Yes / No



1 planar

$$6 - 12 + 8 = 2$$
$$2 = 2$$

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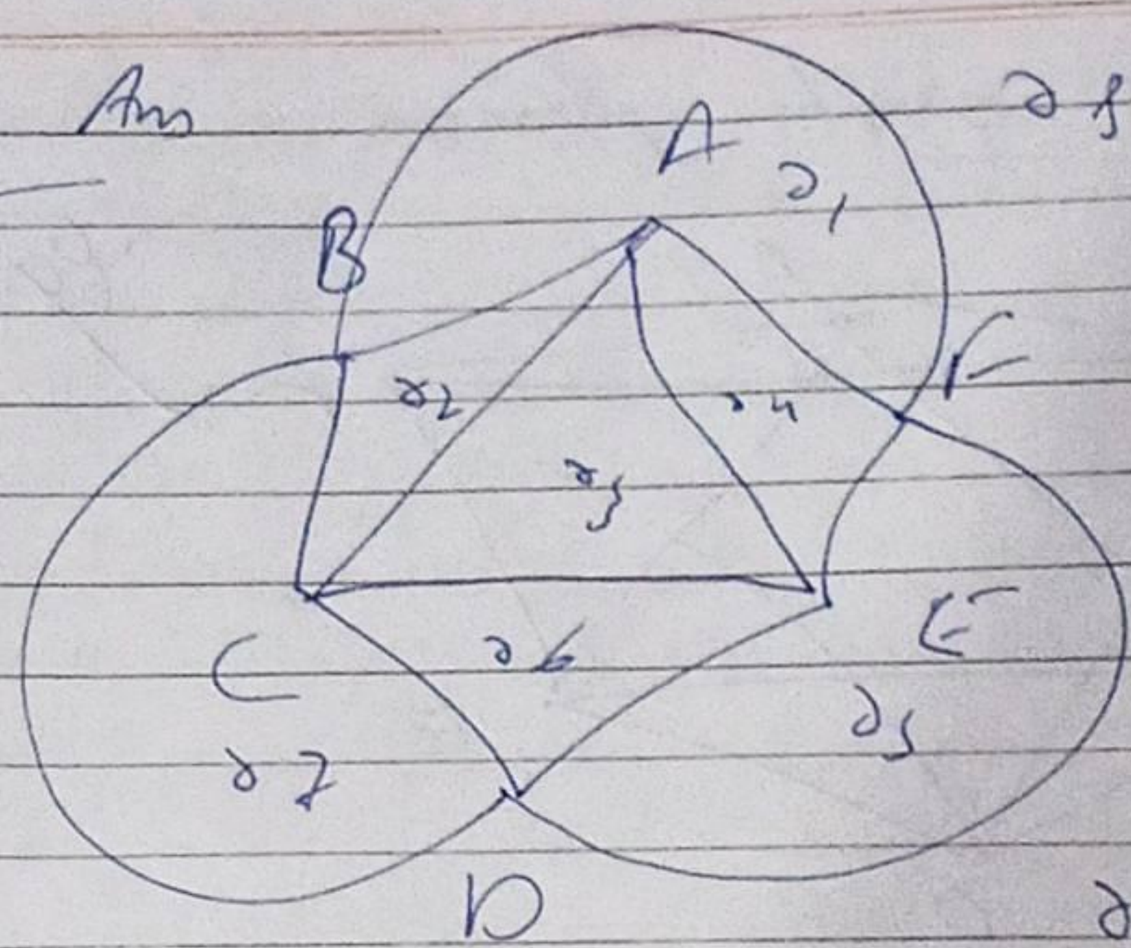
Num Ans



r_8

regions = 8

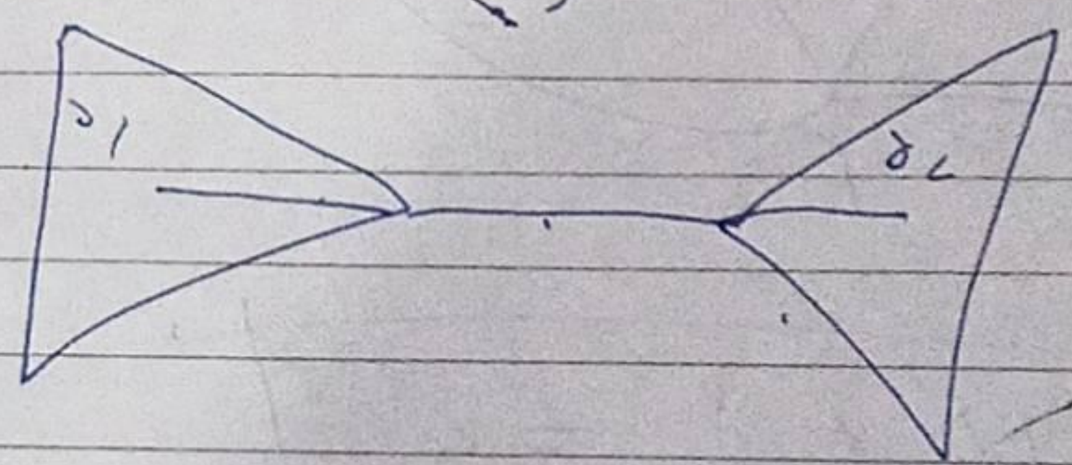
12 Edges

$4 - 12 + 8 = 2$

$2 = 2$

r_3

2:



Planar

Find

① No. of regions

Ans = 3

② Deg of regions R_1 & R_2

3, 4, 5, 6

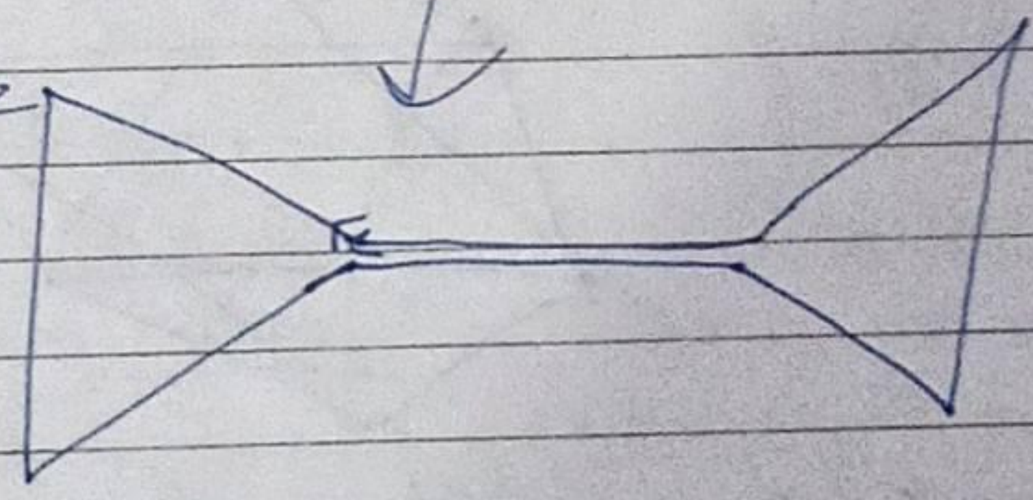
whatever u draw, no. of regions will be same for a planar graph

Q: If G is a connected simple planar

(1) Deg of reg R_3 is , deg-1 closed ckt

6 , 7 , 8 , 9

closed structure



(2) No. of vertices & edges are

(1) 6, 9, (2) 6, 8 (3) 7, 9 (4) 8, 9

$v - E + R =$

- If a graph is planar, then euler formula hold true

- If G is a connected planar simple graph with e edges and v vertices, where $v \geq 3$, then $\boxed{e \leq 3v - 6}$

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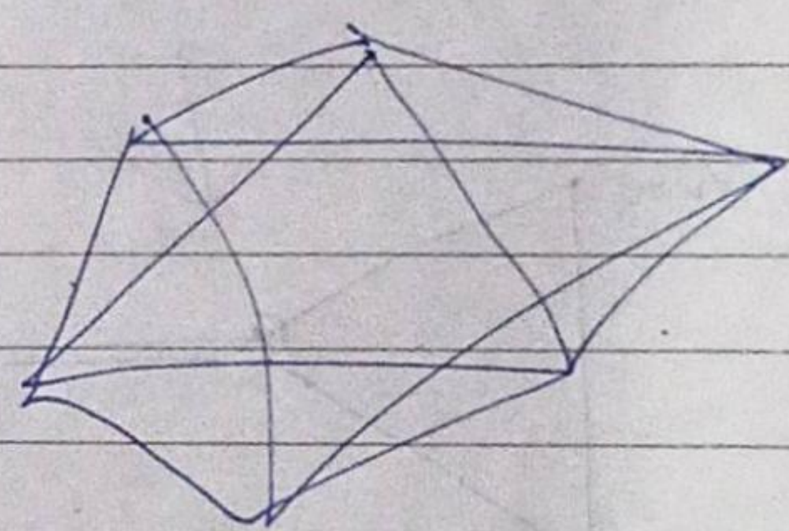
if $e \leq 3v - 6$

, then graph is planar

2. Suppose that a connected planar simple graph has 20 vertices, each of degree 3. Into how many regions does a representation of this planar graph split the plane?

(a) 30 (b) 23 (c) 12 (d) 10



12 edges, 6 vertices

$e \leq 3v - 6$

$12 \leq 3 \times 6 - 6$

$12 \leq 12$

$\therefore$ graph is planar

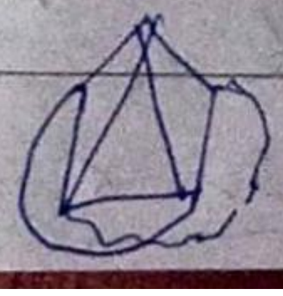
K_4

$v = 4$

$e = 6$

$6 \leq 2(4) - 6$

$6 \leq 6$

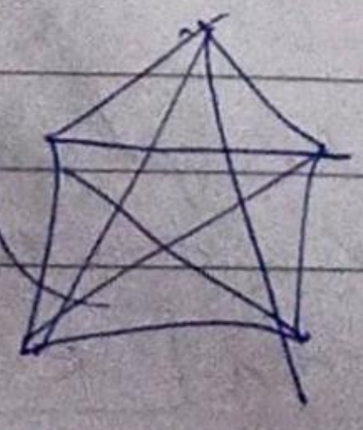


K_5 $v = 5$

$e = \frac{n(n-1)}{2} \Rightarrow \frac{20}{2} = 10$

$10 \leq 15 - 6$

$10 \leq 9$ f, K_5 not planar



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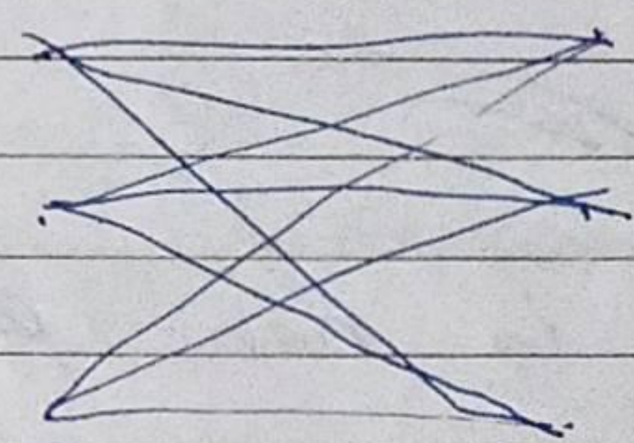
check for $K_{3,3}$ ⇒ Complete Bipartite graph

⇒ have cycle of length 4.



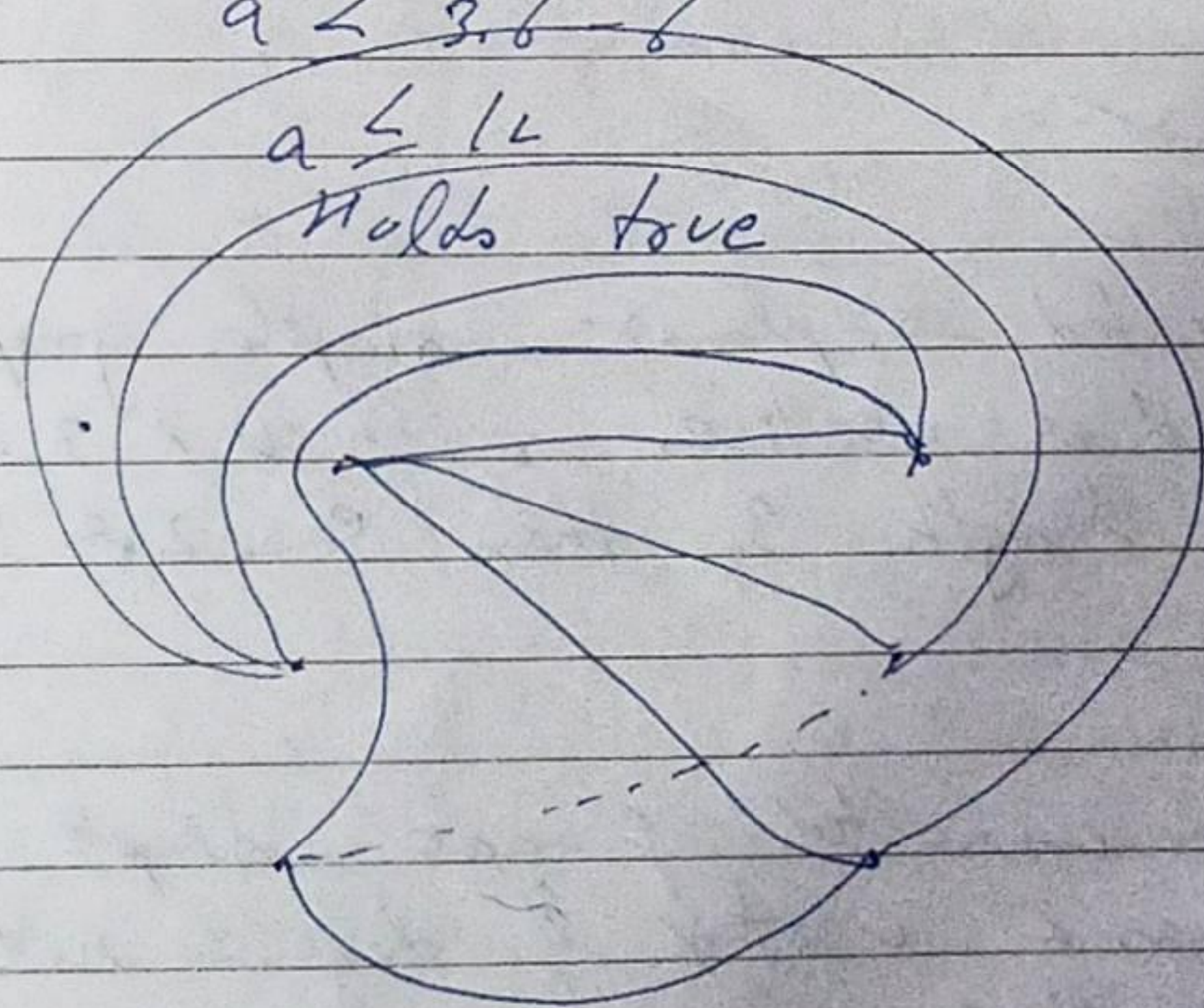
⇒ $K_{3,3}$ is not ~~planar~~ planar

$e \leq 3v - 6$

$9 \leq 3.6 - 6$

$9 \leq 12$

Holds true



∴ Planar representation of $K_{3,3}$ not exist

⇒ No cycle of length 3 exist in $K_{3,3}$

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~~M. 19:00 - 15:00~~

$e \leq 2v - 4$

$9 \leq 2.6 - 4$

$9 \leq 8$

not true //

$\therefore K_{3,3}$ is non-planar

//

Hence there is no cycle of length 3 in a graph.

~~15:00~~

- If a connected ~~ti~~ planar simple graph has e edges and v vertices with $v \geq 3$ and no circuits of length 3, then $e \leq 2v - 4$

If G is a connected planar simple graph, then G has a vertex of degree not exceeding five

$e \le 3v - 6$

$2e \le 4v - 11$

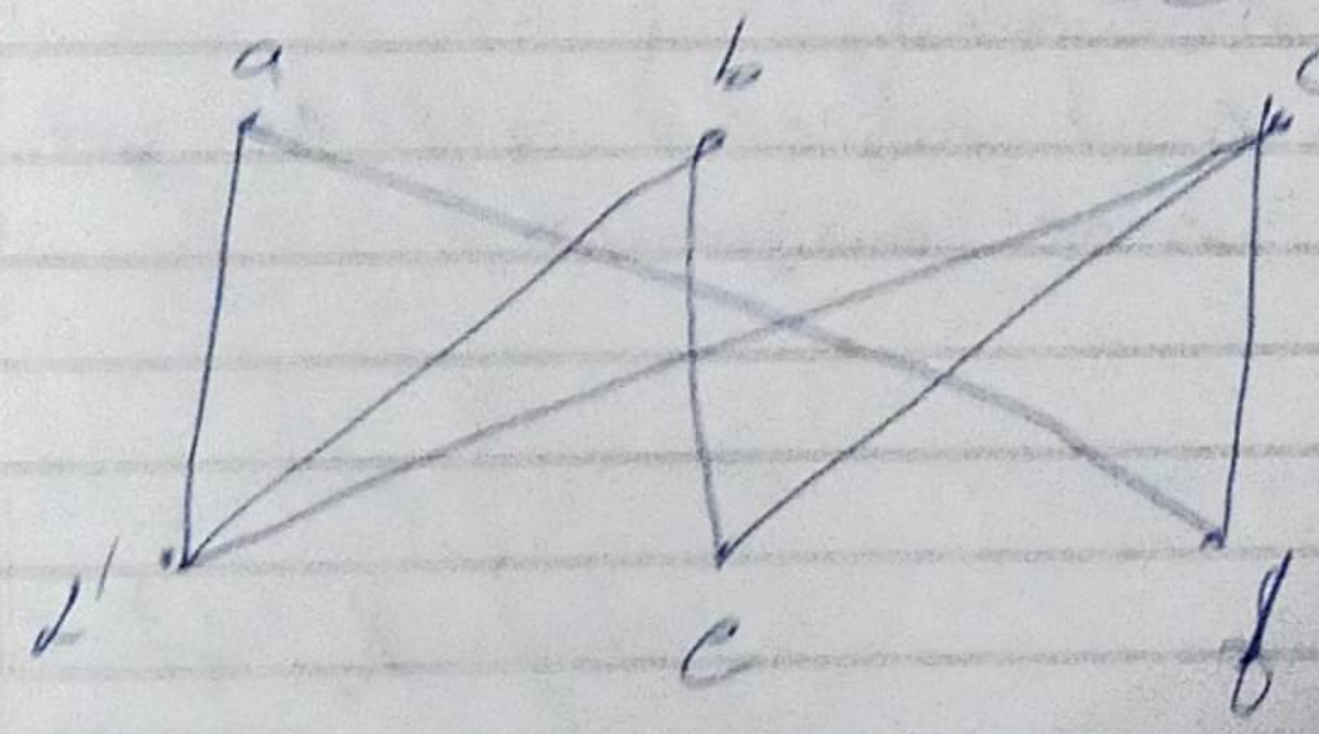
Assume there was a planar with deg ≥ 6
(sum of the deg of vertices
$\sum \deg(v) \ge 6v$

$2e \ge 6v$

Contradiction

a) How many regions are there in the graph if it has planar representation.

(a) 1 (b) 4 (c) 2 (d) 3



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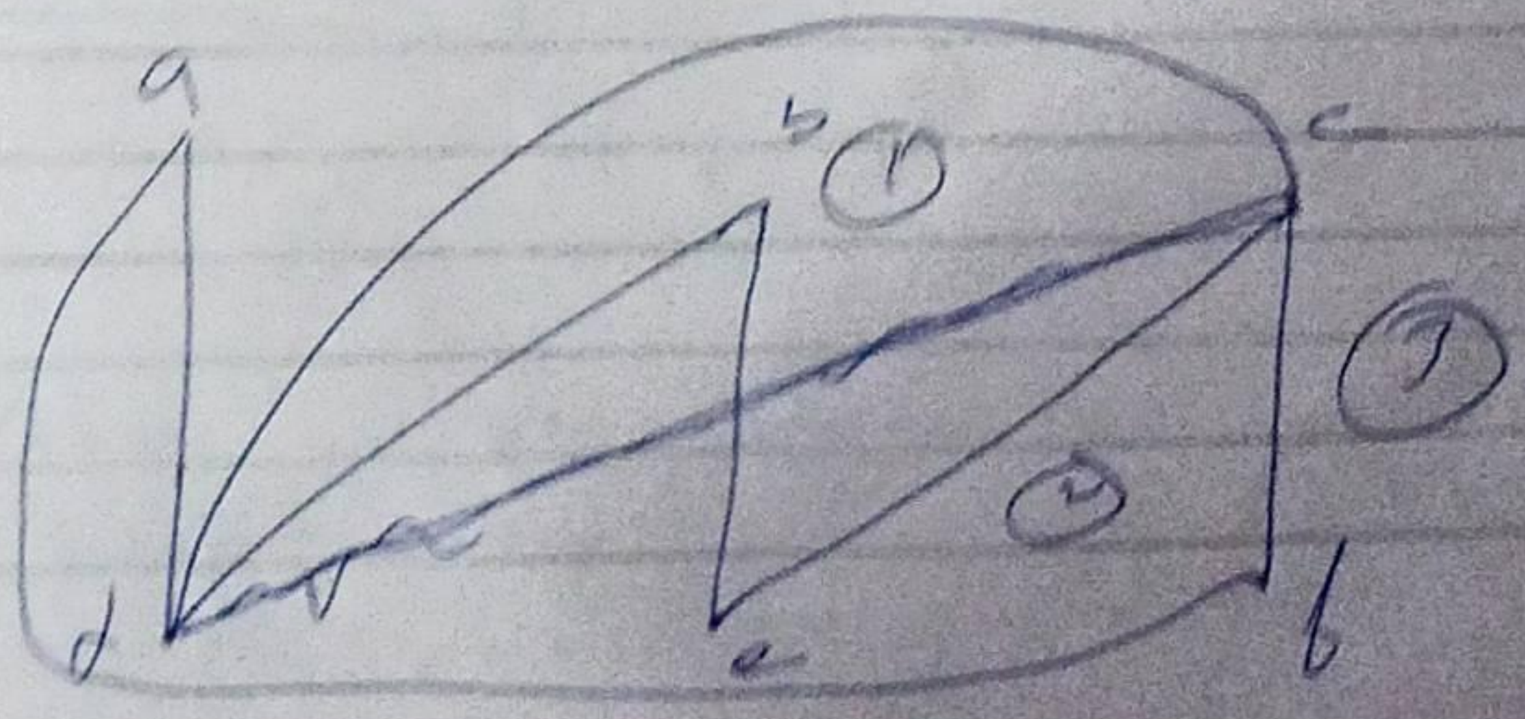

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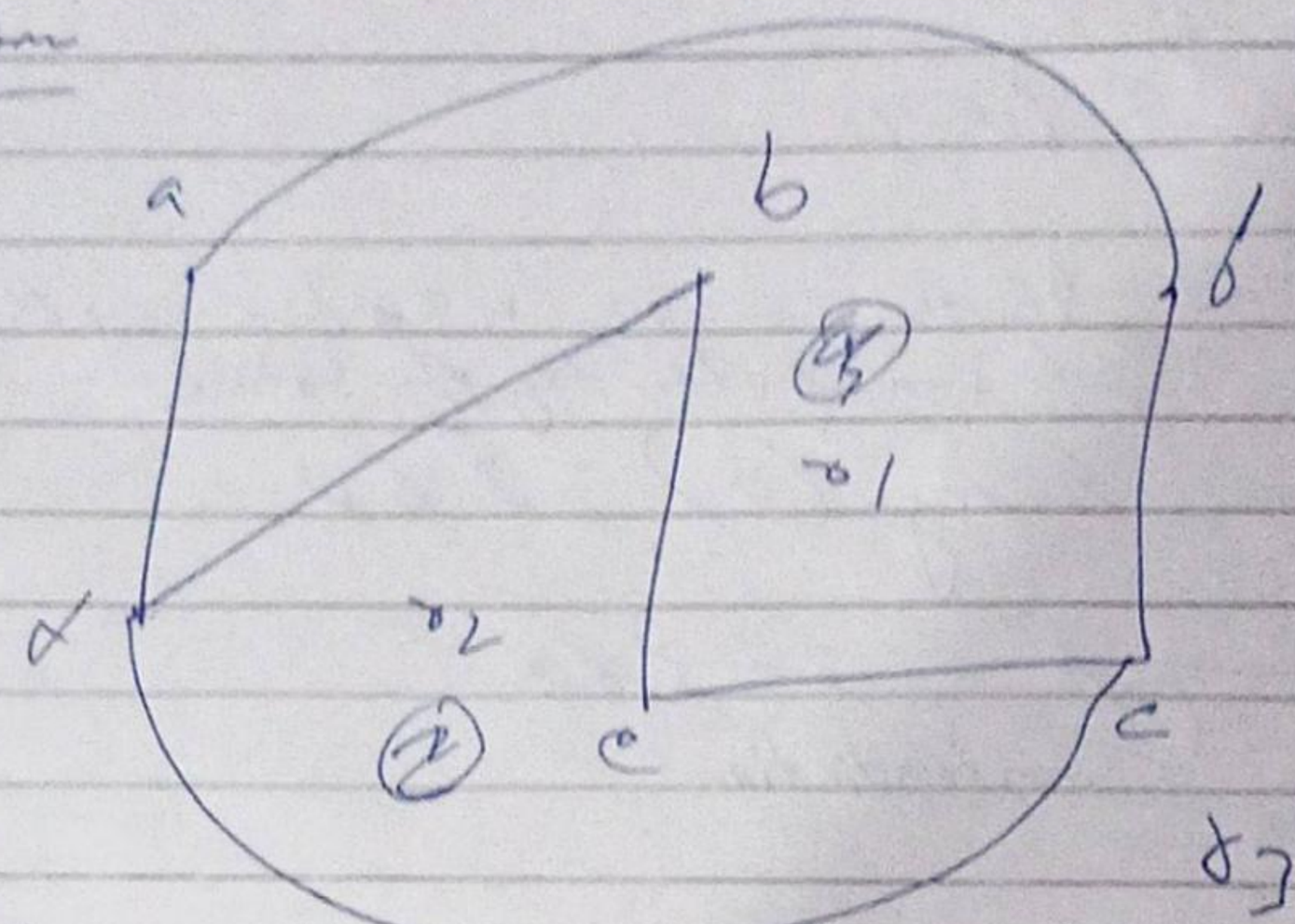

Graph Coloring :/ Color the vertices of the graph

A coloring of a simple graph in such a way that no 2 adjacent vertices enjoy the same color

Chromatic no.

$\chi(G) = 3$

Application: Networking
↳ No 2 adjacent roll No. get same assignment

$$\boxed{\chi(G) = 3}$$

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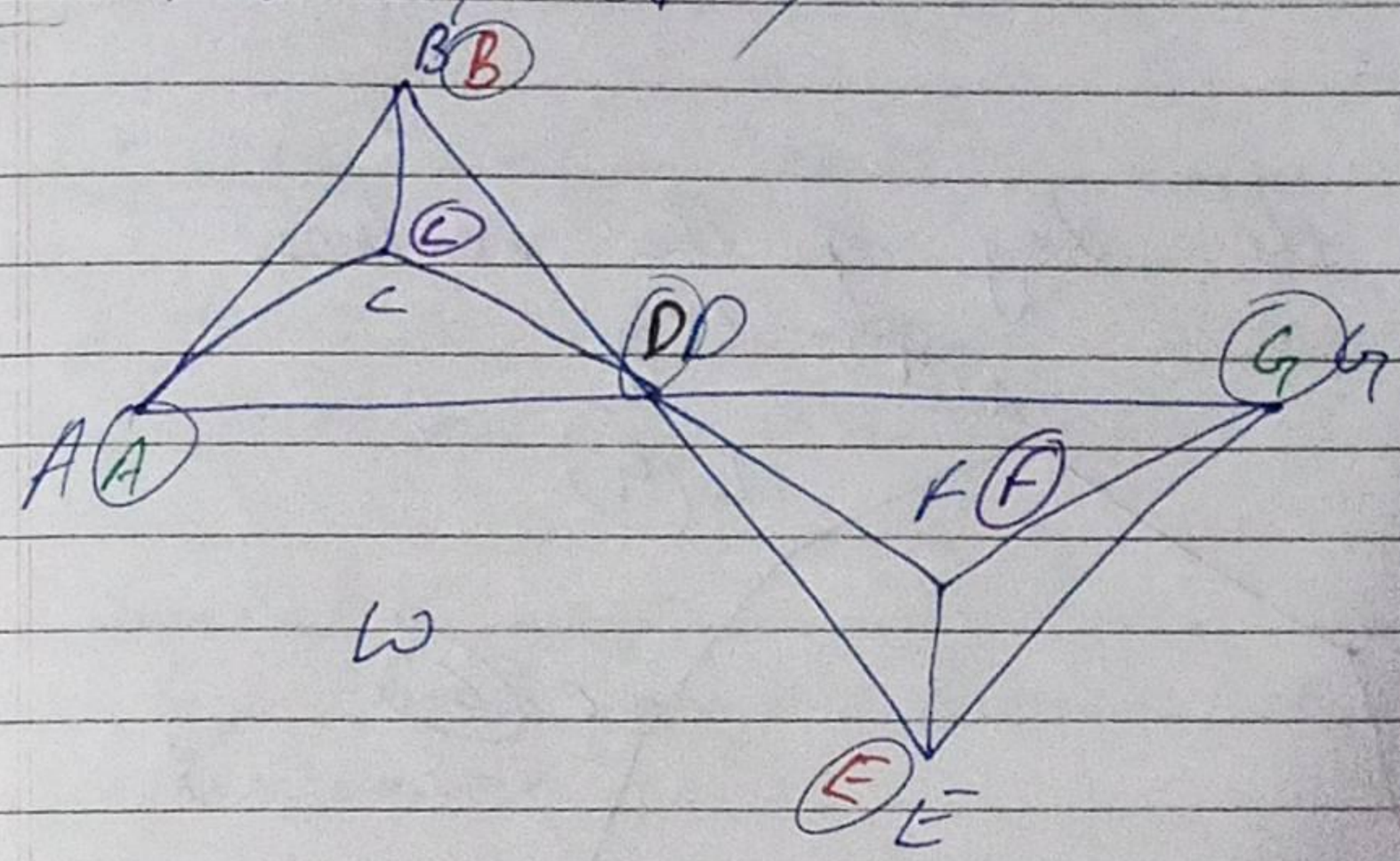
Q: Find $\chi(G)$



(a) 3 (b) 4

(c) 2 (d) 5

$\chi(G) = 4$

Q: what is the chromatic no. of K_n?

(a) 2 (b) n (c) n-1 (d) 5

as every vertex is connected to every other vertex

G N. of Cycle --
Bipartite -

Q: What is the chromatic no. of the complete bipartite graph $K_{m,n}$, where m and n are the +ve integers?

: m+n : 2
: n : m.n

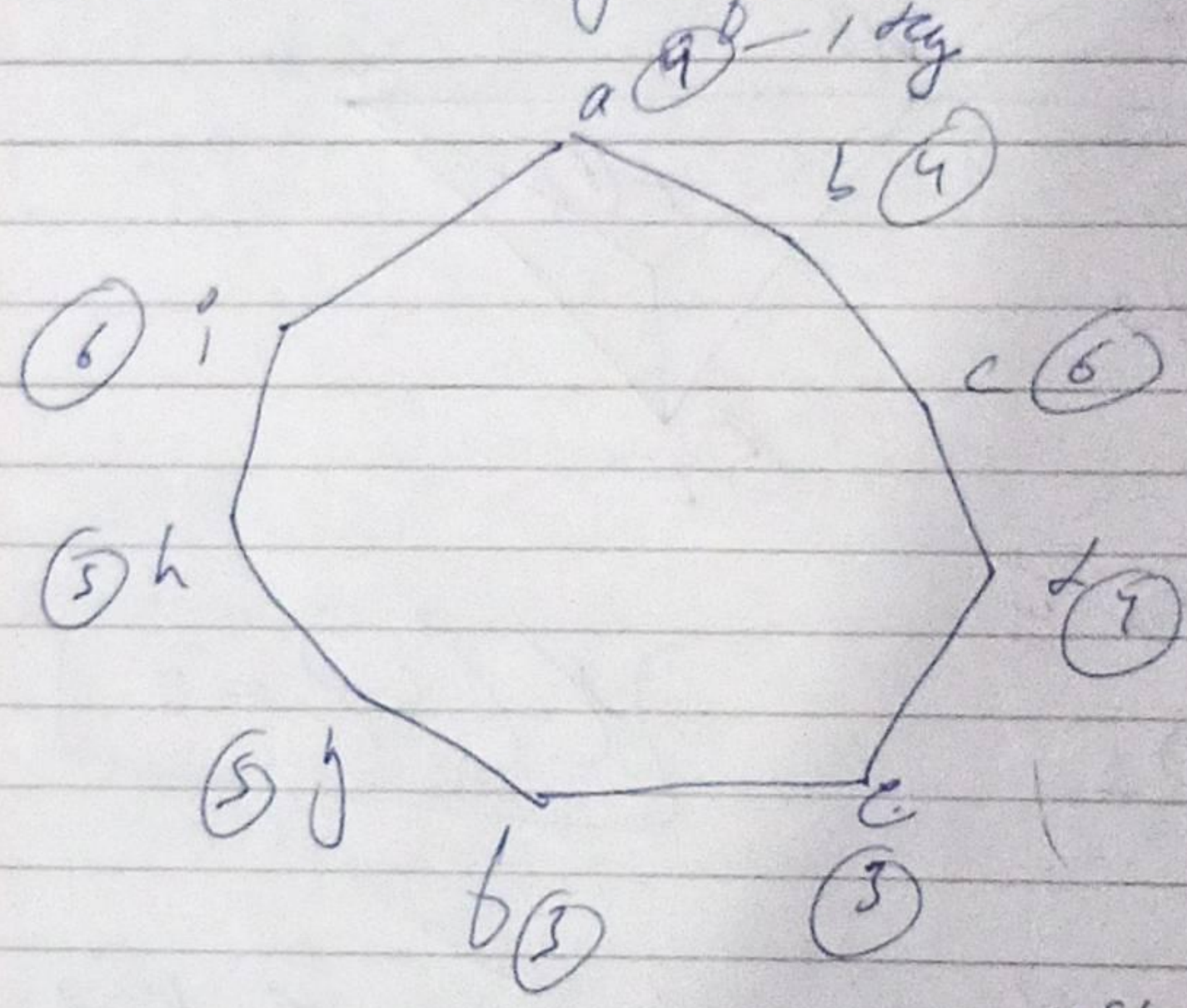
way to find Chromatic

Page No.

Welch - Powell Algo

step 1

① Count the deg of the vertices.



us:00 - -

② Arrange these vertices in descending order of deg

6 ← i	$- c_1$	f $- c_3$
6 ← c	$- c_2$	← e $- c_2$
5 ← g	$- c_1$	j
5 ← h	$- c_3$	
4 ← a	$- c_2$	
4 ← b	$- c_4$	
4 ← d	$- c_3$	

$\chi(G) = 4$

③ Now give a color to highest deg. vertex

④ see to chom is same

Earlier graph were a closed sto, od have a cycle

Tree Graph

G_1 – Tree
G_2 – "
G_3 – Not a Tree
G_4 – "

· not have any cycle

A tree is a connected undirected graph with no simple circuits.

· connected graph
↳ find a path from a vertex to any other vertex

G_3 is not a tree ⇒ bcoz cycle in there
↳ Although it is connected

G_1, G_2 ⇒ Tree

G_4 ⇒ not tree
↳ as not connected

⇒ Unique simple path ⇒ no cycle
· Rooted tree

b is the parent of c

h, i, j → g (parent)

→ give name choose to g c f but also see that they r not connected

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leaf has no children

Some parent → siblings.

d, e, f, k, l, i, m

internal vertices.

which have child

leaf

ancestors of k =, h, g, a

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level Δ height

level

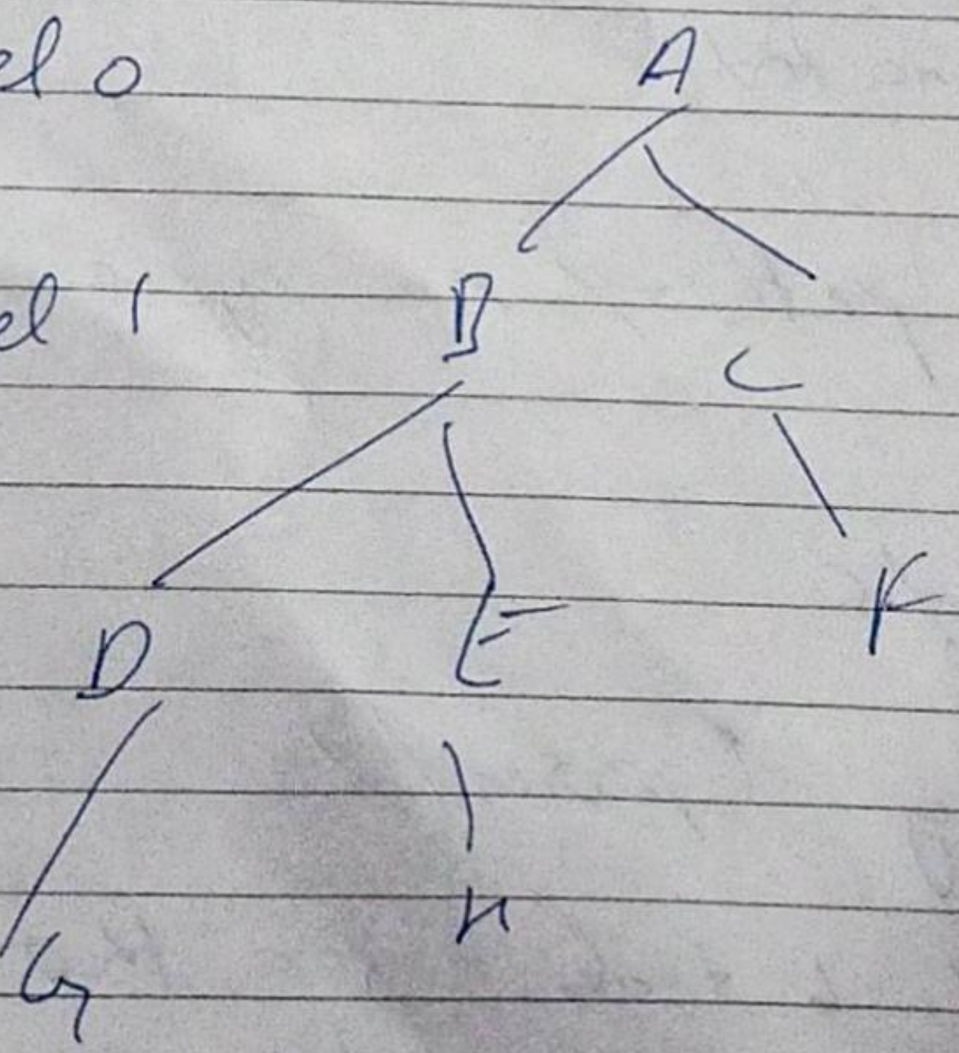
↳ distance of vertices from root

level 0

level 1

level 2

level 3



count level

⇓

height of the tree

⇓

no. of levels

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m-Ary tree

Every internal vertex have not more than m children.

$T_4 \to$ only 2 array tree

$T_3 \to$ Fully 3-array tree

$T_2 \to$ Full 2-ary tree

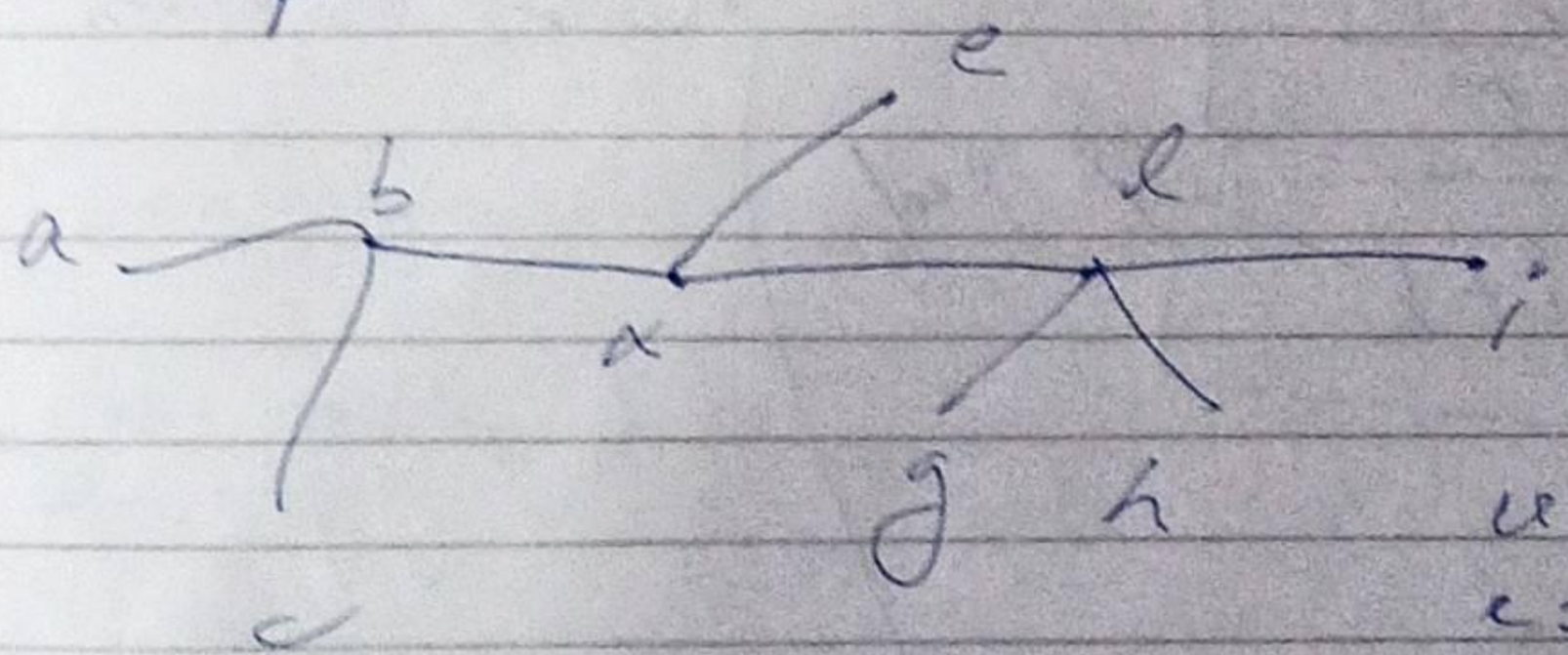
$T_1 \to$ Full 1 ary tree

Internal vertex

not leaf.

A tree with n vertices has n-1 edges

Imp.



$v = 9$

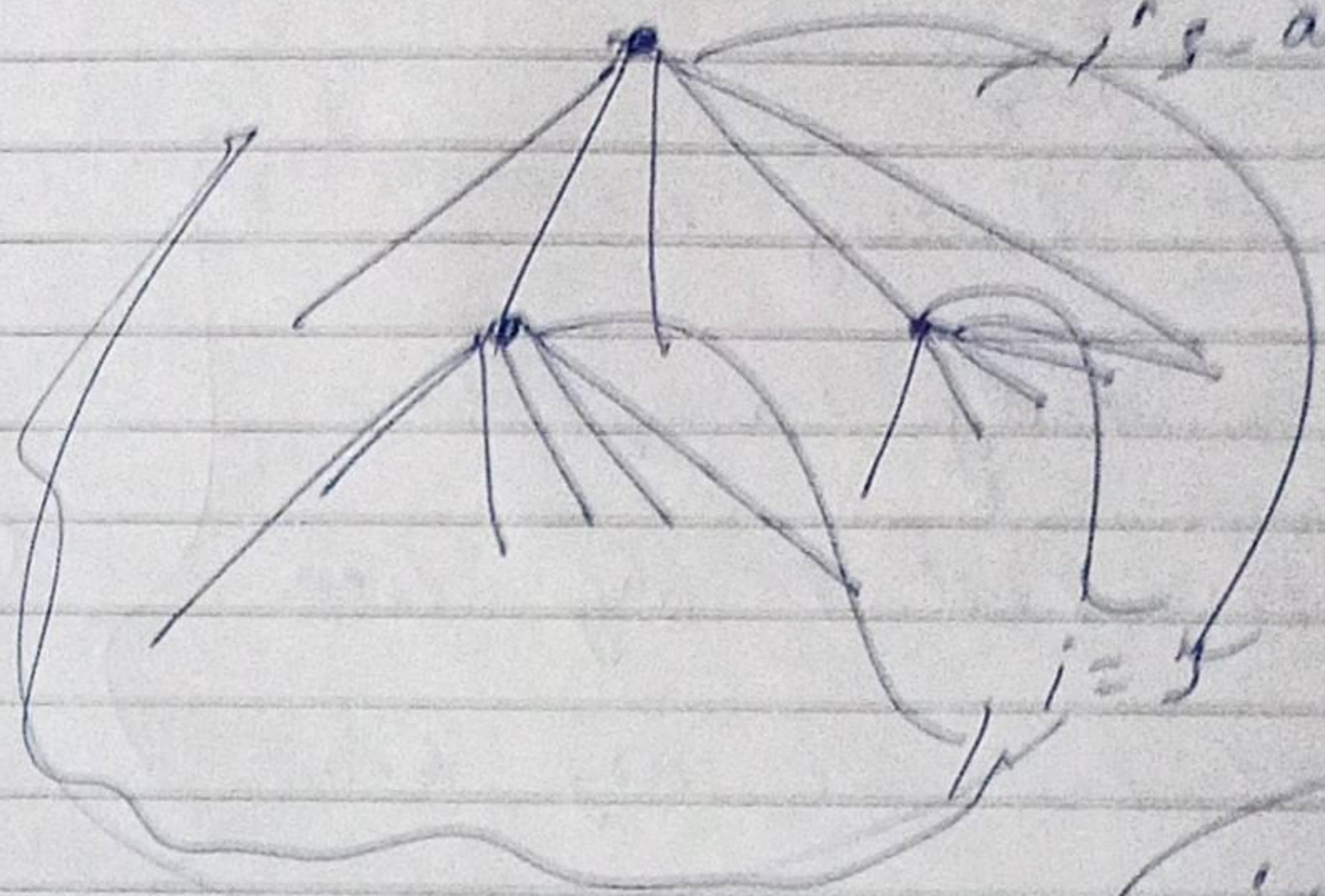
$e = 8$

1. connected
2. no circuits.

vertices

A full m-ary tree with i internal vertices contains (n) = mi + 1 vertices.

full m's - ary tree



i = internal vertices

$n = 3 \times 5 + 1$

$\Rightarrow 16$ vertices.

① $\boxed{\begin{aligned} n &= mi + 1 \\ i &= \frac{n-1}{m} \end{aligned}}$

$\boxed{n = i + l}$

leaf

total no. of vertices.

internal vertices

$l = n - i$

$= n - \left(\frac{n-1}{m}\right)$

one

② $\boxed{l = \frac{[(m-1)n + 1]}{m}}$

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$n = mi + 1$

$i + 1 = mi + 1$

(2) $\rightarrow$ $l = (m-1)i + 1$

Q. 5-11

$m = 4$, $l = 100$ $i = ?$

$l = (m-1)i + 1$

$100 = (4-1)i + 1$

$\frac{100 - 1}{3} = i$

$\boxed{33 = i}$

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(7) $$\boxed{n = \frac{(ml-1)}{m-1}}$$

$$\boxed{i = \frac{l-1}{m-1}}$$

How many edges does a full binary tree with 1000 internal vertices have

1500 ✓ 1000 , 2500 , 500

$m = 2$, $i = 1000$

$n = mi + 1$
$= 2001$

$2001 - 1 = 2000$

Spanning tree = I have to include every vertex of the graph.

Let G be a simple graph. A spanning tree of G is a subgraph of G that is a tree

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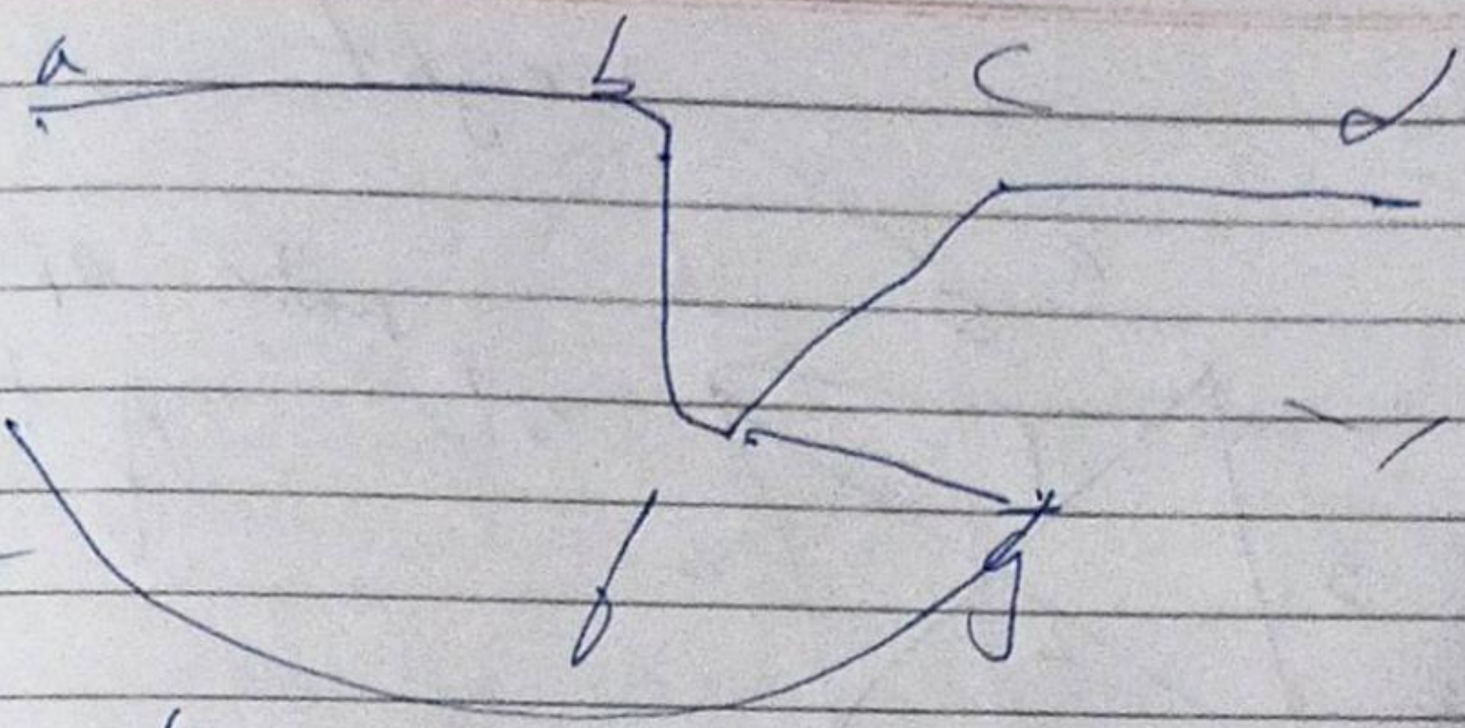
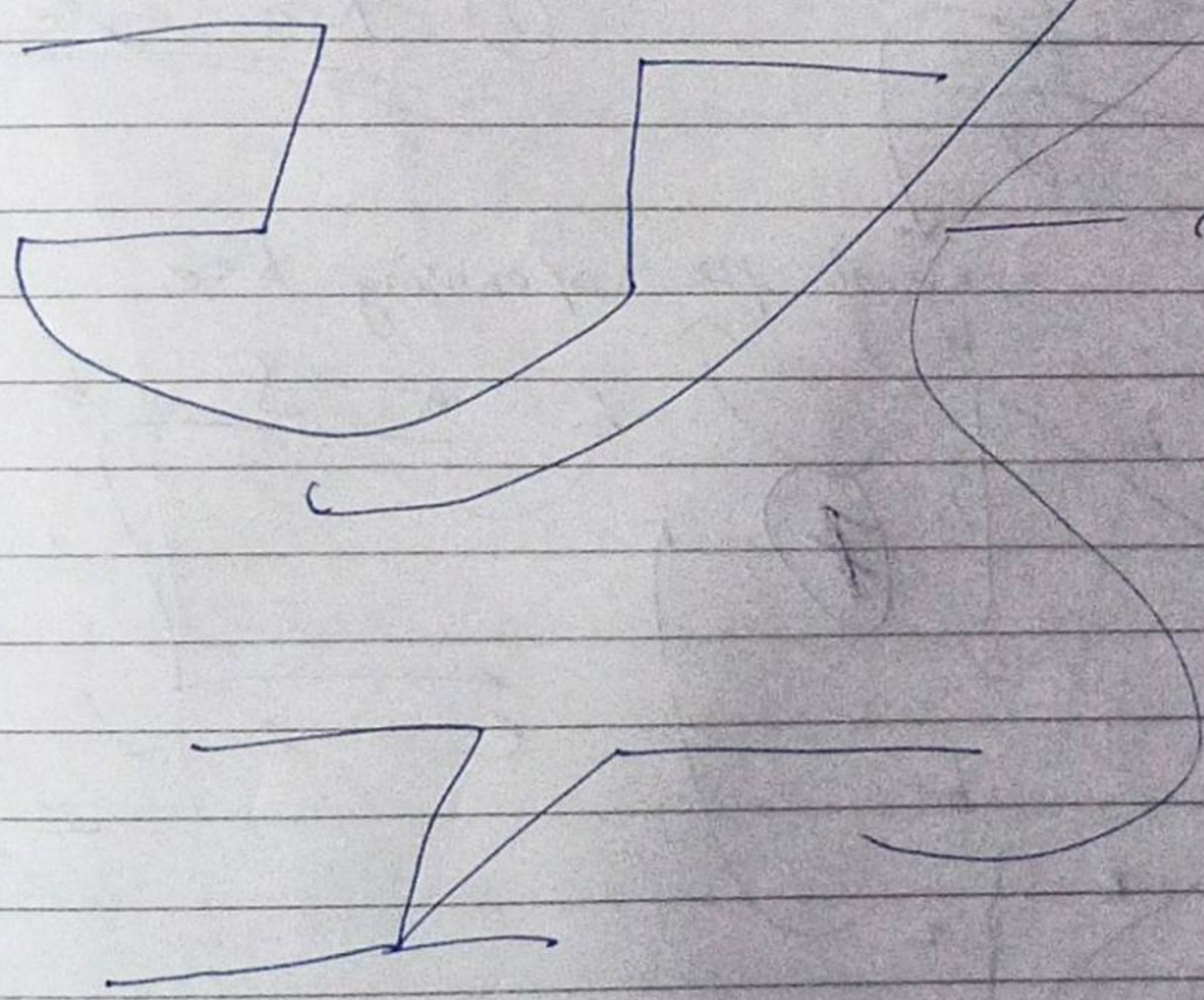


one of the spanning tree

- 7 vertices with 6 edges
- connected with <u>no cycle</u>

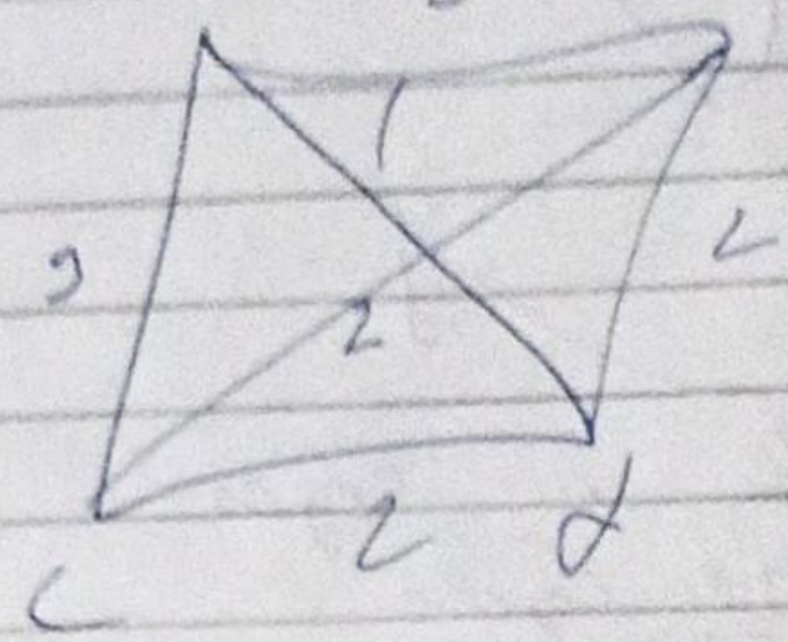


16 edges

There can be no. of spanning tree for a graph.

K_4

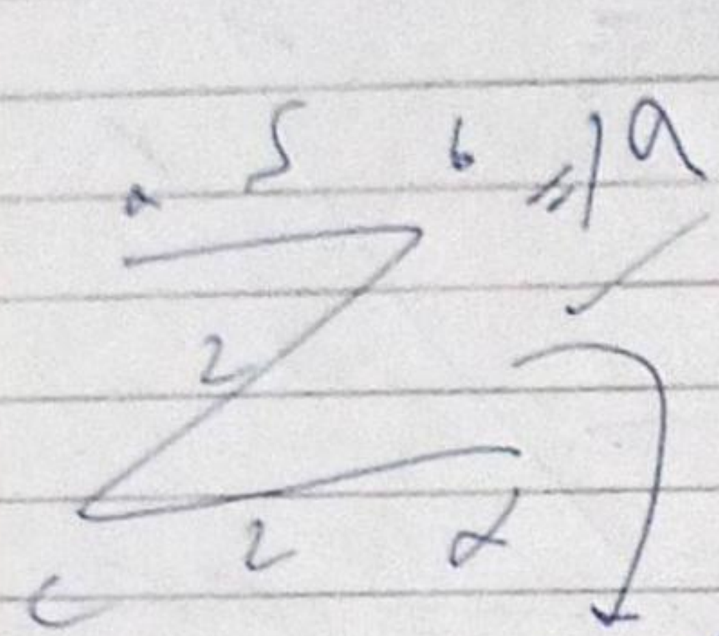
weight



make spanning tree of this

spanning tree

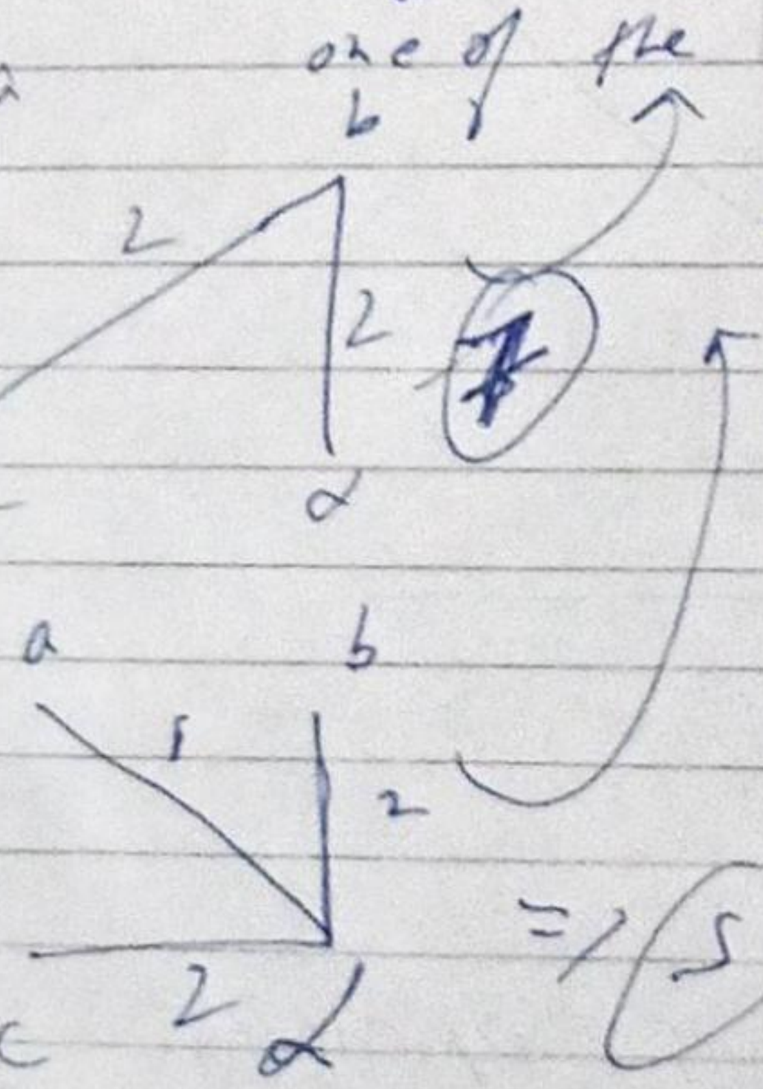
(a) =

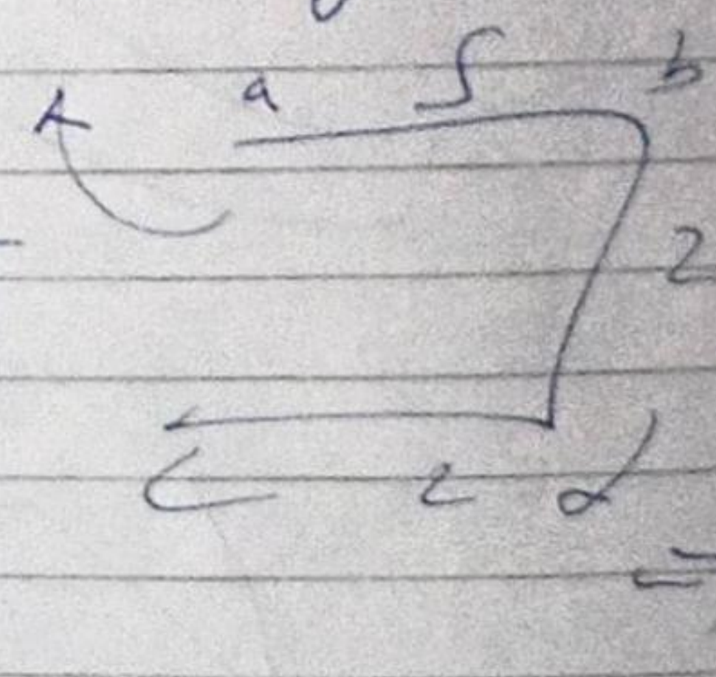


(1) 4 vertices - 3 edges
(2) connected
(1) No cycle

one of the spanning tree

(7) =





=> (a)

=> (5)

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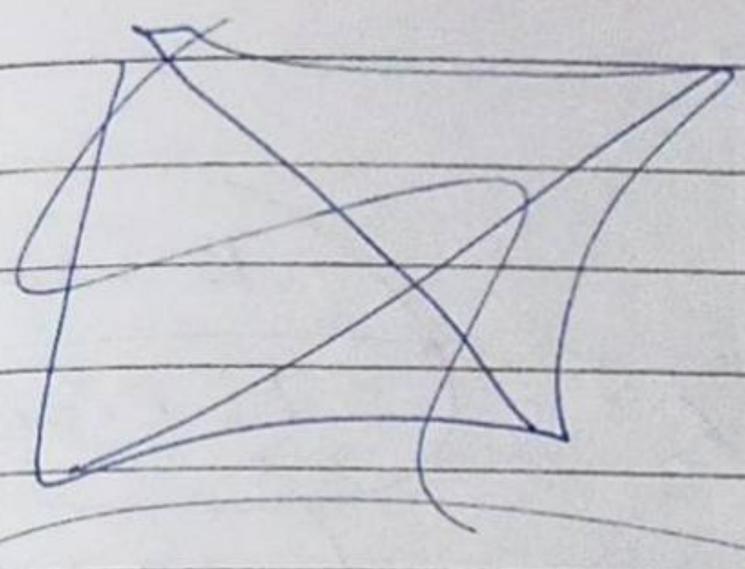
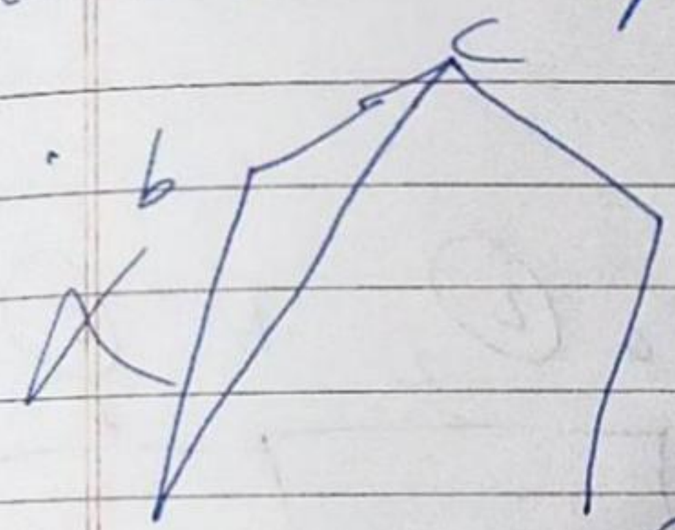


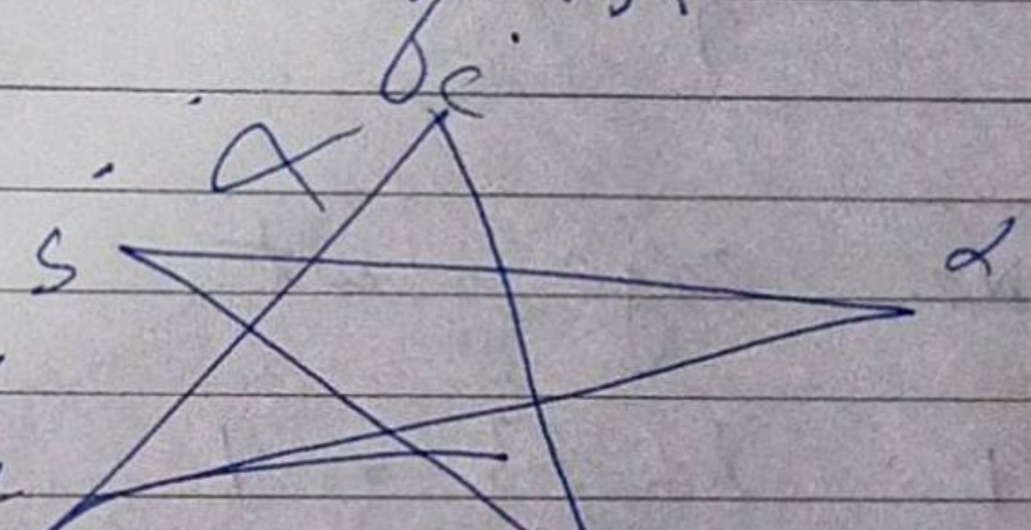
= , value to the spanning tree should be min

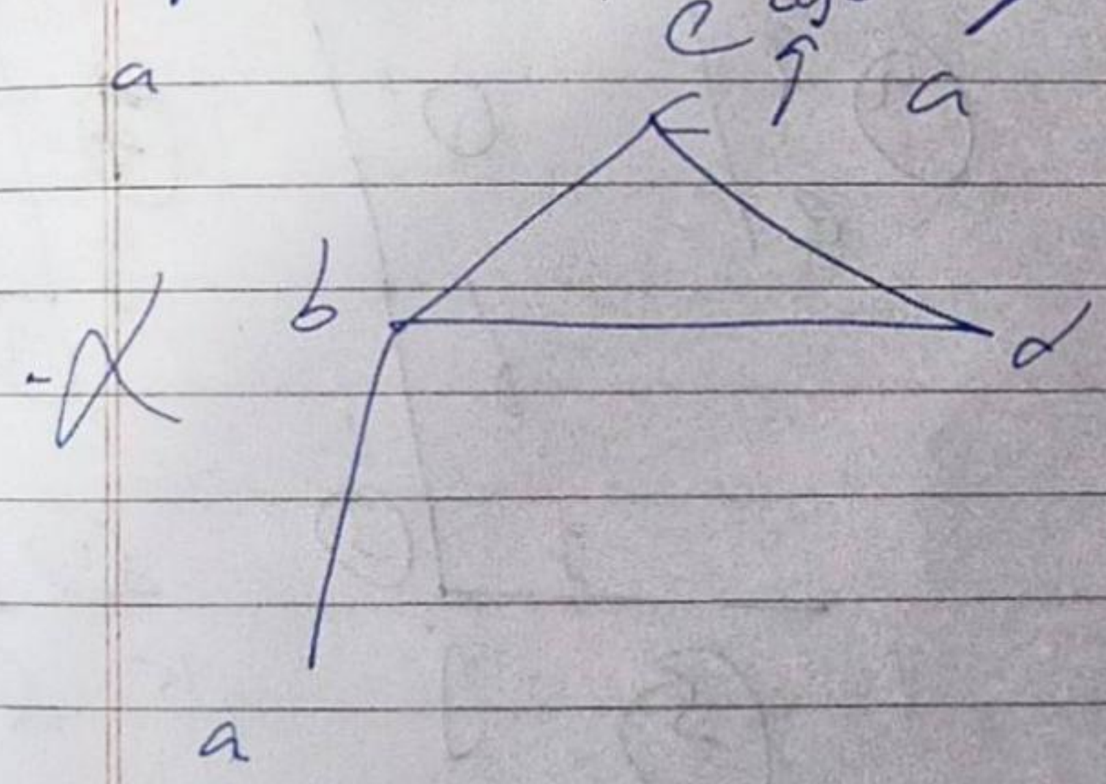
min spanning tree weight

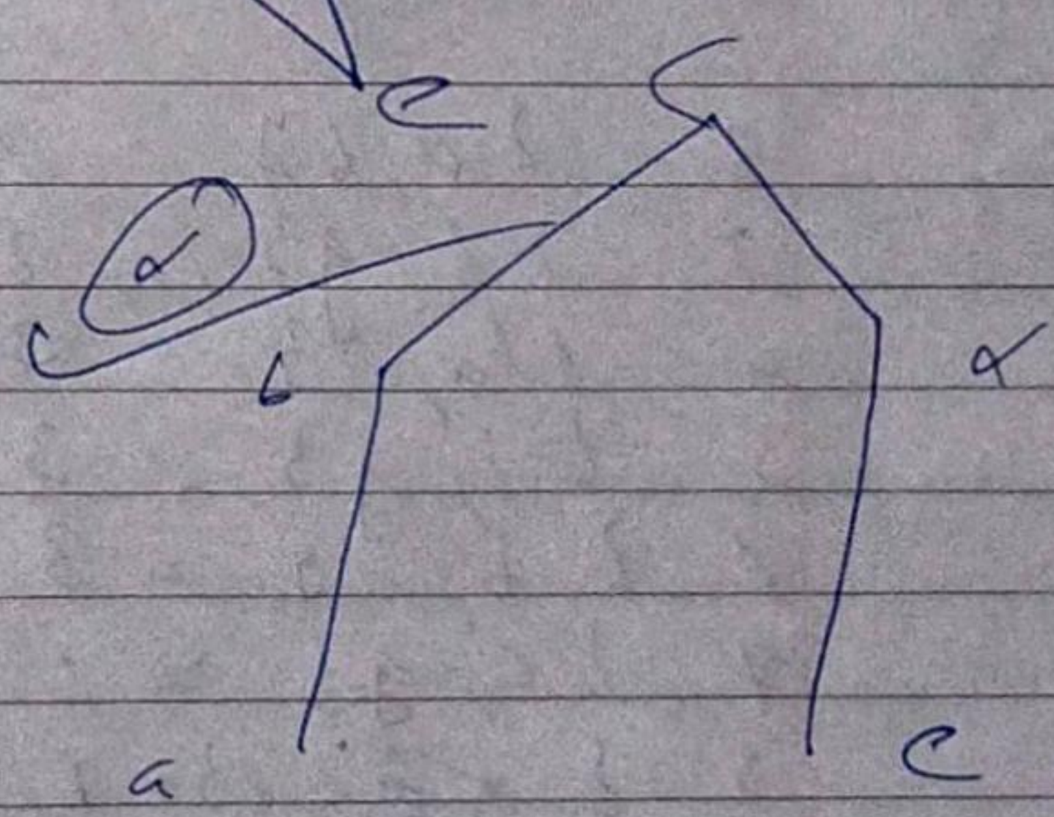
52:00 – 53:00

a: What is spanning tree of ks?









Prim's Algo

1. Choosing initially an edge of min. wt.
2. Successively adding edges of minimum weight that are incident to a vertex in the tree and that do not form cycle.

	a	b	c	d
j	1	2	5	
e		6	8	h
i	4 2 3	4 3	3 1	
	j	k	l	

12 vertex

11 edges

~~trees~~

100 spanning

No. edge wt

1 {b, f} = 1

2 {a, b} = 2

3 {f, j} = 2

4 {a, e} = 3

5 {i, j} = 3

6 {b, c} = 3

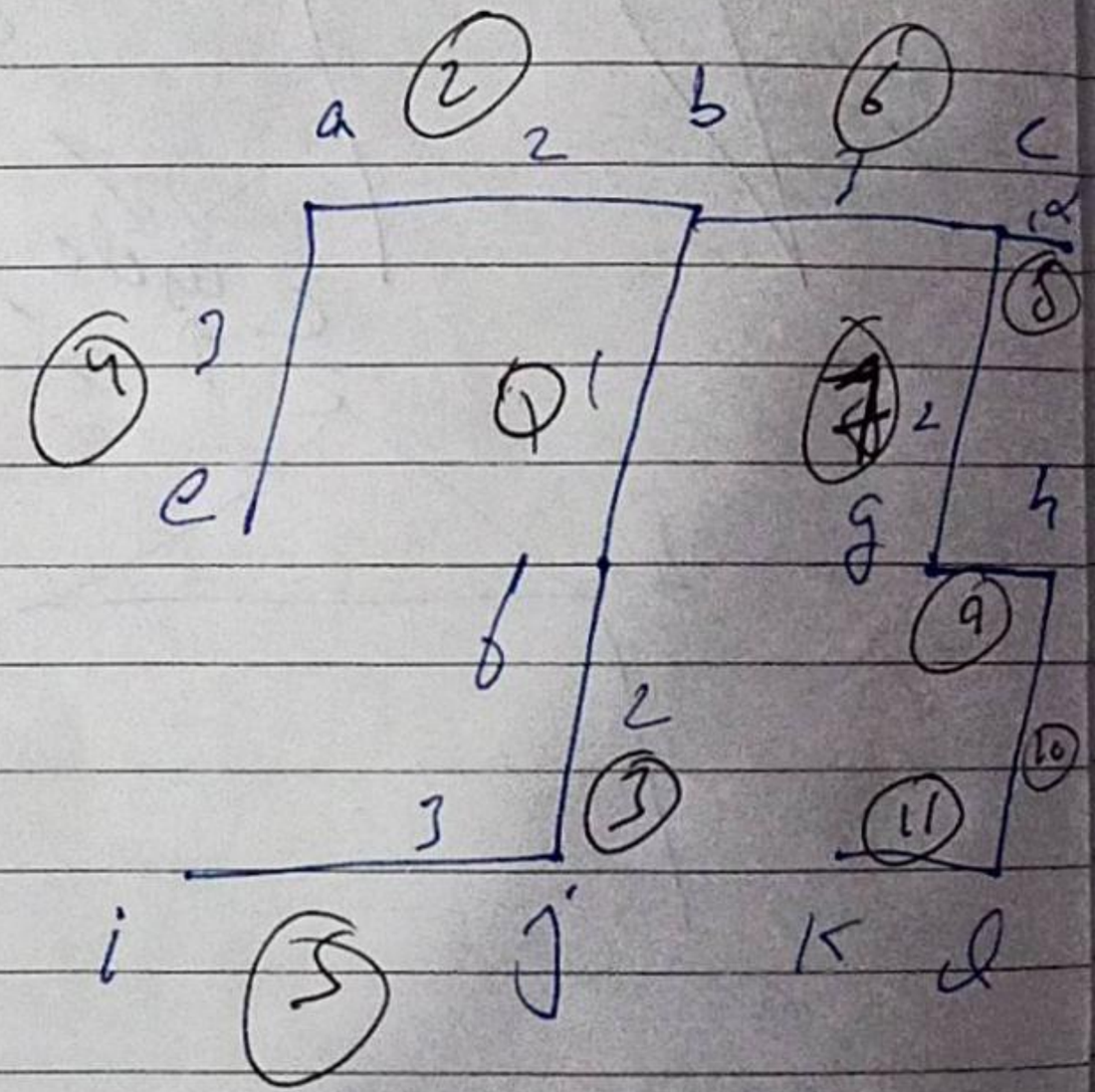
7 {c, g} = 2

8 {c, d} = 1

9 {g, h} = 3

10 {h, l} = 3

11 {k, l} = 1



Min spanning tree wt => 24

over

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19-Oct

Q1 Consider a undirected graph G with vertices (A, B, C, D, E). In graph G, every edge has distinct weight. Edge CD is edge with minimum weight and edge AB is edge with maximum weight. Then which is false?

(a) Every min. spanning tree of G must contain CD

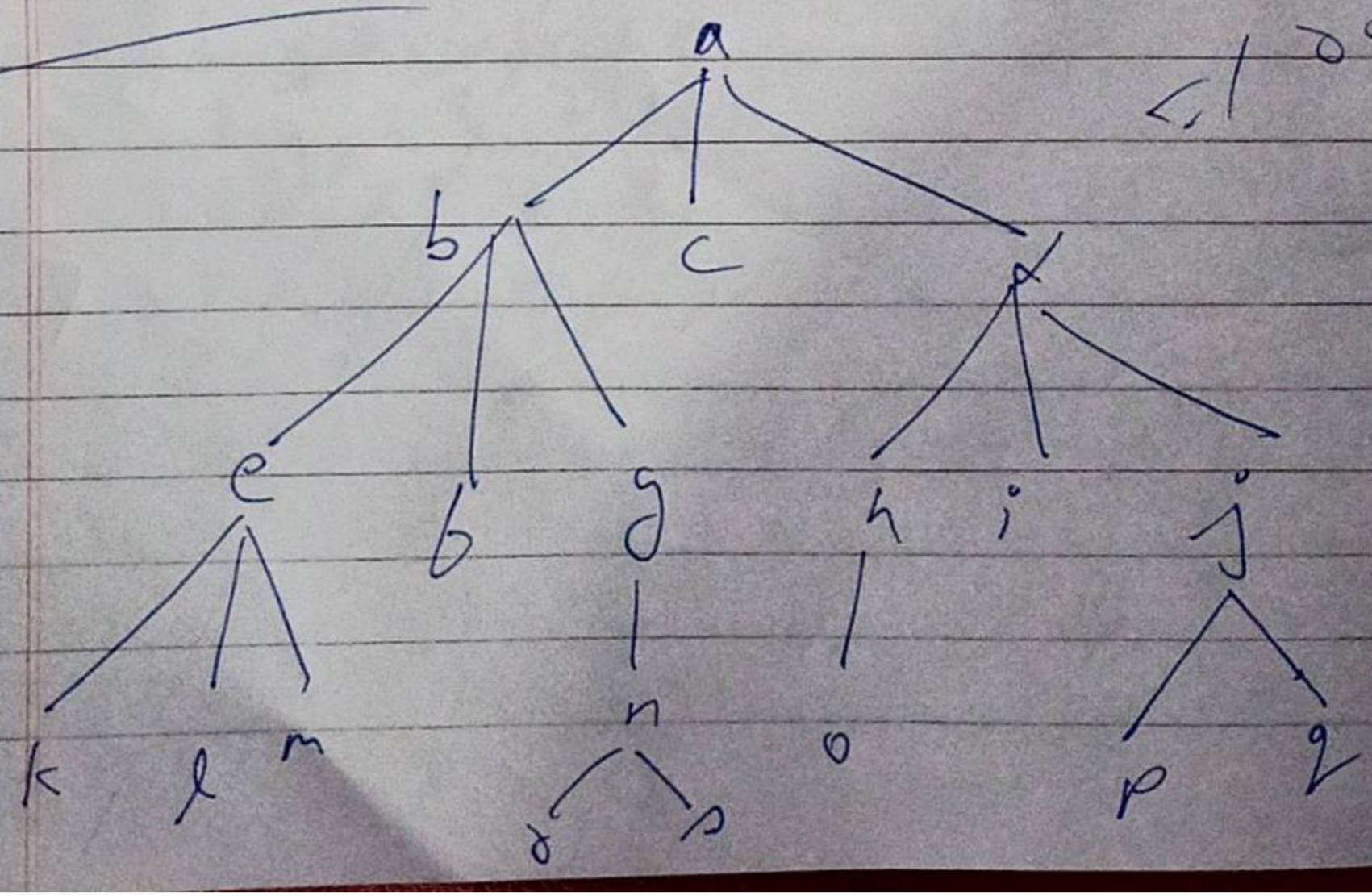
(b) If AB in a minimum spanning tree, then its removal must disconnect G.

(c) No minimum spanning tree contains AB

(d) G has a unique min. spanning tree.

Preorder Traversal — starting from ~~tree~~ root and ~~going~~ visiting every node.

Q: Pre-order Traversal



Rooted tree

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f a b e k l m f g n r s c d h o i j p q
f o

Postorder traversal

visit from right to left

Postorder

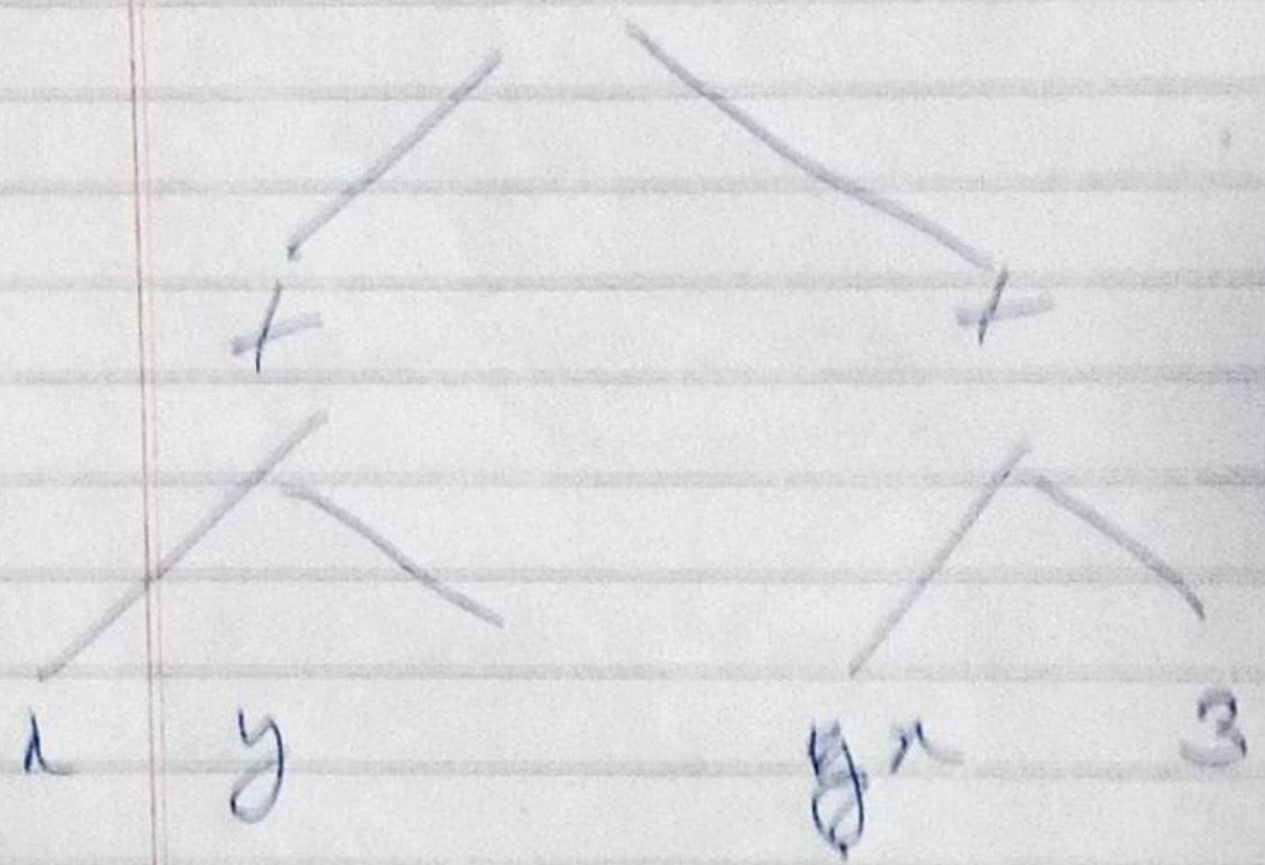
k l m e f r s n g c o h i p q ~~t~~ j d a

20.02.1

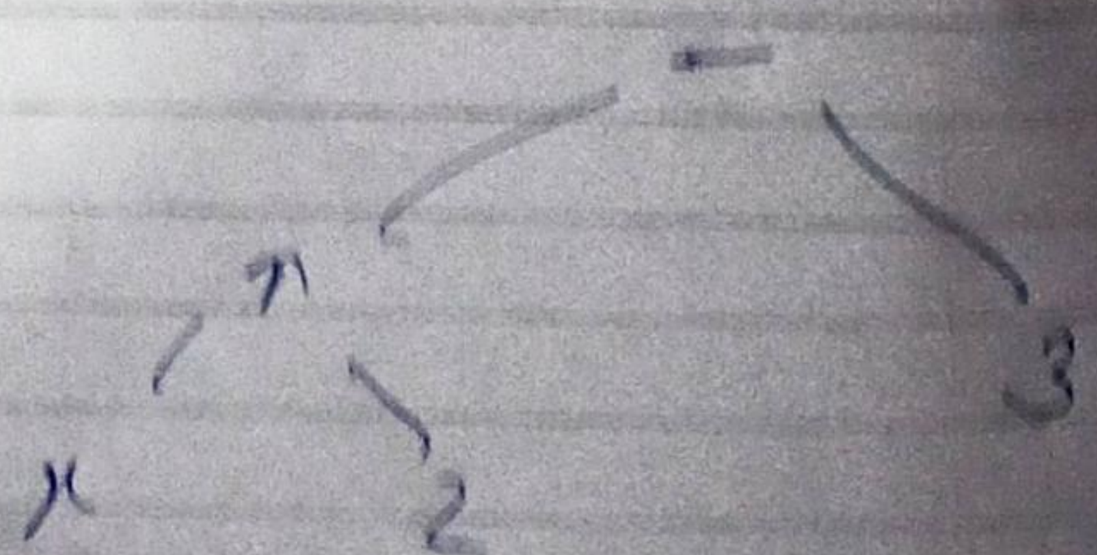
⇒ P.O ⇒ add AR
↳ 1 MB

⇒ (x+y)/(x+3)

operando ⇒ leer
operadores ⇒ internal ↳ costes



⇒ (x*2)-3



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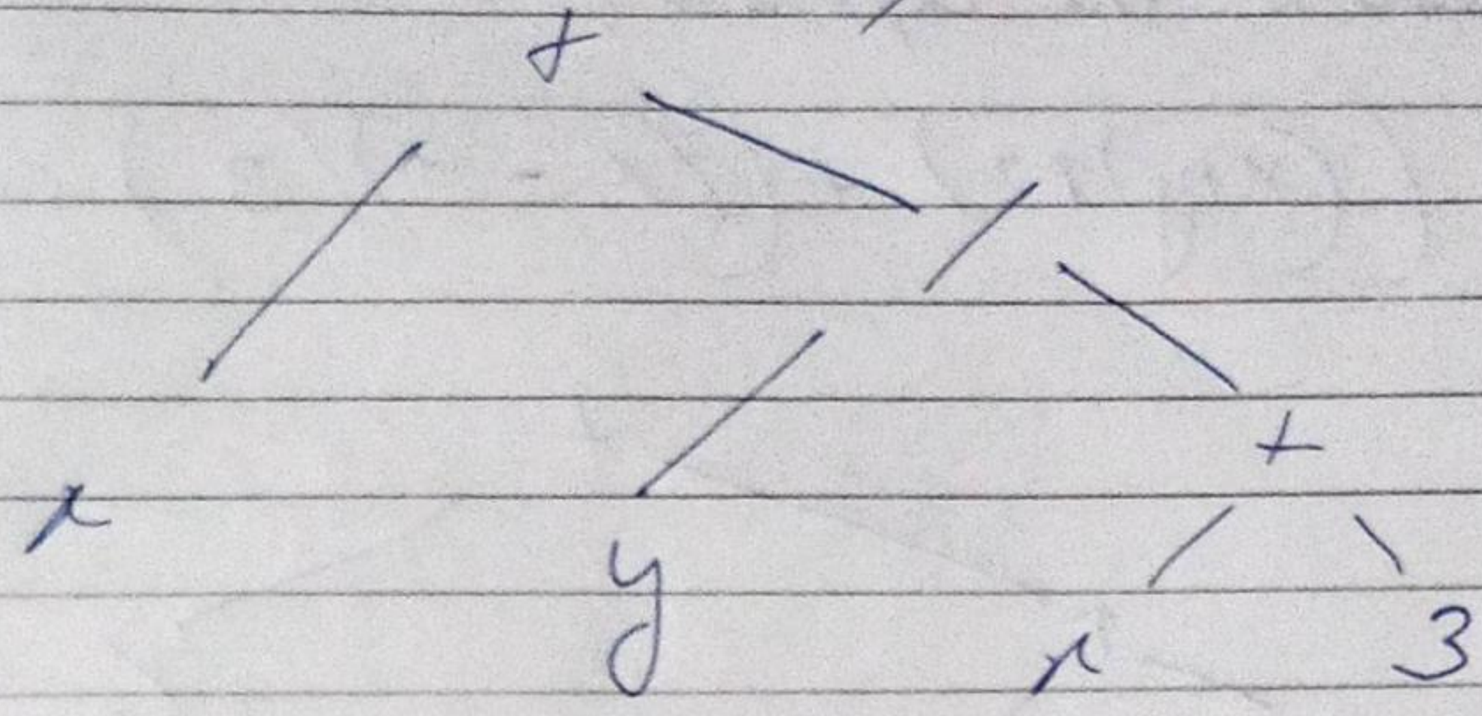
Q:

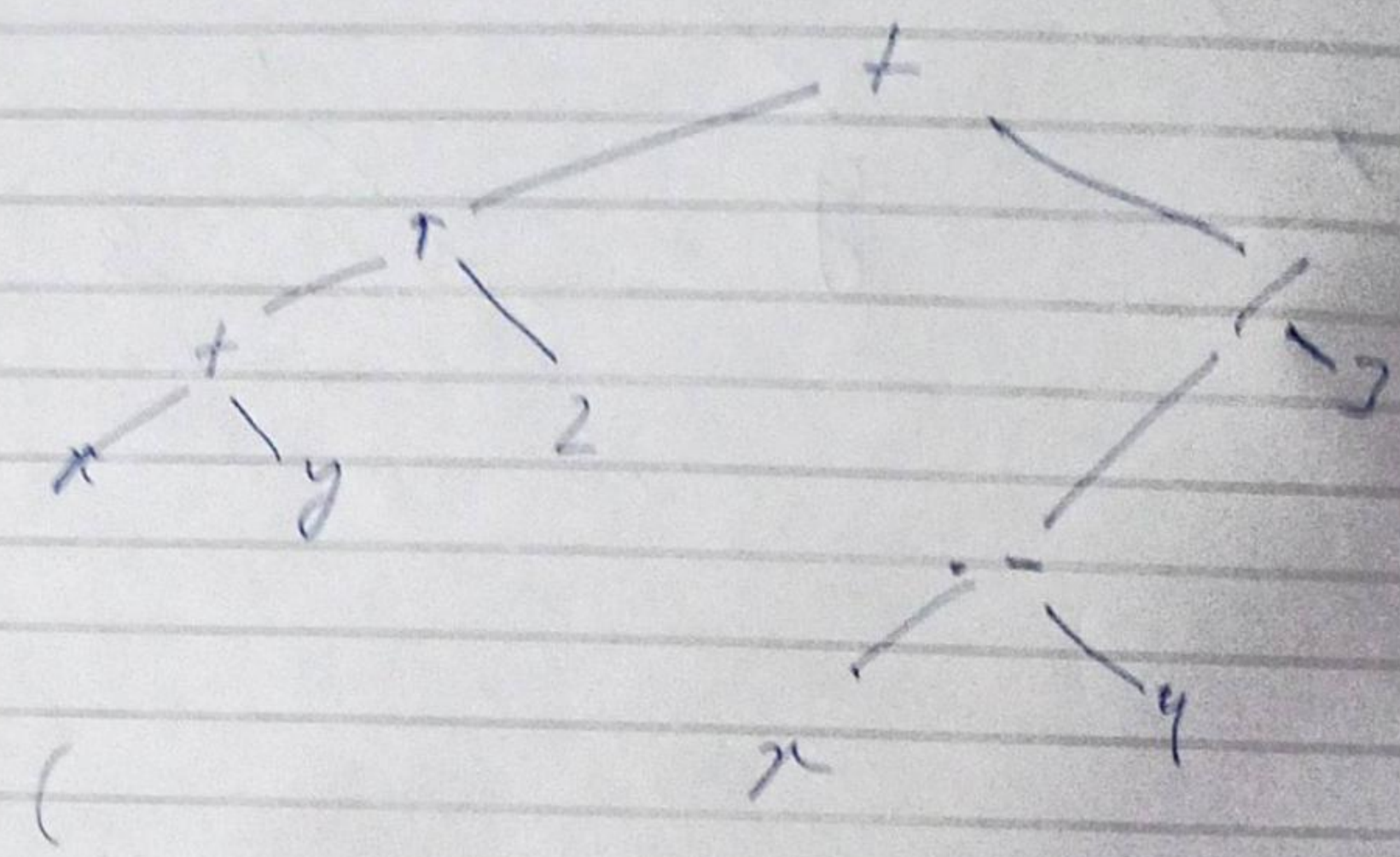
$x + (y / (x+3))$



from $2100

Q: what do the ordered rooted tree that represents the expression?

$$((x+y)\uparrow 2) + ((x-4)/3)$$



Prefix = / + ↑ + x y 2 / - x 4 3

Postfix

x y + 2 ↑ x 4 - 3 / +

postfix

Q: Represent the exp $(x+(xy)\uparrow 2)+(x/y)$ in prefix notation & postfix notation

$(x + *xy) + (/xy)$ root of the expression

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(+x+xy) + (/xy)

+ + x + xy / xy

C, P.C

Q: x + ((xy + x)/y)

Pre -/ +x/+*xyxy

Post -/ xxy+x+y/+

Q: + * + - 53214

↳ Ans

. 4 . 9 . 8 - 6.

((5-3)+2)*1 + 4

(2+2)*1 +4 => 8

21-2c+

2: * + 3 + 3 (+ 3 (+ 33))

(a) 2261
(b) 1453
(c) ~~805~~ 2205
(d) None

(3+3)

[3 + (3 + 3)] * 3

732

735 * 3

2205

2: 723 * - 4 ↑ 93 / +

76 - 4 ↑ 93 / +

14 ↑ 93 / +

193 / +

193 + 8

4

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=/ 5 2 1 - - 3 / 4 + + *

(a) 32 (b) 40 (c) 35 (d) 27

$[5 - (2 - 1)] * [3 + (1 + 4)]$

4 * 8 =/ 32

Q: 3 2 * 2 ↑ 5 3 - 8 4 / * -

(a) 40 (b) 84 (c) 32 (d) 26

$(3 * 2)^2$ $(5 - 3)$

6^2 36 2 2 * -

3 / 4 -

32

End of Unit - 5