

# Linear Algebra Take-home

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May 1, 2017

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# Take Home-1

## Linear Separability

PROBLEM STATEMENT: For the given set of samples,

1.S.T the data is linearly separable

b.S.T  $f_1 = f_2$  is a decision boundary

c.Obtain the weight vector,w. S.T it is orthogonal to the decision boundary.

d.Suggest another decision boundary for the same data.

e.Compute the distance from the origin to the decision boundary

f.By transforming and normalizing the data, reconstruct the table and use perceptron learning algorithm to compute the new weight vector.

label no	f1	f2	class
1	0.5	3	X
2	1	3	X
3	0.5	2.5	X
4	1	2.5	X
5	1.5	2.5	X
6	4.5	1	O
7	5	1	O
8	4.5	0.5	O
9	5.5	0.5	O

Method used: I used perceptron algorithm and if the data converges in some finite amount of time(i.e in 1000 number of iterations) Then the data is said to be linearly separable.

Algorithm:

*Initializing threshold<sub>v</sub> value*  $\leftarrow 0$

*Initializing learnrate*  $\leftarrow 0.2$  i.e any number between 0 and 1

*Initializing ierror*  $\leftarrow -1$

*Initially initializing the weight array with random values using random.randint() with values between 0 and 1*

*Initializing the convergence as 0*  $count \leftarrow 0$

**while**  $count \leq 1000$  and  $ierror \neq 0$  **do**

*count*  $\leftarrow count + 1$

**for** all the given data in training set **do**

*output*  $\leftarrow i[0] * wt[0] + i[1] * wt[1] + wt[2]$

**if** *output*  $>$  *threshold<sub>v</sub> value* **then**

*output*  $\leftarrow 1$

**else**

*output*  $\leftarrow 0$

**end if**

*error*  $\leftarrow expected\ output - output$

*ierror*  $\leftarrow ierror + error * error$

*weight1*  $\leftarrow weight1 + learnrate * error * f1_i$

*weight2*  $\leftarrow weight2 + learnrate * error * f2_i$

*b*  $\leftarrow b + learnrate * error$

**end for**

**if** *ierror*  $= 0$  **then**

*convergence*  $\leftarrow 1$

Exit while loop

**end if**

**end while**

**if** *convergence*  $= 1$  **then**

The given data is linearly separable

```

else
    The given data is not linearly separable
end if

```

CODE

```

1 import math
2 import matplotlib.pyplot as plt
3 import random
4 import numpy as np
5 import statistics as stat
6
7 f=[[0.5,3],[1,3],[0.5,2.5],[1,2.5],[1.5,2.5],[4.5,1],[5,1],[4.5,0.5],[5.5,0.5]]
8 class1=[0,0,0,0,0,1,1,1,1]
9 threshold_value=0
10 learnrate=0.2
11 ierror=-1
12 count=0
13 weight=[]
14 weight.append(random.randint(1,1000)/1000)
15 weight.append(random.randint(1,1000)/1000)
16 weight.append(random.randint(1,1000)/1000)
17
18
19 convergence=0
20 while(count<=1000 and ierror!=0):
21     count=count+1
22     j=0
23     ierror=0
24     for i in f:
25         op=i[0]*weight[0]+i[1]*weight[1]+weight[2]
26         if(op>threshold_value):
27             return 1
28         else:
29             return 0
30
31         error=class1[j]-op
32         j=j+1
33         weight[0]=weight[0]+learnrate*error*i[0]
34         weight[1]=weight[1]+learnrate*error*i[1]
35         weight[2]=weight[2]+learnrate*error
36         ierror=ierror+(error*error)
37     if(ierror==0):
38         convergence=1
39 if(convergence==1):
40     print("The given data is linearly separable")
41     print("The number of iterations are given by:",count)
42 else:
43     print("data is not linearly separable")

```

Result

Output:

The given data is linearly separable The number of iterations are given by:4

part-b

To show that  $f_1=f_2$  is decision boundary

Method Used:

Basically for this problem the weight vector is  $1x-1y=0$  ( $f1-f2=0$ ) with  $weight1=1$  and  $weight2=-1$  and with  $bias=0$ . For each data point in the given sample we find  $.weightvector^{transpose} sample$ . If this value is greater than the threshold value then it belongs to class1 i.e (class 0 in this case) otherwise to class 0 (i.e class X in this case)

Code:

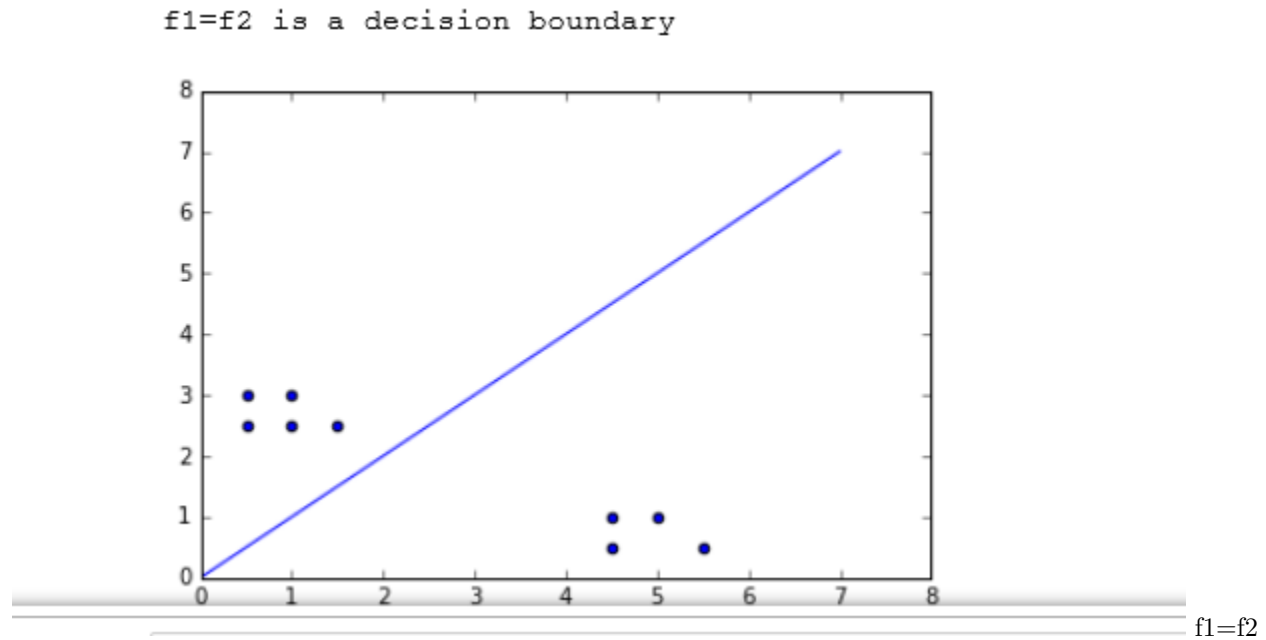
```

1 %matplotlib inline
2 import math
3 import numpy as np
4 import statistics as stat
5 import matplotlib.pyplot as plt
6 import random
7 f=[[0.5,3],[1,3],[0.5,2.5],[1,2.5],[1.5,2.5],[4.5,1],[5,1],[4.5,0.5],[5.5,0.5]]
8 class1=[0,0,0,0,0,1,1,1,1]
9 learnrate=0.2
10 threshold_value=0
11 ierror=-1
12 weight=[]
13 weight.append(random.randint(1,1000)/1000)
14 weight.append(random.randint(1,1000)/1000)
15 weight.append(random.randint(1,1000)/1000)
16 count=0
17 bias=0
18 weight1=1
19 weight2=-1
20
21 decision_boundary=1
22
23
24 j=0
25 for i in f:
26     op=weight1*i[0]+weight2*i[1]+bias
27     if(op>threshold_value):
28         return 1
29     else:
30         return 0
31
32     error=class1[j]-op
33     j=j+1
34     if(error!=0):
35         print("f1=f2 is not a decision boundary")
36         decision_boundary=0
37         break
38 if( decision_boundary==1):
39     print("f1=f2 is a decision boundary")
40     x1_p=np.linspace(0,7,7)
41     y1_p=1*x1_p
42     plt.plot(x1_p,y1_p)
43     plt.scatter(f1,f2)
44     plt.ylim(ymin=0)
45     plt.xlim(xmin=0)
46     plt.show()

```

RESULT: Output:

## Graph



is a decision boundary.

### PART-C

Show that the decision boundary is orthogonal to weight vector.

Method Used:

Basically considering two points  $p$  and  $q$  on the decision boundary  $f = w^T * X + b$ , and if  $w^T(p - q) = 0$  i.e (the inner product of the weight vector with a vector in the direction of  $p$  and  $q$ ) it implies that the weight vector is orthogonal to the decision boundary.

```

1 %matplotlib inline
2 import math
3 import numpy as np
4 import statistics as stat
5 import matplotlib.pyplot as plt
6 import random
7 f = [[0.5, 3], [1, 3], [0.5, 2.5], [1, 2.5], [1.5, 2.5], [4.5, 1], [5, 1], [4.5, 0.5], [5.5, 0.5]]
8 class1 = [0, 0, 0, 0, 0, 1, 1, 1, 1]
9 learnrate = 0.2
10 threshold_value = 0
11 ierror = -1
12 count = 0
13 weight1 = 1
14 weight2 = -1
15 weight = []
16 weight.append(random.randint(1, 1000) / 1000)
17 weight.append(random.randint(1, 1000) / 1000)
18 weight.append(random.randint(1, 1000) / 1000)
19
20 bias = 0
21 #when p=(1, 1) and q=(2,2) which lie on the decision boundary f1=f2
22 point1 = [1, 1]
23 point2 = [2, 2]
24 weight1 = 1
25 weight2 = -1

```

```

26 bias=0
27
28 point2_diff_point1=[1,1]
29 weight_difference=point2_diff_point1[0]*weight1+point2_diff_point1[1]*weight2
30 if (weight_difference==0):
31     print("The weight vector is orthogonal to above decision boundary")
32 else:
33     print("The weight vector is not orthogonal to the above decision boundary")

```

Output:

The weight vector is orthogonal to above decision boundary

part-d

Suggest another decision boundary for the same data.

Method Used:

Actually used the perceptron algorithm to find another decision boundary for the given data points. Algorithm:

```

Initializing thresholdv value  $\leftarrow 0$ 
Initializing learnrate  $\leftarrow 0.2$  i.e any number between 0 and 1
Initializing ierror  $\leftarrow -1$ 
Initially initializing the weight array with random values using random.randint() with values between 0 and 1
Initializing the convergence as 0 count  $\leftarrow 0$ 
while count  $\leq 1000$  and ierror  $\neq 0$  do
    count  $\leftarrow$  count + 1
    for all the given data in training set do
        output  $\leftarrow i[0] * \text{weight}[0] + i[1] * \text{weight}[1] + \text{weight}[2]$ 
        if output  $>$  thresholdv value then
            output  $\leftarrow 1$ 
        else
            output  $\leftarrow 0$ 
        end if
        error  $\leftarrow \text{expected output} - \text{output}$ 
        ierror  $\leftarrow$  ierror + error * error
        weight1  $\leftarrow \text{weight1} + \text{learnrate} * \text{error} * f1_i$ 
        weight2  $\leftarrow \text{weight2} + \text{learnrate} * \text{error} * f2_i$ 
        b  $\leftarrow b + \text{learnrate} * \text{error}$ 
    end for
    if ierror = 0 then
        convergence  $\leftarrow 1$ 
        Exit while loop
    end if
end while
if convergence = 1 then
    The given data is linearly separable
else
    The given data is not linearly separable
end if

```

Code :

```

1 import math
2 import numpy as np
3 import statistics as stat

```

```

4 import matplotlib.pyplot as plt
5 import random
6 f = [[0.5, 3], [1, 3], [0.5, 2.5], [1, 2.5], [1.5, 2.5], [4.5, 1], [5, 1], [4.5, 0.5], [5.5, 0.5]]
7 class1 = [0, 0, 0, 0, 0, 1, 1, 1, 1]
8 learnrate = 0.2
9 threshold_value = 0
10 ierror = -1
11 weight = []
12 weight.append(random.randint(1, 1000) / 1000)
13 weight.append(random.randint(1, 1000) / 1000)
14 weight.append(random.randint(1, 1000) / 1000)
15 count = 0
16
17
18 convergence = 0
19 while (count <= 10000 and ierror != 0):
20     count = count + 1
21     j = 0
22     ierror = 0
23     for i in f:
24         op = i[0] * weight[0] + i[1] * weight[1] + weight[2]
25         if (op > threshold_value):
26             return 1
27         else:
28             return 0
29
30         error = class1[j] - op
31         j = j + 1
32         weight[0] = weight[0] + learnrate * error * i[0]
33         weight[1] = weight[1] + learnrate * error * i[1]
34         weight[2] = weight[2] + learnrate * error
35         ierror = ierror + (error * error)
36     if (ierror == 0):
37         convergence = 1
38
39 if (convergence == 1):
40     print("data is linearly separable")
41     print("decision boundary eqn is: ", weight[0], "*x", weight[1], "*y", "+", weight[2], "=0")
42     f1 = [i[0] for i in f]
43     f2 = [i[1] for i in f]
44     plt.scatter(f1, f2)
45     x_p = np.linspace(0, 6, 10)
46     y_p = (-weight[2] - weight[0] * x_p) / weight[1]
47     plt.plot(x_p, y_p)
48     plt.show()
49 else:
50     print("data is not linearly separable")

```

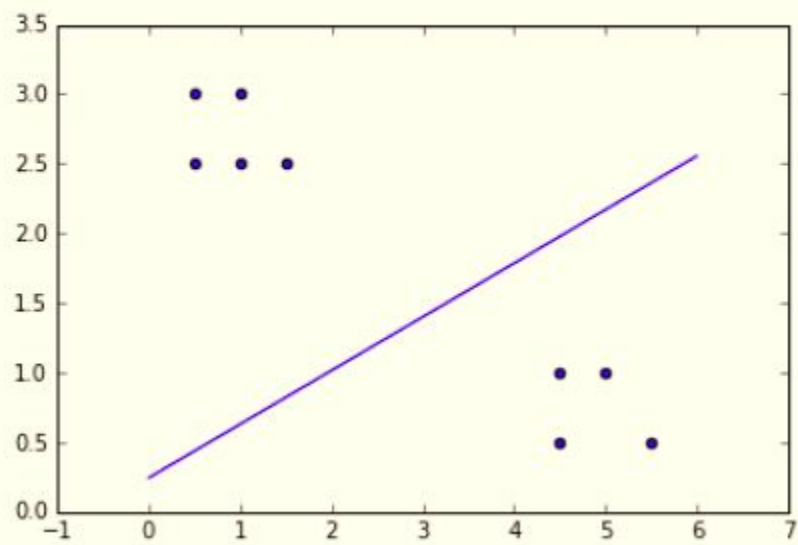
Output:



## Graph

w1= 0.791 0.945 0.784

decision boundary eqn is:  $0.29100000000000004 \cdot x - 0.75500000000000002 \cdot y + 0.18$



part-e

Find the distance from origin to the separating hyperplane.

Method:

If line is given by the equation  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are real constants, the distance from the line to a point  $(x_0, y_0)$  is given by the formula Used the formula of finding distance of a point from a line. If a point is  $(x_0, y_0)$  then distance from a line  $Ax + By + C = 0$  is  $(Ax_0 + By_0 + C)/(\sqrt{A^2 + B^2})$ . Here  $(x_0, y_0)$  is  $(0,0)$  and the hyperplane equation is same as the output of previous question.

Method used: So to find the distance of origin from decision boundary The distance  $d = \text{weight}[2] / \text{math.sqrt}(\text{weight}[0]**2 + \text{weight}[1]**2)$

Output:

The distance is given by 0.38534267477732

part-f

Problem Statement: By transforming and normalising the data, reconstruct the table and use perceptron learning algorithm to find the decision boundary.

Method:

Basically normalizing the data using the formula  $x = (x - \text{mean}) / \text{standard deviation}$  where  $x$  is the data point mean is the mean of the feature to which the datapoint belongs to and standard deviation is also the standard deviation of the feature to which the datapoint belongs to. After normalizing the given data points applying perceptron to find the decision boundary.

Algorithm:

```
Initializing thresholdv value  $\leftarrow 0$ 
Initializing learnrate  $\leftarrow 0.2$  i.e any number between 0 and 1
Initializing ierror  $\leftarrow -1$ 
Initially initializing the weight array with random values using random.randint() with values between 0 and 1
Initializing the convergence as 0 count  $\leftarrow 0$ 
while count  $\leq 1000$  and  $ierror \neq 0$  do
    count  $\leftarrow$  count + 1
    for all the given data in training set do
        output  $\leftarrow i[0] * wt[0] + i[1] * wt[1] + wt[2]$ 
        if output  $>$  thresholdv value then
            output  $\leftarrow 1$ 
        else
            output  $\leftarrow 0$ 
        end if
        error  $\leftarrow$  expected output - output
        ierror  $\leftarrow$  ierror + error * error
        weight1  $\leftarrow$  weight1 + learnrate * error *  $f1_i$ 
        weight2  $\leftarrow$  weight2 + learnrate * error *  $f2_i$ 
         $b \leftarrow b + \text{learnrate} * \text{error}$ 
    end for
    if ierror = 0 then
```

```

        convergence ← 1
    Exit while loop
end if
end while
if convergence = 1 then
    The given data is linearly separable
else
    The given data is not linearly separable
end if

```

Code :

```

1 import math
2 import numpy as np
3 import statistics as stat
4 import matplotlib.pyplot as plt
5 import random
6 f=[[0.5,3],[1,3],[0.5,2.5],[1,2.5],[1.5,2.5],[4.5,1],[5,1],[4.5,0.5],[5.5,0.5]]
7 class1=[0,0,0,0,0,1,1,1,1]
8 f1=[i[0] for i in f]
9 f2=[i[1] for i in f]
10 stddev_of_f1=stat.stdev(f1)
11 stddev_of_f2=stat.stdev(f2)
12 mean_of_f1=stat.mean(f1)
13 mean_of_f2=stat.mean(f2)
14
15 nf=[]
16 for i in range(0,len(f1)):
17     x=[]
18     j=(f1[i]-mean_of_f1)/stddev_of_f1
19     y=(f2[i]-mean_of_f2)/stddev_of_f2
20     x.append(j)
21     x.append(y)
22     nf.append(x)
23 print(nf)
24
25 learnrate=0.2
26 threshold_value=0
27 ierror=-1
28 weight=[]
29 weight.append(random.randint(1,1000)/1000)
30 weight.append(random.randint(1,1000)/1000)
31 weight.append(random.randint(1,1000)/1000)
32 count=0
33
34
35 convergence=0
36 while(count<=10000 and ierror!=0):
37     count=count+1
38     j=0
39     ierror=0
40     for i in f:
41         output=i[0]*weight[0]+i[1]*weight[1]+weight[2]
42
43         if(output>threshold_value):
44             return 1
45         else:
46             return 0
47         error=class1[j]-output
48         j=j+1
49         weight[0]=weight[0]+learnrate*error*i[0]
50         weight[1]=weight[1]+learnrate*error*i[1]
51         weight[2]=weight[2]+learnrate*error
52         ierror=ierror+(error*error)
53     if(ierror==0):

```

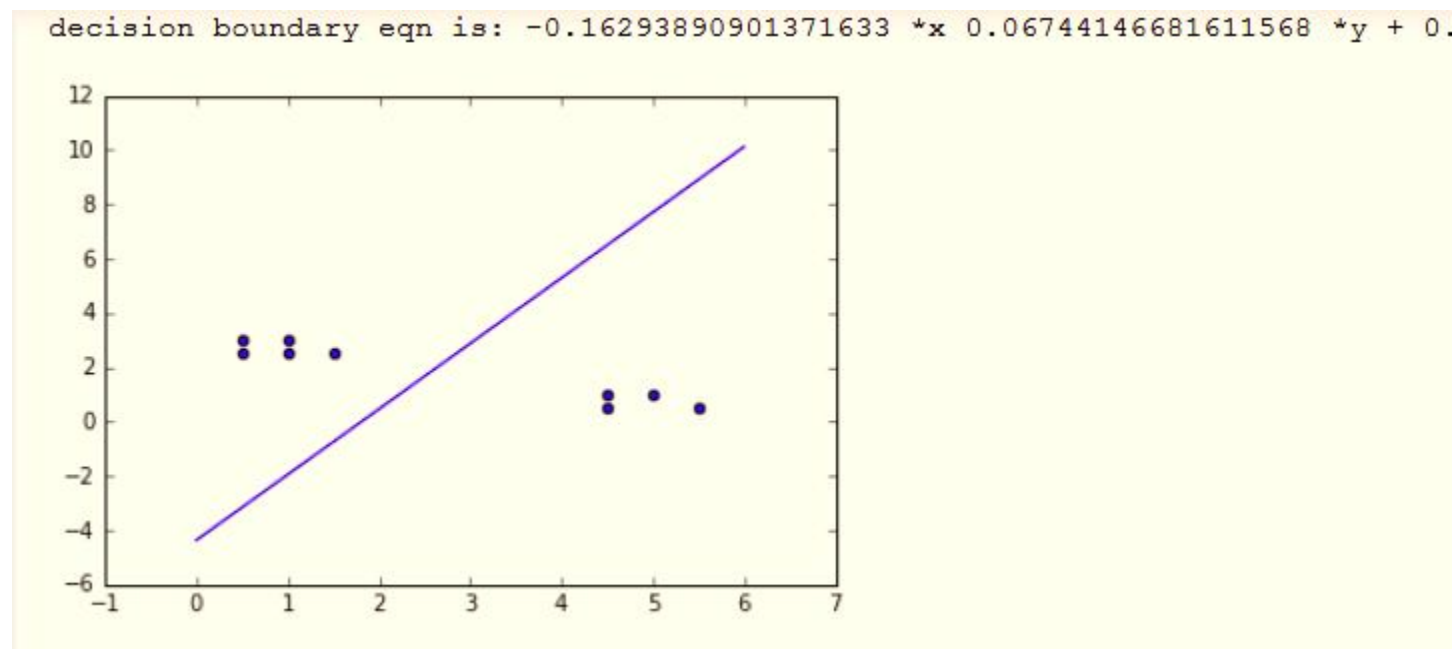
```

54         convergence=1
55     if(convergence==1):
56         print("data is linearly separable")
57         print("decision boundary eqn is:" ,weight[0] ,"*x", weight[1], "*y", "+",weight[2], "=0")
58         f1=[i[0] for i in f]
59         f2=[i[1] for i in f]
60         plt.scatter(f1,f2)
61         x_p=np.linspace(0,6,10)
62         y_p=(-weight[2]-weight[0]*x_p)/weight[1]
63         plt.plot(x_p,y_p)
64         plt.show()
65     else:
66         print("data is not linearly separable")

```

Output:

## Graph



References:

<https://www.youtube.com/watch?v=1XkjVl-j8MM&t=9s>

<https://www.quora.com/How-can-I-know-whether-my-data-is-linearly-separable#!n=18>

<http://mathworld.wolfram.com/Point-PlaneDistance.html>

## Take Home-2

Diagonalization Theorem

PROBLEM STATEMENT:

Prove the Diagonalization Theorem  $P^{-1}AP = D$ . Show that columns of P are eigen vectors of A.

ASSUMPTIONS:

$$1. A \in R^n$$

$$\lambda_1, \lambda_2, \dots, \lambda_n \in R$$

$$v_1, v_2, \dots, v_n \in R^n$$

The theorem holds good only if the matrix A is a square matrix.

SUMMARY:

Suppose A is a n by n matrix. For  $P^{-1}AP$  is a diagonal matrix D. D is a diagonal matrix where the diagonal elements are eigen values of A. P is a matrix containing eigen vectors of A.

Proof:

Basically from eigen-vector and eigen value relation we know that

$$Av_1 = \lambda_1 v_1, Av_2 = \lambda_2 v_2, Av_3 = \lambda_3 v_3, \dots, Av_n = \lambda_n v_n,$$

Putting eigen vectors in the columns of the matrix P.

$$AP = A^*$$

$$\begin{bmatrix} v_1 & v_2 & v_3 & \dots & v_n \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$$\begin{bmatrix} Av_1 & Av_2 & Av_3 & \dots & Av_n \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$$\begin{bmatrix} \lambda_1 v_1 & \lambda_2 v_2 & \lambda_3 v_3 & \dots & \lambda_n v_n \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

Now if we observe the above matrix it is just a scalar multiple of the matrix P. So  $AP =$

$$\begin{bmatrix} \lambda_1 v_1 & \lambda_2 v_2 & \lambda_3 v_3 & \dots & \lambda_n v_n \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

So now  $AP = PD$

$$AP = P \times$$

$$\begin{bmatrix} \lambda_1 & & & & \\ & \ddots & & & \\ & & \lambda_n & & \end{bmatrix}$$

$$P^{-1}AP = D$$

The above given matrix a Diagonal matrix with the eigen values of matrix A on the diagonal. Hence proved that  $P^{-1}AP = D$ . Columns of P are eigen vectors of A.

References:

Linear Algebra and its Applications by Gilbert Strang

## Take Home-3

Problem Statement:  $\|A\|_0 = \sum_{i=1}^n \sum_{j=1}^n |a_{ji}|$  is a matrix norm

Proof:

To prove that  $\|A\|_0$  is a matrix norm, the following properties has to be satisfied:

1. Non - negativity of matrix norm i.e matrix norm is always positive.
2.  $\|\alpha A\|_0 = \alpha \|A\|_0$
3.  $\|A\|_0 = 0$ , if and only if all the elements in the matrix are 0.
4.  $\|A + B\|_0 \leq \|A\|_0 + \|B\|_0$
5.  $\|A.B\|_0 \leq \|A\|_0 \cdot \|B\|_0$

Property number 1: Since the matrix norm is sum of all the elements row-wise and we are considering the absolute value of this. So the matrix norm is always positive.

Property 2:

$$\|\alpha A\|_0 = \sum_{i=1}^n \sum_{j=1}^n |\alpha a_{ij}|$$

Since  $\alpha$  is just a scalar we can take it out.  $\|\alpha A\|_0 = |\alpha| (\sum_{i=1}^n \sum_{j=1}^n |a_{ij}|)$

Property 3:

Since the matrix norm is sum of all the elements row-wise and we are considering the absolute value of this. So the matrix norm is always positive. But it will be 0 if all the elements of the matrix A are 0.

Property 4:

We use Cauchy Schwartz inequality to prove this property.

$$\begin{aligned} \|A + B\|_0 &= \sum_{i=1}^n \sum_{j=1}^n |a_{ij} + b_{ij}| \\ &\leq \sum_{i=1}^n \sum_{j=1}^n |a_{ij}| + \sum_{i=1}^n \sum_{j=1}^n |b_{ij}| \\ &\leq \|A\|_0 + \|B\|_0 \end{aligned}$$

property-5

$$\|A.B\|_0 \leq \|A\|_0 \cdot \|B\|_0$$

$$\begin{aligned}
&= \sum_{i=1}^n \sum_{j=1}^n \left[ \sum_{k=1}^n |a_{jk} b_{ki}| \right] \\
&\leq \sum_{i=1}^n \sum_{j=1}^n \left[ \sum_{k=1}^n |a_{jk}| \left[ \sum_{k=1}^n |b_{ki}| \right] \right] \\
&\leq \sum_{i=1}^n \sum_{k=1}^n |a_{jk}| * \sum_{i=1}^n \sum_{k=1}^n |b_{ki}| \\
&\leq \sum_{k=1}^n \sum_{j=1}^n |a_{jk}| * \sum_{i=1}^n \sum_{k=1}^n |b_{ki}| \\
&\leq \|A\| * \|B\|
\end{aligned}$$

Hence proved that  $\|A\|_0 = \sum_{i=1}^n \sum_{j=1}^n |a_{ji}|$  is a matrix norm since it satisfies all the properties of the matrix norm.

## REFERENCES:

Numerical Analysis R L Burden and J D Faires

## Take Home-4

### Probability problem

#### Problem Statement

Suppose that an object can be at any one of the (n+1) equally spaced points  $x_0, x_1, \dots, x_n$ . When an object is at location  $x_i$ , it is equally likely to move to either  $x_{i-1}$  or  $x_{i+1}$  and cant directly move to any other location. Consider the probabilities  $p_{i=0}^n$  that an object starting at location  $x_i$  will reach the left end point  $x_0$  before reaching the right end point. Clearly  $p_0=1$  and  $p_n=0$ . Also,  $p_i = 0.5p_{i-1} + 0.5p_{i+1}$  for all  $i=1,2,\dots,n-1$ .

Show that:

1.

$$\begin{bmatrix} 1 & -0.5 & 0 & 0 & 0 & \dots & 0 \\ -0.5 & 1 & -0.5 & 0 & 0 & \dots & 0 \\ 0 & -0.5 & 1 & -0.5 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & -0.5 & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_{n-1} \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

2.Solve the system for n=100, and n=1000

3.Change the probabilities  $\alpha$  and  $1 - \alpha$  for movement to left or right. Derive the linear system similar to one in (1).

4.Repeat (2) with  $\alpha = 1/3$ .

a]Part-1:

Multiplying both the matrices we get,

$$\begin{bmatrix} P_1 - 1/2P_2 \\ -1/2P_1 + P_2 - P_3 \\ -1/2P_2 + P_3 \\ -1/2P_{n-1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ -1/2P_{n-2} + P_{n-1} \end{bmatrix}$$

Now from the given equation  $P_i = 1/2P_{i-1} + 1/2P_{i+1}$  Now by solving these simultaneous linear equations we get,

So for i=1  $P_1 = 1/2P_0 + 1/2P_2$

$$P_1 - 1/2P_2 = 1/2P_0$$

Since the value of  $P_0$  is 1

$$\text{So } P_1 - 1/2P_2 = 1/2$$

Then for i=2

$$P_2 = 1/2P_1 + 1/2P_3$$

$$P_2 - 1/2P_1 - 1/2P_3 = 0$$

Similarly for i=3,4.....n

It is 0 So the matrix is

$$\begin{bmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

Hence proved.

part-b:

Assumptions:

Here the input is taken dynamically from the user. So the code below holds good for all the values of n i.e for n=100 and n=1000.

Algorithm:

Used Gaussian elimination to find the to solve the given system.

Basically the program itself generates the given above matrix depending on the input given by the user (i.e n=100 or 1000 in this case)



Gaussian Elimination.

```
for j=0,...,n do
  for i=0,...,n do
    if  $i > j$  then
       $c = a[i][j]/a[j][j]$ 
      Dividing each element by diagonal to generate a upper triangular matrix.
      for k=0,...,n do
         $a[i][k] = a[i][k] - c * a[j][k]$ 
      end for
    end if
  end for
end for
 $x[n-1] = a[n-1][n]/a[n-1][n-1]$ 
for i=n-1,...,0 do
  Initializesum  $\leftarrow 0$ .
  for j=i+1,...,n-1 do
     $sum = sum + a[i][j] * x[j]$ 
  end for
   $x[i] = (a[i][n] - sum)/a[i][i]$ 
end for
```

CODE

```
1 #include<stdio.h>
2 #include<stdlib.h>
3 int input(int n)
4 {
5     int i;
6     int j;
7
8     double **a=(double **) malloc ((n+1)*sizeof(double *));
9     for (i=0;i<=n; i++)
10     {
11         a[i]=(double *) malloc (n*sizeof(double));
12     }
13     //For generating the input matrix
14     for (i=0;i<=n-1; i++)
15     {
16         for (j=0;j<=n; j++)
17         {
18             if (i==j)
19             {
20                 a[i][j]=1;
21             }
22             else if ((j==i+1 || j==i-1) && j!=n)
23             {
24                 a[i][j]=-0.5;
25             }
26             else if (j!=n)
27             {
28                 a[i][j]=0;
29             }
30             else if (j==n && i==0)
31                 a[i][j]=0.5;
32             else if (j==n && i!=0)
33                 a[i][j]=0;
34         }
35     }
36     int k;
```

```

37 float c;
38 //Gaussian elimination-for generating upper triangular matrix.
39 for (j=0; j<=n; j++)
40 {
41     for (i=0; i<=n; i++)
42     {
43         if (i>j)
44         {
45             c=a[i][j]/a[j][j];
46             for (k=0; k<=n+1; k++)
47             {
48                 a[i][k]=a[i][k]-c*a[j][k];
49             }
50         }
51     }
52 }
53 for (i=0; i<=n-1; i++)
54 {
55     for (j=0; j<=n; j++)
56     {
57         printf("%2f ", a[i][j]);
58     }
59     printf("\n");
60 }
61
62 double *x=(double *) malloc((n)*sizeof(double));
63
64 x[n-1]=a[n-1][n]/a[n-1][n-1]; int sum;
65 //Back-substitution :to find the solution i.e to get the values of probabilities in this
66 //case.
67 for (i=n-1; i>=0; i--)
68 {
69     sum=0;
70     for (j=i+1; j<=n-1; j++)
71     {
72         sum=sum+a[i][j]*x[j];
73     }
74     x[i]=(a[i][n]-sum)/a[i][i];
75 }
76 printf("\nThe solution is: \n");
77 for (i=0; i<n; i++)
78     printf("\np%d=%f\t", i+1, x[i]);
79
80
81
82 return 0 ;
83
84 }
85 int main() {
86     int n;
87     int i;
88     scanf("%d", &n);
89     input(n-1);
90
91
92
93 }

```

INPUT: n=100 and n=1000 RESULT : By solving the system for n=100  
The solution is:

p1=0.500000 p2=0.333333 p3=0.250000 p4=0.200000 p5=0.166667 p6=0.142857 p7=0.125000 p8=0.111111  
p9=0.100000 p10=0.090909 p11=0.083333 p12=0.076923 p13=0.071429 p14=0.066667 p15=0.062500  
p16=0.058824 p17=0.055556 p18=0.052632 p19=0.050000 p20=0.047619 p21=0.045455 p22=0.043478  
p23=0.041667 p24=0.040000 p25=0.038462 p26=0.037037 p27=0.035714 p28=0.034483 p29=0.033333

p30=0.032258 p31=0.031250 p32=0.030303 p33=0.029412 p34=0.028571 p35=0.027778 p36=0.027027  
 p37=0.026316 p38=0.025641 p39=0.025000 p40=0.024390 p41=0.023810 p42=0.023256 p43=0.022727  
 p44=0.022222 p45=0.021739 p46=0.021277 p47=0.020833 p48=0.020408 p49=0.020000 p50=0.019608  
 p51=0.019231 p52=0.018868 p53=0.018519 p54=0.018182 p55=0.017857 p56=0.017544 p57=0.017241  
 p58=0.016949 p59=0.016667 p60=0.016393 p61=0.016129 p62=0.015873 p63=0.015625 p64=0.015385  
 p65=0.015152 p66=0.014925 p67=0.014706 p68=0.014493 p69=0.014286 p70=0.014085 p71=0.013889  
 p72=0.013699 p73=0.013514 p74=0.013333 p75=0.013158 p76=0.012987 p77=0.012821 p78=0.012658  
 p79=0.012500 p80=0.012346 p81=0.012195 p82=0.012048 p83=0.011905 p84=0.011765 p85=0.011628  
 p86=0.011494 p87=0.011364 p88=0.011236 p89=0.011111 p90=0.010989 p91=0.010870 p92=0.010753  
 p93=0.010638 p94=0.010526 p95=0.010417 p96=0.010309 p97=0.010204 p98=0.010101 p99=0.010000  
 By solving the system for n=1000

The solution is:

p1=0.500000 p2=0.333333 p3=0.250000 p4=0.200000 p5=0.166667 p6=0.142857 p7=0.125000 p8=0.111111  
 p9=0.100000 p10=0.090909 p11=0.083333 p12=0.076923 p13=0.071429 p14=0.066667 p15=0.062500  
 p16=0.058824 p17=0.055556 p18=0.052632 p19=0.050000 p20=0.047619 p21=0.045455 p22=0.043478  
 p23=0.041667 p24=0.040000 p25=0.038462 p26=0.037037 p27=0.035714 p28=0.034483 p29=0.033333  
 p30=0.032258 p31=0.031250 p32=0.030303 p33=0.029412 p34=0.028571 p35=0.027778 p36=0.027027  
 p37=0.026316 p38=0.025641 p39=0.025000 p40=0.024390 p41=0.023810 p42=0.023256 p43=0.022727  
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 p58=0.016949 p59=0.016667 p60=0.016393 p61=0.016129 p62=0.015873 p63=0.015625 p64=0.015385  
 p65=0.015152 p66=0.014925 p67=0.014706 p68=0.014493 p69=0.014286 p70=0.014085 p71=0.013889  
 p72=0.013699 p73=0.013514 p74=0.013333 p75=0.013158 p76=0.012987 p77=0.012821 p78=0.012658  
 p79=0.012500 p80=0.012346 p81=0.012195 p82=0.012048 p83=0.011905 p84=0.011765 p85=0.011628  
 p86=0.011494 p87=0.011364 p88=0.011236 p89=0.011111 p90=0.010989 p91=0.010870 p92=0.010753  
 p93=0.010638 p94=0.010526 p95=0.010417 p96=0.010309 p97=0.010204 p98=0.010101 p99=0.010000  
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part-c

From changing the probabilities to

$$\alpha \text{ and } (1 - \alpha) \quad (1)$$

for movement to the left or right.

$$P_i = \alpha P_{i-1} + (1 - \alpha) P_{i+1} \quad (2)$$

Solving the equation For i=1

$$P_1 = \alpha P_0 + (1 - \alpha) P_2 \quad (3)$$

For i=2

$$P_2 = \alpha P_1 + (1 - \alpha) P_3 \quad (4)$$

For i=3

$$P_3 = \alpha P_2 + (1 - \alpha) P_4 \quad (5)$$

For i=4

$$P_4 = \alpha P_3 + (1 - \alpha) P_5 \quad (6)$$

For i=n-1

$$P_{n-1} = \alpha P_{n-2} + (1 - \alpha) P_n \quad (7)$$

Since the equations are nothing but linear transformations. Hence writing the above equations in terms of matrix

$$\begin{bmatrix} 1 & -(1 - \alpha) & 0 & \dots & 0 \\ \alpha & -1 & (1 - \alpha) & \ddots & 0 \\ 0 & \alpha & \ddots & 1 & (1 - \alpha) \\ 0 & 0 & \alpha & \ddots & -1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \alpha & \dots & (1 - \alpha) \end{bmatrix}$$

part-d

Assumptions:

Here the input is taken dynamically from the user. So the code below holds good for all the values of n i.e for n=100 and n=1000.

Used Gaussian elimination to find the to solve the given system.

Basically the program itself generates the matrix with the value of alpha as 0.333 depending on the input given by the user (i.e n=100 or 1000 in this case)

Gaussian Elimination.

```

for j=0,...,n do
  for i=0,...,n do
    if i > j then
      c = a[i][j]/a[j][j]
      Dividing each element by diagonal to generate a upper triangular matrix.
      for k=0,...,n do
        a[i][k] = a[i][k] - c * a[j][k]
      end for
    end if
  end for
end for
x[n - 1] = a[n - 1][n]/a[n - 1][n - 1]

```

```

for i=n-1,...,0 do
    Initializesum  $\leftarrow$  0.
    for j=i+1,...,n-1 do
        sum = sum + a[i][j] * x[j]
    end for
    x[i] = (a[i][n] - sum)/a[i][i]
end for

```

CODE

```

1  #include<stdio.h>
2
3
4  #include<stdlib.h>
5
6
7
8
9  int input(int n)
10 {
11     int i;
12     int j;
13     //int *a;
14     double **a=(double **) malloc ((n+1)*sizeof(double *));
15     for (i=0;i<=n; i++)
16     {
17         a[i]=(double *) malloc (n*sizeof(double));
18     }
19     //For generating the input matrix
20     for (i=0;i<=n-1;i++)
21     {
22         for (j=0;j<=n; j++)
23         {
24             if (i==j)
25             {
26                 a[i][j]=1;
27             }
28             else if ((j==i+1) && j!=n)
29             {
30                 a[i][j]=-0.6666;
31             }
32             else if (j==i-1 && j!=n)
33                 a[i][j]=-0.3333;
34             else if (j!=n)
35             {
36                 a[i][j]=0;
37             }
38             else if (j==n && i==0)
39                 a[i][j]=0.3333;
40             else if (j==n && i!=0)
41                 a[i][j]=0;
42         }
43     }
44     for (i=0;i<=n; i++)
45     {
46         for (j=0;j<=n; j++)
47         {
48             printf("%f\t",a[i][j]);
49         }
50         printf("\n");
51     }
52
53     int k;
54     float c;
55     //Gaussian elimination:To generate a upper traingular matrix.
56     for (j=0; j<=n; j++)

```



```

57     {
58         for (i=0; i<=n; i++)
59         {
60             if (i>j)
61             {
62                 c=a[i][j]/a[j][j];
63                 for (k=0; k<=n+1; k++)
64                 {
65                     a[i][k]=a[i][k]-c*a[j][k];
66                 }
67             }
68         }
69     }
70     for (i=0; i<=n-1; i++)
71     {
72         for (j=0; j<=n; j++)
73         {
74             printf("%2f  ", a[i][j]);
75         }
76         printf("\n");
77     }
78
79     double *x=(double *) malloc((n)*sizeof(double));
80
81     x[n-1]=a[n-1][n]/a[n-1][n-1]; int sum;
82     /Back-substitution :to find the solution i.e to get the values of probabilities in this
83     case.
84     for (i=n-1; i>=0; i--)
85     {
86         sum=0;
87         for (j=i+1; j<=n-1; j++)
88         {
89             sum=sum+a[i][j]*x[j];
90         }
91         x[i]=(a[i][n]-sum)/a[i][i];
92     }
93     printf("\nThe solution is: \n");
94     for (i=0; i<n; i++)
95         printf("\np%d=%f\t", i+1, x[i]);
96
97
98
99     return 0 ;
100 }
101
102 int main() {
103     int n;
104     int i;
105     scanf("%d", &n);
106     input(n-1);
107 }
108

```

INPUT: INPUT: n=100 and n=1000 RESULT : By solving the system for n=100

The solution is:

p1=0.333300 p2=0.142820 p3=0.066636 p4=0.032236 p5=0.015858 p6=0.007864 p7=0.003916 p8=0.001953  
p9=0.000975 p10=0.000487 p11=0.000244 p12=0.000122 p13=0.000061 p14=0.000030 p15=0.000015  
p16=0.000008 p17=0.000004 p18=0.000002 p19=0.000001 p20=0.000000 p21=0.000000 p22=0.000000  
p23=0.000000 p24=0.000000 p25=0.000000 p26=0.000000 p27=0.000000 p28=0.000000 p29=0.000000  
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p93=0.000000 p94=0.000000 p95=0.000000 p96=0.000000 p97=0.000000 p98=0.000000 p99=0.000000

For n=1000

The solution is:

p1= 0.333300 p2= 0.142820 p3= 0.066636 p4= 0.032236 p5= 0.015858 p6= 0.007864 p7= 0.003916 p8=  
0.001953 p9= 0.000975 p10= 0.000487 p11= 0.000244 p12= 0.000122 p13= 0.000061 p14= 0.000030 p15=  
0.000015 p16= 0.000008 p17= 0.000004 p18= 0.000002 p19= 0.000001 p20= 0.000000 p21= 0.000000 p22=  
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[illegible]

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References:

<http://www.codewithc.com/gauss-elimination-method-algorithm-flowchart/>