ICP Algorithm for Alignment of Stars from Astronomical Photographic Images

Alexander Marinov, Nadezhda Zlateva

***Abstract**: This article proposes a method for the alignment of stars from astronomical photographic images to stars from a standard astronomical catalogue. Given the coarse celestial coordinates of the image centre and the field of view, we are looking for a mapping between the stars extracted from the image and the stars from the catalogue. The method is based on the Iterative Closest Point (ICP) algorithm, which is robust enough to the presence of noise and the appearance of false stars or such that are missing from the image. Due to the purely geometrical character of the algorithm, the intensity noises of the image pixels have no impact. The result when applying the algorithm is that the original image is translated and rotated so that the stars from the image are aligned with their exact coordinates, given by the star catalogue.*

***Key words:** ICP algorithm, alignment of stars, astronomical photographic images.*

INTRODUCTION

The registration of a photographed sky image is a classical problem and a regular task in every observatory nowadays. The contemporary photographic instruments produce high quality images with high fidelity measurements of their observations, which facilitate the image registration problem. But there are still a large number of photographic plates from the observatories' archives that suffer from the quality of the past instruments. These archives may contain plates with various kinds of defects, such as obscured surface areas due to physical damage, lost information regarding the image celestial coordinates, image noise-artefacts like the réseau grid, introduced to assist the process of visual measurement, etc. The Institute of Astronomy at the Bulgarian Academy of Sciences has prepared a list of such photographic plate archives "Catalogue of Wide-Field Plate Archives" [7] enumerating over 2,000,000 observations obtained since the end of 19th century from observatories worldwide. The problem arising with the registration of such plates involves one or more of the following separate tasks:

1. Filtering the regular noise and artefacts from the input image
2. Extraction of the centres of the stars
3. Searching the stars in a star catalogue to determine the image location on the sky [4]
4. Alignment of the star set from the image with the star set from the catalogue

In our work we assume that the first three tasks are already solved by other methods, so we only focus on solving the last task. We assume that task 3 reduces the catalogue only to the area containing the image. Then we model the problem by considering the star maps from the image and from the catalog as two point sets in the Euclidian 2-dimensional space:

$$A, B \subset E^2, |A| = n, |B| = m \tag{1}$$

and we denote with $a \in A$ and $b \in B$ the elements from the corresponding sets.

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We want to match the stars from the input image with the stars from the catalogue in order to estimate their mutual correspondence and align more precisely their coordinates, including the image centre. This alignment can be achieved by moving the stars from the input image uniformly to optimally adjust them to the corresponding stars from the catalogue. Thus we consider two functions μ and λ as defined below:

$$\mu : \mathrm{A} \to B; \mu(a) = b \tag{2}$$

$$\lambda = (R,t) : \mathrm{A} \to \mathrm{E}^2; \lambda(a) = Ra - t, \tag{3}$$

where $t \in E^2, R \in SO(2)$-special orthonormal 2x2 matrix, i.e. det(R)=1, RRT=I. We measure the adjustment between the two point sets using the sum of squared distances between corresponding points

$$\sum_a \left\| \lambda(a) - \mu(a) \right\|^2 \tag{4}$$

which we are interested to minimize. Thus, our problem is to find such λ, μ which satisfy the criterion:

$$\min_{\lambda,\mu} \sum_a \left\| \lambda(a) - \mu(a) \right\|^2 \tag{5}$$

ITERATIVE CLOSEST POINT ALGORITHM

Since the introduction of the basic concepts of Iterative Closest Point algorithm (ICP) by Chen [3] and Besl [2], there have been developed multiple variants of the algorithm, the classification of which we can find in Rusinkiewicz [6]. An analytical solution finding an optimal rotation matrix R and a translation vector t is developed by Arun [1]. The implementation of the algorithm is based on the work of the authors above with the optimization improvements by Kapoutsis [5].

Basically ICP is expressed by the following algorithm:

1. Initialization: $\lambda := (I,0)$
2. Matching: $\mu \leftarrow \min_{\mu} \sum_a \left\| \lambda(a) - \mu(a) \right\|^2$
3. Transforming: $\lambda \leftarrow \min_{\lambda} \sum_a \left\| \lambda(a) - \mu(a) \right\|^2$
4. Loop to step 2 until convergence or exceeding the iterations limit

Matching step:

The matching algorithm must select the closest point $b \in B$ for each point $a \in A$. Trivial implementation may exhaust all point pairs. Further optimizations can be found in the more efficient algorithm variants reviewed in [6, 5]. Although this step may look simplest to understand, this is the slowest step in the whole algorithm.

Transformation step:

In this step we want to find the orthonormal matrix R and vector t that for given μ minimizes (4).

Let $\bar{a}$ and $\bar{b}$ be the respective point set centroids -

$$\bar{a} = \frac{1}{|A|}\sum_{a \in A} a, \bar{b} = \frac{1}{|B|}\sum_{b \in B} b \quad (5)$$

and the point coordinates relative to them -

$$a' = a - \bar{a}, b' = b - \bar{b} \quad (6)$$

Hence,

$$\sum_a \|\lambda(a) - \mu(a)\|^2 = \sum_a \|Ra - t - b\|^2 = \sum_a \left\|(Ra' - b') + (R\bar{a} - \bar{b} - t)\right\|^2 . \quad (7)$$

From here we can derive t directly by setting the second member of the expression to 0:

$$t = R\bar{a} - \bar{b}. \quad (8)$$

Now the minimization of (4) reduces to minimizing:

$$\sum_a \|Ra' - b'\|^2 = \sum_a (Ra' - b')^T (Ra' - b') =$$
$$\sum_a (a'^T R^T Ra' + b'^T b' - a'^T R^T b' - b'^T Ra') =$$
$$\sum_a \|a'\|^2 + \sum_a \|b'\|^2 - 2Trace(RH), \quad (9)$$

where

$$a'^T R^T b' = b'^T Ra' = Trace(R(a'b'^T) \quad (10)$$

and

$$H = \sum_a a'b'^T . \quad (11)$$

Hence the problem of minimizing (4) is reduced to finding:

$$\max_R Trace(RH). \quad (12)$$

Arun [1] proves that (12) is maximized by

$$R = VU^T \quad (13)$$

where

$$H = U\Sigma V^T \quad (14)$$

is the singular value decomposition of H.

EXPERIMENTAL RESULTS

For our experiment we used a scanned wide-field photographic plate and extracted 1975 stars with stellar magnitudes up to 14, which we treat as the model. From these stars we randomly selected 632 and added rotational, translational and point position noise to them. Our goal is to find the best fit of these 632 stars onto the model. Fig. 1a shows the initial stage of the algorithm, where the stars from the model are displayed in gray points and the noised stars - in black crosses. After the matching phase, the data to model mapping is discovered and is marked with dark lines on the same figure. The target stars to which the data stars point are marked in black. The squared distance (4) monotonically decreases until convergence after 154 iterations (Fig. 2). Fig. 3 shows an intermediate stage of ICP, while the final result of the algorithm is displayed in Fig. 4.

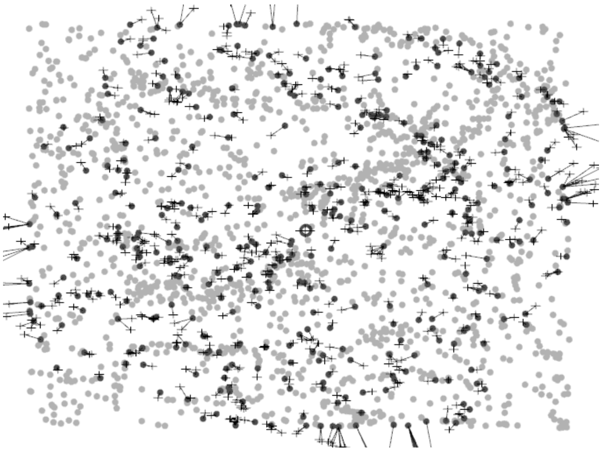

Fig. 1 – initial ICP stage

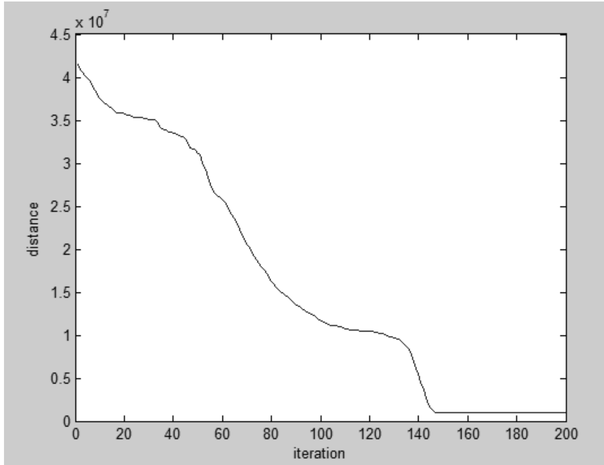

Fig. 2 –square distance between point sets on each iteration

Fig. 3 – intermediate ICP stage

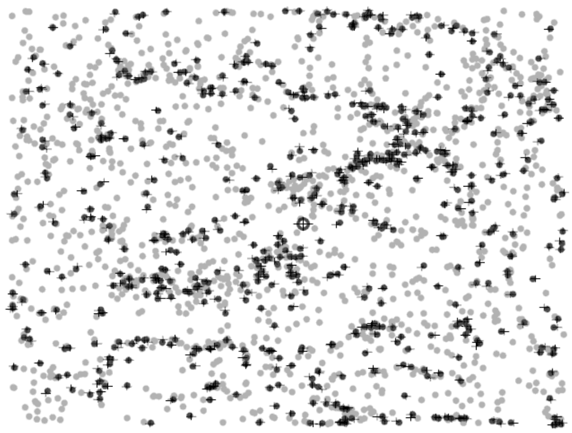

Fig. 4 – final ICP stage

CONCLUSION AND FUTURE WORK

We illustrated the Iterative Closest Point algorithm applied for astronomical image stars alignment, and a successful experiment restoring a shuffled set of stars to their original position. In the common case ICP converges to a local minimum, but this is good enough in the case of astronomical images with known celestial centre, even when it is not quite precise. The present algorithm does not benefit from the stellar magnitudes, which are usually known. A direction for future research is the modification of the suggested algorithm to use assigned weights to the points, which will improve the matching function μ. The authors believe that such improvements will skip inappropriate local minima and decrease the number of iterations until convergence occurs.

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