lecture 7 part 2

(0:00 - 0:26)

So I recall that we have a left-hand side of our equation. And then everything has epsilon 0 and mu 0, which are, if you recall, what is describing the speed of light in vacuum. They are defined this way, as the constants of nature.

(0:30 - 1:44)

So I collect them, which means that what is left is sigma over epsilon 0, conductivity divided by the value of the electrical conductivity, times derivative of the electric field, plus epsilon r, second order times derivative of the electric field. Is this readable? Now, this is one of the seven fundamental constants of nature. So there is no way not to write that in the most canonical form.

(1:45 - 2:13)

And what we are left with is sigma over epsilon 0, times derivative of the electric field, plus epsilon r, second order times derivative of the electric field. I just rewrote the same thing by replacing the definition here. So actually this is the wave equation in the material.

(2:27 - 3:02)

With all the assumptions and simplifications that we made, so that we have no gradients in the electrical charges, and that the material does not have a significant response to the magnetic fields, we are going to add further pieces to this expression here. This is basically the simplest we can get. So, what can we see from here, with respect to this one? Well, we start from epsilon 0, we add an epsilon r, and this epsilon r actually is here.

(3:03 - 3:39)

So we have, let's say, a dilation factor in the electric field, which is entering inside a second order differential equation. We are going to see how this actually pans out, but this is going to introduce some modifications with respect to this case, where we do not have, of course, any effect related to the polarizability of the material. Epsilon r is linked to the polarization field, just to be clear.

(3:40 - 4:14)

So there is going to be some orientation in the microscopic charges, which are affecting the way the wave propagates. And then we have a new term, and this is this case. Then we have a new term, which is proportional to the time derivative of the first order, in the electric field, and which is also proportional to the conductivity of the material.

(4:21 - 4:43)

So, in principle, we already have a hint about what changes when we consider a material which has charges. So here, I was saying, there is the fact that we are dealing with some polarization. It means that an electromagnetic wave will change the orientation of the electrical dipoles in our material.

(4:44 - 5:15)

So it will present an orientation of the electronic orbitals, which are going to change the isotropic configuration of a crystal, for example. And this is having some effect. In which case do you feel that the dipole is going to be affected? So we have different kinds of elementary charges, which luckily are just two.

(5:16 - 5:34)

We have nuclei and electrons. In both cases, nuclei, we consider them as fixed. So they are in a fixed position, and we are just describing the wave function of an electron, which belongs to the entire lattice of material that we are considering.

(5:35 - 5:53)

So the first approach when we deal with an electric field, the first effect is that we are rearranging or moving or having an effect on electrons. Because the nuclei are kept by, let's say, electrons. At this point, we have started to see that we have two kinds of electrons in a crystal.

(5:53 - 6:09)

Ones which are core electrons or even valence electrons. So they are either belonging to an atom, or they participate to a chemical bond. And they form the chemical structure of our material.

(6:10 - 6:25)

And then we have a second kind of electrons, in this regard, which are nearly free. So those who are in the conduction band, for a metal or for a semiconductor. And so they participate in electrical conduction.

(6:26 - 6:53)

So conductivity is going to be affected by, let's say, nearly free electrons. And since they are free to move, they are going to basically oscillate and move around, driven by the alternating field which is propagating, which is our electric field oscillation. These here are fixed to some extent.

(6:53 - 7:03)

They are more localized and participate to chemical bonds. So they are not capable of moving around. They have to respond to some more strict quantization rules.

(7:04 - 7:19)

What they can do is to change, let's say, the shape of the orbitals or redistribute especially the charge. And so they are basically deforming the base charge from a local point of view. They are forming dipoles which are oriented according to the electric fields.

(7:20 - 7:48)

And so this is where we can see, let me say, bound, I say bound, let me say bound electrons. Those which are forced by chemical interactions to stay more or less in a specific energy. And so these two contributions are giving us two different terms in our wave equation.

(7:49 - 8:23)

In the vacuum, this term will not be there because we have no free electrons. And this term here is not getting this multiplying factor because the electric field will just have... As I was saying, we are considering this as the quantity describing the material, which is the contribution of vacuum plus the polarization. If you are dealing with vacuum, the polarization term is not here, so the electric field is just unaffected.

(8:23 - 8:46)

But if you are considering a material, this p here is what is encoded in this epsilon r here. We are going to see that with more detail, but I wanted to give you some hints. Why so? Because now we need some, I wouldn't say calculations, but still, which is the shape of an electromagnetic wave.

(8:46 - 9:07)

This is a differential equation, probably the practice of us already figured it out. But we can get some mathematical expression, which is actually quite simple. So we can see how we can describe an electromagnetic wave and gather some additional information and propagation by finding a solution to this differential equation.

(9:12 - 9:35)

And this is what I am trying to do in the next, let's say, 10 minutes. If something is not clear, interrupt me or during the break ask me. It's crucial that we get to this point together.

(9:39 - 10:02)

So, solution to wave. How can we do that? Ok, we know how a wave travels through a vacuum.

It's something that we are supposed to know from basic algebra courses.

(10:03 - 10:37)

And we can write since electric field and magnetic field are linked together, just by considering the electric field. We consider that as an amplitude for the oscillation k x minus i k x minus i omega t. Here I am just making one assumption. I am working one dimensional case.

(10:38 - 11:12)

So our vector x is going to be in the direction x and our vector t is just going to be a scalar line. In this case, our Laplacian of the electric field is just the second order derivative of the x coordinate. So, this works in vacuum.

(11:17 - 11:30)

Let's make an assumption that this works also in material. This is how a wave propagates. Let's use this.

(11:33 - 11:50)

Why can I do that? Because this is actually an expression. So let me remove these arrows from here. Which is very general because we have not said anything about the two parameters here which are k and omega.

(11:50 - 12:01)

We know the time, we know the space. We do not know how they are made. And actually if there is any relationship between omega and k. Which in vacuum actually is there.

(12:02 - 12:14)

We are just using a functional form. We see if we get to the same constraints or something has to be modified. So if we use that or we plug that here.

(12:15 - 12:35)

There is something which is actually quite simple to be done. I apply this one to this expression here. So this is going to be A. And this is going to be B. So A is going to be the Laplacian of the electric field.

(12:35 - 12:53)

Which is going to be second order special derivative of E. So where is the dependence on x coordinate? It is just in the exponentiation. So what we get. We take the second order special derivative.

(12:54 - 13:04)

We simply pull down the coefficients of the exponent. So we get ik squared. And then we should get the same.

(13:04 - 13:22)

If I am not mistaken. So e to the ikx whole minus i omega t. Am I forgetting anything? Please interrupt me if I am not correct on that. So we save some time just to do that.

(13:22 - 13:38)

What I missed. So the second term is 1 over c squared. Sigma over epsilon 0. Special derivative of the electric field.

(13:40 - 13:52)

Plus epsilon r. This is not special. This is time derivative. Of the electric field.

(13:54 - 14:11)

This is longer to be written in this way than when we apply the solution. So we get 1 over c squared. Then we get sigma over epsilon 0. What we have to pull down here.

(14:12 - 14:25)

This is first order time derivative. So I am pulling down minus i omega. And let me write the electric here.

(14:26 - 14:59)

Epsilon e to the ikx minus i omega t. Then here I get plus epsilon r. And I am taking the second order time derivative here. This is going to be minus i omega to the power of 2. And it is time to figure out whether I made some mistakes. Ok so.

(15:28 - 15:40)

Now. And it has to be equal to b. And so I get ik squared. This is the imaginary number.

(15:40 - 16:07)

So what we end up having is minus k squared. Zero e to the ikx minus i omega t. Which has to be equal to 1 over c squared. Then we have sigma over epsilon 0. Minus i omega.

(16:09 - 16:21)

Plus. Epsilon r. Minus to the power of 2 becomes plus. i to the power of 2 becomes minus.

(16:21 - 16:39)

So we get minus omega squared. If I am not mistaken. And this is going to be equal to e zero e to the ikx minus i omega t. So this choice actually was lucky.

(16:39 - 16:51)

Let's say it is a resilient functional form. Because now we have transformed the differential equation. In something which now actually is an algebraic equation to some extent.

(16:51 - 17:03)

Because these terms here are present at both my hands. And what we get is an equation which gives us a relation. Between all these quantities which are placed here.

(17:03 - 17:08)

And are not dependent on time and space. So there is some relation to this. Fine.

(17:10 - 17:18)

What we get is that we have minus everywhere. So I get k squared. Equal to 1 divided by c squared.

(17:19 - 17:29)

And then I get. Omega squared minus i omega. I am forgetting the coefficients.

(17:29 - 17:38)

So omega squared epsilon r. Plus. i omega. Rondotivity divided by.

(17:38 - 17:51)

The partial derivative permittivity. Sigma over epsilon 0. Now. What I typically do in this case.

(17:52 - 17:56)

Is to collect. This term here. And that's for a good reason.

(17:58 - 18:04)

And so. I can write k squared as. Omega squared divided by c squared.

(18:04 - 18:18)

And we get epsilon r. Plus i. Omega. Sorry i. Sigma over epsilon 0. And since we have to pull. Out an omega squared.

(18:19 - 18:25)

We have to divide that by. Omega. And this is the relation that we get.

(18:29 - 18:35)

Okay. This is the result. What the condition that we have to satisfy.

(18:36 - 18:40)

For this. To be. To describe a wave.

(18:40 - 18:50)

In a material according to the Maxwell's equation. So let's take. A step back and recall.

(18:51 - 18:56)

What happens in vacuum. In vacuum we have. The speed of light.

(19:00 - 19:04)

Which is defined by the speed. Which light travels. In vacuum.

(19:06 - 19:17)

So if you are neglecting epsilon r. Sigma and whatever. We get that. C. Can be written as.

(19:20 - 19:26)

Wave length divided by the period. Of oscillation. And this is also equal.

(19:27 - 19:31)

To. Lambda divided by 2 pi. Times.

(19:32 - 19:41)

2 pi divided by the period. Okay. And so this is equal to.

(19:41 - 19:56)

1 divided by k. And this is going to be omega. So in vacuum. Let me write them as omega 0. And k 0. Because they are referring to vacuum.

(19:57 - 20:04)

This is the relation that we get. When we are dealing. With a wave traveling.

(20:05 - 20:12)

In vacuum. Now. Is this true in this case.

(20:12 - 20:19)

Well if we are dealing with vacuum. Vacuum means. Epsilon r is equal to 1. And we have now.

(20:19 - 20:28)

Moving charge. So productivity is 0. So all these parenthesis here. Becomes equal to 1. We neglect this.

(20:28 - 20:33)

And we get exactly. The definition. Of the speed of light in vacuum.

(20:34 - 20:39)

As it's supposed to be. So there is no. Nothing which does not work.

(20:39 - 20:47)

Let's say in this case. But. In this case.

(20:47 - 21:04)

We have. A direct effect. On the fact that we have.

(21:04 - 21:15)

The speed of light. In the material. We can use this same expression here.

(21:15 - 21:26)

And we can write that as. Omega. Divided by k. So I basically.

(21:27 - 21:31)

Rearrange this equation here. And I take the square root. Okay.

(21:31 - 21:36)

Let's take it one step at a time. So I get 1 omega squared. Divided by k squared.

(21:37 - 21:46)

Which is equal to. The square of the speed of light. Divided by epsilon r. Plus i. Sigma.

(21:47 - 22:02)

Divided by epsilon 0 omega. Which means that. Omega divided by k. Simply c. Divided by epsilon r. Plus i. Sigma divided by epsilon 0 omega.

(22:03 - 22:14)

Under square root. And let me say. That this is v. Which is the speed of light.

(22:14 - 22:27)

In the material. So. If we take the ratio.

(22:27 - 22:31)

Between the speed of light. In vacuum and the speed of light. In the material.

(22:32 - 22:49)

Which is c divided by v. Okay. What we are left with. We are left with.

(22:49 - 22:57)

Epsilon i. Plus i sigma. Divided by epsilon 0 omega. Under square root.

(22:59 - 23:12)

But have you seen this. Expression here already. That's the most basic.

(23:12 - 23:16)

Definition. We can think. About the refractive index.

(23:37 - 23:42)

Which tells us. Which is the ratio. Between the speed of light.

(23:42 - 23:47)

In vacuum and the speed of light. In a given material. And this term.

(23:48 - 23:53)

Of course is always. Larger than one. Except for metal materials.

(23:53 - 23:58)

That's also basic. Effects. We can.

(23:59 - 24:01)

Start back from here. After a break. I would say 10 minutes.

(24:04 - 24:05)

15 past 10.