solid state 9-10 (mp3cut.net)

(0:00 - 1:01)

Last Monday, which was electromagnetic waves to materials. So, we started to see basically the main concepts, which were the fact that we were introducing, let's say, a polarization vector, a polarization field. And this is the microscopic dipole moment, in which we have a positive charge, a negative charge, separated by a distance, which is d, and d is given by q, the charge, times the distance of separation between the two charges.

(1:02 - 1:50)

And this was the density of microscopic dipoles in the material. And the dipoles are typically given by the fact that our material is globally neutral from an electrical point of view, because we have as many nuclei, which are positively charged, and electrons, which are negatively charged. However, if you look at them at a microscopic level, we have a very tiny space charge, which is due to the electrical charge separation, due to the fact that they are discrete, and that electrons move around the lattice, while positively charged nuclei are fixed.

(1:50 - 2:36)

And so this was intended to give us a description of the electric field inside a material, which was given, if I'm not mistaken, by this expression here. Please correct me if there is anything that should be corrected. So, in this sense, we have a displacement field, which is so-called, which is taking into account the electrical contribution, the electrical forces which are acting inside a material, which hosts microscopic dipole moments.

(2:36 - 3:11)

And so the electrical response, the field, the electrical interaction is given by the displacement field, which is the contribution that we would have from the electric field in vacuum, and this is the electric conductivity of vacuum, plus an additional term, which is given up by how this electric field is going to act on the microscopic dipole moments in order to align them. And once they are aligned, they are forming some additional electric field and interaction, which is given by this. So, we had another way to write the displacement field.

(3:11 - 4:11)

I'm just recapping what you should already have in your lecture notes, which I'm also sharing on the Doodle page of the course. And we had that the displacement field could be regarded as ε0 εr electric field, let's say, where this is the material relative permittivity, where εr is larger or equal than 1, and it is equal to 1 when we are dealing with vacuum. And we had some similar considerations also for the magnetic field, since we have this expression here, where this is the magnetic induction field and this is the magnetic strength field, if I'm not mistaken.

(4:13 - 5:35)

So, we wrote and we derived the Maxwell's equation for vacuum, and by applying these two prescriptions here, and basically replacing ε_0 with $\varepsilon_0 \varepsilon_r$ at every step, and then taking advantage of these expressions, we found out the four Maxwell's equations in the material, which are given by, I'm just copying what you should already have in your notes, the Gauss law, which is now written in terms of the displacement field. We just have the fact that with respect to electric field and division by ε_0, now we have the natural unit to express the space charge density ρ. We had that the second of the Maxwell's equations was left unchanged, because it's just describing how a static magnetic field works. We also left unchanged the third one, if I'm not mistaken this was the third one, which was minus d over 3, which is the Maxwell-Faraday-Lenz law.

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And then we have some modifications, at least in the shape, concerning the fourth one, which was written as the curve of the magnetic strength field equal to the current density plus the variation over time of the displacement field. So this is the current density, which comes from the Maxwell-Faraday-Lenz law, and basically this is a way to rewrite what was originally taught in vacuum to be the curve of the magnetic field equal to $\mu_0 J$ plus the time derivative of the electric field, and this was multiplied by $\mu_0 \varepsilon_0$. So if we say that μ_0 now becomes μ, which is bigger than μ_0, μ_r, but in most cases we place it equal to 1, we are considering diamagnetic materials, we are not considering magnetic fields yet in this discussion.

(6:56 - 7:34)

And by ε_0 we replace $\varepsilon_0 \varepsilon_r$, and then we use these two relations here. So d is equal to εe, and h is equal to b divided by μ, and we apply these substitutions here, and we get this final Maxwell law. Now at this point, this is where I went, I started to hint that we actually have the possibility to understand a little bit more, first of all, how a wave propagates in a material.

(7:34 - 8:09)

We are going to see a solution for Maxwell's equation, which corresponds to the wave equation. The other point is that since we are considering a material in which we have charges, ok, let's assume that we are just considering electrons, so in principle also holes are charged, but let's keep it to the microscopic constituents. So the material is globally neutral, but we have charges which in principle are free to move, at least in semiconductors and metals.

(8:10 - 9:06)

So a current density is not just a wire in which we apply current and we observe the magnetic field in the surrounding area. Now we have the charges that move inside the material. And so we know that the Ohm's law, we saw that in the first lecture, of course we can write it as the voltage drop across a resistor is equal to the resistance times the current, which is my given circuit, or, interesting from material science point of view, we know that the current density is equal to the conductivity times the electric field, ok? So there is a direct proportionality between the current density which is flowing in a given section of our material and this is the proportionality factor between the current density and the electric field.

(9:08 - 10:03)

Now we are considering actually an electromagnetic field, so in principle we are expecting that we are going to go to electromagnetic waves, not because this was the title of the lecture. So in principle an electromagnetic wave will have a frequency of oscillation, will have a pulse, which is related to that, ok? And so we will have that the electric field is going to oscillate at certain frequencies and in principle we can expect that this ratio here will depend on the frequency that we apply. So let me start to anticipate that this quantity here is going to depend on the frequency that we apply.

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Why this anticipation? Because the material will have the possibility to absorb or reflect different colors, different wavelengths and these properties depend on how is the behavior, of course to some extent there are other factors, which is the behavior of the conductivity as a function of the oscillation frequency. We are going to have a hint about that. So the task, the goal that I have is to show you basically how the electromagnetic wave behaves inside the material.

(11:04 - 11:27)

So an electromagnetic wave travels in space and travels over time. And this is true in vacuum in any case. Now the task is something which is travelling at the speed of light, see, over space and time.

(11:28 - 12:13)

What is interesting, what basically is the additional part that we are inserting here and this is going to go and travel inside the material. So another way to see that is that we have an electric field which is oscillating and now I erased that, but we have seen that basically we have a displacement field which is describing everything which takes into account also the polarization field. So basically we have an oscillating field which means that we are going to deal with oscillating dipoles.

(12:17 - 12:46)

It's something that we also hinted in the last lecture. So if we have a dipole and we make the dipole oscillate, this dipole will emit an electromagnetic wave in a direction which is perpendicular to the oscillation just because the sense electric field is going to swap over time and over space according to the speed of light at which the information travels. Now we can say also the opposite.

(12:46 - 13:15)

So if we embed our dipole inside an electromagnetic field, the charges will sense the fact that we are swapping the electric field and so they will rearrange accordingly. And so the dipole will start oscillating according to the properties of the electric field itself. So how can I describe that in a medium? Well, we are interested in the Weyl's equation.

(13:21 - 13:50)

In vacuum we already saw that we can get a simple expression which is given by this second order differential equation here. So this is describing basically how a wave propagates in vacuum. What happens in a material? Well, we can do basically the same kind of calculations in order to get that.

(13:51 - 14:18)

So I can use basically these two because we are interested in how there is an interplay between magnetic and electric field to get an electromagnetic wave. And in this case we have something which is supposed to oscillate and these are the two equations which have time dependence. So I'm considering the curve of E which is the time.

(14:18 - 14:53)

I'm just copying these two in order to get rid of this black box. And then we have the curve of the magnetic field strength which is equal to conductivity times electric field plus time derivative of the displacement vector field. So here I replaced the current density with the oscillosync because we are dealing with a material.

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Now, if I couple these two in a smart way I can get something which is going to reproduce more or less this expression here with some fancy use, otherwise we are not doing that. So we proceed the same way we proceeded our equations in vacuum. So I take the curve of the curve of E. So I'm applying a curve to this first equation.

(15:23 - 15:44)

And this is going to be minus time derivative of the curve of the magnetic field. And for the time being I can exploit this expression here. So I just get rid of this to prevent your eyes crossing too much.

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Now, just a few clarifications here in case some of you are interested. So the curve of the curve of the electric field from a mathematical point of view is equal to the gradient of the divergence of the electric field minus the revolution of the electric field. I can write it here.

(16:33 - 16:58)

So, in principle, what's here? This is the first of the Maxwell's equations. So I should write that I have a gradient of rho divided by epsilon zero or epsilon, since we are dealing with a material. And this is going to be minus, sorry, this is not a vector.

(17:07 - 17:25)

So what I'm assuming here is that we can neglect this part here. Let me say that we have a homogeneous material. Which is also neutral.

(17:29 - 18:05)

Because this is going to take into account that we have some gradient in this base charge which is stored inside our material. This is particularly useful specifically for semiconductors where we already know how the structure is and the material is overall neutral. If we are dealing with some glass or some other material which we can, to some extent, inject a charge and have some charge storage or some localized accumulation of charges or interfaces, oxides, this is not necessarily true and things become a little bit more complicated.

(18:06 - 18:34)

But, in general, when you have some space charge distribution this applies very often to the surface, its interfaces. So it's very tiny and small thickness of a material and if we neglect at least this boundary condition, this interface, everything can apply. So if we say that this is equal to zero, we can disregard any special homogeneities in the charge distribution and our material is neutral.

(18:34 - 19:03)

This is going to be the result. Minus the Laplacian of the material. Okay? I just recall that the Laplacian will be regarded as the summation for i from 1 to 3, so the special coordinates of the second derivative with respect to each of the Cartesian axes.

(19:06 - 19:57)

And so, this is more or less what we would get also in the case of the vacuum, the Maxwell's equation of vacuum. We have seen what happens at the right-hand side. And so this equation here becomes minus the Laplacian of the electric field that is equal to minus the time derivative of the curve of V. Okay? This already looks similar to this one here, so what happens to the right-hand side of our equation? Well, we can work as we have done so far.

(19:57 - 20:14)

I'm going to do it here. Please interrupt me if I'm going too fast, if I'm jumping to the results without sufficient justification. You set the pace.

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Okay? Okay. So what happens to this here? We have the partial time derivative of the curve of the magnetic field. Now, we have this expression here regarding the magnetic field and we are dealing with the material.

(20:48 - 22:00)

So the first thing that I do without knowing anything is just to take the time derivative and swap inside the argument mu the curve of H. H. Okay? And at this point, I have this equation here, which is the fourth one of the Maxwell's equation. So this is going to be minus the time derivative of conductivity which multiplies the electric field plus the time derivative of the displacement field. And for the moment, I'd say that that's what we can get by the two sides separately.

(22:01 - 22:27)

So, let's put them together. And since both have a minus sign, I'm just going to cancel them out. And this is going to be plus a fraction of the electric field equal to the time derivative of sigma E plus time derivative of the displacement field.

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And I forgot here that we have mu. Okay? So let me break this somewhere by applying the derivative. Okay? So we will have mu 0 bar I can write it this way sigma time derivative of the electric field plus this is going to be out.

(23:33 - 24:33)

I can write mu here and also here at this moment. Okay? Are you okay with that? Because it comes from this is divergence of H but we start from here and here we have to multiply by mu in order to revert to H. Okay? And here we get the second order time derivative of the displacement field. Now is it okay? Please correct me if I'm forgetting something because jumping from one blackboard to another is not as doing that on a piece of paper.

(24:33 - 25:02)

Don't feel shy or afraid to do something like this. Okay, so at this point mu 0 mu r sigma electric field let me just multiply and divide by epsilon 0. We are going to see why this is going to be useful. And this is the time derivative of the electric field.

(25:02 - 25:26)

Okay, so I'm just adding this little trick. And here I just rewrite this in terms of the electric field. So this is going to be plus we got mu 0 mu r epsilon 0 epsilon r second order time derivative of the electric field.

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Why so? Because I'm trying to get back to something which has this shape here. Now, as I was saying I was considering mu r sorry mu r to be equal to 1 so to consider to neglect any possible contribution from the magnetic field. So if mu r is equal to 1 we can simplify a little bit this expression because we are neglecting this term here.

(26:05 - 26:55)

And so what can I do in order to write more or less this equation here in order to have some way to work around using it. Well first of all I write again the Laplacian of the electric field so I recall that we have a left-hand side of our equation. And then everything has epsilon 0 and mu 0 which are if you recall what is describing the speed of light in vacuum and they are defined this way as the constants of nature by the product.

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So I collect them which means that what is left is sigma over epsilon 0 on the field divided by the value of the electric conductivity times derivative of the electric field. Plus epsilon r second order times derivative of the electric field. Is this readable? Now this is one of the seven fundamental constants of nature so there is no way not to write that in the most canonical form.

(28:12 - 28:40)

And what we are left with is sigma over epsilon 0 times derivative of the electric field plus epsilon r second order times derivative of the electric field. I just rewrote the same thing by replacing the definition here. So actually this is the wave equation in the material.

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With all the assumptions and simplifications that we made so that we have no gradients in the electrical charges and that the material does not have a significant response to the amount of fields which I'm going to add further pieces to this expression here. This is basically the simplest we can get. So what can we see from here with respect to this one? Well we start from epsilon 0 we add an epsilon r and this epsilon r actually is here.

(29:30 - 29:58)

So we have let's say a dilation factor from the electric field which is entering inside a second order differential equation which we are going to see how this actually pans out but this is going to introduce some modifications with respect to this case where we do not have of course any effect related to the polarizability of the material.