1st cut

(0:00 - 0:45)

Okay, so we can continue from the discussion that we set up during the first hour. So, this is our refractive index, and this is the relation between the speed of light and the material. In vacuum, I would expect that in the material, and so this can be written as we have found out here, okay? So, the refractive index, let's say, we can highlight already some features starting from this description here.

(0:46 - 1:28)

The first one is that this is a complex number, and we are going to see, hopefully already by today, what that implies. It is not just a real number, we have some dependence on the imaginary number, which is linked to the conductivity here. The second point, or the point that I want to consider, is that this refractive index depends on the conductivity, okay? As I was mentioning, starting from the weight equation that I now have erased.

(1:29 - 2:20)

The second element is that it also depends on the electric permittivity of the material, the electric permittivity, okay? And the third point is that this also depends on omega, so the frequency of the oscillation of the electric field here, okay? So, we will have a different response based on different energy of the photons. So, this means that we have to deal with the conductivity, which means that there is a contribution, which can be identified with that, related to free electrons. And here, we have a contribution, which is related to bound electrons, and they are summing up together.

(2:21 - 3:06)

So, this is a lot, because we can consider the two contributions separately, and have a hint about what they are telling us, okay? Basically, this can move around and oscillate, this cannot, it can be oriented, there is some change or modification in the behavior, so we have to consider that they occupy quantized energy levels, although they are energy bands in the material, okay? So, step by step. I want to consider this point here. So, the refractive index, as we have found out, is a complex number.

(3:07 - 5:00)

What does this mean? It means that a complex number is a combination of a real term and an imaginary term, okay? I will say that both Nuno and Ki, at this point, are real numbers, so that the fact that it is imaginary is entirely contained in the fact that we are dealing with the imaginary number itself, okay? So, what does this mean, if we look at that? It means that the polarizability gives us the real term, and the conductivity is the imaginary part of the refractive index. Is it like that? Is it like that? Well, it's quite immediate if you look at the mathematics here, because all this term here is under square root, okay? So, actually, if you take the square root of this expression here, which is going to be a nightmare, we will not do it, of course, they are going to be mixed together, okay? So that, if I have to write it in a way that you probably can interpret in a few weeks from now, I will say that this is going to be a function of the polarizability and the conductivity, let's say conductivity divided by omega, and this is going to be, again, a function of both. So, they are going to mix together, and both will have some imaginary and real parts, okay? So, we are not seeing which one is going to end up there, because, I mean, it definitely depends on the material, the expressions are rather complicated, and we are not extracting, let's say, interesting information.

(5:01 - 6:30)

But still, this is important when we consider the wave equation here, okay? Why so? Because we have to pretend that k, so, the speed of light in vacuum is equal to omega 0 divided by k0, okay? Okay. So, if we are considering now a material, here, that k, and I'm using this expression here, is given by omega divided by c times n, okay? But this is equal to k0 times n, if we assume that the path is basically the same. Okay.

(6:35 - 7:09)

So, we do not have an oscillation, a variation in how fast an electric field carries its material, but we will have, so let's say, omega is equal to omega 0, and then at this point, we can write the wave vector k as the multiplication of the one we would have in vacuum times the refractive index. Okay. But now, k is a complex number.

(7:17 - 7:57)

Why so? Because now I'm using this expression here, which becomes 0 to the power of i kx minus i omega 0 t. Okay. Let me write it this way. So, to make it something more practical, because at some point it's for calculations, you need to get some interpretation.

(7:58 - 8:51)

So, we have a material in vacuum, and we have electromagnetic waves, so a ray of light, which is entering our material. It oscillates with a certain e0, okay? And it has its own past oscillation, also the frequency we'll say, which is omega 0. And of course, we'll have k0. So, this is going to be described by d0 e to the i k0x minus i omega 0 t. What happens in the material? Well, at this point, we'll have an oscillation, which is described by, let me call it d prime, d prime 0 e to the i omega 0 minus i omega 0 t plus i k0nx.

(8:54 - 9:10)

Okay. I'm just replacing this here. And I'm just telling that in principle, e0 is not necessarily equal to e0 prime.

(9:10 - 9:36)

So, we might have some modification in the amplitude due to the fact that we are entering inside a vacuum. Now, we can see what happens when a wave propagates along the material, which is what all these calculations are meant for. So, let me write this more explicitly.

(9:36 - 9:55)

So, I call it e prime. So, this is going to be e prime 0 e to the minus i omega 0 t. So, the oscillation frequency does not change. And then we have i k0 and n is a complex number.

(9:56 - 10:19)

So, let's take it the general way. Just write that n here is a complex number. So, this is going to be nu x. And then we are going to have i a0 i k x. Okay.

(10:26 - 10:55)

Is that readable? Or if you prefer, we can just put a plus here. And this is the same explanation. So, this seems a tiny change, but it changes a lot, actually.

(10:56 - 11:45)

Because it is going to be equal to e prime 0 e to the minus i omega 0 t plus i a0 nu x. And what happens here? It happens that we have an imaginary number which is multiplying itself. So, this becomes minus 1. So, we get e to the power of minus k a0 x. Okay. So, here we still have the oscillation over time.

(11:46 - 12:00)

Perfect. As in the case of vector. Here we have a propagation over space i k times x. Nu is a real number.

(12:00 - 12:25)

So, besides the fact that we have some dilation, some multiplication in the wave vector, which is affecting the wavelength, a slight modification in the wavelength when we enter in the material, we basically have no other issues here. Because it has the same shape, so it allows our wave to propagate. Okay.

(12:28 - 12:45)

This term here, however, this last one, is a real exponent. Okay. So, it is a damping factor.

(12:48 - 13:13)

What does this imply? Here we start having e0. Here the propagation might change a little bit. The frequency that the wavelength that we are seeing here, since we are considering x, I don't need to argue about that.

(13:14 - 14:25)

What I'm interested in is that actually, actually, instead of having an amplitude which is fixed over space in the case of vacuum, we are going to material, and here we have what I call the prime, because there might be some component which is neglected due to reflection or whatever. And this is actually going to be the function of the coordinate, in the sense that I can express this as minus k p0 x. Okay. So, this is telling me that if this is my amplitude, now this amplitude is going to decrease exponentially with the distance that the wave is traveling inside the material.

(14:26 - 14:50)

Okay. So, this exponential damping here is describing absorption. How can we see that? I forgot the next slide.

(14:55 - 15:33)

Well, this is expressed in terms of the electric field. If we actually want something which I hope you are already familiar with, absorption is related to the intensity of the light field, which is proportional to how many photons are reaching your eye or eye detector after passing through a certain thickness of the material. The intensity, which I call I, capital letter, can be written as the square modulus of the electric field.

(15:36 - 16:33)

And let's assume that our eye is placed here at a certain distance x. From the interface between vacuum and the material. Well, at this point, we take the square modulus of this expression here and we get the intensity. So, my intensity is going to be the square modulus of T. So, this is the square modulus of, let's say, e to the prime squared.

(16:35 - 17:21)

The square modulus of I minus I over T. The square modulus of e to the I k0 mu x. I don't know if you are capable of reading here as well. e to the minus k0 mu x. These are all multiplying terms, so I take the square modulus of each one. Now, these are easy because these are phases.

(17:22 - 18:03)

This is a dimensional number, so it's just a rotation in a complex plane. And they are equal to 1. So, we end up adding the zero prime square modulus. And this is a real number, so we get e to the minus 2 k0 mu x. Do you recall anything in this notation familiar? Let me say that I call 2 k0 x equal to alpha.

(18:06 - 18:48)

And this is a dimensional, this is centimeters to the minus 1 because it's a wave vector. So, this is centimeters to the minus 1. Then, this is the value we have in our function when x is equal to 0. And so, let's say that I call this I0. Then, what I get actually is a function of the thickness of which the wave travels.

(18:49 - 19:33)

Which can be written as I0 exponential of minus alpha x. Which is known as the Lambert's theorem. Which is simply telling us that the light intensity of a ray of light, could be x-rays, could be visible range, could be infrared, whatever, decreases exponentially with the thickness of the material. And this exponentiation has a coefficient alpha which is a property of the material.

(19:34 - 19:51)

Why so? Because this is basically given by the imaginary part of the refractive index. Okay. Okay, so this is basically the absorption term.

(19:53 - 20:18)

I kept this E' different with respect to this E because when you are taking account for calculations, you will also get that part of intensity is lost due to reflection of this refractive index. And so, you are not just going to get the entire initial intensity entering your material. This is just related to the one which passes through the surface.

(20:19 - 21:16)

Is it clear? Any questions about that? Any requests? I think this is the major consequence of having a complex refractive index which is already taking into account for the fact that the material can absorb. Which is something which in vacuum, of course, cannot occur. So, if I have to summarize what we have seen, can I erase this one? So, we have the refractive index is the ratio between speed of light in vacuum and speed of light in the material.

(21:16 - 21:47)

So, the first consequence of n, n in general is a complex number. So, n in general is larger than 1, which means that light is slower in a material with respect to vacuum. And that we have that k0 has to be replaced by k0 times n, which means that we are going to have absorption.

(21:51 - 22:15)

Actually, we have two limit cases for this that I wanted to give you as a general reference. So, I wrote that in general the refractive index has a real and an imaginary part. Now, let me see that n and these two are real numbers.

(22:15 - 23:21)

So, let me say that n actually is a real number, which means that nu is a real number and E is 0. So, what we get for this expression here? What we would get is that our electric field is traveling as I to minus I omega 0 t plus I k0 nu x. So, we do not have any absorption in this case, still we get a term which is different from 0 or different from k0 in the case of our material. So, the light is traveling still at a lower pace, at a lower speed with respect to vacuum. So, no absorption and this is one case.

(23:40 - 25:18)

What's the opposite case? The opposite case is that n is equal to a pure imaginary number, which means that nu is equal to 0. So, we get E0 e to the minus I omega 0 t plus 100 minus k0 nu x. So, just this second term survives here and we put E. How does a wave behave in this case? Well, a wave travels in space and over time, at least in our relativistic perception. So, traveling over time, we always left it unchanged because of the choice that we made of our, let's say, space-time. We decided that omega 0 is fixed, we are choosing some reference system in which we put all the equations in the wave vector, which I'm sure is the simplest way.

(25:19 - 25:48)

What about traveling in space? Well, traveling in space means this term here. But this term here is not present. So, this means that this expression here cannot describe a wave propagating in optical.

(25:56 - 27:05)

So, when we end up in a situation like this, which is quite technical and we need some engineering switch, and we're not seeing that specific case, in any case, the light is not entitled to travel in our material, so it will be reflected at the interface and will bounce back in that way. Okay. Now, this tells basically everything that I wanted you to know, to have an idea about the fact that refractive index is a complex number, which is telling us that we have a loss in material due to the fact that the photons interact with the charges of a certain lattice, of a certain solid, of a gas.

(27:07 - 27:58)

Whatever includes a charged particle implies that the intensity decreases because of absorption. The other point that I wanted to start to touch today is the contribution of free electrons, which is going to lead us, we're not seeing that today, to the concept of plasma oscillations. Okay, so I erased everything which was possible to erase, so let me rewrite the refractive index in the most general way you can see that.

(28:02 - 28:53)

So, the square root of the sum of damages related to the polarizability, the relative conductivity of the material, plus imaginary number multiplying the conductivity of the material divided by the frequency of the electromagnetic wave. Okay. Now, we are considering free electrons, so let me say that we are going to focus on this term here, which implies that we are considering basically that epsilon r is equal to 1. What does it mean having free electrons? Well, let's start from the Drude model as I promised during the first lecture.

(28:56 - 29:16)

And let me consider a static field. I use this here because it's just two letters and it's simple. What does this mean? It means that in general we have an electric field which has a time dependence.

(29:17 - 29:23)

In this case the time dependence is not that, it's just a constant field. Let's consider that. Okay.

(29:24 - 29:51)

And we have a charge, a point charge, which actually is typically our electron. Let's call it Q. And it is placed at some random position inside the material. It is a free charge, so let's assume that there are no bounds, no lattice keeping that position.

(29:54 - 29:58)

And then we apply an electric field, which is static.