CS132 Homework 9
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6(5.1)) Checking if eigenvector.

It is an eigenvector if there is some value, λ, that satisfies $Ax = \lambda x$

$$\begin{bmatrix} 3 & 6 & 7 \\ 3 & 2 & 7 \\ 5 & 6 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 - 12 + 14 \\ 3 - 4 + 14 \\ 5 - 12 + 8 \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ 1 \end{bmatrix}$$

There is no λ that will satisfy $\lambda x =$, so x is not an eigenvector.

8(5.1)) Checking if eigenvalue & finding eigenvector

nonzero

It is an eigenvalue if there is a vector that satisfies $(A - I)x = 0$

$$\begin{bmatrix} 4-1 & -2 & 3 & 0 \\ 0 & -1-1 & 3 & 0 \\ -1 & 2 & -2-1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 3 & 0 \\ 0 & -2 & 3 & 0 \\ -1 & 2 & -3 & 0 \end{bmatrix} \begin{matrix} -1 \\ \\ +1 \end{matrix}$$

$$\downarrow$$

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & -2 & 3 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} +3 \\ \\ \\ \end{matrix}$$

$$\downarrow$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -2 & 3 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ /-2 \\ /-1 \end{matrix}$$

nonzero solution, is eigenvalue!

$$\downarrow$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{3}{2} & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad \left.\begin{matrix} \leftarrow x_3 \text{ is free} \\ \leftarrow x_2 = \frac{3}{2}x_3 \\ \leftarrow x_1 = 0 \end{matrix}\right\}$$

If we pick $x_3 = 4$, we get

$$x = \begin{bmatrix} 0 \\ 6 \\ 4 \end{bmatrix}$$

↖ eigenvector

10(5.1)) $Ax = -5x \Rightarrow (A+5I)x = 0$

$$\begin{bmatrix} -4+5 & 2 & 0 \\ 3 & 1+5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 6 & 0 \end{bmatrix} \begin{matrix} \\ \cdot -3 \end{matrix}$$

$\downarrow$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \left.\begin{matrix} \Leftarrow x_1 = -2x_2 \\ \Leftarrow x_2 \text{ is free} \end{matrix}\right\} x = \begin{bmatrix} -2x_2 \\ x_2 \end{bmatrix}$$, which means that every

eigenvector is a multiple of $\underline{\underline{\begin{bmatrix} -2 \\ 1 \end{bmatrix}}}$

12(5.1)) $(A-3I)x = 0$

$$\begin{bmatrix} 4-3 & 1 & 0 \\ 3 & 6-3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 3 & 3 & 0 \end{bmatrix} \begin{matrix} \\ -3 \end{matrix}$$

$\downarrow$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \left.\begin{matrix} \Leftarrow x_1 = -x_2 \\ \Leftarrow x_2 \text{ is free} \end{matrix}\right\} x = \begin{bmatrix} -x_2 \\ x_2 \end{bmatrix}$$, which means every

eigenvector is a multiple of $\underline{\underline{\begin{bmatrix} -1 \\ 1 \end{bmatrix}}}$

$(A-7I)x = 0$

$$\begin{bmatrix} 4-7 & 1 & 0 \\ 3 & 6-7 & 0 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 \\ 3 & -1 & 0 \end{bmatrix} \begin{matrix} /-1 \\ +3 \end{matrix}$$

$\downarrow$

$$\begin{bmatrix} 3 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \left.\begin{matrix} \Leftarrow x_1 = \frac{1}{3}x_2 \\ \Leftarrow x_2 \text{ is free} \end{matrix}\right\} x = \begin{bmatrix} 1/3\,x_2 \\ x_2 \end{bmatrix}$$, which means every

eigenvector is a multiple of $\underline{\underline{\begin{bmatrix} 1/3 \\ 1 \end{bmatrix}}}$

14(5.1)) $(A-3I)x=0$

$$\begin{bmatrix} 4-3 & 0 & -1 & 0 \\ 3 & 0-3 & 3 & 0 \\ 2 & -2 & 0-3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 3 & -3 & 3 & 0 \\ 2 & -2 & 2 & 0 \end{bmatrix} /3$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \end{bmatrix} -2$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} -1$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \left.\begin{matrix} x_1 = x_3 \\ x_2 = 2x_3 \\ x_3 \text{ is free} \end{matrix}\right\} x = \begin{bmatrix} x_3 \\ 2x_3 \\ x_3 \end{bmatrix}$$, which means

every eigenvector is a multiple of $\underline{\underline{\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}}}$

22(5.1)) a) x must be nonzero, FALSE

b) ?

c) A steady state vector has a property $Ax=x$, TRUE

d) Only when matrix is triangular, FALSE

e) For $A-\lambda I$, TRUE

24(5.1))
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

30(5.1))
?

2(5.2)) $(A - \lambda I)x = 0$

$$\begin{bmatrix} -4-\lambda & -1 \\ 6 & 1-\lambda \end{bmatrix} = (-4-\lambda)(1-\lambda) - ((-1)6) =$$

$$(-4-\lambda)(1-\lambda) + 6 = -4 + 4\lambda + \lambda^2 - \lambda + 6 =$$

$$\underline{\underline{\lambda^2 + 3\lambda + 2}} = (\lambda+1)(\lambda+2) = 0$$

$$\underline{\underline{\lambda = -1 \vee \lambda = -2}}$$

4(5.2)) $\begin{bmatrix} 8-\lambda & 2 \\ 3 & 3-\lambda \end{bmatrix} = (8-\lambda)(3-\lambda) - (3 \cdot 2) =$

$$(8-\lambda)(3-\lambda) - 6 = 24 - 8\lambda - 3\lambda + \lambda^2 - 6 =$$

$$\underline{\underline{\lambda^2 - 11\lambda - 18}} = (\lambda - 9)(\lambda - 2)$$

$$\underline{\underline{\lambda = 9 \vee \lambda = 2}}$$

22(5.2)) a) It's the abs value, FALSE

b) $\det A^T = \det A$, FALSE

c) TRUE

d) FALSE (?)

25 (5.2)) a) $\det(A - \lambda I) = 0$

$$\begin{bmatrix} 0.6-\lambda & 0.3 \\ 0.4 & 0.7-\lambda \end{bmatrix} = (0.6-\lambda)(0.7-\lambda) - (0.3 \cdot 0.4) =$$

$$(0.6-\lambda)(0.7-\lambda) - 0.12 = 0.42 - 0.6\lambda - 0.7\lambda + \lambda^2 - 0.12 =$$

$$\lambda^2 - 1.3\lambda + 0.3 = (\lambda-1)(\lambda-0.3)$$ Eigenvalues: 1 v 0.3

$\downarrow$

$(A - 0.3I)x = 0$

$$\begin{bmatrix} 0.6-0.3 & 0.3 & 0 \\ 0.4 & 0.7-0.3 & 0 \end{bmatrix} = \begin{bmatrix} 0.3 & 0.3 & 0 \\ 0.4 & 0.4 & 0 \end{bmatrix} \begin{matrix} \cdot \frac{1}{0.3} \\ \cdot \frac{1}{0.4} \end{matrix}$$

$\downarrow$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{matrix} \\ -1 \end{matrix}$$

$\downarrow$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \left.\begin{matrix} x_1 = -x_2 \\ x_2 \text{ is free} \end{matrix}\right\} x = \begin{bmatrix} -x_2 \\ x_2 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 \\ 1 \end{bmatrix} = v_2$$

b) $x_0 = v_1 + cv_2$

$$\begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 3/7 \\ 4/7 \end{bmatrix} + c\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$\downarrow$

$$1/2 = 3/7 - c$$

$$c = 3/7 - 1/2 = -1/14$$

c) ?

38(5.1)) Using numpy we find eigenvalues

$2, -2, -1, 1$

We find the resulting matrices by doing so

$$\begin{bmatrix} x_1-\lambda & x_2 & x_3 & x_4 \\ x_5 & x_6-\lambda & x_7 & x_8 \\ x_9 & x_{10} & x_{11}-\lambda & x_{12} \\ x_{13} & x_{14} & x_{15} & x_{16}-\lambda \end{bmatrix}$$

to each matrix, and row reduce them using code from homework 2.

We get the following matrices:

$$\begin{array}{cc} -2 & \left.\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right\} \end{array} \text{eigenvector: } \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{array}{cc} -1 & \left.\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right\} \end{array} \text{eigenvector: } \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{cc} 1 & \left.\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right\} \end{array} \text{eigenvector: } \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{array}{cc} 2 & \left.\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right\} \end{array} \text{eigenvector: } \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

where each of the vectors are base for the eigenspace

Didn’t have time for any more :(