CS132 Homework 8

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(2.3))

$$\begin{bmatrix} 1 & 4 & 5 & -1 \\ -3 & -4 & -3 & -7 \\ 2 & 5 & 6 & -1 \\ 3 & 7 & 5 & 2 \end{bmatrix} \begin{array}{l} \\ +3 \\ -2 \\ -3 \end{array}$$

$\downarrow$

$$\begin{bmatrix} 1 & 4 & 5 & -1 \\ 0 & 8 & 12 & -10 \\ 0 & -3 & -4 & 1 \\ 0 & -5 & -10 & 5 \end{bmatrix} \begin{array}{l} \\ \\ \\ /-5 \end{array}$$

$\downarrow$

$$\begin{bmatrix} 1 & 4 & 5 & -1 \\ 0 & 8 & 12 & -10 \\ 0 & -3 & -4 & 1 \\ 0 & 1 & 2 & -1 \end{bmatrix} \begin{array}{l} -4 \\ -8 \\ +3 \\ \end{array}$$

$\downarrow$

$$\begin{bmatrix} 1 & 0 & -3 & 3 \\ 0 & 0 & -4 & -2 \\ 0 & 0 & 2 & -2 \\ 0 & 1 & 2 & 1 \end{bmatrix} \begin{array}{l} \\ \\ /2 \\ \end{array}$$

$\downarrow$

$$\begin{bmatrix} 1 & 0 & -3 & 3 \\ 0 & 0 & -4 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 2 & 1 \end{bmatrix} \begin{array}{l} \\ +4 \\ \\ \end{array}$$

$\downarrow$

$$\begin{bmatrix} 1 & 0 & -3 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

← No solution, so $\vec{v}$ is not in $\{v_1, v_2, v_3\}$

8(2.8)

$$\begin{bmatrix} -2 & -2 & 0 & -6 \\ 0 & 3 & -5 & 7 \\ 6 & 3 & 5 & 17 \end{bmatrix} +3$$

$$\downarrow$$

$$\begin{bmatrix} -2 & -2 & 0 & -6 \\ 0 & 3 & -5 & 7 \\ 0 & -3 & 5 & -7 \end{bmatrix} +$$

$$\downarrow$$

$$\begin{bmatrix} -2 & -2 & 0 & -6 \\ 0 & 3 & -5 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow$$ At least one solution, so yes p is in Col A

10(2.8) $Au = 0$

$$\begin{bmatrix} -2 & -2 & 0 \\ 0 & 3 & -5 \\ 6 & 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} -5 \\ 5 \\ 3 \end{bmatrix} = \begin{bmatrix} (-2)\cdot(-5) + (-2)(5) + 3\cdot 0 \\ 0(-5) + 3\cdot 5 + (-5)\cdot 3 \\ 6(-5) + 3\cdot 5 + 5\cdot 3 \end{bmatrix} = \begin{bmatrix} 10 - 10 \\ 15 - 15 \\ -30 + 15 + 15 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow$$ u is in Nul A

12(2.8) The columns of A have 5 entries, so Col A is subspace of R^5, hence $q = 5$.

a

14(2.8) $Ax = 0$

$$\begin{array}{c}\begin{bmatrix} 1 & 2 & 3 & 0 \\ 4 & 5 & 7 & 0 \\ -5 & -1 & 0 & 0 \\ 2 & 7 & 11 & 0 \\ 3 & 3 & 4 & 0 \end{bmatrix}\end{array}\begin{array}{c} \\ -4 \\ +5 \\ -2 \\ -3 \end{array}$$

$$\downarrow$$

$$\begin{array}{c}\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -3 & -5 & 0 \\ 0 & 9 & 15 & 0 \\ 0 & 3 & 5 & 0 \\ 0 & -3 & -5 & 0 \end{bmatrix}\end{array}\begin{array}{c} \\ \\ +3 \\ +1 \\ -1 \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -3 & -5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow x_3 = \text{anything}$$

$x_3 = 6$

$-3x_2 - 30 = 0$
$-3x_2 = 30$
$x_2 = -10$

$x_1 - 20 + 18 = 0$
$x_1 = 2$

$$x = \begin{bmatrix} 2 \\ -10 \\ 6 \end{bmatrix} \rightarrow \text{Nonzero vector in Nul } A$$

All columns of A is nonzero vector in Col A,
for example $\begin{bmatrix} 2 \\ 5 \\ -1 \\ 7 \\ 3 \end{bmatrix}$

16(2.8) $\begin{bmatrix} -2 & 4 & 0 \\ 5 & -10 & 0 \end{bmatrix} \begin{matrix} /-2 \\ /5 \end{matrix}$

$$\downarrow$$

$$\begin{bmatrix} 1 & -2 & 0 \\ 1 & -2 & 0 \end{bmatrix}$$

The rows are equal, hence there is an infinite number of solutions, meaning the vectors are linearly dependent, and they are not bases for $\mathbb{R}^2$

20(1.8) There are four vectors, so no they're not a basis for either.

24(1.8) $\begin{bmatrix} 1 & -2 & 5 & 4 & 0 \\ 0 & 0 & 3 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ x_2, x_4 is free

$3x_3 + 6x_4 = 0$

$3x_3 = -6x_4$

$x_3 = -2x_4$

$x_1 = 2x_2 - 5x_3 - 4x_4$

$x_1 = 2x_2 + 10x_4 - 4x_4$

$x_1 = 2x_2 + 6x_4$

$Ax = 0$

Basis for Nul A

$$x = \begin{bmatrix} 2x_2 + 6x_4 \\ x_2 \\ -2x_4 \\ x_4 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 6 \\ 0 \\ -2 \\ 1 \end{bmatrix} \rightarrow \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$$

Basis of Col A are pivot columns.

$$\left\{ \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 9 \\ 7 \\ 6 \end{bmatrix} \right\}$$

26(1.8)

$$\begin{bmatrix} 3 & -1 & -3 & 0 & 6 & 0 \\ 0 & 2 & 6 & 0 & -4 & 0 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \leftarrow x_3, x_5 \text{ are free}$$

$$-x_4 + 2x_5 = 0$$
$$x_4 = 2x_5$$

$$2x_2 + 6x_3 - 4x_5 = 0$$
$$2x_2 = -6x_3 + 4x_5$$
$$x_2 = -3x_3 + 2x_5$$

$$3x_1 - x_2 - 3x_3 + 6x_5 = 0$$
$$3x_1 = x_2 + 3x_3 - 6x_5$$
$$3x_1 = -3x_3 + 2x_5 + 3x_3 - 6x_5$$
$$3x_1 = -4x_5$$
$$x_1 = -\frac{4}{3}x_5$$

$$Ax = 0$$

$$x = \begin{bmatrix} -\frac{4}{3}x_5 \\ -3x_3 + 2x_5 \\ x_3 \\ 2x_5 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -\frac{4}{3} \\ 2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \rightarrow \left\{ \begin{bmatrix} 0 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{4}{3} \\ 2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}$$

Basis for Nul A

Basis of Col A are pivot columns

$$\left\{ \begin{bmatrix} 3 \\ 3 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -1 \\ -2 \end{bmatrix} \right\}$$

20 (4.5) a) $\mathbb{R}^2$ has two entries, $\mathbb{R}^3$ has three, FALSE

b) The number of free variables, FALSE

c) It must be impossible to span it with finite set, FALSE

d) S must have n vectors, FALSE

e) If it's in 3D, it must be spanned by 3 vectors, and they span $\mathbb{R}^3$. TRUE

10 (2.9) $\begin{bmatrix} 1 & -2 & -1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

$x_5 = 0$
$x_4 = 0$
x_3 is free

$x_2 + x_3 + 3x_5 = 0$
$x_2 = -x_3 - \cancel{3x_5}$
$x_2 = -x_3$

$x_1 - 2x_2 - x_3 + 2x_4 = 0$
$x_1 = -x_3$

$Ax = 0$

$$x = \begin{bmatrix} -x_3 \\ -x_3 \\ x_3 \\ 0 \\ 0 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Basis for Nul A:

$$\left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

Basis of Col A are pivot columns

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ -7 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ -6 \\ 6 \end{bmatrix} \right\}$$

12(2.9))

$$\begin{bmatrix} 1 & 2 & 8 & 4 & -6 & 0 \\ 0 & 2 & 3 & 4 & -1 & 0 \\ 0 & 0 & 5 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

x_4 is free
x_5 is free

$5x_3 - 5x_5 = 0$
$x_3 = x_5$

$2x_2 + 3x_3 + 4x_4 - x_5 = 0$
$2x_2 = -2x_5 - 4x_4$
$x_2 = -x_5 - 2x_4$

$x_1 + 2x_2 + 8x_3 + 4x_4 - 6x_5 = 0$
$x_1 = -2x_2 - 8x_3 - 4x_4 + 6x_5$
$x_1 = 2x_5 + 4x_4 - 8x_5 - 4x_4 + 6x_5$
$x_1 = 0$

$Ax = 0$

$$x = \begin{bmatrix} 0 \\ -x_5 - 2x_4 \\ x_5 \\ x_4 \\ x_5 \end{bmatrix} = x_4 \begin{bmatrix} 0 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

Basis for Nul A:

$$\left\{ \begin{bmatrix} 0 \\ -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

14(2.9))

$$\begin{array}{c}\begin{bmatrix} 1 & 2 & 0 & -1 & 3 \\ -1 & -3 & -1 & 4 & -7 \\ -2 & -1 & 3 & -7 & 6 \\ 3 & 4 & -2 & 7 & -9 \end{bmatrix} \begin{array}{l} \\ +1 \\ +2 \\ -3 \end{array}\end{array}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 2 & 0 & -1 & 3 \\ 0 & -1 & -1 & 3 & -4 \\ 0 & 3 & 3 & -9 & 12 \\ 0 & -2 & -2 & 10 & -18 \end{bmatrix} \begin{array}{l} \\ \\ +3 \\ -2 \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 2 & 0 & -1 & 3 \\ 0 & -1 & -1 & 3 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & -10 \end{bmatrix}$$

Basis is pivot columns:

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ -7 \\ 7 \end{bmatrix} \right\}$$, dimension is three

20(2.9)) ?

24 (2.9) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

14 (4.6) Both cases only possible with 4 pivots, so largest dimension is 4 for both

16 (4.6) Smallest dimension = Largest rank = 5 [illegible]
7×5 ntx, $n = 5$

$n - \text{rank } A = 5 - 5 = \underline{\underline{0}}$

18 (4.6) a) the corresponding columns of A form the column space, FALSE

b) FALSE (?)

c) It's the same as number of columns not a pivot, TRUE

d) TRUE

e) TRUE