CS132

HW 3

2) $\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \end{bmatrix}$ Undefined:

there aren't enough numbers in the second matrix!

4) $\begin{bmatrix} 1 & 3 & -4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 3 \end{bmatrix} + 2\begin{bmatrix} 3 \\ 2 \end{bmatrix} + 1\begin{bmatrix} -4 \\ 1 \end{bmatrix} =$

$\begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 6 \\ 4 \end{bmatrix} + \begin{bmatrix} -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$

6) $\begin{bmatrix} 2 & -3 \\ 3 & 2 \\ 8 & -5 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 5 \end{bmatrix} = \begin{bmatrix} -21 \\ 1 \\ -49 \\ 11 \end{bmatrix}$

$-3\begin{bmatrix} 2 \\ 3 \\ 8 \\ -2 \end{bmatrix} + 5\begin{bmatrix} -3 \\ 2 \\ -5 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -9 \\ -24 \\ 6 \end{bmatrix} + \begin{bmatrix} -15 \\ 10 \\ -25 \\ 5 \end{bmatrix} = \begin{bmatrix} -21 \\ 1 \\ -49 \\ 11 \end{bmatrix}$

8) $z_1\begin{bmatrix} 2 \\ -4 \end{bmatrix} + z_2\begin{bmatrix} -1 \\ 5 \end{bmatrix} + z_3\begin{bmatrix} -4 \\ 3 \end{bmatrix} + z_4\begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

$\begin{bmatrix} 2 & -1 & -4 & 0 \\ -4 & 5 & 3 & 2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$ (?)

10)

$$4x_1 - x_2 = 8$$
$$5x_1 - 3x_2 = 2$$
$$3x_1 - x_2 = 1$$

$$\begin{bmatrix} 4 & -1 \\ 5 & -3 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \\ 1 \end{bmatrix}$$

12)

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -3 & -4 & 2 \\ 5 & 2 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ -3 & -4 & 2 & 2 \\ 5 & 2 & 3 & -3 \end{bmatrix} \cdot 3$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 2 & -1 & 5 \\ 5 & 2 & 3 & -3 \end{bmatrix} + -5$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 2 & -1 & 5 \\ 0 & -8 & 8 & -8 \end{bmatrix} /-8$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 2 & -1 & 5 \\ 0 & 1 & -1 & 1 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 2 & -1 & 5 \end{bmatrix} -2$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix} +$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix} +$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{bmatrix} -2$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\downarrow$$

$$x_1 = -4, \quad x_2 = 4, \quad x_3 = 3$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \\ 3 \end{bmatrix}$$

14) $v = \begin{bmatrix} 4 \\ -1 \\ 4 \end{bmatrix}$ $A = \begin{bmatrix} 2 & 5 & -1 \\ 0 & 1 & -1 \\ 1 & 2 & 0 \end{bmatrix}$

$$\begin{bmatrix} 2 & 5 & -1 & 4 \\ 0 & 1 & -1 & -1 \\ 1 & 2 & 0 & 4 \end{bmatrix}$$

$$\downarrow$$

$$\begin{array}{cc} \begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 1 & -1 & -1 \\ 2 & 5 & -1 & 4 \end{bmatrix} & \begin{array}{c} \\ \\ -2 \end{array} \end{array}$$

$$\downarrow$$

$$\begin{array}{cc} \begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 1 & -1 & -4 \end{bmatrix} & \begin{array}{c} \\ \\ -1. \end{array} \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

It is inconsistent, and thus v is not in the subset.

16) $A=\begin{bmatrix}1 & -2 & -1\\ -2 & 2 & 0\\ 4 & -1 & 3\end{bmatrix}$, $b=\begin{bmatrix}b_1\\ b_2\\ b_3\end{bmatrix}$

$$\left[\begin{array}{cccc}1 & -2 & -1 & b_1\\ -2 & 2 & 0 & b_2\\ 4 & -1 & 3 & b_3\end{array}\right] -2$$

$$\downarrow$$

$$\left[\begin{array}{cccc}1 & -2 & -1 & b_1\\ 0 & -2 & -2 & 2b_1+b_2\\ 4 & -1 & 3 & b_3\end{array}\right] -4$$

$$\downarrow$$

$$\left[\begin{array}{cccc}1 & -2 & -1 & b_1\\ 0 & -2 & -2 & 2b_1+b_2\\ 0 & 7 & 7 & -4b_1+b_3\end{array}\right] /-2$$

$$\downarrow$$

$$\left[\begin{array}{cccc}1 & -2 & -1 & b_1\\ 0 & 1 & 1 & \frac{2b_1+b_2}{2}\\ 0 & 7 & 7 & -4b_1+b_3\end{array}\right] -7$$

$$\downarrow$$

$$\left[\begin{array}{cccc}1 & -2 & -1 & b_1\\ 0 & 1 & 1 & \frac{2b_1+b_2}{2}\\ 0 & 0 & 0 & \frac{7(2b_1+b_2)}{2}-4b_1+b_3\end{array}\right]$$

$$\downarrow$$

$$\frac{7(2b_1+b_2)}{2}-4b_1+b_3=0$$

$$7(2b_1+b_2)-8b_1+2b_3=0$$

$$6b_1+7b_2+2b_3=0$$

There will be no solution if there's a row in the matrix of [0 0 0 k], therefore the last row must be [0 0 0 0]. It only has a solution when $6b_1+7b_2+2b_3=0$

18) $A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 2 & 6 & 7 \\ 2 & 9 & 5 & -7 \end{bmatrix}$

$$\begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 2 & 6 & 7 \\ 2 & 9 & 5 & -7 \end{bmatrix} -2 \cdot$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 2 & 6 & 7 \\ 0 & 1 & 3 & -11 \end{bmatrix} -2$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 15 \\ 0 & 1 & 3 & -11 \end{bmatrix} -1 \cdot$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 15 \\ 0 & 0 & 0 & -7 \end{bmatrix} /-7$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 15 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot 15$$

$$\begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} -4$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} -2$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 4 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 4 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} -4 \cdot$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & -11 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the last row is all 0's, B spans $\mathbb{R}^3$ but not all $\mathbb{R}^4$

20) No, from problem 18 we saw that β could not span $\mathbb{R}^4$.