CS132 - Homework 4

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1 (12)

Intersection	Flow in	Flow out
A	$x_1 + x_4$	x_2
B	x_2	$x_3 + 100$
C	$x_3 + 80$	x_4

Knowing that the total flow into the network must equal the total flow out of the network, we know $x_1 = (100 - 80) = \underline{20}$. Since x_4 is the only output for the 80 inputted at C, its smallest value must also be $\underline{\underline{80}}$. We can then infer that $x_2 = 100$ and $x_3 = 0$ (free)

2 (14) a)

Intersection	Flow in	Flow out
A	80	$x_1 + x_5$
B	$x_1 + x_2 + 100$	x_4
C	x_3	$x_2 + 90$
D	$x_4 + x_5$	$x_3 + 90$

$$\begin{aligned}
x_1 \qquad\qquad + x_5 &= 80 \\
x_1 + x_2 \quad - x_4 \quad &= -100 \\
-x_2 + x_3 \qquad\quad &= 90 \\
-x_3 + x_4 + x_5 &= 90
\end{aligned}$$

$$\downarrow$$

$$\begin{aligned}
x_1 \qquad\qquad + x_5 &= 80 \\
x_2 \quad - x_4 - x_5 &= -180 \\
x_3 - x_4 - x_5 &= -90
\end{aligned}$$

?

3(2) $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -8 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$

They are independent if:

$V_1 + V_2 + V_3 = 0$

$$x_1 \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ -8 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & -1 & 0 \\ 2 & 0 & 3 & 0 \\ 3 & -8 & 1 & 0 \end{bmatrix}$$

$\downarrow$

$$\begin{bmatrix} 3 & -8 & 1 & 0 \\ 2 & 0 & 3 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} -1\cdot$$

$\downarrow$

$$\begin{bmatrix} 1 & -8 & -2 & 0 \\ 2 & 0 & 3 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} -2\cdot$$

$\downarrow$

$$\begin{bmatrix} 1 & -8 & -2 & 0 \\ 0 & 16 & 7 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{matrix} \\ /16 \\ /-1 \end{matrix}$$

$\downarrow$

$$\begin{bmatrix} 1 & -8 & -2 & 0 \\ 0 & 1 & \frac{7}{16} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} -\frac{7}{16}\cdot$$

$\downarrow$

$$\begin{bmatrix} 1 & -8 & -2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} 2\cdot$$

$$\begin{bmatrix} 1 & -8 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} 8\cdot$$

$\downarrow$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$x_1 = 0$

$x_2 = 0$

$x_3 = 0$

The vectors are linearly independent

3(4) $\begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ -9 \end{bmatrix}$

$$\begin{bmatrix} -1 & -3 & 0 \\ 3 & -9 & 0 \end{bmatrix} \begin{array}{l} \\ -3\cdot \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} -1 & -3 & 0 \\ 0 & -18 & 0 \end{bmatrix} \begin{array}{l} /-1 \\ /-18 \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{array}{l} -3\cdot \\ \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\left.\begin{array}{l} x_1 = 0 \\ x_2 = 0 \end{array}\right\}$$ Yes, they are independent!

3(b)

$$\begin{bmatrix} -4 & -3 & 0 & 0 \\ 0 & -1 & 5 & 0 \\ 1 & 1 & -5 & 0 \\ 2 & 1 & -10 & 0 \end{bmatrix} \begin{array}{l} 4\cdot \\ \\ \\ \\ \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} 0 & 1 & -20 & 0 \\ 0 & -1 & 5 & 0 \\ 1 & 1 & -5 & 0 \\ 2 & 1 & -10 & 0 \end{bmatrix} \begin{array}{l} \\ \\ \\ -2 \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} 0 & 1 & -20 & 0 \\ 0 & -1 & 5 & 0 \\ 1 & 1 & -5 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 1 & -5 & 0 \\ 0 & -1 & 5 & 0 \\ 0 & 1 & -20 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \begin{array}{l} \\ \\ + \\ \\ \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 1 & -5 & 0 \\ 0 & -1 & 5 & 0 \\ 0 & 0 & -15 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \begin{array}{l} \\ \\ \\ - \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 1 & -5 & 0 \\ 0 & -1 & 5 & 0 \\ 0 & 0 & -15 & 0 \\ 0 & 0 & -5 & 0 \end{bmatrix} \begin{array}{l} \\ 1/-1 \\ 1/-15 \\ 1/-5 \end{array}$$

$$\begin{bmatrix} 1 & 1 & -5 & 0 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{array}{l} \\ -5 \\ \\ \\ \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 1 & -5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{array}{l} -5 \\ \\ \\ \\ \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{array}{l} \\ \\ \\ - \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} - \\ \\ \\ \\ \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\left.\begin{array}{l} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{array}\right\} \text{Yes, they're independent}$$

3(8) We know that if a set contains more vectors than entries in each vector, it is linearly dependent. Thus this set is not linearly independent

3(10) a)

$$\begin{array}{cc} \begin{bmatrix} 1 & -3 & 2 \\ -3 & 9 & -5 \\ -5 & 15 & h \end{bmatrix} & \begin{array}{c} \\ \cdot 3 \\ \\ \end{array} \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -3 & 2 \\ 0 & 0 & 1 \\ -5 & 15 & h \end{bmatrix}$$ ← Inconsistent, hence V_3 is not in the span!

b)

$$\begin{array}{cc} \begin{bmatrix} 1 & -3 & 2 & 0 \\ -3 & 9 & -5 & 0 \\ -5 & 15 & h & 0 \end{bmatrix} & \begin{array}{c} \\ \cdot 3 \\ \\ \end{array} \end{array}$$

$$\downarrow$$

$$\begin{array}{cc} \begin{bmatrix} 1 & -3 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ -5 & 15 & h & 0 \end{bmatrix} & \begin{array}{c} -2 \\ \\ \\ \end{array} \end{array}$$

$$\downarrow$$

$$\begin{array}{cc} \begin{bmatrix} 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -5 & 15 & h & 0 \end{bmatrix} & \begin{array}{c} \\ \\ -5 \end{array} \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & h & 0 \end{bmatrix}$$ ← It is consistent for all values of h

3(12) $\begin{bmatrix} 3 & -6 & 9 & 0 \\ -6 & 4 & h & 0 \\ 1 & -3 & 3 & 0 \end{bmatrix} \div 3$

$\downarrow$

$\begin{bmatrix} 1 & -2 & 3 & 0 \\ -6 & 4 & h & 0 \\ 1 & -3 & 3 & 0 \end{bmatrix}$

$\downarrow$

$\begin{bmatrix} 1 & -3 & 3 & 0 \\ -1 & 2 & -3 & 0 \\ -6 & 4 & h & 0 \end{bmatrix} +$

$\downarrow$

$\begin{bmatrix} 1 & -3 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ -6 & 4 & h & 0 \end{bmatrix}$ 3

$\downarrow$

$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ -6 & 4 & h & 0 \end{bmatrix}$ -4

$\downarrow$

$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ -6 & 0 & h & 0 \end{bmatrix}$ 6

$\downarrow$

$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & h+18 & 0 \end{bmatrix}$

$\underline{\underline{h = -18}}$

3(14) $$\begin{bmatrix} 1 & -3 & 2 & 0 \\ -2 & 7 & 1 & 0 \\ -4 & 6 & h & 0 \end{bmatrix} \begin{matrix} \\ 2 \\ \\ \end{matrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -3 & 2 & 0 \\ 0 & 1 & 5 & 0 \\ -4 & 6 & h & 0 \end{bmatrix} \begin{matrix} \\ \\ 4 \end{matrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -3 & 2 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & -6 & h+8 & 0 \end{bmatrix} \begin{matrix} 3 \\ \\ \\ \end{matrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 17 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & -6 & h+8 & 0 \end{bmatrix} \begin{matrix} \\ \\ 6 \end{matrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 17 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 1 & h+38 & 0 \end{bmatrix}$$

$$\underline{\underline{h = -38}}$$

3(24) It is independent if

$x_1 v_1 + x_2 v_2 + x_3 v_3 = 0$

only has the trivial solution:

$$\begin{bmatrix} \blacksquare & * & * \\ 0 & \blacksquare & * \\ 0 & 0 & \blacksquare \end{bmatrix}$$

3(26) $$\begin{bmatrix} \blacksquare & * & * \\ 0 & \blacksquare & * \\ 0 & 0 & \blacksquare \\ 0 & 0 & 0 \end{bmatrix}$$

3(28) It needs $\underline{4}$ pivot columns to span R^4

$$\begin{bmatrix} \underline{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \underline{1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \underline{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \underline{1} & 0 & 0 \end{bmatrix}$$