CS132 HW6
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16(1.) a) True, that's how it is...

b) False, no '+'-signs

c) True, $(A^2)^T = (AA)^T = A^T A^T = (A^T)^2$

d) False, the order is wrong $(ABC)^T = C^T B^T A^T$

e) True, because of theorem 3

18(1) It's also all zero's

20(1) They are also equal

22(1) If they are dependent, there is not only the trivial solution $Bx = 0$, but we can multiply both sides by $A$ and get $ABx = 0$ so $AB$ is also linearly dependent.

24(1.) First find independent columns (by row reduction), then build a new 3x3 matrix with those columns.

28(1.) $u^T v = (u^T v)^{T} = v^T (u^T)^T = v^T u$
$(uv)^T = (v^T)^T u^T = v u^T$

NB! Reversed

26(1.) $AD = I$ ~~multiply both sides by b~~
~~25(1.) A~~ $\downarrow$ ~~linear~~

$(AD)b = (I)b$

$\downarrow$

$A(Db) = b$

$\downarrow$

$Db = x = b$ It would cause an inconsistent system, not enough pivots!

14(2.) $(B-C)D=0$

$(B-C)DD^{-1}=0 \cdot D^{-1}$

$(B-C)I=0$

$B-C=0$

$B=C$

16(2.) $C=AB$

$CB^{-1}=ABB^{-1}$

$CB^{-1}=AI$

$CB^{-1}=A$

18(2.) $AB=BC$

$ABB^{-1}=BCB^{-1}$

$AI=BCB^{-1}$

$A=BCB^{-1}$

20(2.) a) $(A-AX)^{-1}=X^{-1}B$ b)

$X(A-AX)^{-1}=X \cdot X^{-1}B$

$X(A-AX)^{-1}=B$

QED

26(2.) $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^{-1}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$A \cdot A^{-1}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}=\frac{1}{ad-bc}\begin{bmatrix} ad-bc & -ab+ab \\ cd-cd & ad-bc \end{bmatrix}=$

$\frac{1}{ad-bc}\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ QED

18(3.) If there are 2 identical rows, you can subtract one from the other, leaving all 0's. If so, it's not invertible

22(3.) $EF = I$ implies $F$ is invertible

$FEF = F$

$FEFF^{-1} = FF^{-1}$

$FEI = I$

$FE = I$

24(3.) They are not linearly independent, and they don't span $\mathbb{R}^n$, this is because if $G$ can't be reduced to $I_n$, all statements of the theorem must be false about $G$.

26(3.) Due to the theorem, $A$ is invertible, and therefore $A \cdot A = A^2$ is also invertible. Therefore the columns of $A^2$ must span $\mathbb{R}^n$