CS132 Homework 4

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10(1.8)

$$\begin{bmatrix} 3 & 2 & 10 & -6 \\ 1 & 0 & 2 & -4 \\ 0 & 1 & 2 & 3 \\ 1 & 4 & 10 & 8 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 10 & -6 & 0 \\ 1 & 0 & 2 & -4 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 1 & 4 & 10 & 8 & 0 \end{bmatrix} \begin{matrix} \cdot 2 \\ \\ \\ \\ \end{matrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 2 & 6 & 2 & 0 \\ 1 & 0 & 2 & -4 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 1 & 4 & 10 & 8 & 0 \end{bmatrix} \begin{matrix} \\ -1 \\ \\ \\ \end{matrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 2 & 6 & 2 & 0 \\ 0 & -2 & -4 & -6 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 1 & 4 & 10 & 8 & 0 \end{bmatrix} \begin{matrix} \\ \\ \\ -1 \end{matrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 2 & 6 & 2 & 0 \\ 0 & -2 & -4 & -6 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 2 & 4 & 6 & 0 \end{bmatrix} \begin{matrix} \\ /-2 \\ \\ /2 \end{matrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 2 & 6 & 2 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 3 & 0 \end{bmatrix} \begin{matrix} \\ \\ -1 \cdot \\ \\ \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 6 & 2 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 \end{bmatrix} \begin{matrix} \\ \\ \\ -1 \cdot \end{matrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 2 & 6 & 2 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} -2 \cdot \\ \\ \\ \\ \end{matrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 2 & -4 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = -2x_3 + 4x_4$

$x_2 = -2x_3 - 3x_4$

$x_3 = \text{free}$

$x_4 = \text{free}$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2x_3 + 4x_4 \\ -2x_3 - 3x_4 \\ x_3 \\ x_4 \end{bmatrix} =$$

$$\begin{bmatrix} -2x_3 \\ -2x_3 \\ x_3 \\ 0 \end{bmatrix} + \begin{bmatrix} 4x_4 \\ -3x_4 \\ 0 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -2 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 4 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

12(1.8) $$\left[\begin{array}{ccccc} 3 & 2 & 10 & -6 & -1 \\ 1 & 0 & 2 & -4 & 3 \\ 0 & 1 & 2 & 3 & -1 \\ 1 & 4 & 10 & 8 & 4 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccccc} 1 & 4 & 10 & 8 & 4 \\ 1 & 0 & 2 & -4 & 3 \\ 0 & 1 & 2 & 3 & -1 \\ 3 & 2 & 10 & -6 & -1 \end{array}\right]\begin{array}{l} \\ -1 \\ \\ \\ \end{array}$$

$$\downarrow$$

$$\left[\begin{array}{ccccc} 1 & 4 & 10 & 8 & 4 \\ 0 & -4 & -8 & -12 & -1 \\ 0 & 1 & 2 & 3 & -1 \\ 3 & 2 & 10 & -6 & -1 \end{array}\right]\begin{array}{l} \\ \\ \\ -3 \end{array}$$

$$\downarrow$$

$$\left[\begin{array}{ccccc} 1 & 4 & 10 & 8 & 4 \\ 0 & -4 & -8 & -12 & -1 \\ 0 & 1 & 2 & 3 & -1 \\ 0 & -10 & -20 & -30 & -13 \end{array}\right]\begin{array}{l} \\ \\ \\ -2.5 \end{array}$$

$$\downarrow$$

$$\left[\begin{array}{ccccc} 1 & 4 & 10 & 8 & 4 \\ 0 & -4 & -8 & -12 & -1 \\ 0 & 1 & 2 & 3 & -1 \\ 0 & 0 & 0 & 0 & -10.5 \end{array}\right]$$

No solution (because)

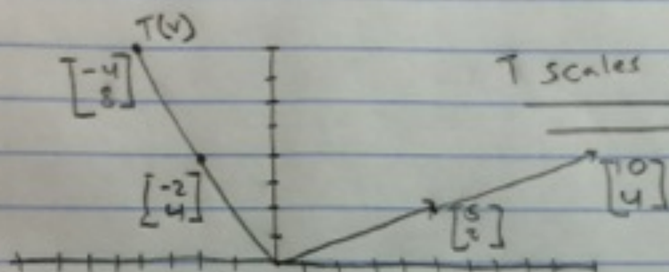
Therefore, b is not in the range $x \to Ax$

14 (1.8) $u = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ $v = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$

$$T(x) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$T(u) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 4 \end{bmatrix}$$

$$T(v) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} -4 \\ 8 \end{bmatrix}$$

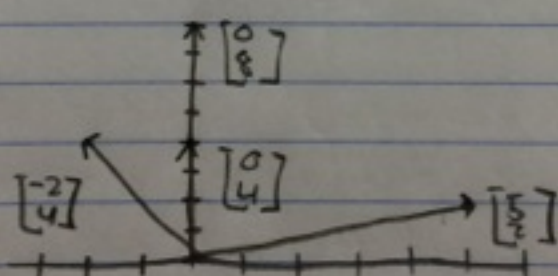


T scales u and v by 2

16 (1.8) $T(x) = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

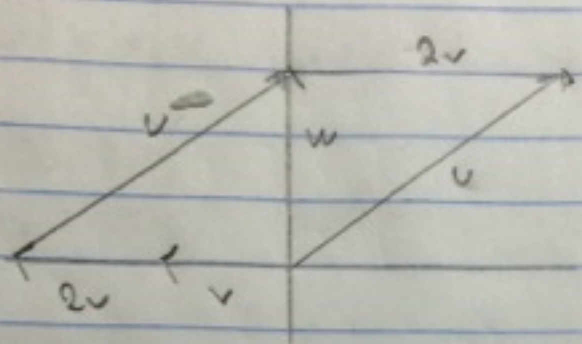
$$T(u) = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$T(v) = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \end{bmatrix}$$



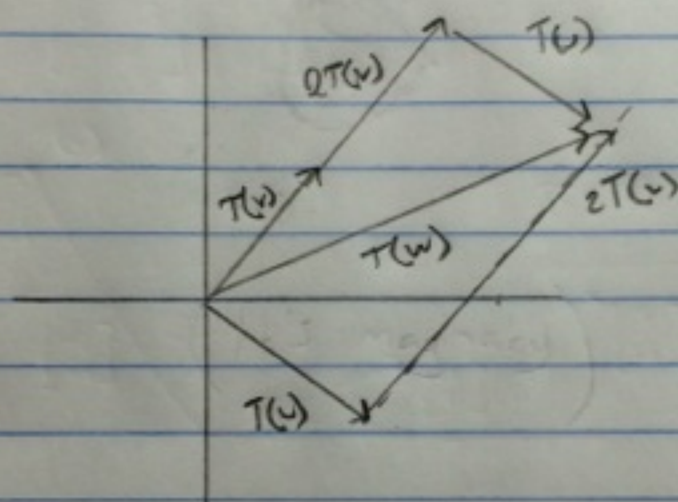
T projects u and v on the y axis and scales by 2

18(1.8)



$w = u + 2v$

$$T(w) = T(u + 2v) = T(u) + T(2v) = T(u) + 2T(v)$$



20(1.8) $T(x) = x_1 v_1 + x_2 v_2 = \begin{bmatrix} v_1 & v_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\begin{bmatrix} -3 & 7 \\ 5 & -2 \end{bmatrix} x = Ax$$

$$A = \begin{bmatrix} -3 & 7 \\ 5 & -2 \end{bmatrix}$$

2(1.9) $\begin{bmatrix} T(e_1) & T(e_2) & T(e_3) \end{bmatrix}$

$\downarrow$

$$\begin{bmatrix} 1 & -2 & 3 \\ 4 & 9 & -8 \end{bmatrix}$$

4(1.9) $T(e_1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$T(e_2) = e_2 + 2e_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \cancel{t(e)}_1 T(e_2) \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

6(1.9) rotate $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ gives $\begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow T(e_1)$

rotate $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ gives $\begin{bmatrix} -1 \\ 0 \end{bmatrix} \rightarrow T(e_2)$

$\begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$

$\downarrow$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

14(1.9) Wat

18(1.9) $T(x_1, x_2) = (x_1 + 4x_2, 0, x_1 - 3x_2, x_1)$

$$T(x) = \begin{bmatrix} x_1 + 4x_2 \\ 0 + 0 \\ x_1 - 3x_2 \\ x_1 + 0 \end{bmatrix} = \begin{bmatrix} x_1 \\ 0 \\ x_1 \\ x_1 \end{bmatrix} + \begin{bmatrix} 4x_2 \\ 0 \\ -3x_2 \\ 0 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 0 \\ -3 \\ 0 \end{bmatrix} =$$

$$\underline{\underline{\begin{bmatrix} 1 & 4 \\ 0 & 0 \\ 1 & -3 \\ 1 & 0 \end{bmatrix}}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

20(1.9) $T(x_1, x_2, x_3, x_4) = (3x_1 + 4x_3 - 2x_4)$

$$T(x) = 3x_1 + 0x_2 + 4x_3 - 2x_4 =$$

$$\underline{\underline{\begin{bmatrix} 3 & 0 & 4 & -2 \end{bmatrix}}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

22(1.1) $T(x_1, x_2) = (2x_1 - x_2, -3x_1 + x_2, 2x_1 - 3x_2)$

$T(x) = (0, -1, -4)$

$$\begin{bmatrix} 2x_1 - x_2 = 0 \\ -3x_1 + x_2 = -1 \\ 2x_1 - 3x_2 = -4 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 2 & -1 & 0 \\ -3 & 1 & -1 \\ 2 & -3 & -4 \end{bmatrix} +2$$

$$\downarrow$$

$$\begin{bmatrix} 2 & -1 & 0 \\ 1 & -1 & -1 \\ 2 & -3 & -4 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -1 & -1 \\ 2 & -1 & 0 \\ 2 & -3 & -4 \end{bmatrix} -1$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 4 \\ 2 & -3 & -4 \end{bmatrix} -2$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 4 \\ 0 & -1 & -2 \end{bmatrix} /2$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix} +$$

$$\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} +$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 1$

$x_2 = 2$

$x_3 =$ free

$x = (1, 2)$

$$10(1.10)\quad \begin{array}{cc} \text{City} & \text{Suburbs} \\ \end{array}$$

$$\begin{bmatrix} 0.94 & 0.04 \\ 0.06 & 0.96 \end{bmatrix} \begin{array}{l} \text{To city} \\ \text{To suburbs} \end{array}$$

$x_{k+1} = Mx_k$ where $x_0 = \begin{bmatrix} 10\,000\,000 \\ 800\,000 \end{bmatrix}$

Population 2012 $(k=2) = x_2 = Mx_1 + Mx_0$

$$Mx_1 = \begin{bmatrix} 0.94 & 0.04 \\ 0.06 & 0.96 \end{bmatrix}\begin{bmatrix} 10\,000\,000 \\ 800\,000 \end{bmatrix} = \begin{bmatrix} 9\,432\,000 \\ 1\,368\,000 \end{bmatrix}$$

$$Mx_2 = \begin{bmatrix} 0.94 & 0.04 \\ 0.06 & 0.96 \end{bmatrix}\begin{bmatrix} 9\,432\,000 \\ 1\,368\,000 \end{bmatrix} = \begin{bmatrix} 8\,920\,800 \\ 1\,879\,200 \end{bmatrix}$$

City pop: 8 920 000

Suburb pop: 1 879 200

$$12(1.10)\quad \begin{array}{ccc} \text{Airport} & \text{East} & \text{West} \end{array}$$

$$\begin{bmatrix} 0.97 & 0.05 & 0.10 \\ 0.00 & 0.90 & 0.05 \\ 0.03 & 0.05 & 0.85 \end{bmatrix} \begin{array}{l} \text{To airport} \\ \text{To East} \\ \text{To west} \end{array}$$

$x_{k+1} = Mx_k$ where $x_0 = \begin{bmatrix} 295 \\ 55 \\ 150 \end{bmatrix}$

Cars on wednesday $(k=2) = x_2 = Mx_1 + Mx_0$

$$Mx_1 = \begin{bmatrix} 0.97 & 0.05 & 0.10 \\ 0.00 & 0.90 & 0.05 \\ 0.03 & 0.05 & 0.85 \end{bmatrix}\begin{bmatrix} 295 \\ 55 \\ 150 \end{bmatrix} = \begin{bmatrix} 303.9 \\ 57.0 \\ 139.1 \end{bmatrix}$$

$$Mx_2 = \begin{bmatrix} 0.97 & 0.05 & 0.10 \\ 0.00 & 0.90 & 0.05 \\ 0.03 & 0.05 & 0.85 \end{bmatrix}\begin{bmatrix} 303.9 \\ 57.0 \\ 139.1 \end{bmatrix} = \begin{bmatrix} 311.5 \\ 58.3 \\ 130.2 \end{bmatrix}$$

No cars at airport ≈ 312

No cars at east ≈ 58

No cars at west ≈ 130

```
In [1]: migrationMatrix = np.array([[0.94, 0.04], [0.06, 0.96]])

In [2]: migrationMatrix
Out[2]:
array([[ 0.94,  0.04],
       [ 0.06,  0.96]])

In [3]: populationMatrix0 = np.array([10000000, 800000])

In [4]: populationMatrix0
Out[4]: array([10000000,   800000])

In [7]: AxIP(migrationMatrix, populationMatrix0)
Out[7]: array([ 9432000.,  1368000.])

In [8]: populationMatrix1 = AxIP(migrationMatrix, populationMatrix0)

In [9]: AxIP(migrationMatrix, populationMatrix1)
Out[9]: array([ 8920800.,  1879200.])
```

See previous page for explanation of results. Should be pretty self explanatory.

```
In [11]: deliveryMatrix = np.array([[0.97, 0.05, 0.10], [0.00, 0.90, 0.05],
[0.03, 0.05, 0.85]])

In [12]: deliveryMatrix
Out[12]:
array([[ 0.97,  0.05,  0.1 ],
       [ 0.  ,  0.9 ,  0.05],
       [ 0.03,  0.05,  0.85]])

In [13]: carMatrix0 = np.array([295.0, 55.0, 150.0])

In [14]: carMatrix0
Out[14]: array([ 295.,   55.,  150.])

In [15]: AxIP(deliveryMatrix, carMatrix0)
Out[15]: array([ 303.9,   57. ,  139.1])

In [16]: carMatrix1 = AxIP(deliveryMatrix, carMatrix0)

In [17]: AxIP(deliveryMatrix, carMatrix1)
Out[17]: array([ 311.543,   58.255,  130.202])
```