Lecture 2: Optimisation

DTU Course 46770: Integrated Energy Grids

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Problem 2.1. Given the following optimization problem,

$$
\begin{aligned}
\max_{x,y} \quad & 2x + 5y \\
\text{s.t.} \quad & -x + y \geq -3, \\
& 2x + y \leq 14, \\
& -\frac{1}{2}x + y \leq \frac{3}{2}, \\
& x \geq 1.
\end{aligned}
$$

a) Solve the optimisation problem graphically (pen and paper or on your laptop). Note that it is a maximisation problem, whereas we will mostly work with minimizations. Reformulate as a minimisation problem.

b) Return to the original formulation. Indicate which constraints are binding and calculate the values of the Lagrange multipliers.

c) In the above set-up, we have a unique solution to our maximisation problem (existence and uniqueness). Adapt the exercise such that this is no longer the case.

Problem 2.2. For this problem you will need to use the Python package linopy. Start with the introduction to linopy: `https://aleks-g.github.io/integrated-energy-grids/intro-linopy.html`.

Consider the following economic dispatch problem:

- we have three generators: solar, wind and gas
- solar and wind have no marginal costs, and gas has fuel costs of 60 EUR/MWh.
- we need to cover demand of 13.2 MWh
- the installed capacities are 15 MW, 20 MW and 20 MW for wind, solar, and gas, respectively
- assume the capacity factor for solar is 0.17 and for wind 0.33.

a) Use linopy to define and solve the LP and find the optimal solution as well as reading out the Lagrange multipliers as defined in the lecture.

b) Open `problem2_2b.csv`, and use the values as inputs for capacity factors as well as demand in the dispatch problem. Solve the LP with linopy.

c) Open `problem2_2c.csv`, and use the values as inputs for capacity factors as well as demand in the dispatch problem. Solve the LP with linopy.

d) Compare the share of renewable generation, the dual variables, and the objective from a)-c) [average, median, min, max] and interpret the differences. Compute the curtailment from renewables.

e) Plot the supply and demand duration curves for the different resources in c). Also consider `demand - renewable generation` ("net load"). Could transmission or storage be useful for this system? Why or why not?

Problem 2.3. We assume the system from Problem 2.2.

a) Another system that is connected to ours — assuming copper-plate — decides dispatch just before we do and can export utility solar. The results from that dispatch optimization are saved in `problem2_3a.csv`. Create a time series of available imports and their price.

b) Solve the updated problem with the available imports. Think of the imports as a generator with variable, marginal prices corresponding to the dual variables.