FINA 4320: Investment Management
Efficient Diversification Part V

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Index models

- We started this chapter with the distinction between **systematic** and **firm-specific risk**. Systematic risk is macroeconomic, affecting all securities, while firm-specific risk factors affect only one particular firm or at most, a cluster of firms
- **Index models** are statistical models designed to estimate these two components of risk for a particular security or portfolio.

A Single-Index Stock Market

- **Index models** are statistical models designed to estimate **systematic risk** and **firm-specific risk** for a particular security or portfolio
- Assume that **one common factor** is responsible for all the covariability of stock returns, with all other variability due to firm-specific factors. Then, a Single-Index Stock Market model decomposes the excess return of a security into three components:

$$R_i = \beta_i R_M + e_i + \alpha_i$$

  - R_i: Excess return of the firm's stock (stock return minus risk-free rate $R_i = r_i - r_f$)
  - R_M: Excess return on a broad market index (e.g S&P 500)

A Single-Index Stock Market

The three components in the Single-Index Stock Market

$$R_i = \beta_i R_M + e_i + \alpha_i$$

	Symbol
1. The component of return due to movements in the overall market (as represented by the index R_M); β_i is the security's responsiveness to the market.	$\beta_i R_M$
2. The component attributable to unexpected events that are relevant only to this security (firm-specific).	e_i
3. The stock's expected excess return if the market factor is neutral, that is, if the market-index excess return is zero.	α_i

A Single-Index Stock Market

- The expected value of e_i is zero, as the impact of unexpected events must average out to zero. The e_i are uncorrelated across securities.
- α_i is not a risk measure. It represents the expected return on the stock beyond any return induced by movements in the market index. This term is called security **alpha**.

Variance of the Excess Return of the Stock

$$\begin{aligned}
\text{Variance}(R_i) &= \text{Variance}(\beta_i R_M + e_i + \alpha_i) \\
&= \text{Variance}(\beta_i R_M) + \text{Variance}(e_i) \\
&= \beta_i^2 \sigma_M^2 + \sigma^2(e_i) \\
&= \textbf{Systematic risk} + \textbf{Firm-specific risk}
\end{aligned}$$

The total variance of the rate of return of each security is a sum of two components:

1. The variance attributable to the uncertainty of the entire market. This variance depends on both the variance of R_M, denoted by σ_M^2, and the β of the stock on R_M.
2. The variance of the firm-specific return, e_i, which is independent of market performance.

How do we estimate systematic and firm-specific risk in the single index stock market model?

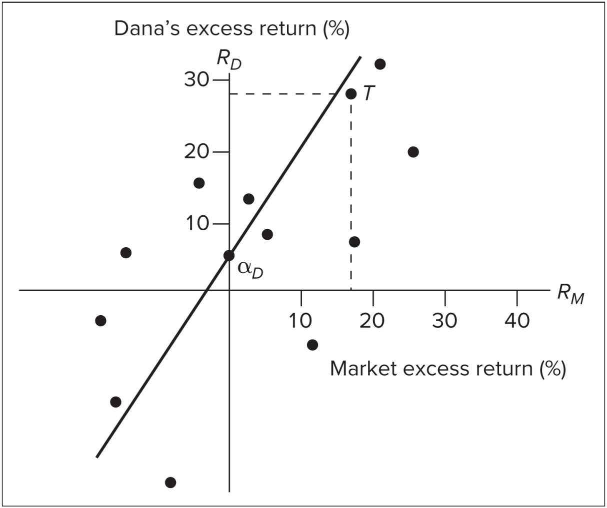
Statistical and Graphical Representation of Single-Index Model

- The equation

$$R_i = \beta_i R_M + e_i + \alpha_i$$

  may be interpreted as a **single-variable regression equation** of R_i on the market excess return R_M
  - R_i is the dependent variable
  - α_i is the intercept
  - R_M is the independent variable
  - β_i is the regression (slope) coefficient
- The regression line is called the **Security Characteristic Line (SCL)**

Scatter Diagram for Dana Computer Corp.



The Security Characteristic Line

- The SCL is the regression line that best fits the data in the scatter diagram
- Given a sample of pairs of returns (R_M, R_D), the regression line is the line that minimizes the sum of squared deviations of the observations from the line
- The regression line does not represents actual returns; points on the scatter diagram almost never lie exactly on the regression line. Rather, the line represents average tendencies; it shows the **expectation** of R_D given the market excess return, R_M:

$$E(R_i|R_M) = \alpha_i + \beta_i R_M$$

- The actual returns (points in the scatter diagram) also include a residual e_i, reflecting the firm-specific component of return. This surprise is measured by the vertical distance between the point of the scatter diagram and the regression line.

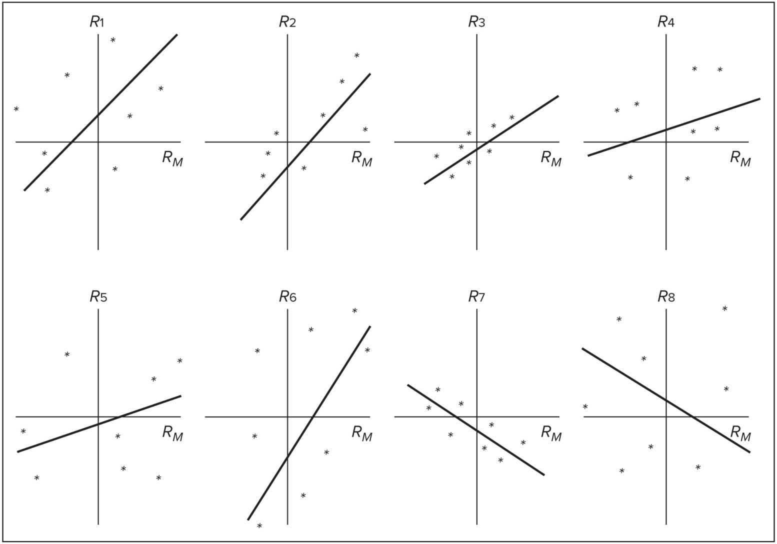
Relative Importance of Systematic Risk

- The dispersion of the scatter of actual returns about the regression line is measured by the **variance of the residuals**, $\sigma^2(e)$.
- One way to measure the relative importance of systematic risk is to measure the **ratio of systematic variance to total variance**. This is called the **R-square** of the regression line:

$$
\begin{aligned}
R - square &= \frac{\text{Systematic (or explained) variance}}{\text{Total variance}} \\
&= \frac{\beta_i^2 \sigma_M^2}{\sigma_i^2} = \frac{\beta_i^2 \sigma_M^2}{\beta_i^2 \sigma_M^2 + \sigma_{e_i}^2}
\end{aligned}
$$

- A **large R-square** means systematic variance dominates the total variance; that is, firm-specific variance is relatively unimportant
- When **R-square is small**, the market factor plays a relatively unimportant part in explaining the variance of the asset, and firm-specific factors dominate

Various Scatter Diagrams



Diversification in a Single-Index Security Market

SYSTEMATIC RISK: NO DIVERSIFICATION EFFECTS

- Imagine a portfolio P that is divided equally among securities whose returns follow the single-index model
- The β of the portfolio is a simple average of the individual security β
- The systematic component of each security return, $\beta_i R_M$, is driven by the market factor and therefore is perfectly correlated with the systematic part of any other security's return
- Hence, there are **no diversification effects on systematic risk** no matter how many securities are involved
- A single security has the same systematic risk as a diversified portfolio with the same beta. The number of securities makes no difference.

Diversification in a Single-Index Security Market

NONSYSTEMATIC RISK: DIVERSIFICATION BENEFITS

- Consider a portfolio P of n securities with weights w_i $(\sum_{i=1}^{n} w_i = 1)$
- Nonsystematic risk of each security is $\sigma^2_{e_i}$
- Nonsystematic portion of portfolio P's return is

$$e_P = \sum_{i=1}^{n} w_i e_i$$

- Portfolio P's nonsystematic variance is

$$\sigma^2_{e_P} = \sum_{i=1}^{n} w_i^2 \sigma^2_{e_i}$$

- The sum is far less than the average firm-specific variance of the stocks in the portfolio

Diversification in a Single-Index Security Market

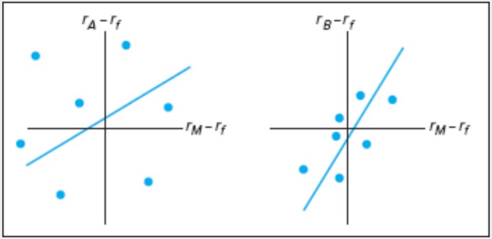
- **The impact of nonsystematic risk becomes negligible as the number of securities grows** and the portfolio becomes more diversified
  - The number of securities counts more than the size of their nonsystematic variance
  - Sufficient diversification can virtually eliminate firm-specific risk. Only systematic risk (market risk) remains.
- **For diversified investors, the relevant risk measure for a security is the security's** β, since firms with higher β have greater sensitivity to market risk
  - The systematic risk, $\beta^2\sigma_M^2$, will be determined by both the market volatility, σ_M^2, and the firm's β

Exercise

21. The following figure shows plots of monthly rates of return and the stock market for two stocks. *(LO 6-5)*

    a. Which stock is riskier to an investor currently holding a diversified portfolio of common stock?

    b. Which stock is riskier to an undiversified investor who puts all of his funds in only one of these stocks?



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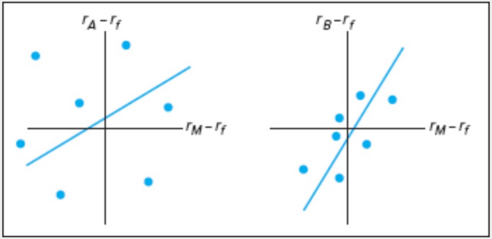

ANSWERS: i) Stock B because it has a higher β ii) Stock A because it has a higher variance