FINA 4320: Investment Management
Capital Asset Pricing and Arbitrage Pricing Theory
Part I

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CHAPTER 7: Capital Asset Pricing and Arbitrage Pricing Theory

- The **Capital Asset Pricing Model**
  - Assumptions
  - Implications
  - Applications
- Implementing the CAPM: **Index models**
- **Multifactor models** and the CAPM
- **Arbitrage Pricing Theory**

Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM)

THE PROBLEM:

- We know that investors want to be compensated for risk.
- The riskier the security, the more the return an investor would require
- What is the relevant risk given that all investors act this way and given that they can invest in many securities?

THE MODEL:

- The relevant risk turns out to be **the covariance with the market portfolio, not variance**
- Securities whose returns have a high covariance with the return of the market portfolio have to pay a high expected return

The CAPM Assumptions

INVESTOR ASSUMPTIONS

- Investors plan for the **same horizon** (single-period)
- Investors have **homogeneous expectations** and so, they agree on outlook and beliefs. This means that if two investors examine the same investment opportunity, they will have identical beliefs about the expected returns, variance of returns and correlations with other investments.(The wealth and risk tolerance of each investor need not be the same.)
- Investors are **rational**, mean-variance optimizers (normality or quadratic utility). This means that all investors attempt to construct efficient frontier portfolios
- Investors are efficient users of analytical methods

The CAPM Assumptions

MARKET ASSUMPTIONS

- All information relevant to security analysis is free and publicly available
- All relevant assets are publicly owned and traded (no investment in human capital). Thus, all risky assets are in the investment universe.
- Risk-free rate available to all (**unlimited lending/borrowing**)
- **No taxes or transaction costs**
- All investors are price takers and there is **perfect competition** in the market: no investor is sufficiently wealthy that his/her action alone can affect the market prices

In short: Perfect capital markets

CAPM Results

RESULT 1: All investors hold the market portfolio

- All investors hold risky assets proportionally to market portfolio
- Intuition: **Homogeneous expectations** $\Longrightarrow$ all investors have the same CAL and hold the same risky portfolio. In equilibrium, this portfolio must be market portfolio.
- **Passive strategy** optimal for all investors: Hold market portfolio + riskless asset
  - Optimal risky portfolio = Market Portfolio
  - CML is the best attainable CAL
  - "Mutual fund" or "separation" theorem: only one mutual fund of risky assets is sufficient to satisfy the investment demands of all investors.

RESULT 2: The market risk premium

- **Formula**: Risk premium on market portfolio is proportional to variance of market portfolio and average investor's risk aversion

$$E[r_m] - r_f = \overline{A}\sigma_m^2$$

- **Proof**: The risk-free investments involve borrowing & lending. Hence the net borrowing = 0 so that avg. position in risky portfolio is 100% $\Longrightarrow$ y = 1

$$\begin{aligned} y^* &= \frac{E[r_p] - r_f}{A\sigma_p^2} \Longrightarrow \\ 1 &= \frac{E[r_m] - r_f}{\overline{A}\sigma_m^2} \\ E[r_m] - r_f &= \overline{A}\sigma_m^2 \end{aligned}$$

CAPM Results

RESULT 3: The risk premium on individual assets

- **Formula (No proof)**. The expected return on individual securities (and portfolios) is given by the following mean-beta relationship:

$$E[r_i] = r_f + \beta_i(E[r_m] - r_f)$$

where

$$\beta_i = \frac{Cov(r_i, r_m)}{\sigma_m^2}$$

- Rearranging this equation gives

$$\frac{E[r_i] - r_f}{\beta_i} = \frac{E[r_m] - r_f}{1}$$

The ratio of risk premium to β is the same for all assets and is equal to that for the market portfolio

- The risk that matters is the **systematic risk**

CAPM Results: implications

- **Portfolio β and Risk Premium.**
  If the mean-beta relationship holds for any individual asset, it must hold for any combination of assets. The β of a portfolio is simply the weighted average of the betas of the stocks in the portfolio, using as weights the portfolio proportions. $\beta_P = \sum w_i \beta_i$

  EXERCISE: If the market risk premium is 7.5%, the risk premium on the portfolio is 0.84 x 7.5%=6.3%

Asset	Beta	Risk Premium	Portfolio Weight
Microsoft	1.2	9.0%	0.5
American Electric Power	0.8	6.0	0.3
Gold	0.0	0.0	0.2
Portfolio	0.84	?	1.0

CAPM Results: implications

- **Security Market Line**
  - Graph of expected return vs. beta (reward vs. risk):

$$E[r_i] = r_f + (E[r_M - r_f])\beta_i$$

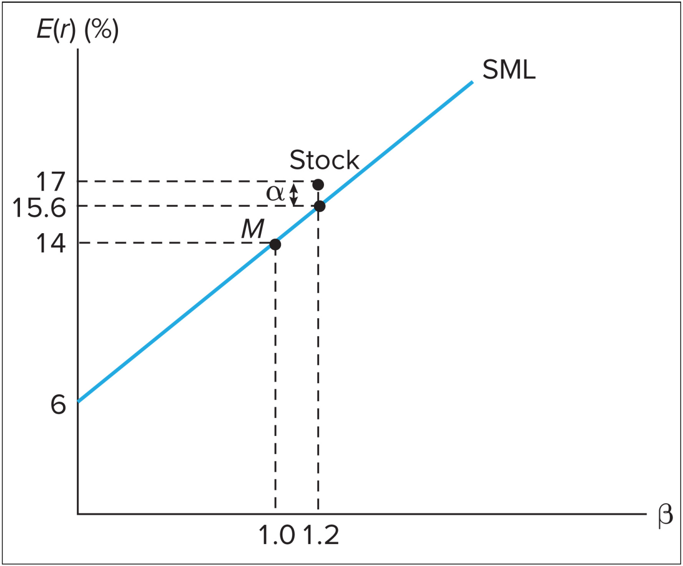
  - For individual assets: Relevant risk is beta (contribution of asset to the risk of efficient portfolio)
  - The expected return of the CAPM relation is the "fair price". Underpriced stocks plot above the SML $\longrightarrow$ they have positive α. The difference between fair and actual expected return is the **alpha**. The expected return on a mispriced security is given by

$$E[r_i] = \alpha_i + r_f + \beta_i(E[r_M - r_f])$$

    And the **alpha** is defined as:

$$\alpha_i = \underbrace{E[r_i]}_{\text{Actual expected return}} - \underbrace{[r_f + \beta_i(E[r_M - r_f])]}_{\text{Expected return implied by the CAPM}}$$

Security Market Line (SML)



Security Market Line (SML) and Capital Market Line (CML)

- The SML is different than the **Capital Market Line** (Graph of expected return vs. standard deviation of the complete portfolios made up of the market portfolio and the risk-free asset). In the CML case, for portfolios that are candidates for an investor's complete portfolio the relevant risk is standard deviation

Applications of the CAPM

- **Capital budgeting**: Determine discount rates (hurdle rates) to evaluate projects ($\Longrightarrow$ NPV)
- **Find "mispriced" securities**: Alpha (α) of security = "abnormal return"
  - $\alpha =$ actual expected return - expected return predicted by the CAPM
  - $\alpha < 0$: security over-priced
  - $\alpha > 0$: security under-priced
- **Performance measurement**
  - A positive alpha for a fund can be taken as evidence of the manager's stockpicking skill
  - In this context, alpha is often referred to as Jensen's alpha

EXERCISE 1

- Two portfolio managers are comparing performance. One averaged a 19% rate of return and the other a 16% rate of return. However, the beta of the first manager was 1.5, whereas that of the second was 1.
  1. Which investor was a better stock-picker?
  2. If the T-bill rate were 6% and the market return during the period were 14%, which investor would be the superior stock selector?

SOLUTION

1. We know that: $R_1 = 19\%$, $R_2 = 16\%$, $\beta_1 = 1.5$, and $\beta_2 = 1$.
    - To tell which investor was a better picker of individual stocks, we should look at their abnormal return, which is the ex-post α (alpha). The abnormal return is the difference between the actual return and the return predicted by the SML.
    - Without information about the parameters of this equation (risk-free rate and the market rate of return) we cannot tell which manager is more accurate.
2. If $R_f = 6\%$ and $R_m = 14\%$, then (using the notation of alpha for the abnormal return):
    - $\alpha_1 = 19\%$ - $[6\% + 1.5(14\%$ - $6\%)] = 19\%$ - $18\% = 1\%$
    - $\alpha_2 = 16\%$ - $[6\% + 1(14\%$ - $6\%)] = 16\%$ - $14\% = 2\%$.
    - The second manager has the larger abnormal return: better stock-picker

EXERCISE 2

- The expected rates of return on stocks A and B are 11% and 14%, respectively.
- The beta of stock A is 0.8, while that of stock B is 1.5.
- The T-bill rate is currently 6%. The expected rate of return of the market is 12%.
- The standard deviation of stock A is 10% annually, while that of stock B is 11%.
  1. If you currently hold a well-diversified portfolio (and are happy with it), would you choose to add either of these stocks to your holdings?
  2. If instead you could invest only in T-bills plus only one of these stocks, which stock would you choose?

SOLUTION

1. The alpha of Stock A is:

$$\begin{aligned}\alpha_A &= r_A - [r_f + \beta_A(r_m - r_f)] \\ &= 11 - [6 + 0.8(12 - 6)] = 0.2\%\end{aligned}$$

$$\begin{aligned}\alpha_B &= r_B - [r_f + \beta_B(r_m - r_f)] \\ &= 14 - [6 + 1.5(12 - 6)] = -1\%\end{aligned}$$

   - Thus, Stock A would be a good addition. Taking a short position in B may be desirable.

2. The reward to variability ratio of the stocks is:

$$\begin{aligned}S_A &= (11 - 6)/10 = 0.50 \\ S_B &= (14 - 6)/11 = 0.73\end{aligned}$$

   - Stock B is superior when only one can be held.

EXERCISE 3: Verifying the Validity of the CAPM

If the simple CAPM is valid, which of the situations below are possible? Explain. Consider each situation independently.

1.

Portfolio	Expected Return	Beta
A	20%	1.4
B	25%	1.2

EXERCISE 3: Verifying the Validity of the CAPM

If the simple CAPM is valid, which of the situations below are possible? Explain. Consider each situation independently.

1.

Portfolio	Expected Return	Beta
A	20%	1.4
B	25%	1.2

SOLUTION

1. Not possible. Portfolio A has a higher beta than Portfolio B, but the expected return for Portfolio A is lower.

EXERCISES

2.

Portfolio	Expected Return	Beta
Risk-free	10%	0
Market	18%	1
A	16%	1.5

2.

Portfolio	Expected Return	Beta
Risk-free	10%	0
Market	18%	1
A	16%	1.5

SOLUTION

2. Not possible. Given these data, the SML is:

$$E(r) = 10\% + \beta(18\% - 10\%)$$

A portfolio with beta of 1.5 should have an expected return of:

$$E(r) = 10\% + 1.5(18\% - 10\%) = 22\%$$

The expected return for Portfolio A is 16% so that Portfolio A plots below the SML (i.e., has an alpha of $16\% - 22\% = -6\%$), and hence is an overpriced portfolio. This is inconsistent with the CAPM which predicts an alpha of 0 (with all securities plotting on the SML).

EXERCISES

3.

Portfolio	Expected Return	Standard Deviation
Risk-free	10%	0%
Market	18%	24%
A	16%	12%

EXERCISES

3.

Portfolio	Expected Return	Standard Deviation
Risk-free	10%	0%
Market	18%	24%
A	16%	12%

SOLUTION

3. Not possible. The reward-to-variability ratio (Sharpe ratio) for Portfolio A is better than that of the market, which is not possible according to the CAPM, since the CAPM predicts that the market portfolio is the most efficient portfolio. Using the numbers supplied:

$$S_A = \frac{16 - 10}{12} = 0.5$$
$$S_M = \frac{18 - 10}{24} = 0.33$$

These figures imply that Portfolio A provides a better risk-reward tradeoff than the market portfolio.