FINA 4320: Investment Management
Efficient Diversification Part IV

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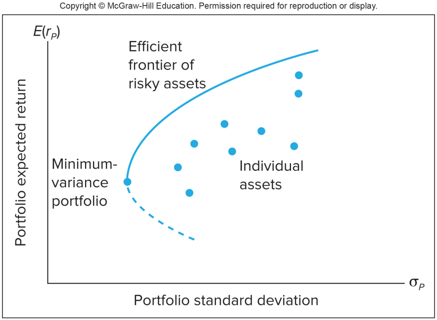
Asset allocation with many risky assets

The Efficient Frontier of Risky Assets



- The **efficient frontier** looks like the previous opportunity set with 2 assets.
- But now the efficient frontier is only a subset of the opportunity set, it is the set of portfolios that are not dominated. These portfolios provide the optimal risk return trade-offs.

The Efficient Frontier of Risky Assets in Practice

How do we mathematically find the efficient frontier? There are two equivalent ways to find the efficient frontier:

- For any level of standard deviation find the portfolio with that level of standard deviation and the maximum possible expected return.
- For any level of expected return find the portfolio with that level of expected return and the minimum possible standard deviation.

The Efficient Frontier of Risky Assets in Practice

Example.

- Assume we have N risky asset with expected return $E[r_1], ..., E[r_N]$, variances $Var(r_1), ..., Var(r_N)$ and pairwise covariances $Cov(r_1, r_2), ..., Cov(r_{N-1}, r_N)$.
- What is the portfolio with $E[r_P] = 10\%$ and the minimum possible standard deviation?
- We need two formulas to solve the problem:
  - The expected return of a portfolio composed by N assets is given by

$$E[r_P] = \sum_{i=1}^{N} w_i E[r_i]$$

  - The variance of a portfolio composed by N assets is given by

$$\sigma_P^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j Cov(r_i, r_j)$$

Example continued.

- The problem is then solved by finding the weights $w_1, ..., w_N$ that satisfy the following **optimization problem**:

$$\min_{w_1,\dots w_N} \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j Cov(r_i, r_j)$$

such that

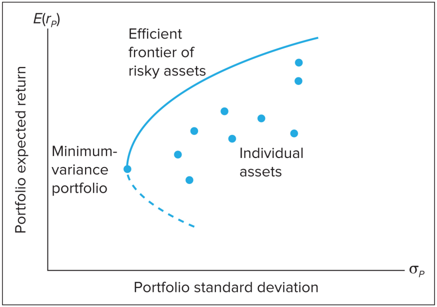
$$E[r_P] = \sum_{i=1}^{N} w_i E[r_i] = 10\%.$$

- To solve the problem we need a programming software with an **optimizer**. For example in Excel the optimizer is called **Solver**.

Asset allocation with many risky assets + Risk-free asset

Asset Allocation with Many Risky Assets and a Risk-free Asset

What happens if we add the risk-free asset in the opportunity set?



Asset Allocation with Many Risky Assets and a Risk-free Asset

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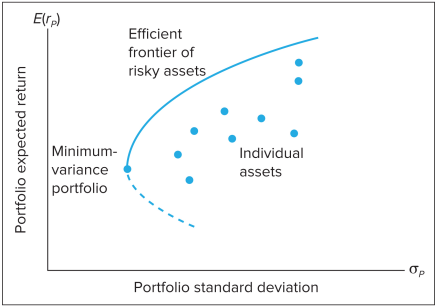
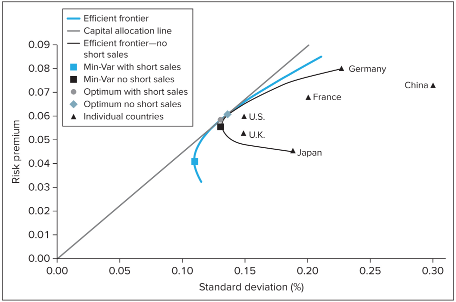


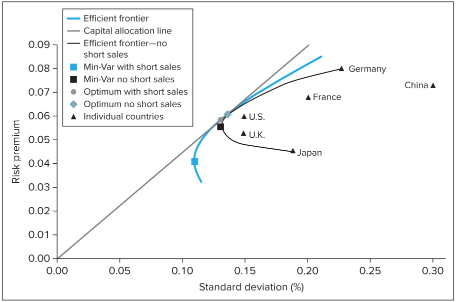
- Combinations of riskless asset + any risky portfolio = Straight line (CAL)
- Each risky portfolio $\longrightarrow$ Different CAL

Which is the steepest possible CAL?

Asset Allocation with Many Risky Assets and a Risk-free Asset



Asset Allocation with Many Risky Assets and a Risk-free Asset



- Optimal risky portfolio $\longrightarrow$ Point of tangency between CAL & efficient frontier
- Gives highest feasible reward-to-variability ratio (slope of CAL)
- Note: Optimal risky portfolio is independent of risk aversion!!! Portfolio manager offers same risky portfolio to all = **separation property**

Summary: Asset Allocation with Many Risky Assets and a Risk-free Asset

Three steps:

- Specify risk-return characteristics of securities and calculate the **efficient frontier of risky assets**
  - Statistical task
- Find the **optimal risky portfolio** P (same for all investors)
  - Maximize reward-to-variability ratio
- **Combine the optimal risky portfolio P and the riskless asset**
  - This is the optimal CAL. Finally, the investor will choose a complete portfolio C on the CAL that is the best for his level of risk-aversion, or for his constraints.

Discussion

Mean-variance analysis is one of the crown jewels of finance theory (It got Harry Markowitz the Nobel Prize). But:

- Mean-variance analysis sometimes leads to large short positions in some assets. This might not be appropriate if investors cannot short-sell.
  - Solution: Constrain the weights to be positive
- Implementation problems (see next slide)

Discussion

Implementation Problems:

- Precision in estimating the inputs
  - What if you have an asset with interesting risk/return trade-off or correlation properties but these inputs are estimated imprecisely?
  - Also, large covariance matrices are hard to work with.
- Time variation in the inputs
  - For variances, historical averages are usually okay
  - Covariances are trickier and change over the business cycle. They also change in periods of market turmoil.
  - The hardest part is to estimate expected returns !!! Expected returns are not constant over time, so historical averages are not always helpful.