FINA 4320: Investment Management
Risk, Return, and the Historical Record. Part II

Paola Pederzoli

University of Houston

C. T. Bauer College of Business

Risk and Risk premiums

Risk Premiums definitions

- The **risk-free rate** (r_f) is the rate of return that can be earned with certainty (T-bill rate)
- The **risk premium** of a risky asset is the difference between the expected return of the risky asset and the risk-free rate

$$RP_{asset} = E[r_{asset}] - r_f$$

- The **excess return** is the return in excess of the risk-free rate, i.e. $r_{asset} - r_f$
- A reasonable forecast of an asset's risk premium is the average of its historical excess returns

Degree of Risk Aversion

- Risk-neutral investor cares only about expected return, level of risk irrelevant
- **Risk-averse Investor**: Requires compensation (higher expected return) for risk-taking
- A positive risk-premium distinguishes speculation from gambling. Speculation: investors take risk to earn a premium. Gamblers take on risk even without a premium, just for the enjoyment of the risk itself.

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EXAMPLE

Investor A is very risk-averse and he is happy with a fixed income portfolios with expected excess return of 2.16% and SD=8.44%. Investor B is less risk-averse and he is happy with a portfolio of equity with expected excess return of 8% and SD=20%.

- How do we measure the different attitudes toward risk?

Degree of Risk Aversion

- If the investor puts his entire wealth in a portfolio, say C, with expected return $E[r_C]$ and variance σ_C^2, then

$$\textbf{The degree of risk aversion } A = \frac{E[r_C] - r_f}{\sigma_C^2}$$

- Measured by the risk premium required (per unit of variance) to compensate the risk-averse investor for taking on risk
- The ratio of the portfolio's risk premium to its variance is also called the **price of risk**
- **EXAMPLE continued** Investor A risk aversion is equal to $\frac{0.0216}{(0.0844)^2} = 3.03$, while Investor B risk aversion is equal to $\frac{0.08}{(0.2)^2} = 2$

Sharpe Ratio (Reward-to-volatility Ratio)

- A related measure, the **Sharpe Ratio**, is commonly used to rank portfolios in terms of their risk-return trade-off

$$\text{Sharpe Ratio } S = \frac{E[r_C] - r_f}{\sigma_C}$$

- In a mean-variance portfolio analysis, the Sharpe ratio is commonly used to rank portfolios
- Careful: the price of risk does not depend on the holding period, while the Sharpe ratio is horizon-dependent
  - Annual return $= 12 \times$ monthly return
  - Annual VAR $= 12 \times$ monthly variance, Annual SD $= \sqrt{12} \times$ monthly standard dev

Asset Allocation between 1 Risky Asset and 1 Riskless Asset

Asset Allocation between 1 Risky Asset and 1 Riskless Asset

- Split investment funds between safe and risky assets
- Risk-free asset gives a return r_f and the risky asset P gives a return r_P
- The **complete portfolio** is the entire portfolio including risky and risk-free assets
- Think of the following choice of assets
  - risky (r_P): portfolio of stocks, bonds, real estate
  - riskless (r_f): cash, T-bills, MM funds
- Example: Your total wealth is $10,000. You put $2,500 in risk-free T-Bills and $7,500 in a stock portfolio P invested as follows:
  - Stock A you put $2,500
  - Stock B you put $3,000
  - Stock D you put $2,000

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How much should you invest in the risky asset? 10%, 50%, 100%, 110%? (110 is taking a loan and investing (on margin))

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ANSWER: **You might have a target return, or a maximum risk you can take, or you can optimize according to your level of risk-aversion:**

- The more you invest in the risky asset, the higher the return *and* the higher the risk!
- The key issue in setting the weights is the risk and return trade-off you prefer among all possible options
- Different degrees of risk aversion will lead to different preferences and hence affect the optimal allocation between risky and risk-free assets
- The more risk-averse you are, the more you invest in the risk-free asset

Asset Allocation between 1 Risky Asset and 1 Riskless Asset

Mathematically:

- Risk-free asset gives a return r_f and the risky asset P gives a return r_P with σ_P
- Let $\mathbf{y}$ = weight of risky portfolio P in the complete Portfolio C. This is the variable we want to find!
- Let $1 - y$ = weight of risk-free asset in the complete Portfolio C
- Overall Return of Portfolio C, $r_C = yr_P + (1 - y)r_f$
- Expected Return and Risk of Portfolio C are

$$
\begin{aligned}
E[r_C] &= yE[r_P] + (1 - y)r_f \\
Var[r_C] &= \sigma_C^2 = y^2\sigma_P^2
\end{aligned}
$$

Asset Allocation between 1 Risky Asset and 1 Riskless Asset

We equate the Reward-to-variance of the complete portfolio to the risk aversion A

$$A = \frac{E[r_C] - r_f}{\sigma_C^2} = \frac{y^*(E[r_P] - r_f)}{{y^*}^2 \sigma_P^2}$$

Rearranging this equation gives $y^* = \dfrac{E[r_P] - r_f}{A\sigma_P^2}$

Asset Allocation between 1 Risky Asset and 1 Riskless Asset

EXAMPLE

What is the optimal allocation **y** in a risky portfolio of an investor with a risk-aversion of 2.3? The expected excess return of the risky portfolio is 8% and SD=20%.

ANSWER:

$$y = \frac{E[r_P] - r_f}{A\sigma_P^2} = \frac{0.08}{2.3 \times (0.2)^2} = 0.87$$

Capital Allocation Line (CAL)

A GRAPHICAL REPRESENTATION OF THE PROBLEM

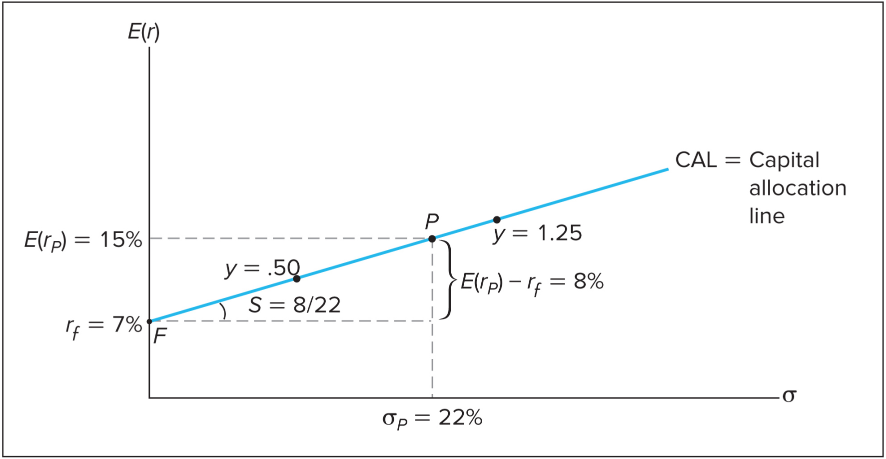
- The choice between 1 riskless and 1 risky asset is sometimes referred to as the "capital allocation problem"
- Graphically, the investment opportunity set of risk-return combinations available = Capital Allocation Line
- To see why the relationship between return $E[r_C]$ and risk σ_C is linear, substitute $\frac{\sigma_C}{\sigma_P}$ for y in $E[r_C] = yE[r_P] + (1-y)r_f$ (slide 10) and you obtain the CAL line:

$$\begin{aligned} E[r_C] &= \frac{\sigma_C}{\sigma_P}E[r_P] + (1 - \frac{\sigma_C}{\sigma_P})r_f \\ &= r_f + \frac{E[r_P] - r_f}{\sigma_P}\sigma_C \end{aligned}$$

Remember the standard equation for a line is : $Y = a + mX$.

Capital Allocation Line: Example

CAL Equation: $E[r_C] = r_f + \frac{E[r_P]-r_f}{\sigma_P}\sigma_C$.
Assume that $r_f = 7\%$, $E[r_P] = 15\%$ and $\sigma_P = 22\%$. The CAL equation is $E[r_C] = 7\% + 0.36 \times \sigma_C$



Capital Allocation Line: Example

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CAL equation: $E[r_C] = 7\% + 0.36 \times \sigma_C$

Consider 4 possibilities of the complete portfolio C with 0%, 50%,100% and 125% invested in the risky portfolio P respectively

y	$E[r_C]$	σ_C
0	0(15%)+1(7%)=7%	0(22%)=0%
0.5	0.5(15%)+0.5(7%)=11%	0.5(22%)=11%
1	1(15%)+0(7%)=15%	1(22%)=22%
1.25	1.25(15%)+(-0.25)(7%)=17%	1.25(22%)=27.5%

As the return of the portfolio increases, the risk also increases. This is the trade-off. All these portfolios lie on the Capital Allocation Line.

Interpreting the weight y

- $0 < y < 1$: Some lending, some investing
- $y < 0$: Short sales
- $y > 1$: Invest own $+$ borrowed resources $=$ buying on margin, to increase risk and return via leverage

Capital Allocation Line (CAL) key takeaways

- Slope of the CAL, $\frac{E[r_P]-r_f}{\sigma_P}$, is the Sharpe ratio (Reward-to-variability ratio) and measures the extra return per extra risk
- The Sharpe ratio is the same for all complete portfolios on the CAL.
  - From the CAL Equation: $E[r_C] = r_f + \frac{E[r_P]-r_f}{\sigma_P}\sigma_C$,
  - we get $\frac{E[r_C]-r_f}{\sigma_C} = \frac{E[r_P]-r_f}{\sigma_P}$.
- The risk-averse investor chooses to allocate his funds between the risk-free rate and the risky portfolio with the **highest Sharpe-ratio**

The Capital Market Line (CML)

- The CML is the CAL provided by 1-month T-bills and a broad index of common stocks (for example, the S&P 500)
- Often referred to as a "passive" investment strategy
- Attainable using
  - S&P 500 Mutual Fund (Vanguard, Fidelity)
  - ETF
- A passive portfolio strategy implies

$$y^*_{passive} = \frac{E[r_M] - r_f}{A\sigma^2_M}$$

where M denotes the Market portfolio (for example, the S&P 500)

Active versus Passive Strategies

- Active strategies entail more trading costs than passive strategies
- Passive involves investment in two passive portfolios
  - Short-term T-bills
  - Fund of common stocks that mimics a broad market index
  - Vary combinations according to investor's risk aversion

Exercise

EXERCISE

You manage a risky portfolio with $E[r_P] = 17\%$ and $\sigma_P = 27\%$. the T-bill rate is $r_f = 7\%$.

Your client chooses to invest 70% of a portfolio in your fund and 30% in a T-bill money market fund.

- What is the expected return and standard deviation of your client's portfolio?
- What is the Sharpe-Ratio of your risky portfolio and your client's overall portfolio?
- Draw the CAL of your portfolio on an expected return/ standard deviation diagram. What is the slope of the CAL? Show the position of your client on your fund's CAL.

EXERCISE - SOLUTION

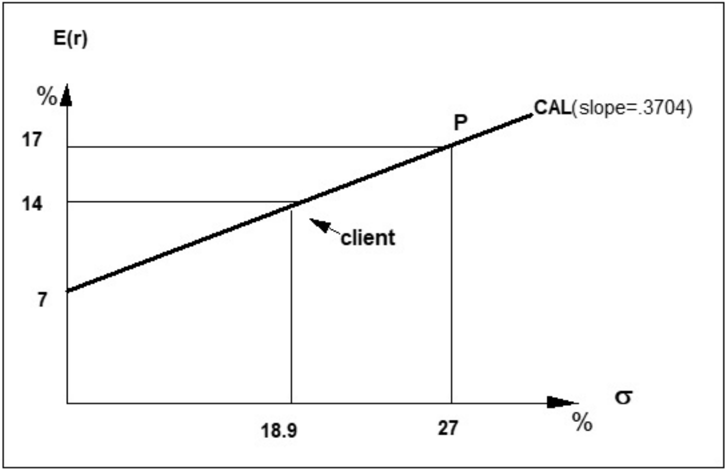
- What is the expected return and standard deviation of your client's portfolio?

$$
\begin{aligned}
E[r_C] &= yE[r_P] + (1-y)r_f \\
&= (0.7 \times 0.17) + (0.3 \times 0.07) = 0.14 \\
\sigma_C &= y \times \sigma_P = 0.7 \times 0.27 = 0.189
\end{aligned}
$$

- What is the Sharpe-Ratio of your risky portfolio and your client's overall portfolio?

$$
\begin{aligned}
S_P &= \frac{E(r_P) - r_f}{\sigma_P} = \frac{0.17 - 0.07}{0.27} = 0.3704 \\
S_C &= \frac{E(r_C) - r_f}{\sigma_C} = \frac{0.14 - 0.07}{0.189} = 0.3704
\end{aligned}
$$

EXERCISE - SOLUTION



EXERCISE

Suppose the same client in the previous problem decides to invest in your risky portfolio a proportion y of his total investment budget so that his overall portfolio will have an expected rate of return of 15%.

- What is the proportion y?
- What is the standard deviation of the rate of return on your client's portfolio?

EXERCISE - SOLUTION

- $$\begin{aligned} E[r_C] &= yE[r_P] + (1-y)r_f \\ &= y \times 0.17 + (1-y) \times 0.07 = 0.15 \end{aligned}$$

  Solving for y we get $y = \frac{0.15-0.07}{0.10} = 0.8$
- $\sigma_C = y \times \sigma_P = 0.8 \times 0.27 = 0.216$

EXERCISE

Suppose the same client as in the previous problem prefers to invest in your portfolio a proportion y that maximizes the expected return on the overall portfolio subject to the constraint that the overall portfolio's standard deviation will not exceed 20%.

- What is the investment proportion y?
- What is the expected rate of return on the overall portfolio?

SOLUTION:

- $\sigma_C = y \times 0.27$, hence if the client wants a standard deviation to be equal or less than 20% then $y = (0.20/0.27) = 0.7407$
- $E[r_C] = r_f + y \times (E(r_P) - r_f) = 0.07 + 0.7407 \times 0.10 = 0.1441$

EXERCISE

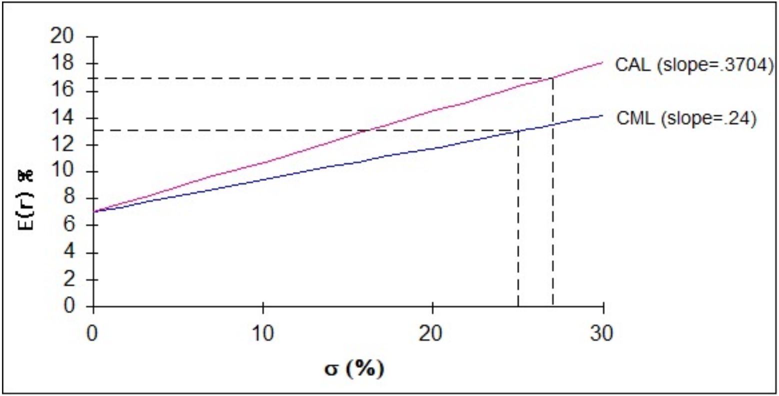
You estimate that a passive portfolio invested to mimic the S&P500 stock index yields an expected rate of return of 13% with a standard deviation of 25%. Draw the CML and your fund's CAL on an expected return/standard deviation diagram.

- What is the slope of the CML?
- Characterize shortly the advantage of your fund over the passive fund

SOLUTION:

- Slope of the CML $= \frac{E(r_M)-r_f}{\sigma_M} = \frac{0.13-0.07}{0.25} = 0.24$
- Your fund allows an investor to achieve a higher expected rate of return for any given standard deviation than would a passive strategy

EXERCISE - SOLUTION



EXERCISE

Your client (see previous problem) wonders whether to switch the 70% that is invested in your fund to the passive portfolio.

- Explain to your client the disadvantage of the switch
- Show your client the maximum fee you could charge (as a percentage of the investment in your fund deducted at the end of the year) that would still leave him at least as well off investing in your fund as in the passive one. (*Hint:* the fee will lower the slope of your client's CAL by reducing the expected return net of the fee.)

EXERCISE - SOLUTION

- With 70% of his money in your fund's portfolio, the client has an expected rate of return of 14% per year and a standard deviation of 18.9% per year. If he switches to the market portfolio, in order to achieve the same average return he needs to accept a standard deviation of

$$\sigma_C = y \times \sigma_M$$

y is found by solving for

$$\begin{aligned} E[r_C] &= yE[r_M] + (1-y)r_f \\ &= y \times 0.13 + (1-y) \times 0.07 = 0.14 \end{aligned}$$

Solving for y we get $y = \frac{0.14-0.07}{0.06} = 1.16$ and $\sigma_C = 1.16 \times 0.25 = 0.29$.

EXERCISE - SOLUTION

- The fee would reduce the reward-to-variability ratio, i.e., the slope of the CAL. Clients will be indifferent between your fund and the passive portfolio if the slope of the after-fee CAL and the CML are equal. Let f denote the fee:

$$\begin{aligned}\text{Slope of the CAL with fee} &= \frac{17\% - 7\% - f}{27\%} = \frac{10\% - f}{27\%} \\ \text{Slope of the CML} &= \frac{13\% - 7\%}{25\%} = 0.24\end{aligned}$$

Setting these slopes equal and solving for f:

$$\begin{aligned}\frac{10\% - f}{27\%} &= 0.24 \\ 10\% - f &= 27\% \times 0.24 = 6.48\% \\ f &= 10\% - 6.48\% = 3.52\%\end{aligned}$$