Posterior Approximation for Normal Normal Using Markov Chains

Aleksa Kostic

2/20/2022

```
library(bayesplot)
library(bayesrules)
library(tidyverse)
library(rstan)
```

Model

$$Y_i|\mu \sim N(\mu, 1.3^2)\mu \sim N(10, 1.2^2)$$

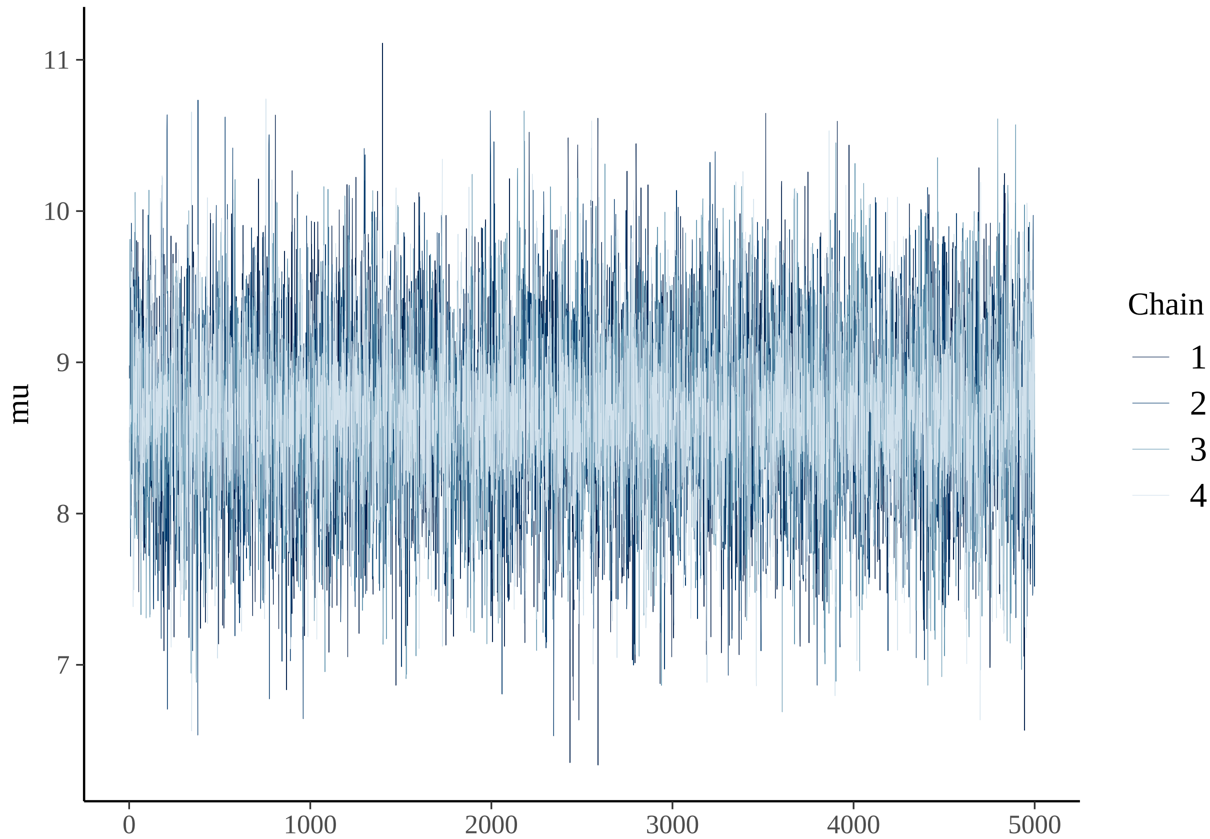
Suppose that on n = 4 independent observations, you observe $(Y_1, Y_2, Y_3, Y_4) = (7.1, 8.9, 8.4, 8.6)$

```
nn_model <- "
data{
  real Y[4];
}
parameters {
  real mu;
}
model{
  mu ~ normal(10, 1.2);
  Y ~ normal(mu, 1.3);
}
"
```

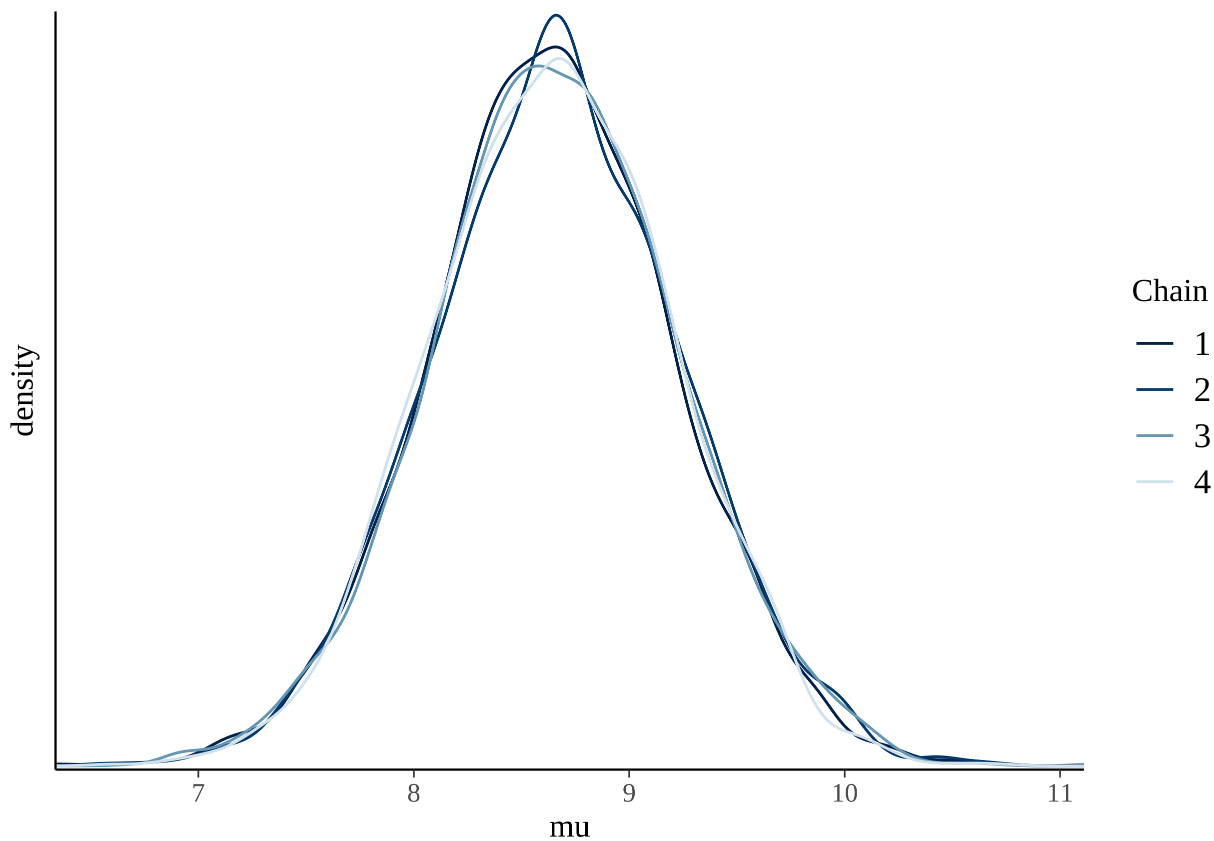
```
nn_sim  <- stan(
  model_code = nn_model, data = list(Y = c(7.1, 8.9, 8.4, 8.6)),
  chains = 4, iter = 5000*2, seed = 84735, refresh = FALSE
)
```

Checking for convergence and consistency

```
mcmc_trace(nn_sim, pars="mu", size = 0.1)
```




```r
mcmc_dens_overlay(nn_sim, pars = "mu") +
  ylab("density")
```



The most plausible value of mu seems to be at around 8.8.

Checking that the formula basis for the posterior matches the simulation (which is pretty close)

```
summarize_normal_normal(mean = 10, sd = 1.2, sigma = sd(c(7.1, 8.9, 8.4, 8.6)), y_bar = mean(c(7.1, 8.9
```

```
##       model      mean      mode       var        sd
## 1     prior 10.000000 10.000000 1.4400000 1.2000000
## 2 posterior  8.422535  8.422535 0.1419718 0.3767915
```

$\mu | Y_i \sim \mathcal{N}(8.422535, 0.3767915^2)$