Posterior Approximation for Gamma Poisson Using Markov Chains

Aleksa Kostic

2/3/2022

```
library(bayesplot)
library(bayesrules)
library(rstan)
library(tidyverse)
```

$$\lambda \sim \text{Gamma}(20,5) Y_i|\lambda \sim \text{Pois}(\lambda) n = 3 (Y_1, Y_2, Y_3) = (0,1,0)$$

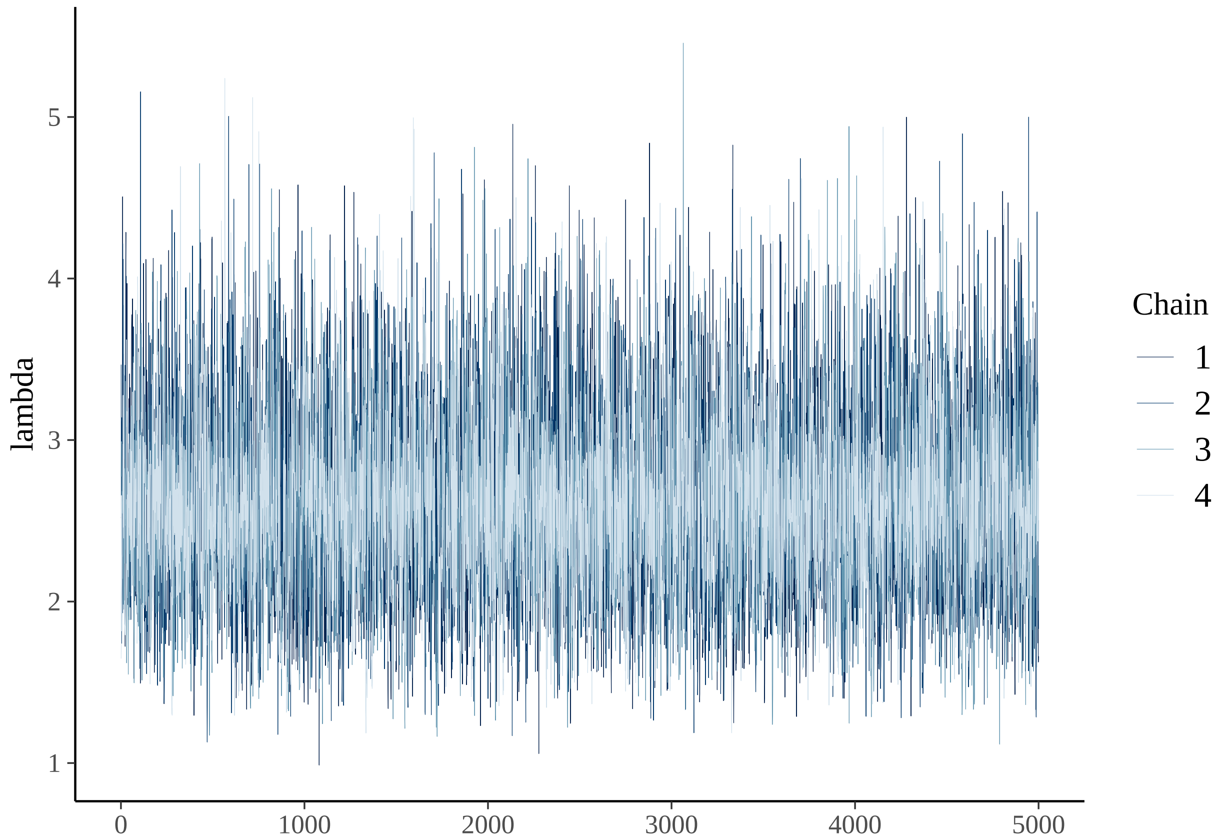
Model

```
gp_model <- "
data{
  int<lower=0> Y[3];
}
parameters {
  real<lower=0> lambda;
}
model{
  lambda ~ gamma(20,5);
  Y ~ poisson(lambda);
}
"
```

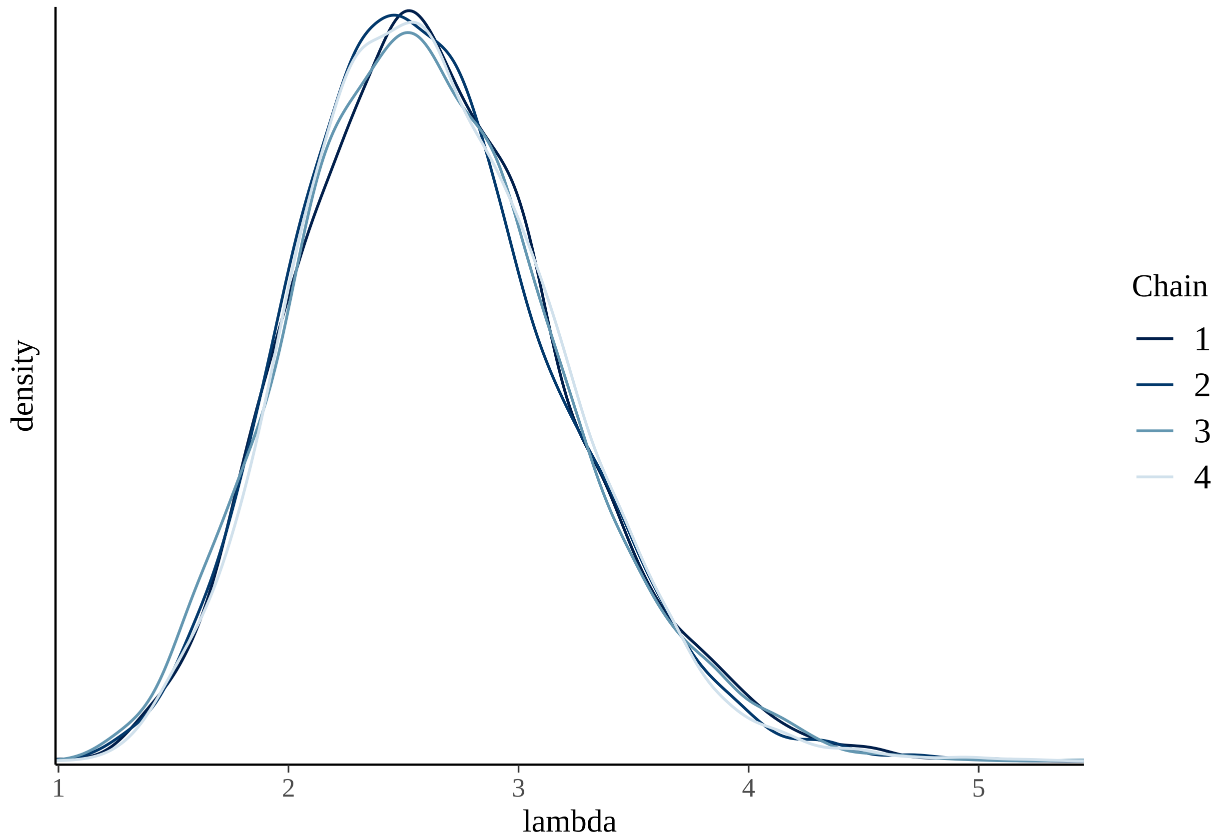
```
gp_sim <- stan(
  model_code = gp_model, data = list(Y = c(0,1,0)),
  chains = 4, iter = 5000*2, seed = 84735, refresh=FALSE
)
```

Check for convergence

```
mcmc_trace(gp_sim, pars="lambda", size = 0.1)
```




```
mcmc_dens_overlay(gp_sim, pars = "lambda") +
  ylab("density")
```



The most plausible value for lambda seems to be around 2.5.

Seeing if formula calculation matches simulation

The posterior distribution of $\lambda|\vec{Y}$ also follows a gamma distribution but one that has a smaller variance and a mean at around 2.5.

```
summarize_gamma_poisson(shape=20,rate=5,sum_y=1,n=3)
```

```
##       model shape rate  mean mode      var        sd
## 1     prior    20    5 4.000  3.8 0.800000 0.8944272
## 2 posterior    21    8 2.625  2.5 0.328125 0.5728220
```

$\lambda|\vec{y} \sim \text{Gamma}(21, 8)$