A Unique Equilibrium in Cournot Coordination Markets with Cost Saving Communication

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1 Introduction

In markets where producers can engage in cost saving communication price competition erodes away cooperation incentives. In regular markets cooperative outcomes tend to be unstable and even completely excluded off equilibrium path. However, in situations where producers have a stake in consumer surplus, cooperation incentives are not easily quantified due to the trade off between consumer surplus and profit on each unit sold. Traditional economic reasoning would dictate that an expert worker will not share their expertise to other workers maintaining a comparative advantage. In modern economies, expert employees almost certainly own shares of their company creating an incentive to reduce costs of other employees. In this paper, we examine incentives in a market with many sellers who are able to lower costs for other sellers and a single buyer who attempts to maximize surplus by leveraging that communication. We demonstrate that under certain assumptions, the market converges to a unique equilibrium where the buyer misreports their demand and sellers produce-cournot like quantities.

2 Market Model

The model design is inspired by the beauty contest variation present by Morris and Shin (2002)[1]. There are n sellers and one buyer, we assume that the buyer has a linear demand $M^{'} - P_q$. Seller i has a constant marginal cost of u_i per unit. Costs are assigned from an i.i.d. well defined distribution with mean μ. Costs are privately known by each seller only. The specifics and parameters of the distribution are publicly known by both the buyer and sellers. The buyer announces a demand curve $M - P_q$ and sales are allocated using a regular market clearing procedure. Each seller chooses a quantity q_i to supply. The buyer receives surplus $\frac{(M'-p_q)*p_q}{2}$ based on the quantity sold and the price is determined based on the announced demand curve. Given $q = (q_1...q_n)$, seller i's profit π_i is given by the function $\pi_i(q, u_i) = q_i(M - \sum q) - q_i u_i$. We define seller payoff as:

$$U_i(q, u_i) = (1-r)\pi_i(q, u_i) + r\frac{(M - p_q) * p_q}{2} \tag{1}$$

The variable $r \in (0, 1)$ is a publicly known constant that captures the substitutability of utility gained from the buyer surplus and profit on units sold. When $r = 0$, sellers have no stake in improving buyer surplus which leads to regular cournot outcomes. At $r = 1$, sellers only care about buyer surplus and will produce the maximum quantity possible. Values in between capture nuanced cases where sellers have some stake in the buyer's welfare. In a Bayesian Nash equilibrium each seller selects q_i while best responding to q_{-i} the total amount produced by all other sellers.

$$\text{We take the first order condition } \forall i \in (1..n)\frac{\delta U(q, u_i)}{\delta q_i} = 0$$

$$\implies (1-r)(M - 2q_i - q_{-i} - u_i) + \frac{r}{2}(M - 2q) = 0 \implies (1-\frac{r}{2})M - (1-r)u_i + (1-r)(-2q_i - q_{-i}) - rq = 0$$

$$(1-\frac{r}{2})M - (1-r)u_i + (1-r)(-2q_i - q_{-i}) - rq = 0 \implies (1-\frac{r}{2})M - (1-r)u_i - (1-\frac{r}{2})q_i - q_{-i} = 0$$

We apply expectation twice, since types are independent expectation is invariant under iterated expectations:

$$(1-\frac{r}{2})M - (1-r)\mu - (1-\frac{r}{2})E(q_i) - (n-1)E(q_i) = 0$$

$$(1-\frac{r}{2})M - (1-r)\mu + E(q_i)(-(1-\frac{r}{2}) - (n-1)) = 0$$

$$E(q_i)((1-\frac{r}{2}) + (n-1)) = (1-\frac{r}{2})M - (1-r)\mu$$

$$E(q_i) = \frac{(1-\frac{r}{2})M - (1-r)\mu}{n - \frac{r}{2}} \tag{2}$$

We obtain equation (2) which represent the expected quantity produced by one seller.

By rearranging previous equations we get $q_i = M - \frac{(1-r)u_i + (n-1)E_q}{1-\frac{r}{2}}$

$$\implies q_i = M\frac{1-\frac{r}{2}}{(n-\frac{r}{2})} - \frac{(1-r)u_i}{1-\frac{r}{2}} + (n-1)\frac{(1-r)\mu}{(1-\frac{r}{2})(n-\frac{r}{2})} \tag{3}$$

In equation (3) we have each seller's best response quantity as a mapping from their cost and game parameters to a specific value.

3 Expertise Communication and Preferences

After the buyer's demand curve is revealed, each seller gets to send a signal $s_i \in S$ which impacts every other seller's marginal cost $u_j \forall j \neq i$, we will constrain our analysis to public communication where every seller selects a single signal and that signal is sent to all other sellers. The choice of s_i only impacts the expected value of u_j conditional on u_i and s_i. This can be viewed as changing the distribution mean μ

by some amount holding u_i constant conditional on s_{-i}. In order to reason about player's preferences over different values of μ, we derive a potential function $\phi_i(\mu^{'}, \mu)$ which is ordinally equivalent to:

$$\text{seller } i \text{ 's payoff with distribution mean } \mu^{'} \text{ - seller } i\text{'s payoff with distribution mean } \mu; \forall \mu^{'} \neq \mu$$

Therefore, $\phi_i(\mu^{'}, \mu)$ will be positive if and only if seller i is better off in a state where $E(u_j) = \mu^{'}$ as opposed to $E(u_j) = \mu$. Using equations $(1) - (3)$ we obtain the following potential function. (Derivation is detailed in appendix A.1.)

$$\phi_i(\mu^{'}, \mu) = -sgn(\mu^{'} - \mu) * (\mu^{'} - \mu - M * C_m + u_i * C_u) \tag{4}$$

Where $sgn(x)$ is the sign function

$$C_m = \frac{(2-r)(2n^2(r^2 - 3r - 2) + (r^2 - 4r - 4)(2r - 5n))}{4n(n-1)(1-r)(r^2 - 4r - 4)} \tag{5}$$

$$C_u = \frac{-(2n - r)(4r^3 - 19r^2 + 6r + 16)}{(n-1)r(1-r)(r^2 - 4r - 4)} \tag{6}$$

We note that both constants are positive given $r \in (0, 1)$

$$C_m > 0 \ \forall n \in (5, \infty)$$

$$C_u > 0 \ \forall n \in (1, \infty)$$

Generally, if the new mean is determined by some function $g(s_i|s_{-i})$ then seller i will pick a utility maximizing signal:

$$s_i \in \underset{x}{argmax} \ \phi(g(x|s_{-i}), \mu) \tag{7}$$

4 Binary Communication Game

Going further we will analyze a specific communication game where $s_i \in \{0, -\gamma\}$. In addition we will assume that u_i is drawn from a uniform distribution over the range $(\underline{u}, \overline{u})$. After demand is announced sellers choose their signals. Finally players choose to supply q_i and each seller has a marginal cost equal to $u_i + \sum_{j \neq i} s_j$. Communication in this game offers each seller the ability to reduce everyone else's marginal cost by γ. In expectation after seller i sets s_i, every other seller will receive s_i and $n-2$ other signals, therefore $E(u_j) = \mu + s_i + (n-2)E(s_k)$.

Following the logic of equation (7) $s_i = \gamma$ if and only if $\phi(\mu + (n-2)E(s_j) + s_i, \mu + (n-2)E(s_j)) > 0$

$$\implies ((n-2)E(s_j) + s_i + \mu - (n-2)E(s_j) - \mu - M * C_m + (u_i + (n-2)E(s_j) * C_u)) > 0 \tag{8}$$

Theorem 1 *There exists a unique equilibrium where for some value* $u^* \in \mathrm{R}$ $s_i = -\gamma \iff u_i \geq u^*$, $s_i = 0 \iff u_i < u^*$ *and sellers with* $u_i = u^*$ *are indifferent between both outcomes*

Proof:

Let u^* be the value at which sellers are indifferent:

$$\phi_{u^*} = -\gamma - M * C_m + (u^* + (n-2)E(s_j)) * C_u = 0 \ \& \ C_u > 0$$

$$\implies \phi_i((n-2)E(s_j) - \gamma + \mu, \mu + (n-2)E(s_j)) > \phi_{u^*} = 0 \ \forall i \in \{i; u_i > u^*\}$$

Symmetrically,

$$\phi_i((n-2)E(s_j) - \gamma + \mu, \mu + (n-2)E(s_j)) < \phi_{u^*} = 0 \ \forall i \in \{i; u_i < u^*\}$$

Since all higher types will choose $s_i = -\gamma$ and all lower types will choose $s_i = 0$.

$$\implies E(s_j) = \frac{\overline{u} - u^*}{\overline{u} - \underline{u}}(-\gamma)$$

$$\implies -\gamma - M * C_m + (u^* + (-\gamma)(n-2)(\frac{\overline{u}}{\overline{u} - \underline{u}} - \frac{u^*}{\overline{u} - \underline{u}})) * C_u = 0$$

$$\implies u^*(1 + \frac{\gamma(n-2)}{\overline{u} - \underline{u}})) * C_u = M * C_m + \gamma + (\gamma)(n-2)(\frac{\overline{u}}{\overline{u} - \underline{u}}) * C_u$$

$$u^* = \frac{M * C_m + \gamma(1 + (n-2)(\frac{\overline{u}}{\overline{u} - \underline{u}}) * C_u)}{(1 + \frac{\gamma(n-2)}{\overline{u} - \underline{u}}) * C_u} \tag{9}$$

Only one unique value of u^* satisfies equation (9), therefore u^* is unique. QED.

5 Buyer Welfare

Although quantity and price are determined by the buyer's demand curve, the introduction of the communication game prior to sales may create an incentive for her misrepresent her type.

Theorem 2 *A buyer with demand* $M' - q$*, given constant game parameters, will choose to report a unique surplus maximizing demand curve* $M - q$ *where* M *and* M' *could be distinct.*

Proof: Let $\mu^*(M)$ be the expected average cost given announced demand $M - q$. By equation (9) from theorem 1, this function is well defined, continuous and differentiable. The expected supplied quantity can be derived from equation (2).

$$Eq(M) = n\frac{(1 - \frac{r}{2})M - (1 - r)\mu^*(M)}{n - \frac{r}{2}}$$

A buyer with true demand $M^{'} - q$ who reports $M - q$ gain utility $U(M^{'}, M)$ where:

$$U(M^{'}, M) = \frac{(M^{'} - Eq(M))Eq(M)}{2}$$

Taking a first order condition

$$0 = \frac{\delta U(M^{'}, M)}{\delta M} = \frac{1}{2}\frac{\delta Eq(M)}{\delta M}(M^{'} - 2Eq(M))$$
$$= \frac{1}{2}\frac{\delta Eq(M)}{\delta M}(M^{'} - 2n\frac{(1-\frac{r}{2})M - (1-r)\mu^*(M)}{n - \frac{r}{2}})$$

$$\implies M^{'} = \frac{2n}{n-\frac{r}{2}}((1-\frac{r}{2})M - (1-r)\mu^*(M)) \quad (10)$$

Equation (9) $\implies \mu^*(M) = \mu - (n-1)\gamma(\frac{\overline{u} - u^*(M)}{\overline{u} - \underline{u}})$ then we can recude (10) to closed form

$$M^{'} = \frac{2n}{n-\frac{r}{2}}\left(M\big(1 - \frac{r}{2} - \frac{C_m(n-1)(1-r)}{C_u(\overline{u} - \underline{u} + \gamma(n-2))}\big) + (1-r)\big(\mu - (n-1)(\gamma - \frac{\gamma + (n-2)\frac{\overline{u}}{\overline{u}-\underline{u}}C_u}{C_u(\overline{u} - \underline{u} + \gamma(n-2))})\right) \quad (11)$$

When $r < 1$, the relationship is linear with a constant term independent from M, therefore, there can exist at most one value where $M = M^{'}$. QED.

6 Results and Comparative Statics

The result from theorem 2 shows a unique surplus maximizing strategy for the buyer and under linear demand and uniform costs the buyer will almost always misrepresent their demand. Due to the nature of the linear demand curve, the buyer always tries to set quantity close to $\frac{M^{'}}{2}$ adjusted with r, as $r \to 0$ market begins to behave like a regular competitive monopsony.

Since $C_m \to -\infty$ as $n \to 1$ and $C_m \leq 0 \ \forall \ n \leq 3$ we note that an increase in demand acts as an incentive to cooperate and lower average costs only when n is small. If $n > 5$ an increase in demand will always make sellers less likely to cooperate in expectation by increasing the value of u^*.

As n increases the second term in equation (11) increases $\implies$ M decreases holding $M^{'}$ constant. The buyer increasingly under-reports their type as the number of sellers increases.

Appendix A Potential Function

This section details the derivation of the potential function *phi* defined in equation (4).

First we define the following variables as a tool to shorten notation: let $A = \frac{1-\frac{r}{2}}{n-\frac{r}{2}}, B = \frac{1-r}{1-\frac{r}{2}}, C = \frac{1-r}{n-\frac{r}{2}}, D = \frac{n-1}{1-\frac{r}{2}}$. We suppress notation such that $Eq = E(q_i)$, then we start from best response quantities derived above.

$$E_q = AM - C\mu \text{ from (2)}$$

$$q_i = AM - Bu_i + CD\mu \text{ from (3)}$$

We precompute the quantity $\sum q = q_i + (n-1)E_q$

$$q_i + (n-1)E_q = nAM - Bu_i + \mu(CD - (n-1)C) = nAM - Bu_i + C\mu(D - (n-1)C)$$

$$q_i + (n-1)E_q = nAM - Bu_i - (\frac{r}{2})CD\mu$$

We plug those values in into the utility function defined in (1):

$$U(\mu, u_i) = (1-r)(M - q_i - (n-1)E_q - u_i)q_i + \frac{r}{2}(M - q_i - (n-1)E_q)(q_i + (n-1)E_q)$$

$$= (M - nAM + Bu_i + (\frac{r}{2})CD\mu + u_i)((1-r)(AM - Bu_i + CD\mu) + \frac{r}{2}(nAM - Bu_i - (\frac{r}{2})CD\mu))$$

$$= (M(1-nA) + (B+1)u_i + (\frac{r}{2})CD\mu)(AM(1 + \frac{r(n-2)}{2}) - Bu_i(1-\frac{r}{2}) + CD\mu(1 - r - (\frac{r^2}{4})))$$

$$= (M(1-nA) + (B+1)u_i + (\frac{r}{2})CD\mu)(AM(1 + \frac{r(n-2)}{2}) - Bu_i(1-\frac{r}{2}) - CD\mu(-r + \frac{(r-2)(r+2)}{4}))$$

$$= (M(1-nA) + (B+1)u_i + (\frac{r}{2})CD\mu)(AM(1 + \frac{r(n-2)}{2}) - Bu_i(1-\frac{r}{2}) - CD\mu(-r + \frac{(r-2)(r+2)}{4}))$$

$$= (M(1-nA) + (B+1)u_i + (\frac{r}{2})CD\mu)(AM(1 + \frac{r(n-2)}{2}) - Bu_i(1-\frac{r}{2}) - \frac{1}{2}CD\mu(-2r + \frac{(r-2)(r+2)}{2}))$$

We define Δ_i which captures the changes in utility as μ changes to $\mu^{'}$

$$\Delta_i(\mu^{'}, \mu) = U(\mu^{'}, u_i) - U(\mu, u_i)$$

Let

$$E1 = (\frac{r}{2})CD(AM(1 + \frac{r(n-2)}{2}) - Bu_i(1-\frac{r}{2}))$$

$$E2 = -\frac{1}{2}CD(-2r + \frac{(r-2)(r+2)}{2})(M(1-nA) + (B+1)u_i + \frac{r}{2}CD * (\mu^{'} - \mu))$$

$$\Delta_i(\mu^{'}, \mu) = U(\mu^{'}, u_i) - U(\mu, u_i) = (\mu^{'} - \mu)(E1 + E2)$$

We refactor Δ_i using a series of algebriac manipulations:

$$= CD\frac{(\mu^{'} - \mu)}{2}(rAM(1+\frac{r(n-2)}{2}) - rBu_i(1-\frac{r}{2}) + (2r - \frac{(r-2)(r+2)}{2})(M(1-nA) + (B+1)u_i + \frac{r}{2}CD*(\mu^{'} - \mu)))$$

$$= CD\frac{(\mu^{'} - \mu)}{2}(M(rA(1 + \frac{r(n-2)}{2}) + (2r - \frac{(r-2)(r+2)}{2})(1-nA))$$

$$-rBu_i(1-\frac{r}{2})+(2r-\frac{(r-2)(r+2)}{2})(M+(B+1)u_i+\frac{r}{2}CD*(\mu^{'}-\mu)))$$

$$=CD\frac{(\mu^{'}-\mu)}{2}(M(rA+\frac{rAr(n-2)}{2}+2r-2rnA-\frac{(r-2)(r+2)}{2}+\frac{nA(r-2)(r+2)}{2})-rBu_i(1-\frac{r}{2})$$
$$+(2r-\frac{(r-2)(r+2)}{2})(M+(B+1)u_i+\frac{r}{2}CD*(\mu^{'}-\mu)))$$

$$=CD\frac{(\mu^{'}-\mu)}{2}(\frac{M}{2}(2r\frac{1-\frac{r}{2}}{n-\frac{r}{2}}+\frac{1-\frac{r}{2}}{n-\frac{r}{2}}r^2(n-2)+4r-4rn\frac{1-\frac{r}{2}}{n-\frac{r}{2}}+4-r^2+n\frac{1-\frac{r}{2}}{n-\frac{r}{2}}(r^2-4))$$
$$-rBu_i(1-\frac{r}{2})+(2r-\frac{(r-2)(r+2)}{2})(M+(B+1)u_i+\frac{r}{2}CD*(\mu^{'}-\mu)))$$

$$=CD\frac{(\mu^{'}-\mu)}{2}(\frac{M}{2}(2r\frac{1-\frac{r}{2}}{n-\frac{r}{2}}+\frac{1-\frac{r}{2}}{n-\frac{r}{2}}r^2(n-2)+(\frac{r(1-n)}{2n-r})(r^2-4-4r))-rBu_i(1-\frac{r}{2})$$
$$+(2r-\frac{(r-2)(r+2)}{2})(M+(B+1)u_i+\frac{r}{2}CD*(\mu^{'}-\mu)))$$

$$=CD\frac{(\mu^{'}-\mu)}{2}(\frac{rM}{2}((\frac{2-r}{2n-r}rn+\frac{(1-n)}{2n-r}(r^2-4-4r))-rBu_i(1-\frac{r}{2})+(r^2-4-4r)(M+(B+1)u_i+\frac{r}{2}CD*(\mu^{'}-\mu)))$$

$$=CD\frac{(\mu^{'}-\mu)}{2}(\frac{rM}{2}((\frac{2-r}{2n-r}rn+\frac{(1-n)}{2n-r}(r^2-4-4r))$$
$$-r(1-r)u_i+(r^2-4-4r)M+(r^2-4-4r((\frac{1-r}{1-\frac{r}{2}}+1)u_i+\frac{r}{2}\frac{1-r}{n-\frac{r}{2}}\frac{n-1}{1-\frac{r}{2}}*(\mu^{'}-\mu)))$$

$$=CD\frac{(\mu^{'}-\mu)}{2}(\frac{rM}{2}((\frac{2-r}{2n-r}rn+(\frac{(1-n)}{2n-r}+\frac{2}{n})(r^2-4-4r))+(-r(1-r)+(r^2-4-4r)(\frac{1-r}{1-\frac{r}{2}}+1))u_i)$$
$$+\frac{2(n-1)r(1-r)(r^2-4r-4)}{(2-r)(2n-r)}*(\mu^{'}-\mu)))$$

$$=\frac{CD}{(2n-r)(2-r)}\frac{(\mu^{'}-\mu)}{2}(2(n-1)r(1-r)(r^2-4r-4)*(\mu^{'}-\mu)-\frac{rM}{2n}((2-r)(2n^2(r^2-3r-2)+(r^2-4r-4)(2r-5n)))$$
$$-((2n-r)*(4r^3-19r^2+6r+16))u_i))$$

$$= ((n-1)r(1-r)(r^2-4r-4))\frac{CD}{(2n-r)(2-r)}(\mu^{'}-\mu)((\mu^{'}-\mu)-M*C_m+u_i*C_u))$$

The values of C_m and C_u follow their definitions from equations (5) and (6) respectively. Then we note we compare the sign of $(\mu^{'}-\mu-M*C_m+u_i*C_u)$ to Δ_i. Since

$$((n-1)r(1-r)(r^2-4r-4))\frac{CD}{(2n-r)(2-r)} < 0 \ \forall \ n > 1 \ \& \ r \in (0,1)$$

Therefore,

$$sgn(\Delta_i(\mu', \mu)) = (-)sgn(\mu^{'}-\mu)sgn(\mu^{'}-\mu-M*C_m+u_i*C_u) = sgn(\phi_i(\mu^{'}, \mu))$$

$$\implies \phi_i(\mu^{'}, \mu) > 0 \iff \Delta_i(\mu^{'}, \mu) > 0$$

The potential function ϕ captures all ordinal rankings between different values of μ and $\mu^{'}$.

References

[1] Stephen Morris and Hyun Song Shin. Social value of public information. *american economic review*, 92(5):1521–1534, 2002.