# $\alpha$ Rby—An Embedding of Alloy in Ruby

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**Abstract.** We present  $\alpha$ Rby—an embedding of the Alloy language in Ruby—and demonstrate the benefits of having a declarative modeling language (backed by an automated solver) embedded in a traditional object-oriented imperative programming language. This approach aims to bring these two distinct paradigms (imperative and declarative) together in a novel way. We argue that having the other paradigm available within the same language is beneficial to both the modeling community of Alloy users and the object-oriented community of Ruby programmers. In this paper, we primarily focus on the benefits for the Alloy community, namely, how  $\alpha$ Rby provides elegant solutions to several well-known, outstanding problems: (1) mixed execution, (2) specifying partial instances, (3) staged model finding.

### 1 Introduction

A common approach in formal modeling and analysis is to use (1) a declarative language (based on formal logic) for writing *specifications*, and (2) an automated constraint solver for finding valid *models* of such specifications<sup>1</sup>. Such models are most often either examples (of states or execution traces), or counterexamples (of correctness claims).

In many practical applications, however, the desired analysis involves more than a single model finding step. At the very least, a tool must convert the generated model into a form suitable for showing to the user; in the case of Alloy [8], this includes projecting higher-arity relations so that the model can be visualized as a set of snapshots. In some cases, the analysis may involve repeating the model finding step, e.g., to find a minimal model by requesting a solution with fewer tuples [13].

To date, these additional analyses have been hard-coded in the analysis tool. The key advantage of this approach is that it gives complete freedom to the tool developer. The disadvantage is that almost no freedom is given to the modeler, who must make do with whatever additional processing the tool developer chose to provide.

This paper explores a different approach, in which, rather than embellishing the analysis in an ad hoc fashion in the implementation of the tool, the modeling language itself is extended so that the additional processing can be expressed directly by the end user. An imperative language seems most suitable for this purpose, and the challenge

<sup>&</sup>lt;sup>1</sup> Throughout this paper, we use the term 'model' in its mathematical sense, and never to mean the artifact being analyzed, for which we use the term 'specification' instead.

therefore is to find a coherent merging of two languages, one declarative and one imperative. We show how this has been achieved in the merging of Alloy and Ruby.

This challenge poses two questions, one theoretical and one practical. Theoretically, a semantics is needed for the combination: what combinations are permitted, and what is their meaning? Practically, a straightforward way to implement the scheme is needed. In particular, can a tool be built without requiring a new parser and engine that must handle both languages simultaneously?

This project focuses on the combination of Alloy and Ruby. In some respects, these choices are significant. Alloy's essential structuring mechanism, the signature [7], allows a relational logic to be viewed in an object-oriented way (in just the same way that instance variables in an object-oriented language can be viewed as functions or relations from members of the class to members of the instance variable type). So Alloy is well suited to an interpretation scheme that maps it to object-oriented constructs. Ruby is a good choice because, in addition to being object-oriented and providing (like most recent scripting languages) a powerful reflection interface, it offers a syntax that is flexible enough to support an almost complete embedding of Alloy with very few syntactic modifications.

At the same time, the key ideas in this paper could be applied to other languages; there is no reason, in principle, that similar functionality might not be obtained by combining the declarative language B [1] with the programming language Racket, for example.

The contributions of this paper include: (1) an argument for a new kind of combination of a declarative and an imperative language, justified by a collection of examples of functionality implemented in a variety of tools, all of which are subsumed by this combination, becoming expressible by the end-user; (2) an embodiment of this combination in  $\alpha$ Rby, a deep embedding of Alloy in Ruby, along with a semantics, a discussion of design challenges, and an open-source implementation [3] for readers to experiment with; and (3) an illustration of the use of the new language in a collection of small but non-trivial examples (out of 23 examples available on GitHub [4], over 1400 lines of specification in total).

### 2 Motivations

Analysis of a declarative specification typically involves more than just model finding. In this section, we outline the often needed additional steps.

**Preprocessing** The specification or the analysis command may be updated based on user input. For example, in an analysis of Sudoku, the size of the board must be specified. In Alloy, this size would be given as part of the 'scope', which assigns an integer bound to each basic type. For Sudoku, we would like to ensure that the length of a side is a perfect square; this cannot be specified directly in Alloy.

**Postprocessing** Once a model has been obtained by model finding, some processing may be needed before it is presented to the user. A common application of model finding in automatic configuration is to cast the desired configuration constraints as the specification, then perform the configuration steps based on the returned solution.

**Partial instances** A partial instance is a partial solution that is known (given) upfront. In solving a Sudoku problem, for example, the model finder must be given not only the rules of Sudoku but also the partially filled board. It is easy to encode a partial solution as a formula that is then just conjoined to the specification. But although this approach is conceptually simple, it is not efficient in practice, since the model finder must work to reconstruct from this formula information (namely the partial solution) that is already known, thus needlessly damaging performance.

Kodkod (the back-end solver for Alloy) explicitly supports partial instances: it allows the user to specify relation *bounds* in terms of tuples that <u>must</u> (*lower bound*) and <u>may</u> (*upper bound*) be included in the final value. Kodkod then uses the bounds to shrink the search space, often leading to significant speedups [18]. At the Alloy level, however, this feature is not directly available<sup>2</sup>.

**Staged model finding** Some analyses involve repeated calls to the model finder. In the simplest case, the bounds on the analysis are iteratively increased (when no counterexample has been found, to see if one exists within a larger scope), or decreased (when a counterexample has already been found, to see if a smaller one exists). Sometimes this is used as a workaround when a desired property cannot be expressed directly because it would require higher order logic.

**Mixed execution** Model finding can be used as a step in a traditional program execution. In this case, declarative specifications are executed 'by magic', as if, in a conventional setting, the interpreter could execute a program assertion by making it true despite the lack of any explicit code to establish it [10,9]. Alternatively, flipping the precedence of the two paradigms, the interpreter can be viewed as a declarative model finder that uses imperative code to setup a declarative specification to be solved. In this paper, we are primarily concerned with the latter direction, which has not been studied in the literature as much.

The Alloy Analyzer—the official and the most commonly used IDE for Alloy—does not currently provide any scripting mechanisms around its core model finding engine. Instead, its Java API must be used to automate even the most trivial scripting tasks. Using the Java API, however, is inconvenient; the verbosity and inflexibility of the Java language leads to poor transparency between the API and the underlying Alloy specification, making even the simplest scripts tedious and cumbersome to write. As a result, the official API is rarely used in practice, and mostly by expert users and researchers building automated tools on top of Alloy. This is a shame, since a simple transparent scripting shell would be, in many respects, beneficial to the typical Alloy user—the user who prefers to stay in an environment conceptually similar to that of Alloy and not have to learn a second, foreign API.

This is exactly what  $\alpha$ Rby provides—an embedding of the Alloy language in Ruby. Thanks to Ruby's flexibility and a very liberal parser, the  $\alpha$ Rby language manages to offer a syntax remarkably similar to that of Alloy, while still being syntactically correct Ruby. To reduce the gap between the two paradigms further, instead of using a separate AST,  $\alpha$ Rby maps the core Alloy concepts are onto the corresponding core concepts in

<sup>&</sup>lt;sup>2</sup> The Alloy Analyzer recognizes certain idioms as partial instances; some extensions (discussed in Section 6) support explicit partial instance specification.

Ruby (e.g., sigs are classes, fields are instance variables, atoms are objects, functions and predicates are methods, most operators are the same in both languages).  $\alpha$ Rby automatically interoperates with the Alloy back end, so all the solving and visualization features of the Alloy Analyzer can be easily invoked from within an  $\alpha$ Rby program. Finally, the full power of the Ruby language is at the user's disposal for other tasks unrelated to Alloy.

# 3 $\alpha$ Rby for Alloy Users

A critical requirement for embedding a modeling language in a programming language is that the embedding should preserve enough of the syntax of the language for users to feel comfortable in the new setting. We first introduce a simple example to illustrate how  $\alpha$ Rby achieves this for Alloy. Next, we address the new features brought by  $\alpha$ Rby, highlighted in Section 2, which are the primary motivation for the embedding.

Consider using Alloy to specify directed graphs and the *Hamiltonian Path* algorithm. *Signatures* are used to represent unary sets: Node, Edge, and Graph. *Fields* are used to represent relations between the signatures: val mapping each Node to an integer value; src and dst mapping each Edge to the two nodes (source and destination) that it connects; and nodes and edges mapping each Graph to its sets of nodes and edges.

A standard Alloy model for this is shown in Fig. 1(b), lines 2–4; the same declarations are equivalently written in  $\alpha$ Rby as shown in Fig. 1(a), lines 2–4.

To specify a Hamiltonian path (that is, a path visiting every node in the graph exactly once), a predicate is defined; lines 6–12 in Figs. 1(b) and 1(a) show the Alloy and  $\alpha$ Rby syntax, with equivalent semantics. This predicate asserts that the result (path) is a sequence of nodes, with the property that it contains all the nodes in the graph, and that, for all but the last index i in that sequence, there is an edge in the graph connecting the nodes at positions i and i+1. A *run* command is defined for this predicate (line 18), which, when executed, returns a satisfying instance.

Just as a predicate can be run for examples, an assertion can be checked for counterexamples. Here we assert that starting from the first node in a Hamiltonian path and transitively following the edges in the graph reaches all other nodes in the graph (lines 13–17). We expect this check (line 19) to return no counterexample.

From the model specification in Fig. 1(a),  $\alpha$ Rby dynamically generates the class hierarchy in Fig. 1(c). The generated classes can be used to freely create and manipulate graph instances, independent of the Alloy model.

In Alloy, a command is executed by selecting it in the user interface. In  $\alpha$ Rby, execution is achieved by calling the exe\_cmd method. Fig. 1(d) shows a sample program that calls these methods, which includes finding an arbitrary satisfying instance for the hampath predicate and checking that the reach assertion indeed cannot be refuted.

This short example is meant to include as many different language features as possible and illustrate how similar  $\alpha$ Rby is to Alloy, despite being embedded in Ruby. We discuss syntax in Section 5.1; a summary of main differences is given in Table 1.

```
(a) Graph specification in \alphaRby
                                                        (b) Equivalent Alloy specification
                                                         module GraphModel
   alloy : GraphModel do
    sig Node [val: (lone Int)]
                                                         sig Node {val: lone Int}
                                                         sig Edge {src, dst: one Node}{src != dst}
    sig Edge [src, dst: (one Node)] {src != dst}
    sig Graph[nodes:(set Node), edges:(set Edge)]
                                                         sig Graph{nodes: set Node, edges: set Edge}
    pred hampath[g: Graph, path: (seq Node)] {
                                                         pred hampath[g: Graph, path: seq Node] {
    path[Int] == g.nodes and
                                                         path[Int] = g.nodes
     path.size == g.nodes.size and
                                                          \#path = \#q.nodes
                                                          all i: Int | i >= 0 && i < minus[#path,1] => {
     all(i: 0...path.size-1) |
                                                    ⇔ 9
      some(e: g.edges) {
                                                          some e: g.edges |
                                                     10
      e.src == path[i] && e.dst == path[i+1] }
                                                           e.src = path[i] && e.dst = path[plus[i,1]] }
                                                     11
12
                                                     12
                                                         assert reach {
    assertion reach {
                                                     13
     all(g: Graph, path: (seq Node)) |
                                                          all g: Graph, path: seq Node |
     if hampath(g, path)
                                                            hampath[g, path] =>
      g.nodes.in? path[0].*((~src).dst)
                                                     16
                                                             g.nodes in path[0].*(~src.dst)
16
                                                     17
    run :hampath, 5, Graph=>exactly(1), Node=>3
                                                         run hampath for 5 but exactly 1 Graph, 3 Node
                                                     18
                                                        check reach for 5 but exactly 1 Graph, 3 Node
   check :reach, 5, Graph=>exactly(1), Node=>3
   module GraphModel
                                                        # find an instance satisfying the :hampath pred
    class Node; attr_accessor :val end
                                                        sol = GraphModel.run hampath
    class Edge: attr_accessor :src. :dst end
                                                        assert sol.satisfiable?
    class Graph; attr_accessor :nodes, :edges end
                                                        q, path = sol["$hampath_q"], sol["$hampath_path"]
                                                        puts g.nodes # => e.g., {<Node$0>, <Node$1>}
                                                        puts g.edges # => e.g., {<Node$1, Node$0>}
    def self.hampath(g, path) #same as above end
                                                        puts path # => {<0, Node$1>, <1, Node$0>}
# check that the "reach" assertion holds
    def self.reach()
                              #same as above end
    def self.run_hampath() exe_cmd :hampath end
                                                        sol = GraphModel.check reach
    def self.check_reach() exe_cmd :reach end
   end
                                                        assert !sol.satisfiable?
                                                       (d) Running hampath, checking reach
 (c) Automatically generated Ruby classes
```

Fig. 1. Hamiltonian Path example.

### 4 Beyond Standard Analysis

Sudoku has become a popular benchmark for demonstrating constraint solvers. The solver is given a partially filled  $n \times n$  grid (where n must be a square number, so that the grid is perfectly divided into n times  $\sqrt{n} \times \sqrt{n}$  sub-grids), and is required to fill the empty cells with integers from  $\{1,\ldots,n\}$  so that all cells within a given row, column, and sub-grid have distinct values.

Implementing a Sudoku solver directly in Alloy poses a few problems. A practical one is that such an implementation cannot easily be used as a stand-alone application, e.g., to read a puzzle from some standard format and display the solution in a user-friendly grid. A more fundamental problem is the inability to express the information about the pre-filled cell values as a partial instance; instead, the given cell values have to be enforced with logical constraints, resulting in significant performance degradation [18]. The  $\alpha$ Rby solution in Fig. 2 addresses both of these issues: on the left is the formal  $\alpha$ Rby specification, and on the right is the Ruby code constructing bounds and invoking the solver for a concrete puzzle.

```
allov : SudokuModel do
                                                   class SudokuModel::Sudoku
    SudokuModel::N = 9
                                                    def pi
                                                     bnds = Arby::Ast::Bounds.new
   sia Sudoku[arid: Int ** Int ** (lone Int)]
                                                     inds = (0...N)**(0...N) - self.grid.project(0..1)
                                                                          = self
                                                     bnds[Sudoku]
   pred solved[s: Sudoku] {
                                                     bnds.lo[Sudoku.grid] = self ** self.grid
        = Integer(Math.sqrt(N))
                                                     bnds.hi[Sudoku.grid] = self ** inds ** (1..N)
     rng = lambda{|i| m*i...m*(i+1)}
                                                    bnds.bound_int(0..N)
     all(r: 0...N) {
                                                    def solve() SudokuModel.solve :solved, self.pi end
     s.grid[r][Int] == (1..N) and
                                                    def display() puts grid end
                                                    def self.parse(s) Sudoku.new grid:
      s.grid[Int][r] == (1..N)
                                                      s.split(/;\s*/).map\{|x|\ x.split(/,/).map(\&:to_i)\}
     all(c, r: 0...m) {
      s.grid[rng[c]][rng[r]] == (1..N)
16
                                                   SudokuModel.N = 4
                                                   s = Sudoku.parse "0,0,1; 0,3,4; 3,1,1; 2,2,3"
17
                                                   s.solve(); s.display(); \# => \{<0,0,1>,<0,1,3>,
   end
```

**Fig. 2.** A declarative *Sudoku* solver using  $\alpha$ Rby with partial instances.

**Mixed execution** The imperative statements (lines 2, 7, 8) used to dynamically produce a Sudoku specification for a given size would not be directly expressible in Alloy. A concrete Ruby variable N is declared to hold the size, and can be set by the user before the specification is symbolically evaluated. Another imperative statement calculates the square root of N (line 6); that value is later embedded in the symbolic expression specifying uniqueness within sub-grids (line 13). For illustration purposes, a lambda function is defined (line 7) and used to compute sub-grid ranges (line 14).

Fig. 2(b) shows how the bounds are computed for a given Su-Partial instances doku puzzle, using a Ruby function pi (for "partial instance"). Remember that bounds are just tuples (sequences of atoms) that a relation must or may include; since signature definitions in  $\alpha$ Rby are turned into regular Ruby classes, instances of those classes will be used as atoms. The Sudoku signature is bounded by a singleton set containing only the self Sudoku object (line 5). Tuples that must be included in the grid relation are the values currently present in the puzzle (line 6); additional tuples that may be included are values from 1 to N for the empty cells (line 7; empty cell indexes computed in line 4). We also bound the set of integers to be used by the solver; Alloy, in contrast, only allows a cruder bound, and would include all integers within a given bitwidth. Finally, a Sudoku instance can be parsed from a string, and the solver invoked to find a solution satisfying the solved predicate (lines 17–18). When a satisfying solution is found, if a partial instance was given, fields of all atoms included in that partial instance are automatically populated to reflect the solution (confirmed by the output of line 18). This particular feature makes for seamless integration of executable specifications into otherwise imperative programs, since there is no need for any manual back and forth conversion of data between the program and the solver.

**Staged model finding** Consider implementing a Sudoku puzzle generator. The goal is now to find a partial assignment of values to cells such that the generated puzzle has a unique solution. Furthermore, the generator must be able to produce various dif-

ficulty levels of the same puzzle by iteratively decrementing the number of filled cells (while maintaining the uniqueness property). This is a higher-order problem that cannot be solved in one step in Alloy. With  $\alpha$ Rby, however, it takes only the following 8 lines to achieve this with a simple search algorithm on top of the already implemented solver:

```
def dec(sudoku, order=Array(0...sudoku.grid.size).shuffle)
return nil if order.empty? # all possibilities exhausted
s_dec = Sudoku.new grid: sudoku.grid.delete_at(order.first) # delete a tuple at random position
sol = s_dec.clone.solve() # clone so that "s_dec" doesn't get updated if a solution is found
(sol.satisfiable? && !sol.next.satisfiable?) ? s_dec : dec(sudoku, order[1..-1])
end
def min(sudoku) (s1 = dec(sudoku)) ? min(s1) : sudoku end
s = Sudoku.new; s.solve(); s = min(s); puts "local minimum found: #{s.grid.size}"
```

The strategy here is to generate a solved puzzle (line 8), and keep removing one tuple from its grid at a time until a local minimum is reached (line 7); the question is which one can be removed without violating the uniqueness property. The algorithm first generates a random permutation of all existing grid tuples (line 1) to determine the order of trials. It then creates a new Sudoku instance with the chosen tuple removed (line 3) and runs the solver to find a solution for it. It finally calls next on the obtained solution (line 5) to check if a different solution exists; if it does not, a decremented Sudoku is found, otherwise moves on to trying the rest of the tuples. On a commodity machine, on average it takes about 8 seconds to minimize a Sudoku of size 4 (generating 13 puzzles in total, number of filled cells ranging from 16 down to 4), and about 6 minutes to minimize a puzzle of size 9 (55 intermediate puzzles), number of filled cells ranging from 81 down to 27).

## 5 The $\alpha$ Rby Language

 $\alpha$ Rby is implemented as a domain-specific language in Ruby, and is (in standard parlance) "deeply embedded". *Embedded* means that all syntactically correct  $\alpha$ Rby programs are syntactically correct Ruby programs; *deeply* means that  $\alpha$ Rby programs exist as an AST that can be analyzed, interpreted, and so on. Ruby's flexibility makes it possible to create embedded languages that look quite different from standard Ruby.  $\alpha$ Rby exploits this, imitating the syntax of Alloy as closely as possible. Certain differences are unavoidable, mostly because of Alloy's infix operators that cannot be defined in Ruby.

The key ideas behind our approach are: (1) mapping the core Alloy concepts directly to those of object-oriented programming (OOP), (2) implementing keywords as methods, and (3) allowing mixed (concrete and symbolic) execution in  $\alpha$ Rby programs.

**Mapping Alloy to OOP** is aligned with the general intuition, encouraged by Alloy's syntax, that signatures can be understood as classes, atoms as objects, fields as instance variables, and all function-like concepts (functions, predicates, facts, assertions, commands) as methods [2].

Implementing keywords as methods works because Ruby allows different formats for specifying method arguments.  $\alpha$ Rby defines many such methods (e.g., sig, fun, fact, etc.) that (1) mimic the Alloy syntax and (2) dynamically create the underlying Ruby class structure using the standard Ruby metaprogramming facilities. For

an example of the syntax mimicry, compare Figs. 1(a) and 1(b)); for an example of metaprogramming, see Fig. 1(c).

Note that the meta information that appears to be lost in Fig. 1(c) (for example, the types of fields) is actually preserved in separate *meta* objects and made available via the *meta* methods added to each of the generated modules and classes (e.g., Graph.meta.field("nodes").type).

**Mixed execution**, implemented on top of the standard Ruby interpreter, translates  $\alpha$ Rby programs into symbolic Alloy models. Using the standard interpreter means adopting the Ruby semantics of name resolution and operator precedence (which is inconvenient when it conflicts with Alloy's); a compensation, however, is the benefit of being able to mix symbolic and concrete code. We override all the Ruby operators in our symbolic expression classes to match the semantics of Alloy, and using a couple of other tricks (Section 5.3), are able to keep both syntactic (Section 5.1) and semantic (Section 5.2) differences to a minimum.

# 5.1 Syntax

A grammar of  $\alpha$ Rby is given in Fig. 3 and examples of principal differences in Table 1. In a few cases (e.g., function return type, field declaration, etc.) Alloy syntax has to be slightly adjusted to respect the syntax of Ruby (e.g., by requiring different kind of brackets). More noticeable differences stem from the Alloy operators that are illegal or cannot be overridden in Ruby; as a replacement, either a method call (e.g., size for cardinality) or a different operator (e.g., \*\* for cross product) is used.

The difference easiest to overlook is the equality sign: == versus =. Alloy has no assignment operator, so the single equals sign always denotes boolean equality;  $\alpha R$ by, in contrast, supports both concrete and symbolic code, so we must differentiate between assignments and equality checks, just as Ruby does.

The tokens for the join and the two closure operators (., ^ and \*) exist in Ruby, but have fundamentally different meanings than in Alloy (object dereferencing and an infix binary operator in Ruby, as opposed to an infix binary and a prefix unary operator in Alloy). Despite this,  $\alpha$ Rby preserves Alloy syntax for many idiomatic expressions. Joins in Alloy are often applied between an expression and a field whose left-hand type matches the type of the expression (ie, in the form e.f, where f is a field from the type of e). This corresponds closely to object dereferencing, and is supported by  $\alpha$ Rby (e.g., g.nodes in Fig. 1(a)). In other kinds of joins, the right-hand side must be enclosed in parentheses. Closures are often preceded by a join in Alloy specifications. Those constructs yield *join closure* expressions of the form x.\*f. In Ruby, this translates to calling the \* method on object x passing f as an argument, so we simply override the \* method to achieve the same semantics (e.g., line 15, Fig. 1(a)).

This grammar is, for several reasons, an under-approximation of programs accepted by  $\alpha Rby$ : (1) Ruby allows certain syntactic variations (e.g., omitting parenthesis in method calls, etc.), (2)  $\alpha Rby$  implements special cases to enable the exact Alloy syntax for certain idioms (which do not always generalize), and (3)  $\alpha Rby$  provides additional methods for writing expression that have more of a Ruby-style feel.

```
::= "alloy" cname "do" [open*] paragraph* "end"
spec
           ::= "open" cnameID
open
paragraph ::= factDecl | funDecl | cmdDecl | sigDecl
          ::= "abstract"
                            "lone" | "one"
                                                   | "some" | "ordered"
sigQual
           ::= sigQual* "sig" cname,+ ["extends" cnameID] ["[" rubyHash "]"] [block]
sigDecl
factDecl
          ::= "fact"
                                 [fname] block
           | "rubyHash "]" "[" expr "]" block | "pred" | fname "[" rubyHash "]" | block | "crum"|"check") fname "," scope
funDecl
cmdDecl
           | ("run"|"check") "(" scope ")" block
::= ID | rubyInt | rubyBool | "(" expr ")"
expr
                                    | unMeth "(" expr ")"
               unOp expr
                                  expr "[" expr "]"
                                                               | expr "if" expr
                expr binOp expr
               expr "." "(" expr ")" // relational join
expr "." (binMeth | ID) "(" expr,* ")" // function/predicate call
             | "if" expr "then" expr ["else" expr] "end" | quant "(" rubyHash ")" block
           quant
bin0p
binMeth
           ::= "!" | "-" | "not"
::= "no" | "some" | "lone" | "one" | "set" | "seq"
::= "{" stmt* "}" | "do" stmt* "end"
un0n
unMeth
block
           ::= expr | rubyStmt
::= rubyInt "," rubyHash // global scope, individual sig scopes
stmt
scope
           ::= cnameID | fnameID
::= cnameID | '"'cnameID'"' | "'"cnameID"'" | ":"cnameID
ID
cname
           ::= fnameID | '"'fnameID'"' | "'"fnameID"'" | ":"fnameID
fname
           ::= \textit{constant identifier in Ruby (starts with upper case)}
cnameID
fnameID
           ::= function identifier in Ruby (starts with lower case)
```

**Fig. 3.** Core  $\alpha$ Rby syntax in BNF. Productions starting with: ruby are defined by Ruby.

description	Alloy	$\alpha$ <b>Rby</b>
equality	x = y	x == y
sigs and fields	<pre>sig S {    f: lone S -&gt; Int }</pre>	<pre>sig S [    f: lone(S) ** Int ]</pre>
fun return type declaration	<pre>fun f[s: S]: set S {}</pre>	<pre>fun f[s: S][set S] {}</pre>
set comprehension	{s: S   p1[s]}	S.select{ s  p1(s)}
quantifiers	<pre>all s: S {   p1[s]   p2[s] }</pre>	all(s: S) {   p1(s) and   p2(s) }
illegal Ruby operators	<pre>x in y, x !in y x !&gt; y x -&gt; y function funct</pre>	<pre>x.in?(y), x.not_in?(y) not x &gt; y x ** y x.(y) x.size y if x if x then y else z S.&lt; f, f.&gt; Int</pre>
operator arity mismatch	^x, *x	x.closure, x.rclosure
fun/pred calls	f1[x]	f1(x)

**Table 1.** Examples of differences in syntax between  $\alpha$ Rby and Alloy.

#### 5.2 Semantics

This section formalizes the translation of  $\alpha$ Rby programs into Alloy. We provide semantic functions (summarized in Fig. 5) that translate the syntactic constructs of Fig. 3 to Alloy AST elements defined in Fig. 4. A store, binding names to expressions or declarations, is maintained throughout, representing the current evaluation context.

**Expressions** The evaluation of the  $\alpha$ Rby expression production rules (expr) into Alloy expressions (Expr) is straightforward for the most part (Fig. 6). Most of the unary and binary operators have the same semantics as in Alloy; exceptions are \*\* and if, which translate to -> and => (lines 5–11). For the operators that do not exist in Ruby, an equivalent substitute method is used (lines 12–20). A slight variation of this approach is taken for the ^ and \* operators (lines 21–22), to implement the "join closure" idiom (explained in Section 5.1).

The most interesting part is the translation of previously undefined method calls (lines 23–27). We first apply the  $\tau$  function to obtain the type of the left-hand side expression, and then the  $\oplus$  function to extend the current store with that type (line 23). In a nutshell, this will create a new store with added bindings for all fields and functions defined for the range signature of that type (the  $\oplus$  function is formally defined in Fig. 8 and discussed in more detail shortly). Afterward, we look up meth as an identifier in the new store (line 24) and, if an expression is found (line 25), the expression is interpreted as a join; if a function declaration is found (line 26), it is interpreted a function call; otherwise, it is an error.

For quantifiers (lines 29-30), quantification domains are evaluated in the context of the current store (using the  $\delta$  helper function, defined in Fig. 8) and the body is evaluated (using the  $\beta$  function, defined in Fig. 7) in the context of the new store with added bindings for all the quantified variables (returned previously by  $\delta$ ).

**Blocks** The semantics of  $\alpha$ Rby blocks differs from Alloy's. An Alloy block (e.g., a quantifier body) containing a sequence of expressions is interpreted as a conjunction of all the constituent constraints (a feature based on Z [17]). In  $\alpha$ Rby, in contrast, such as sequence evaluates to the meaning of the last expression in the sequence. This was a design decision, necessary to support mixed execution (as in Fig. 2(a), lines 7–16). Since Ruby is not a pure functional language, previous statements can affect the result of the last statement by mutating the store, which effectively gives us the opportunity to easily mix concrete and symbolic execution.

This behavior is formally captured in the  $\beta$  function (Fig. 7). Statements  $(s_1, ..., s_n)$  are evaluated in order (line 32). If a statement corresponds to one of the expression rules from the  $\alpha$ Rby grammar (line 34), it is evaluated using the previously defined  $\mathcal E$  function; otherwise (line 35), it is interpreted by Ruby (abstracted as a call to the  $\mathcal R$  functions). Statements interpreted by Ruby may change the store, which is then passed on to the subsequent statements.

**Function declarations** The evaluation function ( $\phi$ , Fig. 7, lines 37–38) is similar to quantifier evaluation, except that the return type is different. The semantics of other function-like constructs (predicates, facts, etc.) is analogous, and is omitted for brevity.

Fig. 4. Overview of the semantic domains. (Expr and Decl correspond directly to the Alloy AST)

Fig. 5. Overview of the semantic functions which translate grammar rules to semantic domains.

```
\mathcal{E}: \mathsf{expr} \to \mathsf{Store} \to \mathsf{Expr}
       \mathcal{E}[\![ D ]\!] \sigma
                                                                       \equiv \sigma[ID]
       \mathcal{E}[\![\mathsf{rubyInt}]\!]\sigma
                                                                       ■ IntExpr(rubyInt)
       \mathcal{E}\llbracket\mathsf{rubyBool}\rrbracket\sigma
                                                                       ■ BoolExpr(rubyBool)
       \mathcal{E}[\![(e)]\!]\sigma
                                                                       \equiv \mathcal{E}[\![e]\!]\sigma
       \mathcal{E}\llbracket\mathsf{un0p}\,e
rbracket\sigma
                                                                       ≡ UnExpr(unOp,
       \mathcal{E}[\mathbf{unMeth}(e)]\sigma
                                                                       \equiv UnExpr(unMeth, \mathcal{E}[\![e]\!]\sigma)
                                                                       \equiv BinExpr("->", \mathcal{E}[e_1]\sigma, \mathcal{E}[e_2]\sigma)
       \mathcal{E}\llbracket e_1 ** e_2 \rrbracket \sigma
       \mathcal{E}\llbracket e_1 \; \mathsf{binOp} \; e_2 
rbracket \sigma
                                                                       \equiv BinExpr(binOp, \mathcal{E}[e_1]\sigma, \mathcal{E}[e_2]\sigma)
                                                                       \equiv \, \mathsf{BinExpr}(\texttt{"[]",} \, \mathcal{E}[\![e_1]\!]\sigma, \, \mathcal{E}[\![e_2]\!]\sigma)
       \mathcal{E}\llbracket e_1[e_2] \rrbracket \sigma
10. \mathcal{E}\llbracket e_1 \text{ if } e_2 \rrbracket \sigma
                                                                       \equiv BinExpr("=>", \mathcal{E}[\![e_2]\!]\sigma, \mathcal{E}[\![e_1]\!]\sigma)
                                                                       \equiv BinExpr(".", \mathcal{E}[\![e_1]\!]\sigma, \mathcal{E}[\![e_2]\!]\sigma)
11. \mathcal{E}[[e_1.(e_2)]]\sigma
12. \mathcal{E}[e.binMeth]\sigma
                                                                       ≡ match binMeth with
13.
                                                                            l closure → UnExpr("^",
                                                                                                                                                \mathcal{E}[e]\sigma
                                                                            I rclosure → UnExpr("*",
14.
                                                                                                                                                \mathcal{E}[\![e]\!]\sigma
                                                                            I size
                                                                                                      → UnExpr("#",
                                                                                                                                                \mathcal{E}[\![e]\!]\sigma)
15.
       \mathcal{E}[\![e.\mathsf{binMeth}(a_1)]\!]\sigma
                                                                       match binMeth with
16.
                                                                                                     \rightarrow BinExpr("in", \mathcal{E}[\![e]\!]\sigma, \mathcal{E}[\![a_1]\!]\sigma)
17.
                                                                            l in?
                                                                                                      \rightarrow BinExpr(">>>", \mathcal{E}[e]\sigma, \mathcal{E}[a_1]\sigma)
18.
                                                                            l shr

ightarrow BinExpr("<:", \mathcal{E}[\![e]\!]\sigma, \mathcal{E}[\![a_1]\!]\sigma)
                                                                            | <
19.
                                                                                                      \rightarrow BinExpr(":>", \mathcal{E}[\![e]\!]\sigma, \mathcal{E}[\![a_1]\!]\sigma)
20.
                                                                                                      \rightarrow \mathcal{E}[e.(a_1.\text{closure})]\sigma
21.
                                                                                                      \rightarrow \mathcal{E}[e.(a_1.\text{rclosure})]\sigma
                                                                            1 *
22
       \mathcal{E}[\![e.\mathsf{ID}(a_1,\ldots)]\!]\sigma
                                                                       \equiv let \sigma_{sub} = \sigma \oplus \tau(e) in
23.
24.
                                                                             match \sigma_{sub}[{	t ID}] as x with
                                                                                                       \rightarrow BinExpr(".", \mathcal{E}[\![e]\!]\sigma, x)
25.
26.
                                                                             | FunDecl \rightarrow CallExpr(\mathcal{E}[\![e]\!]\sigma, x, \mathcal{E}[\![a_1]\!]\sigma, ...)
2.7
                                                                                                        \rightarrow fail
28. \mathcal{E}[\![if e_1 then e_2 else e_3 end\![]\sigma \equiv \mathcal{E}[\![(e_2 if e_1) and (e_3 if !e_1)\![]\sigma
29. \mathcal{E}[[\mathsf{quant}(d_*) \, \mathsf{block}]] \sigma
                                                                       \equiv let v_*, \sigma_b = \delta(d_*)\sigma in
                                                                               QuantExpr(quant, v_*, \beta[block]\sigma_b)
```

**Fig. 6.** Evaluation of  $\alpha$ Rby expressions (expr production rules) into Alloy expressions (Expr).

```
\beta: block \rightarrow Store \rightarrow Expr
     \beta[\![\operatorname{do} s_1;\ldots;s_n\operatorname{end}]\!]\sigma\equiv\beta[\![\{s_1;\ldots;s_n\}]\!]\sigma\equiv\sigma_{curr}=\sigma,res=\operatorname{nil}
32
                                                                           for s_i: \{s_1, \ldots, s_n\} do
33.
                                                                               match s_i with
34.
                                                                               l \exp r \rightarrow res \leftarrow \mathcal{E}[s_i] \sigma_{curr}
35.
                                                                                                res, \sigma_{curr} \leftarrow \mathcal{R}(s_i)\sigma_{curr}
36.
                                                                            return \ res
\phi: \mathsf{funDecl} \to \mathsf{Store} \to \mathsf{FunDecl}
     \phi \llbracket \text{fun fname}[d_*][e_{ret}] \text{ block} \rrbracket \sigma \equiv \text{let } v_*, \ \sigma_b = \delta(d_*) \sigma \text{ in}
                                                           FunDecl(fname, v_*, \mathcal{E}[e_{ret}]\sigma, \beta[block]\sigma_b)
\xi: sigDecl \rightarrow Store \rightarrow SigDecl
      \xi \llbracket \mathbf{sig} \text{ cname extends sup } [d_*] \text{ block} \rrbracket \sigma \equiv
                             = \tau(\sigma[\sup]) in
40.
         let s_p
41.
         let fld_*, _ = \delta(d_*)\sigma in
         let this
                             = VarExpr("this", cname) in
42.
43.
         let tfld_*
                            = map(\lambda f_i \cdot BinExpr(".", this, f_i), fld_*) in
                             = \sigma \oplus \mathrm{SigDecl}(\mathrm{cname}, s_p, tfld_*, \mathrm{BoolExpr}(\mathrm{true})) in
44
             SigDecl(cname, s_p, fld_*, \beta[block]\sigma_s["this" \mapsto this])
45.
A: \operatorname{spec} \to \operatorname{Store} \to \operatorname{Spec}
       \mathcal{A}[\![alloy cname do open* paragraph* end\![\![]\sigma]
46.
         let opn_*
                             = map(\lambda cnameID \cdot \sigma[cnameID], open*) in
47.
                             = map(\xi, filter(sigDecl, paragraph*)) in
48
         let sig_*
                             = map(\phi, filter(funDecl, paragraph*)) in
         let fun_*
49
50.
                             = Spec(cname, opn_*, sig_*, fun_*) in
             if resolved(a) then a
             elsif \sigma \oplus a \neq \sigma then \mathcal{A}[alloy cname do open* paragraph* end] <math>\sigma \oplus a else fail
                                Fig. 7. Evaluation of blocks and all declarations.
\delta: decl* \rightarrow Store \rightarrow (VarExpr*, Store)
     \delta(v_1:e_1, ..., v_n:e_n)\sigma \equiv let vars = \bigcup_{1<=i<=n} \mathsf{VarExpr}(v_i,~\mathcal{E}[\![e_i]\!]\sigma) in
53
                                                 [vars, \sigma \oplus vars]
54.
\oplus: Store \rightarrow Any \rightarrow Store
     \sigma \oplus x \equiv \mathsf{match}\ x \ \mathsf{with}
                 56.
                 | VarExpr* | FunDecl*
                                                                   \rightarrow fold(\oplus, \sigma, x)
57
                 I ProductType(_-, rhs)
                                                                   \rightarrow \sigma \oplus rhs
58
59.
                 | SigDecl(n, s_p, fld_*, -)
                                                                   \rightarrow let \sigma_p = \sigma \oplus s_p in
                                                                       let \sigma_s = \sigma_p[n \mapsto VarExpr(n,x)] in
60.
                                                                          fold(\oplus, \sigma_s, funs(x) + fld_*)
61.
                 I Spec(n, opn_*, sig_*, fun_*)
62
                                                                   \rightarrow fold(\oplus, \sigma, opn_* + fun_* + sig_*)
                                                                   \rightarrow \sigma
63
\mathcal{R}
               : rubyStmt \rightarrow Store \rightarrow (Object \rightarrow Store)
                                                                                         Executes arbitrary Ruby code
               : Expr → Type
                                                                                          Type of the given expression
funs
               : SigDecl → FunDecl*
                                                                           Functions where given sig is first param
resolved: Spec \rightarrow Bool
                                                                                 Whether all references are resolved
```

Fig. 8. Helper functions.

**Signature declarations** The evaluation function (function  $\xi$ , Fig. 7, lines 39–45) is conceptually straightforward: as before, functions  $\delta$  and  $\beta$  can be reused to evaluate the field name-domain declarations and the appended facts block, respectively. The caveat is that appended facts in Alloy must be evaluated in the context of Alloy's *implicit this*, meaning that the fields from the parent signature should be implicitly joined on the left with an implicit this keyword. To achieve this, we create a variable corresponding to this and a new list of fields with altered domains (lines 42–43). A temporary SigDecl containing those fields is then used to extend the current store (line 44). A binding for this is also added and the final store is used to evaluate the body (line 45). The temporary signature is created just for the convenience of reusing the semantics of the  $\oplus$  operator (explained shortly).

**Top-level specifications** Evaluation of an  $\alpha$ Rby specification (function  $\mathcal{A}$ , Fig. 7, lines 46–52) uses the previously defined semantic functions to evaluate the nested signatures and functions. Since declaration order does not matter in Alloy, multiple passes may be needed until everything is resolved or a fixed point is reached (lines 51–52).

Name-domain declaration lists Name-domain lists are used in several places (for fields, method parameters, and quantification variables); common functionality is extracted and defined in the  $\delta$  function (Fig. 8). It simply maps the input list into a list of VarExpr expressions, each having name the same as in the declaration list and domain equal to the evaluation of the declared domain against the current store (line 53). It returns that list and the current store extended with those variables (line 54).

Store extension The  $\oplus$  operator (Fig. 8) is used to extend a store with one or more VarExpr or FunDecl, a Type, and a Spec. If a VarExpr or a FunDecl is given, its name is bound to itself. If a list is given, the operation is folded over the entire list. Extending with a Type reduces to extending with the range of that type. Extending with a SigDecl means recursively adding bindings for its parent signature, adding a binding for the name of that signature, bindings for all the functions that take that signature as the first argument (an auxiliary function funs(x) discovers such functions), and bindings for all its fields. Extending with a Spec adds bindings for all the sigs and functions defined in it, including those from all opened specifications.

#### 5.3 Implementation Considerations

**Symbolic Execution** Using the standard Ruby interpreter to symbolically execute  $\alpha$ Rby programs relieves us from having to keep an explicit representation of the store; instead, the store is implicit in the states of the object in which the execution takes place. Having signatures, fields, and functions represented directly as classes, instance variables, and methods, means having most of the bindings (as defined in Section 5.2) already in place for all sigs and atoms; for all other expressions, missing methods are dynamically forwarded to the signature class corresponding to the expression's type.

One technical challenge is that the semantics of quantifiers requires a new scope to be created, which, for our syntax, Ruby does not already ensure. Consider the following  $\alpha$ Rby code: all(s: S){some s}. This is just a hash and a block passed to our domain-specific-language method. When the block is eventually executed (to obtain the

symbolic body for this universal quantifier), s must be available as a symbolic variable inside of that block. We do that by first dynamically defining a method with the same name in the context of that block, then calling the block, and, finally, redefining the same method to call super:

```
ctx = block.binding.eval("self")
ctx.define_singleton_method :s, lambda{VarExpr.new(:s, S)}
begin block.call ensure ctx.define_singleton_method(:s) do super() end end
```

Responding to Missing Methods and Constants To avoid requiring strings instead of identifiers for every new definition (e.g., sig: Graph instead of sig Graph, where Graph is previously undefined),  $\alpha$ Rby overrides const\_missing and method\_missing and instead of failing returns a MissingBuilder instance. Furthermore, MissingBuilder instances also accept a block at creation time, and respond to several operator methods, making constructs like fun f[s: S][set S] {} possible. To guard against unintended conversions (e.g., typos),  $\alpha$ Rby raises a syntax error every time a MissingBuilder is not "consumed" (by certain DSL methods, like sig and fun) by the end of its scope block.

Online Source Instrumentation For the purpose of symbolic evaluation, the source code of every  $\alpha R$  by function/predicate is instrumented before it is turned into a Ruby method. The need for instrumentation arises because certain operators and control structures, which we would like to treat symbolically, cannot be overridden; examples include all the if-then-else variants, as well as the logic operators. Our instrumentation uses an off-the-self parser and implements a visitor over the generated AST to replace these constructs with appropriate  $\alpha R$  by expressions (e.g., x if y gets translated to BinExpr.new(IMPLIES, proc{y}, proc{x})). This "traverse and replace" algorithm is far simpler than implementing a full parser for the entire Alloy grammar.

**Distinguishing Equivalent Ruby Constructs** Ruby allows different syntactic constructs for the same underlying operation. For example, some built-in infix operators can be written with or without a dot between the left-hand side and the operator (e.g., a\*b is equivalent to a.\*b). Since  $\alpha$ Rby already performs online source instrumentation, it additionally detects the following syntactic nuances for the purpose of assigning different semantics: (1) in Ruby, "<b2>if <b1>" is equivalent to "b1 and b2", but our instrumenter always rewrites and and or to boolean conjunction and disjunction; (2) when prefixed with a dot, operators \*, < and > are translated to join closure, domain restriction, and range restriction, respectively (.\*, <:, and :> in Alloy).

 $\alpha$ **Rby to Alloy Bridge** All model-finding tasks are delegated to a slightly modified version of the official Alloy Analyzer Java implementation. The main modification we made was adding an extra API method, which additionally accepts a partial instance (represented in a simple textual format independent of the Alloy language). The Alloy Analyzer already has a complex heuristic for computing bounds from the scope specification and certain (automatically detected) idioms; we retain all those features, and on top of them use the  $\alpha$ Rby-provided partial instance to shrink the bounds further (formalization of which is beyond the scope of this paper). To interoperate between Ruby and Java, we use RJB [14], which conveniently automates most of the process.

### 6 Related Work

Montaghami and Rayside [11] extended the Alloy language with special syntax for specifying partial instances. They argue convincingly for the importance of having partial instances for Alloy, giving use cases such as test-driven model development, regression testing of models, modeling by example etc. They also provide experimental evidence that staged model finding can lead to better scalability. Their approach is limited to partial instances only, and it does not provide any scripting mechanisms for automating such tasks. Thus to carry out their staged model finding experiments, after obtaining an instance in the first stage, they manually inspected it (e.g., in the visualizer), rewrote it using the new syntax, and then solved in the second stage. Using  $\alpha Rby$  would automate the whole process, since an  $\alpha Rby$  instance can provide a set of exact bounds for all included relations, and can handle all the use cases discussed.

A number of tools built on top of Alloy have implemented (often in an ad hoc fashion) one or more features that can now be provided by  $\alpha R$ by. Aluminum [13] implements an interesting heuristic for minimizing Alloy instances and by default showing the minimal one first. It also allows the user to augment the current instance by selecting one or more tuples to be included in the next instance.  $\alpha R$ by provides a more generic mechanism that lets the user provide an arbitrary formula (possibly involving atoms from the current instance) to be satisfied in the next solution. TACO [5] is a bounded verifier for Java that achieves scalability by relying heavily on the Alloy Analyzer to recognize certain idioms as partial instances; we believe  $\alpha R$ by would have made their implementation much simpler.

Our mixed execution was inspired by Rubicon's [12] symbolic evaluator, which also uses the standard Ruby interpreter. Unlike  $\alpha$ Rby, Rubicon stubs the library code with custom expressions in order to symbolically execute and verify existing web apps.

Many research projects explore the idea of extending a programming language with symbolic constraint-solving features (e.g., [15,10,9,20,19,16]).  $\alpha$ Rby can be understood as a kind of dual, with the opposite goal. While these efforts aim to bring declarative features in imperative programming,  $\alpha$ Rby aims to bring imperative features to declarative modeling. Although the basic idea of combining declarative model finding and imperative model finding is shared, the research challenges are very different. As this paper has explained,  $\alpha$ Rby addresses the challenge of embedding an entire modeling language in a programming language, whereas these related projects instead tend to use a constraint language that is only a modest extension of the programming language's existing expression sublanguage.  $\alpha$ Rby also addresses the challenge of reconciling two different views of a data structure: one as objects on a heap, and the other as relations (and in this respect is related to work on relational data representation, such as [6]).

### 7 Conclusion

On the one hand,  $\alpha R$  by addresses a collection of very practical problems in the use of a model finding tool. This paper's contribution can thus be regarded as primarily architectural, in demonstrating a different way to build an analysis tool that uses a DSL embedding to allow end-user scripting, rather than a closed compiler-like tool that can be extended only by one of the tool's developers.

On the other hand,  $\alpha R$  by suggests a new way to think about a modeling language. The constructs of the language are not treated as functions that generate abstract syntax trees only in a mathematical sense, but are implemented as these functions in a manner that the end user can exploit. This leads us to wonder whether it might be possible to use this style of embedding in the very design of the modeling language. Perhaps, had this approach been available when Alloy was designed, an essential core might have been more cleanly separated from a larger collection of structuring idioms, implemented as functions on top of the core's functions.

Practically speaking, we hope that the developers of tools that use Alloy as a backend will be able to use  $\alpha$ Rby in their implementations, at the very least making it easier to prototype new functionality. And perhaps the implementors of tools for other declarative languages will find ideas here that they can exploit in similar embeddings.

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