FYS2140 - Oblig 5

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Oppgave 1

Forventningsverdien til x er:

$$< x >= \int_0^a x\psi(x)_n^*\psi(x)_n dx$$

$$= \int_0^a \frac{2}{a}\sin^2(\frac{n\pi}{a}x)xdx$$

$$\frac{2}{a}\int_0^a x\sin^2(\frac{n\pi}{a}x)dx$$

Vi substituerer $\frac{n\pi}{a}x = u \Rightarrow x = \frac{a}{n\pi}u$, $dx = \frac{a}{n\pi}du$, $u(0) = 0, u(a) = n\pi$:

$$< x >= \frac{2}{a}\int_{u(0)}^{u(a)} \frac{a}{n\pi}u\sin^2(u)\frac{a}{n\pi}du$$

$$= \frac{2a}{n^2\pi^2}\int_0^{n\pi} u\sin^2(u)du$$

Vi løser integralet $\int u\sin^2(u)du$ ved delvis integrasjon:

$$\int u\sin^2(u)du = \frac{1}{2}u^2\sin^2(u) - \int \frac{1}{2}u^2 \cdot 2\sin(u)\cos(u)du$$

$$= \frac{1}{2}u^2\sin^2(u) - \frac{1}{2}\int u^2\sin(2u)du$$

Fra Rottmann finner vi løsningen av integraler på formen:

$$\int u^2\sin(au)du = \frac{2}{a^2}u\sin(au) - \frac{2-a^2u^2}{a^3}\cos(au) + C$$

Vi har altså:

$$\int u\sin^2(u)du = \frac{1}{2}u^2\sin^2(u) - \frac{1}{2}\left(\frac{2}{2^2}u\sin(2u) + \frac{2-2^2u^2}{2^3}\cos(2u) + c\right)$$

$$= \frac{1}{2}u^2\sin^2(u) - \frac{1}{4}u\sin(2u) - \frac{1}{8}\cos(2u) + \frac{1}{4}u^2\cos(2u) + C$$

$$= \frac{1}{2}u^2\left(\sin^2(u) + \frac{1}{2}\cos(2u)\right) - \frac{1}{4}u\sin(2u) - \frac{1}{8}\cos(2u) + C$$

$$\frac{1}{2}u^2\left[\sin^2(u) + \frac{1}{2}\left(1-2\sin^2(u)\right)\right] - \frac{1}{4}u\sin(2u) - \frac{1}{8}\cos(2u) + C$$

$$= \frac{1}{4}u^2 - \frac{1}{4}u\sin(2u) - \frac{1}{8}\cos(2u) + c$$

Forventningsverdien til x blir altså:

$$< x >= \frac{2a}{n^2\pi^2}\left[\frac{1}{4}u^2 - \frac{1}{4}u\sin(2u) - \frac{1}{8}\cos(2u) + c\right]_0^{n\pi}$$

$$= \frac{2a}{n^2\pi^2}\left(\frac{n^2\pi^2}{4} - \frac{n\pi}{4}\sin(2n\pi) - \frac{1}{8}\cos(2n\pi) + \frac{1}{8}\right)$$

Siden $\sin(2n\pi) = 0$ og $\cos(2n\pi) = 1$ for alle $n \in \mathbb{N}$, får vi:

$$< x >= \frac{2a}{n^2\pi^2}\left(\frac{n^2\pi^2}{4} - 0 - \frac{1}{8} + \frac{1}{8}\right) = \frac{a}{2}$$

Forventningsverdien til x^2 er:

$$< x^2 >= \int x^2|\psi_n(x)|^2 dx$$

Oppgave 2

a)

b)

c)

d)

e)