Oblig 4: oppg. 4.1 – 4.5
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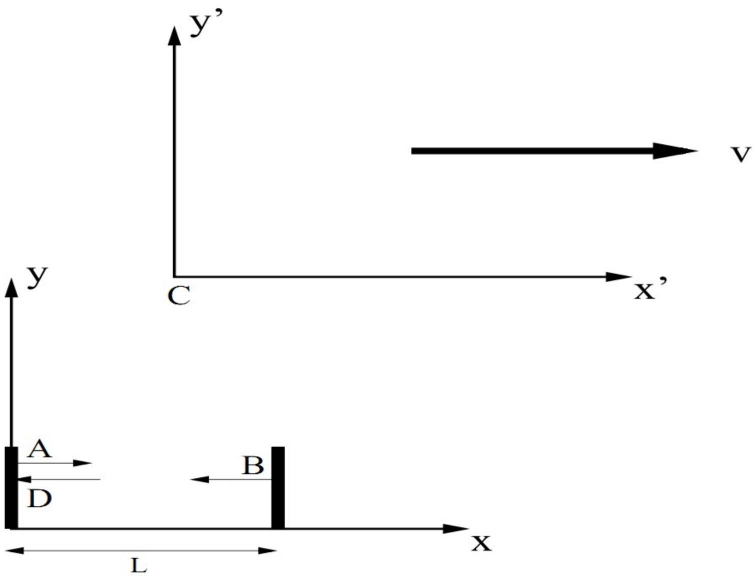
Oppg. 4.1
In the reference frame of the clock, the time between each tick is equal to the time light takes to travel the distance between the mirrors. If we use units where time and space is measured in the same units (c=1), then the time it takes between each tick equals, $L_{0,}$ the distance between the mirrors.

Oppg. 4.2



The figure show the setup. The primed axes moves with a speed v along the unprimed x-axis. At event A a light beam is emitted, at event B the light beam is reflected in a mirror. Event C happens at the same time seen from the laboratory frame which means that it's position on the unprimed x-axis is vt_B. We can explicitly read all the other coordinates from the problem text, and they are listed below.

<u>Event A:</u>
$x = 0$ $\quad$ $t = 0$
$x' = 0$ $\quad$ $t' = 0$

<u>Event B:</u>
$x = L_0$ $\quad$ $t = L_0$
$x' = x'_B$ $\quad$ $t' = t'_B$

<u>Event C:</u>
$x = vt_B = vL_0$ $\quad$ $t = L_o$
$x' = 0$ $\quad$ $t' = t'_c$

Oppg. 4.3
The space-time interval between the events A and B in the two frames are:

$$\Delta s_{AB}^2=\Delta t_{AB}^2-\Delta x_{AB}^2=(L_0-0)^2-(L_0-0)^2=0$$

and,

$$(\Delta s'_{AB})^2=(\Delta t'_{AB})^2-(\Delta x'_{AB})^2=(t'_B-0)^2-(x'_B-0)^2=(t'_B)^2-(x'_B)^2$$

from the invariance of the space-time interval, we get:

$$\Delta s_{AB}^2=(\Delta s'_{AB})^2$$
$$0=(t'_B)^2-(x'_B)^2$$
$$(x'_B)^2=(t'_B)^2$$
$$\underline{x'_B=t'_B}$$

We could also realised this without doing the calculation above, by considering the fact that light travels at the same speed in all frames of reference and that we are using units where time and space have the same units. Naturally, the time light takes to travel the distance x'_B is equal to the distance the light travels in the time t'_B.

Oppg. 4.4
The space-time intervals are:

$$\Delta s_{AC}^2=\Delta t_{AC}^2-\Delta x_{AC}^2=L_0^2-(vL_0)^2=L_0^2(1-v^2)$$

and,

$$(\Delta s'_{AC})^2=(\Delta t'_{AC})^2-(\Delta x'_{AC})^2=(t'_C)^2-0=(t'_C)^2$$

from the invariance we get:

$$\Delta s_{AC}^2=(\Delta s'_{AC})^2$$
$$L_0^2(1-v^2)=(t'_C)^2$$
$$t'_c=\sqrt{L_0^2(1-v^2)}=L_0\sqrt{(1-v^2)}=\underline{\frac{L_0}{\gamma},\quad when\ \gamma=\frac{1}{\sqrt{(1-v^2)}}}$$

Oppg. 4.5
Again, the space-time intervals are:

$$\Delta s_{BC}^2=\Delta t_{BC}^2-\Delta x_{BC}^2=(L_0-L_0)^2-(vL_0-L_0)^2=-L_0^2(v-1)^2$$

and,

$$(\Delta s'_{BC})^2=(\Delta t'_{BC})^2-(\Delta x'_{BC})^2=(t'_C-t'_B)^2-(0-x'_B)^2=(t'_C-t'_B)^2-(x'_B)^2$$

from the invariance of the space-time interval, we get:

$$-L_0^2(v-1)^2=(t'_C-t'_B)^2-(x'_B)^2$$

we then use the results we obtained in 4.3 and 4.4 for x'_B and t'_C

$$-L_0^2(v-1)^2=(\frac{L_0}{\gamma}-t'_B)^2-(t'_B)^2$$
$$-L_0^2(v-1)^2=\frac{L_o^2}{\gamma^2}-2\frac{L_0}{\gamma}t'_B$$
$$t'_B=\frac{\frac{L_0^2}{\gamma^2}+L_0^2(v-1)^2}{2\frac{L_0}{\gamma}}=\frac{L_0\gamma}{2}(\frac{1}{\gamma^2}+(v-1)^2)$$
$$t'_B=\frac{L_0\gamma}{2}((1-v^2)+(v-1)^2)=\frac{L_0\gamma}{2}(2-2\mathrm{v})=\underline{L_0\gamma(1-v)}$$