FYS1120 Oblig: Cyclotron

This is a compulsory problem set in FYS1120 at UiO for the Autumn 2012 semester. The assignment starts with some introductory exercises that consider particles in electric and magnetic fields separately. It goes on to combine electric and magnetic fields into a cyclotron.

Practical information

The oblig is due Friday 26 October 2012 at 14:00. It is to be handed in through Devilry, which you'll find at `https://devilry.ifi.uio.no/`. Log in with your UiO username and password.

You must hand in a single pdf report and all your code files for the assignment.

Figures All figures must have figure numbers and captions. All axes must be named; however, as we will be using dimensionless quantities, you do not need to specify units. Multiple plots in the same axes should have different colours and markers / line styles and an accompanying legend.

Code You may use any computer language you want to solve the exercises, including Python and MATLAB.

The code for each exercise should be in a separate file; all of them are to be handed in. Do not include your code in the report. You may include *snippets* of code, of only a few lines at the time, if you want to explain something in your algorithm.

Writing style Each exercise should be answered with a few sentences. It is considered good style to keep your answers concise and to the point, rather than writing long discussions that may only touch the right answer by chance.

Cooperation Do cooperate, but write your own code and report. Each student *must hand in their own code and their own report.* This way you will benefit from cooperating with others and from using your own words and skills to solve the exercises.

Things that do not work If you are unable to solve an exercise for any reason, including programs that don't work and equations you are unable to solve, explain what you have tried and what you would like to try next. If the code isn't working, explain what you have done and what error messages you get. Include some discussion about what you would expect to happen if the code worked.

An empty answer or comments like "I couldn't do it" gives zero credit and may result in an immediate failure of the report. You're always able to say *something* about what you have been trying to do and what you have learned. A wrong answer is much better than no answer.

Asking for help There are many requirements listed above, but we are here to help you. If you ask for help early on, you should have all exercises answered before the assignment is due. We are available online, at the group sessions, and at our offices most of the time. See our Support and Feedback page for more information about how to get in touch.

Also, make use of your fellow students and ask them if they would like to cooperate. Don't be afraid to ask others even if you don't know them: most are happy to help and the group session is a great place to meet fellow students.

Exercise 1: Particle in electric field

In this exercise you will write a program to predict the trajectory of a particle moving in an electric field $\mathbf{E}$. For simplicity we'll perform the time integration with the Euler–Cromer method. This scheme uses the most recent value of the velocity $\mathbf{v}$ to calculate the next value of the position $\mathbf{r} = (x, y, z)$.

a) Consider a particle of mass $m = 2$ and charge $q = 3$ (we'll stick with dimensionless units) in a constant electric field $\mathbf{E} = (5, 0, 0)$. Let $\mathbf{r}(t = 0) = (0, 0, 0)$ and $\mathbf{v}(t = 0) = (0, 0, 0)$. Integrate to find the trajectory from $t = 0$ to $t = 1$ with time step length $\mathrm{d}t = 10^{-4}$. Plot the x-component of the particle's position as a function of t, for all t.

b) Find the analytical solution and show the numerical and analytical solutions in the same plot.

c) Now let $\mathbf{E} = (1, 2, -5)$. Plot $x(t)$, $y(t)$ and $z(t)$ in the same axes with different colours. Show $v_x(t)$, $v_y(t)$ and $v_z(t)$ in a different axes. How can you tell that this is ballistic motion?

d) Show the 3D trajectory of the motion in exercise c).

Exercise 2: Particle in magnetic field

In this exercise you will exchange the electric field for a magnetic field $\mathbf{B}$. The particle is acted upon by a magnetic force

$$\mathbf{F}_B = q\left(\mathbf{v} \times \mathbf{B}\right),$$

where q is the charge and $\mathbf{v}$ is the velocity of the particle. If you want to learn more about magnetic fields we recommend *OpenCourseWare* from MIT. Lecture 11 in the course "Electricity and magnetism" from the Spring 2002 semester is superb. Lecture 13 of the same series deals, among other things, with the cyclotron.[1]

a) Let $m = 2$, $q = 3$, $\mathbf{r}(t) = (0, 0, 0)$ and $\mathbf{v}(t = 0) = (5, 0, 0)$. Use a magnetic field $\mathbf{B} = (0, 0, 3)$. Study the motion from $t = 0$ to $t = 5$. Plot $x(t)$, $y(t)$ and $z(t)$ in the same axes with different colours and show $v_x(t)$, $v_y(t)$ and $v_z(t)$ in a different axes. Also show the 3D trajectory of the particle.

b) Measure the period T of the particle's motion.

c) Show analytically that the cyclotron frequency of this system is

$$\omega_c = \frac{qB}{m},$$

where $B = |\mathbf{B}|$, and use this to show that

$$T = \frac{2\pi m}{qB}.$$

Comment on your numerical result from exercise b).

d) Return to exercise a) and change the initial velocity to $\mathbf{v}(t = 0) = (5, 0, 2)$. Show the 3D trajectory of the particle.

Exercise 3: Particle in a cyclotron

A cyclotron is a particle accelerator for charged particles. Energy is provided by an electric field, while a magnetic field keeps the particles inside the cyclotron. A simple cyclotron is made by placing two closed, metallic half-cylinders next to each other as shown in Figure 1. The half-cylinders, which are often called D's because of their shape, lie in a constant magnetic field perpendicular to the field of view. A proton source supplies protons to the region between the half-cylinders; we will study the motion of one of these protons. Between the half-cylinders, an oscillating **E**-field acts in the x-direction. Inside the half-cylinders the electric field is zero. If the voltage across the half-cylinders is changing in accordance with the cyclotron frequency of the **B**-field the protons will be accelerated every time they travel from one half-cylinder to the other. Consequently, the speed and radius will increase with every pass. The particles leave the cyclotron when the radius of their trajectories exceeds the radius of the half-cylinders.

A particle in a cyclotron can reach relativistic speeds. This will give relativistic corrections that we will ignore in our modelling.

The University of Oslo has a cyclotron that is used for research and for nuclear medicine. You can learn more about it at `ocl.uio.no`.

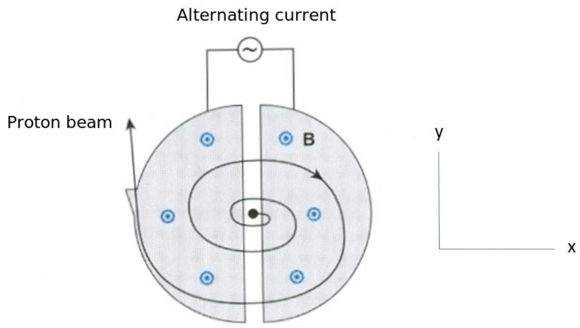

Figure 1: Sketch of a cyclotron. Taken from Lillestøl et al. [2].

a) Taking code from the programs you now have, make a program to study motion in the general case of a combined **E**- and **B**-field. Use $m = 1$, $q = 1$, $\mathbf{v}(t = 0) = (0, 0, 0)$ and $\mathbf{r}(t = 0) = (0, 0, 0)$. Let $\mathbf{B} = (0, 0, 1)$ and

$$\mathbf{E} = \begin{cases} \cos(\omega t)\hat{\mathbf{e}}_x & \text{for } x \in [-0.1, 0.1], \\ 0 & \text{otherwise.} \end{cases}$$

The angular frequency ω must equal the cyclotron frequency ω_c. Time should start at 0 and end at 50. Plot $x(t)$ against $y(t)$. Why does the radius increase less with each revolution?

b) In reality the D's have a radius r_D: the particle leaves the cyclotron when it reaches this radius. Let $r_D = 2.6$ and implement this addition to the model. Plot $x(t)$, $y(t)$ and $z(t)$ as functions of t, in the same axes. Additionally, show $v_x(t)$, $v_y(t)$ and $v_z(t)$ as functions of t, in another axes.

c) At what speed does the particle leave the cyclotron?

d) Our model misrepresents the energy gained by the particle on each pass of the gap between the D's, and the particle leaves the cyclotron after a few revolutions. In this and the next exercise we will consider a real cyclotron. Show that the kinetic energy of the particle is

$$E_k = \frac{1}{2}\frac{q^2 B^2 r^2}{m},$$

where $r = |\mathbf{r}|$. Find the energy of a proton at $r = 1$ m if $\mathbf{B} = 1$ T. Express your answer in MeV. Compare it to the mass of a proton and comment on the non-relativistic approximation that we have made in our model.

e) The increase in the energy of the particle after each complete revolution is determined by the voltage across the D's as the particle passes from one to the other. Estimate an experimentally accessible value of the voltage and determine the number of revolutions required for a proton to exit a cyclotron with $r = 1$ m and $\mathbf{B} = 1$ T. What frequency must the electric field have? Give your answer in Hz.

Bibliography

[1] Walter Lewin. Massachusetts Institute of Technology: MIT OpenCouseWare. http://ocw.mit.edu/courses/physics/8-02-electricity-and-magnetism-spring-2002/video-lectures/. Accessed 09.09.2010.

[2] Egil Lillestøl, Ola Hunderi, and Jan R. Lien. *Generell fysikk for universiteter og høgskoler*, volume 2. Universitetsforlaget, 2001.