Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
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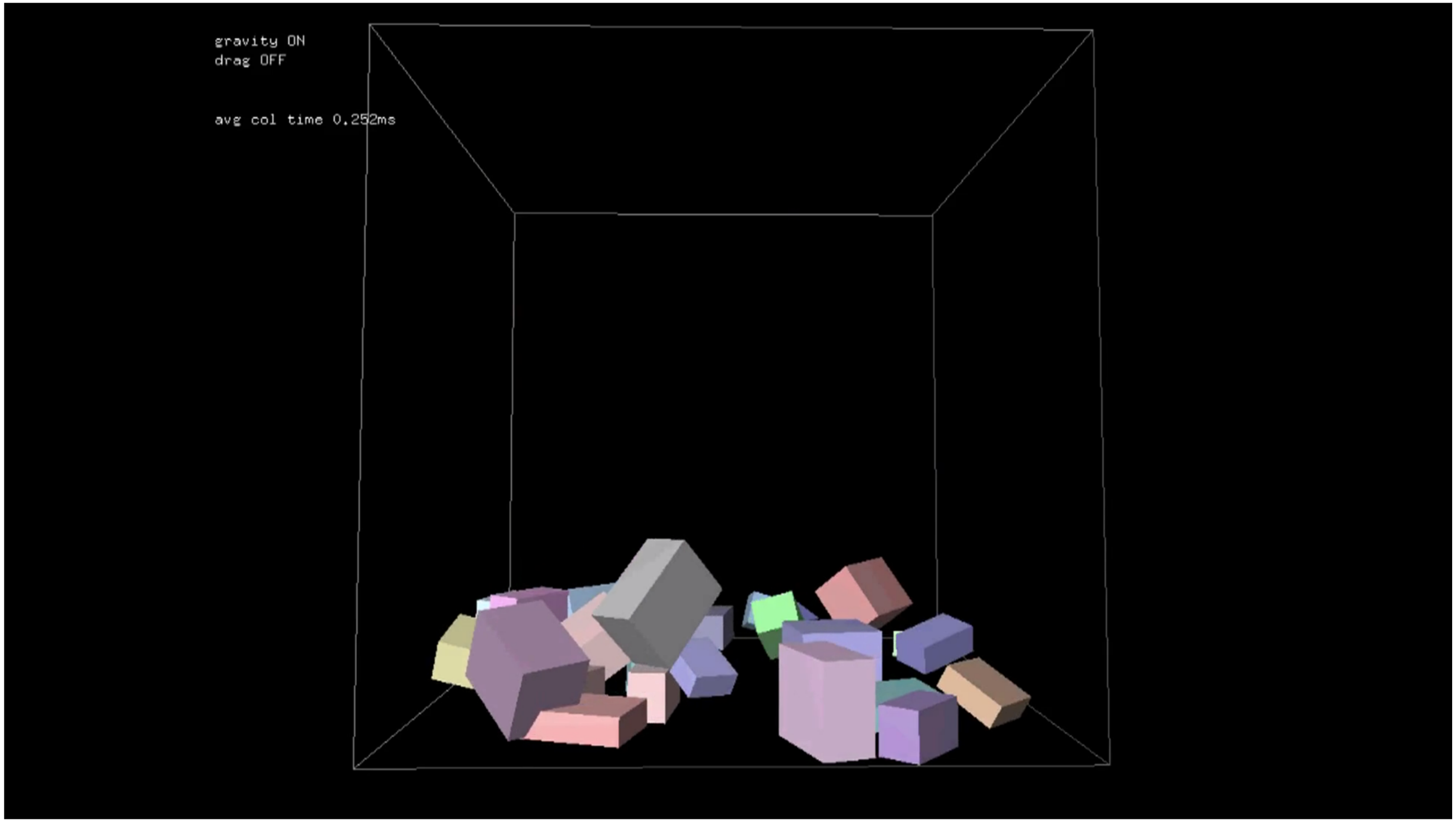
09: RIGID BODY COLLISION RESPONSE

18/02/2013

COLLISION RESPONSE

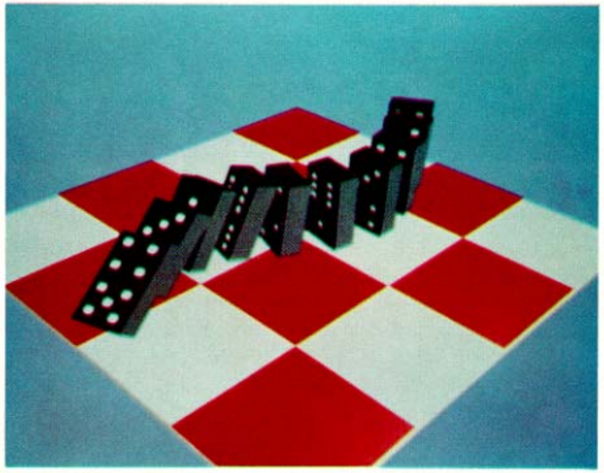


http://www.youtube.com/watch?v=P9kndaQNCDg

COLLISION RESPONSE

[Moore and Wilhelms 88]: *"automatic suggestions about the motions of objects immediately following a collision; animation systems using dynamical simulation inherently must respond to collisions automatically and realistically"*

- Intersection/contact information alone is useful for interaction, picking, collision avoidance etc.
- In dynamic simulations collisions are the basis for interaction between different objects, between users and objects
  - ... "accurate" response required



Matthew Moore and Jane Wilhelms. 1988. **Collision Detection and Response for Computer Animation**. In *Proceedings* SIGGRAPH '88.

PROBLEMS OF COLLISION RESPONSE

In Physically Based Animation, objects need to behave in ways similar to their real-world counterparts

With regard to collisions:

- Detect interpenetration and model contact
- Ensure that objects react according to believable dynamics
- Enforce non-interpenetration

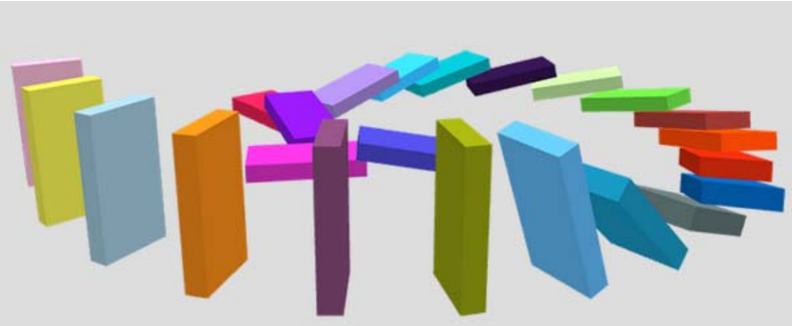


Image © Marc Ten Bosch 2006

RIGID BODY REPRESENTATION

RigidBody

Mass = m

InertialTensor = $\mathbf{I}_{body}$

$\mathbf{X}(t)$= {
Position = $\mathbf{x}(t)$
Orientation = $\mathbf{R}(t)$
LinearMomentum = $\mathbf{P}(t)$
AngularMomentum = $\mathbf{L}(t)$ }

Force = $\mathbf{F}(t)$

Torque = $\boldsymbol{\tau}(t)$

Rigid body geometry	
Dynamics Proxy	Display Mesh

Affects: collision detection, Inertial tensor **I**, center of mass

Simulation involves finding the change of **X** over time:

$$\frac{d}{dt}\mathbf{X}(t) = \frac{d}{dt}\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{R}(t)\\ \mathbf{P}(t)\\ \mathbf{L}(t)\end{bmatrix} = \begin{bmatrix}\mathbf{v}(t)\\ \boldsymbol{\omega}(t) * \boldsymbol{R}(\mathrm{t})\\ \mathbf{F}(t)\\ \boldsymbol{\tau}(t)\end{bmatrix}$$

[Baraff, Witkin, Kass '01]

SPRING BASED RESPONSE

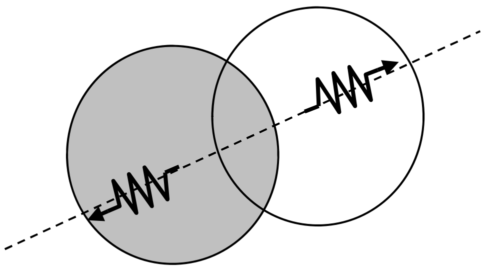
Places a virtual spring at each collision point to push two objects apart

Easy to understand and program, applies equally well to rigid and non-rigid bodies

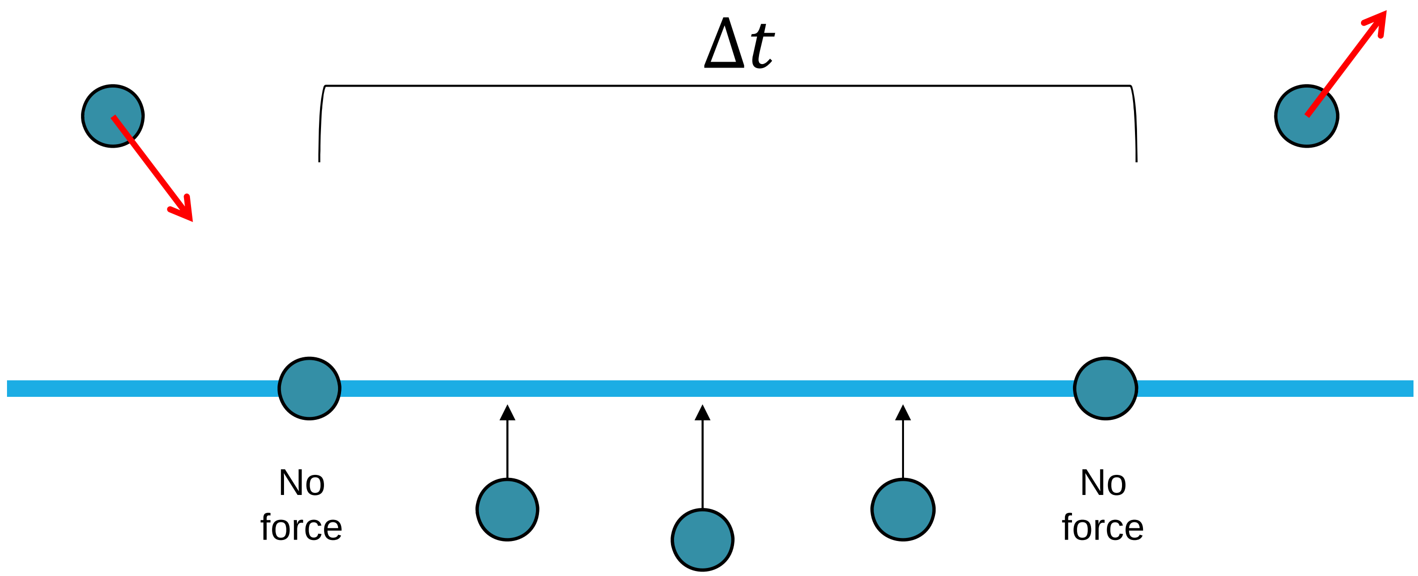
Numerical effort increases with violence of collision (smaller time-steps required)

A.K.A. Penalty methods

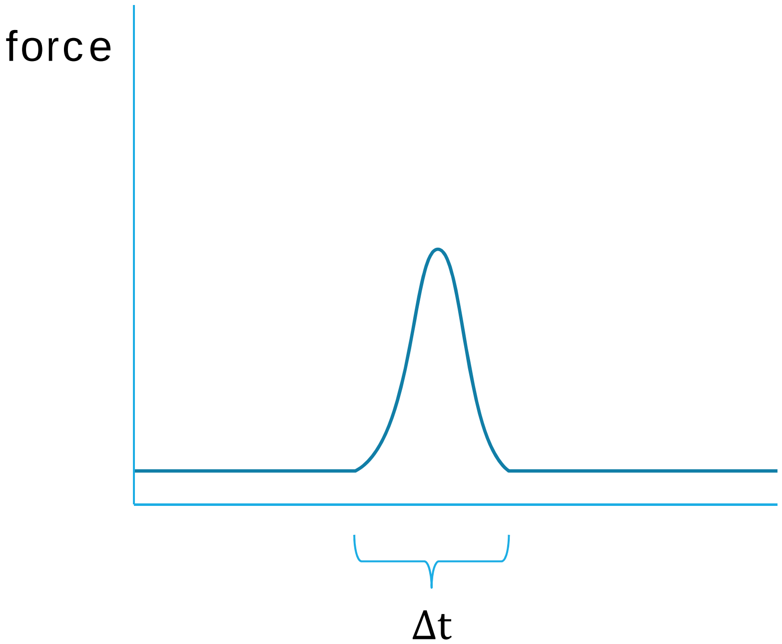
- Objects allowed to penetrate
- Spring force proportional to interpenetration
- Self correcting

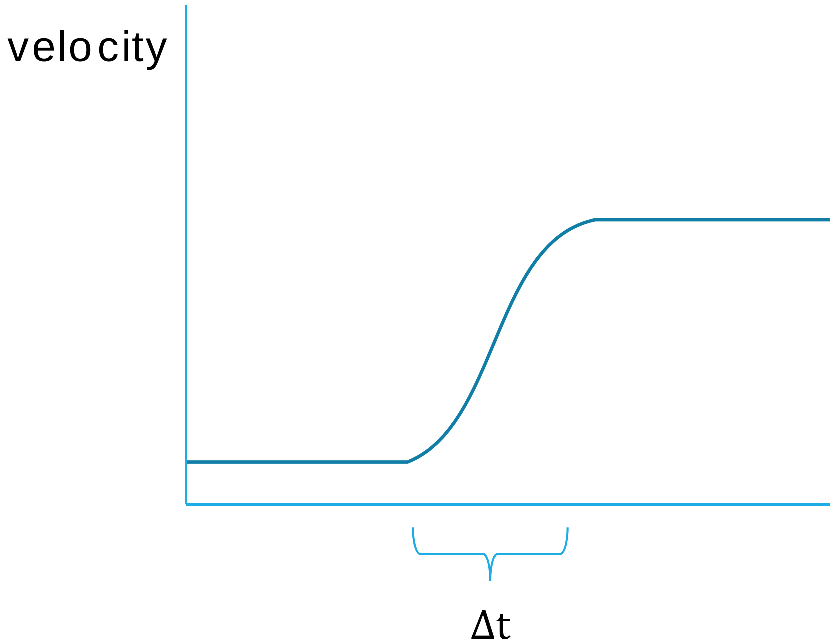


COLLISION EVENTS

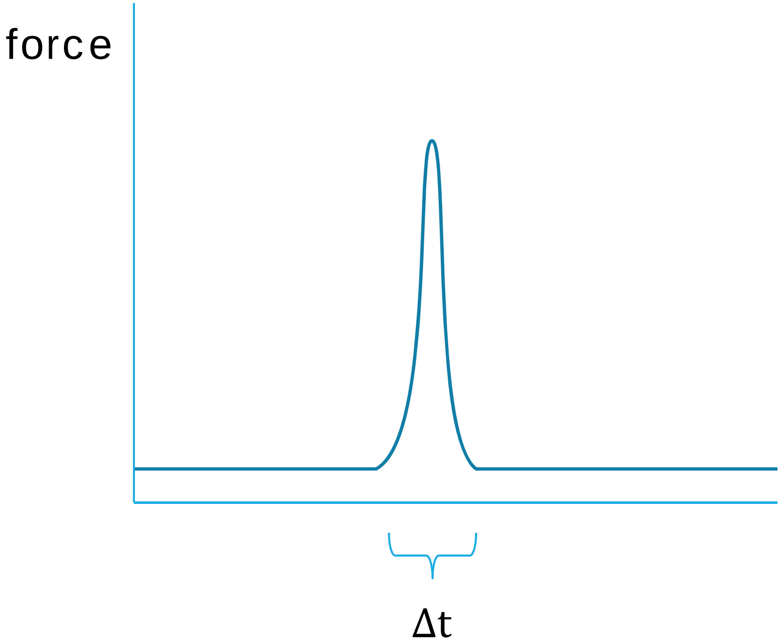


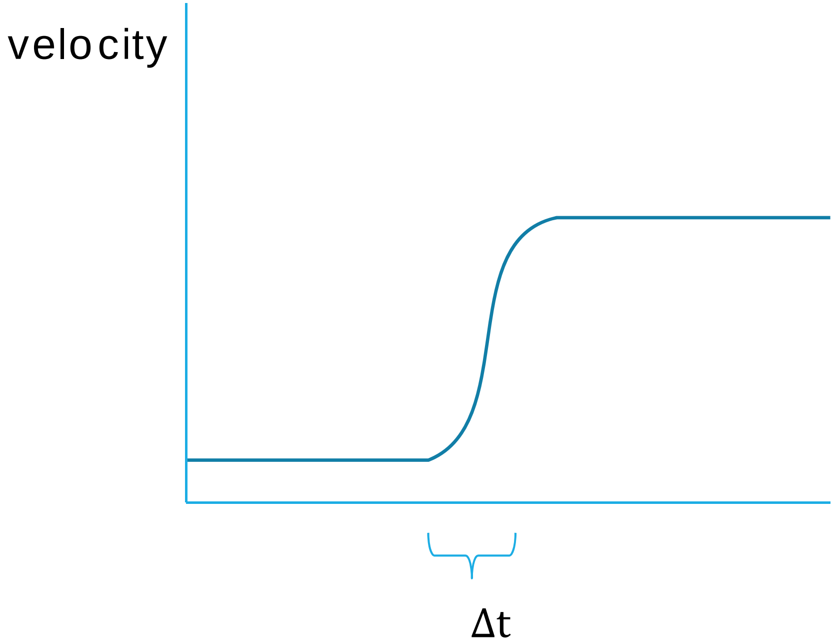
SOFT COLLISION



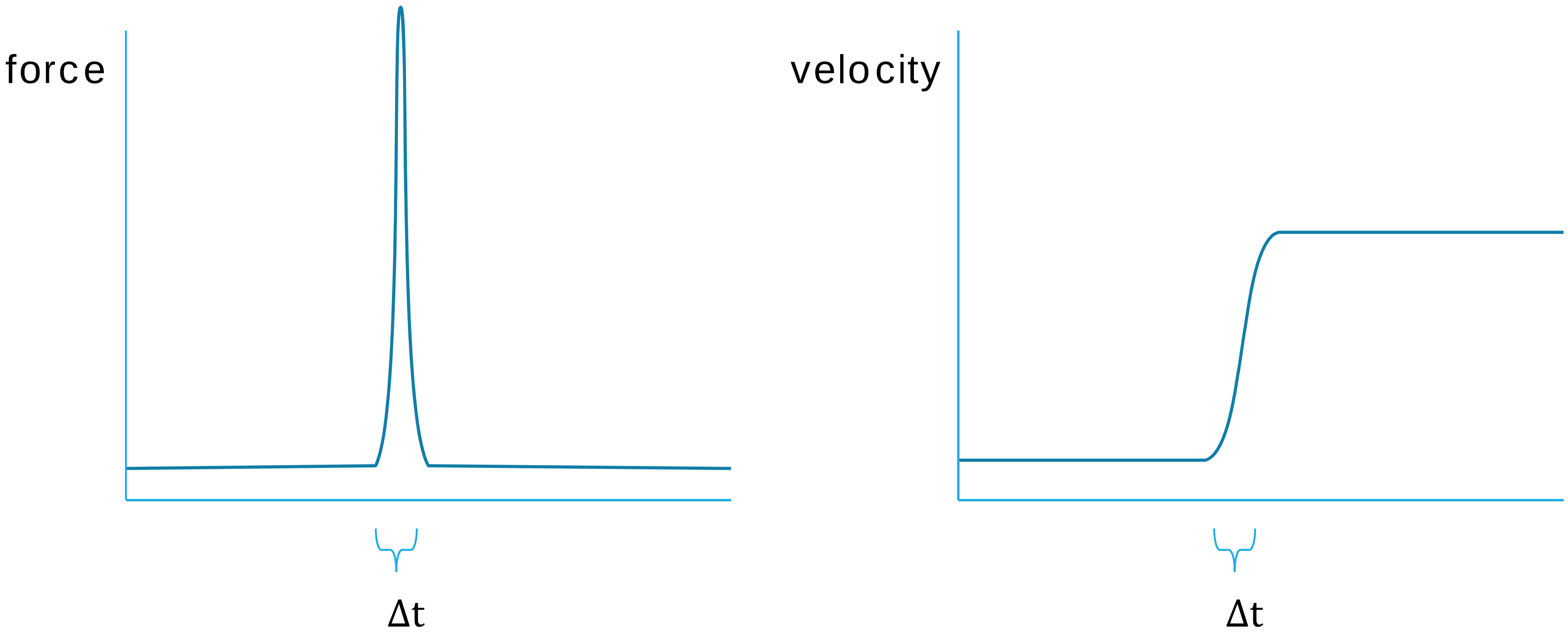


HARD COLLISION

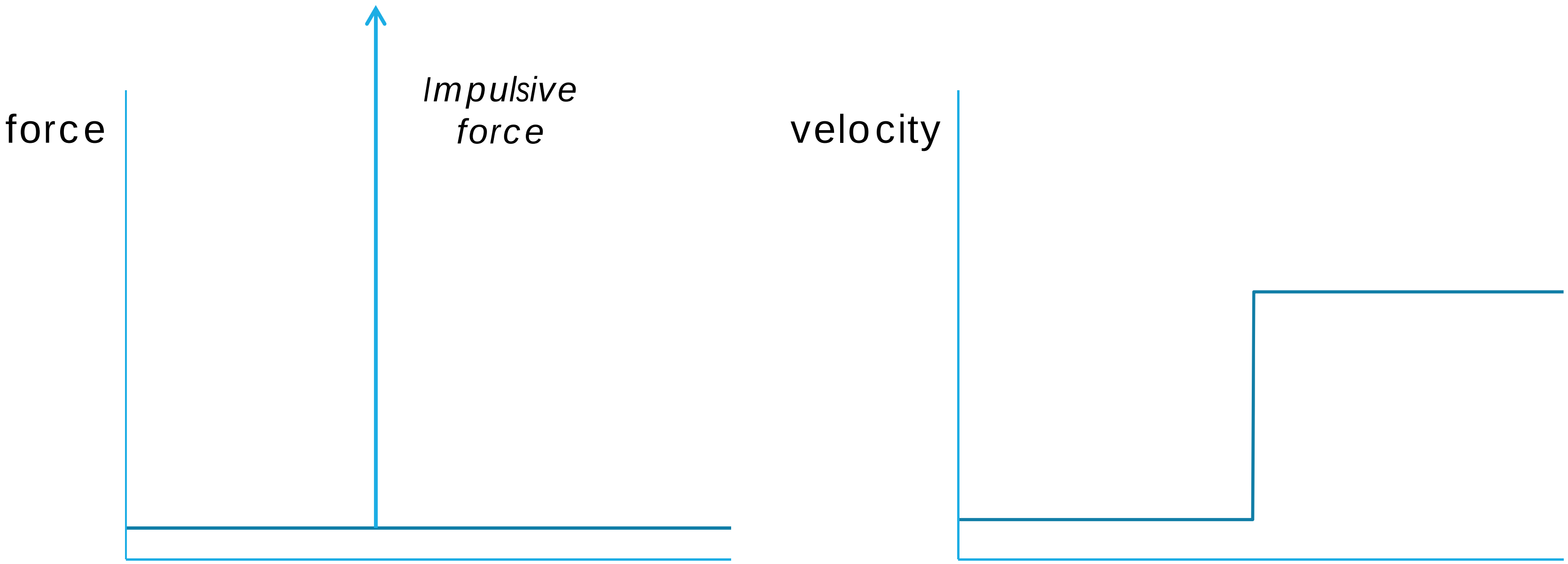




VERY HARD COLLISION



INFINITELY HARD COLLISION

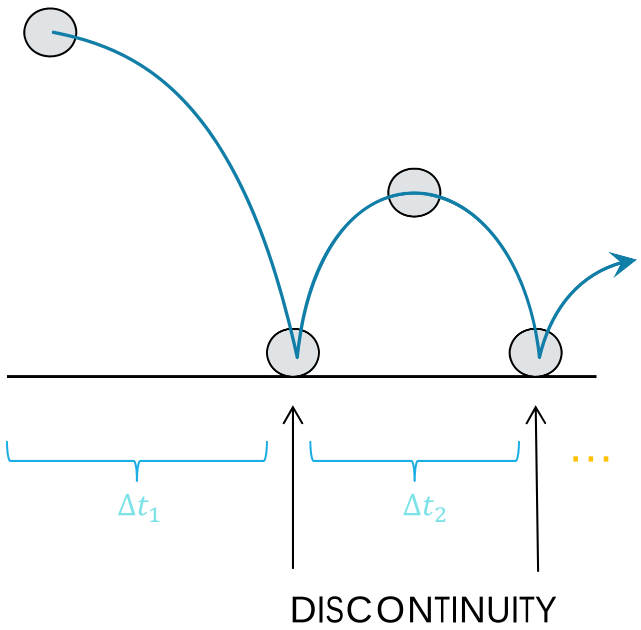


$$\boldsymbol{f}_{imp} = \infty$$

RIGID BODY COLLISIONS

Collisions are a source of discontinuity in the simulation process

- Particularly in the case of rigid bodies we need to compute an (almost) instantaneous change in state at the time of collision to prevent colliding objects from penetrating



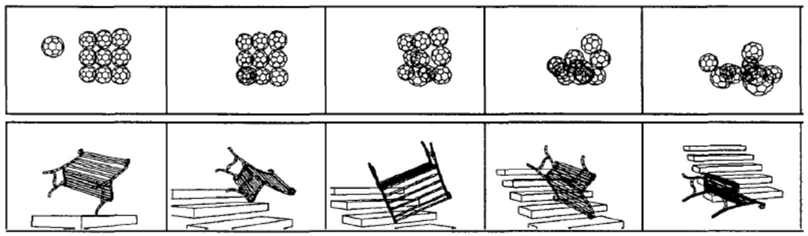
Unconstrained simulation is interrupted by discontinuity due to collision

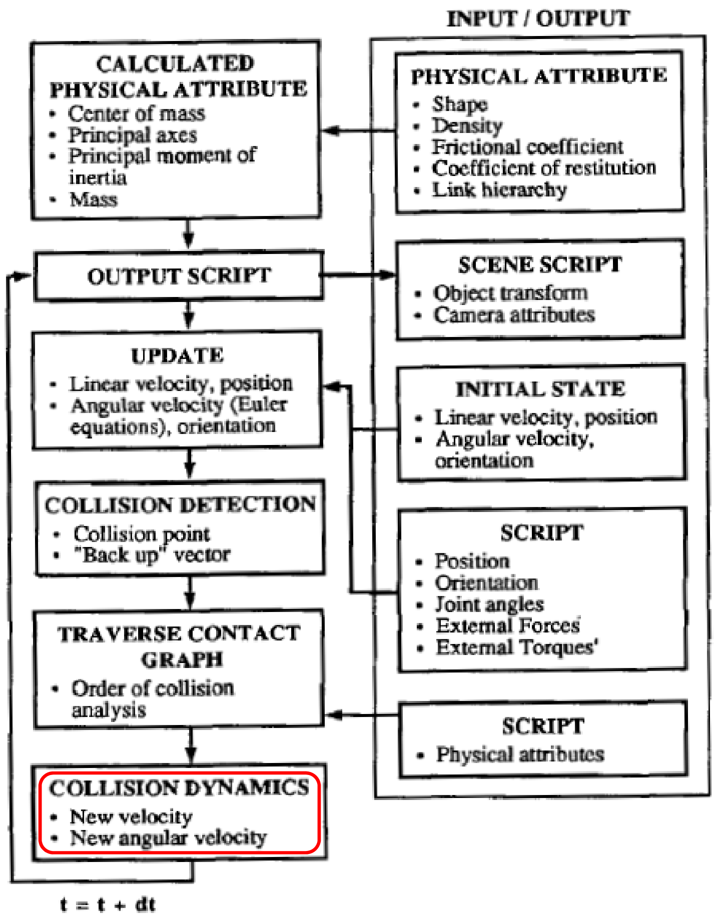
IMPULSE-BASED COLLISION RESPONSE

[Hahn88] Collision events are discontinuities in simulation Loop

- Instantaneous change of Rigid-Body state
- Quick local calculation of instantaneous change in velocities is possible at contact points
- Good solution for rigid body impact

Challenge is accurately modelling resting contact and static friction





James K. Hahn. 1988. **Realistic animation of rigid bodies.** In *Proceedings of* SIGGRAPH '88

IMPULSE RESPONSE CONTACT MODELS

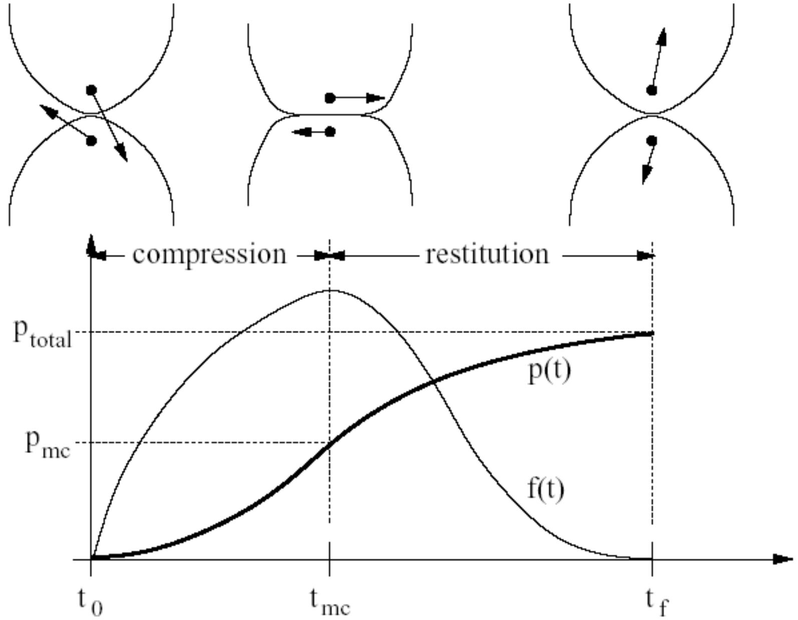
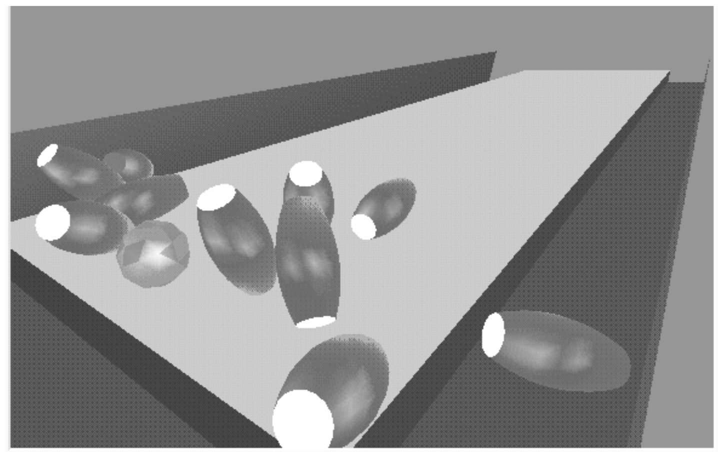


Figure 3: *A collision consists of a compression and a restitution phase. The boundary between these phases is the point of maximum compression, at which the relative contact velocity in the normal direction vanishes.* $f(t)$ *and* $p(t) = \int f(t)dt$ *are the force and total impulse delivered at time* t *in the collision.*

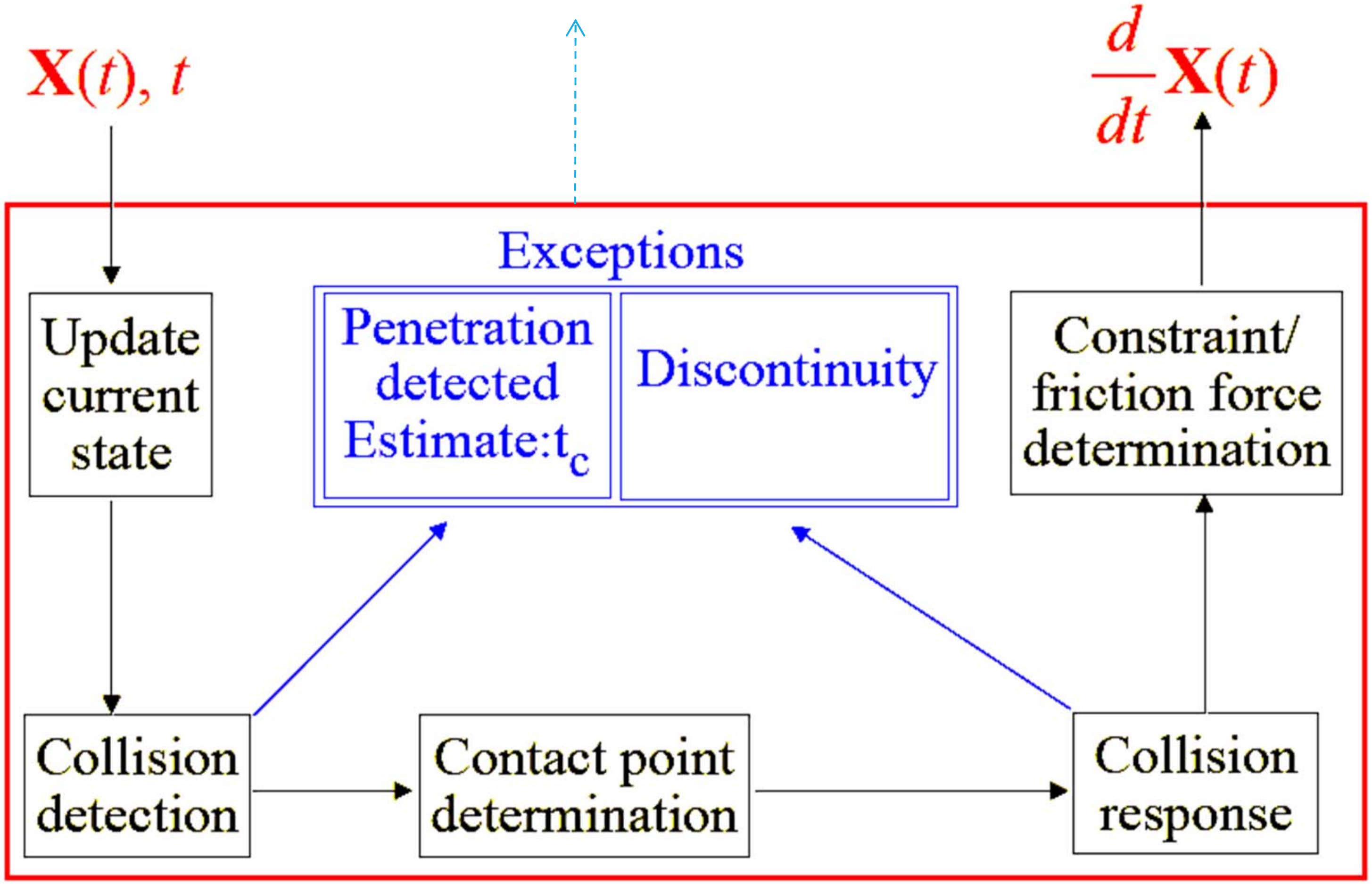
Improved by [Mirtich and Canny 95]:

- Collision Detection
- Friction
- Resting contact
- Multiple contacts



Mirtich, B. and Canny, J: **Impulse-based Simulation of Rigid Bodies** – Symposium on Interactive 3D Graphics I3D 1995

PHYSICS PIPELINE WITH COLLISIONS

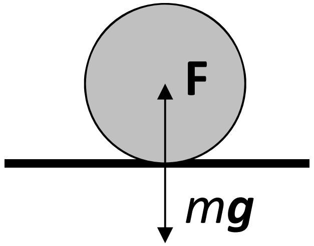


From: Baraff, Witkin & Kass. Course Notes on Physically Based Modelling, SIGGRAPH 2001

CONSTRAINT BASED METHODS

Makes a distinction between contact and collisions

For two objects in contact (relative velocity 0) ensure that the right force is applied to prevent them from interpenetrating



For objects moving towards each other (negative relative velocities), force applied over time is often not enough to prevent interpenetration

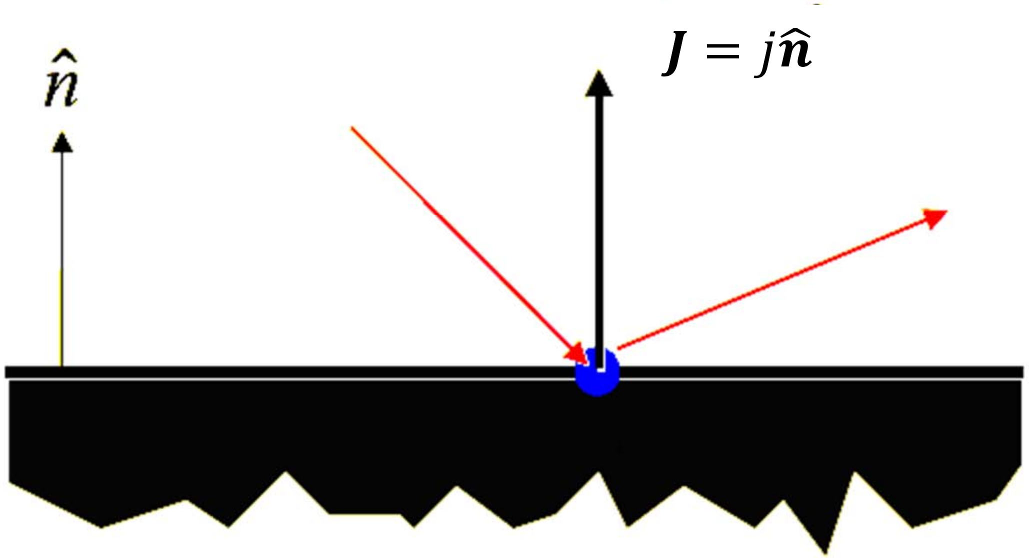
Changing nature of constraints can cause high computational demand

PARTICLE-PLANE: FRICTIONLESS IMPULSE RESPONSE

Change in velocity caused by applying an Impulse in direction normal to plane, and of magnitude j

$$\boldsymbol{v}_\perp^+ = -\epsilon \boldsymbol{v}_\perp^- \qquad \boldsymbol{v}_\parallel^+ = \boldsymbol{v}_\parallel^-$$

Invert and scale perpendicular component of velocity



$$\hat{\boldsymbol{n}}.\boldsymbol{v}^- < 0$$

$$\boldsymbol{v}^+ = \frac{\boldsymbol{J}}{m} + \boldsymbol{v}^-$$

$$\boldsymbol{J} = j\hat{\boldsymbol{n}}$$

$$j = 1 + \epsilon$$

Not this easy for Rigid Bodies

NOTE: MATHEMATICAL NOTATION USED IN THESE SLIDES

vectors and matrices in boldface e.g. $\mathbf{J}, \mathbf{v}, \mathbf{w}$

scalars are in italics e.g. j, v, ω

^ (hat) denotes unit vector/normalised vector (a vector one in length) e.g $\hat{\mathbf{n}}$

where vector multiplication is not specified as cross "×", then it is assumed to be a dot product or scalar product e.g.

$$v_{rel} = \hat{\mathbf{n}}(\dot{\mathbf{p}}_A - \dot{\mathbf{p}}_B)$$

dot indicates derivative e.g. $\dot{\mathbf{p}}_B$

superscript "+" denotes post-collision value e.g. $\mathbf{v}^+$

superscript "-" denotes pre-collision value e.g $\mathbf{v}^-$

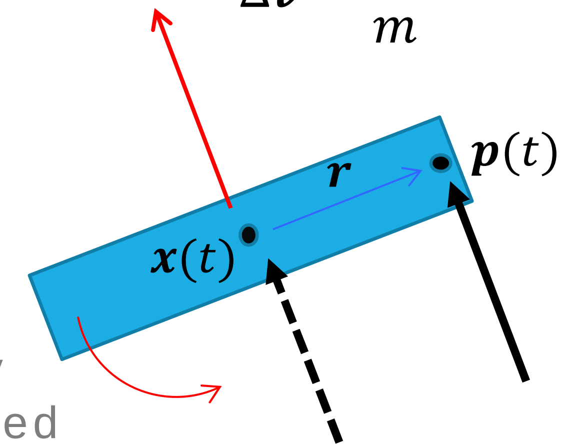
RIGID BODY IMPULSE

Impulse causes change in Linear and Angular velocity

The effect of an impulse (or for that matter a force) on an object's linear and angular momentum are independent

$$\Delta \boldsymbol{v} = \frac{\boldsymbol{J}}{m}$$

- Linear component: similar to particles
  - Causes a change in velocity inversely proportional to mass
  - As if force was applied at c.o.m.
- Angular component: impulsive torque
  - Causes change in angular velocity inversely proportional to moment of inertia (determined from inertial tensor)
  - Dependent on position of impulse



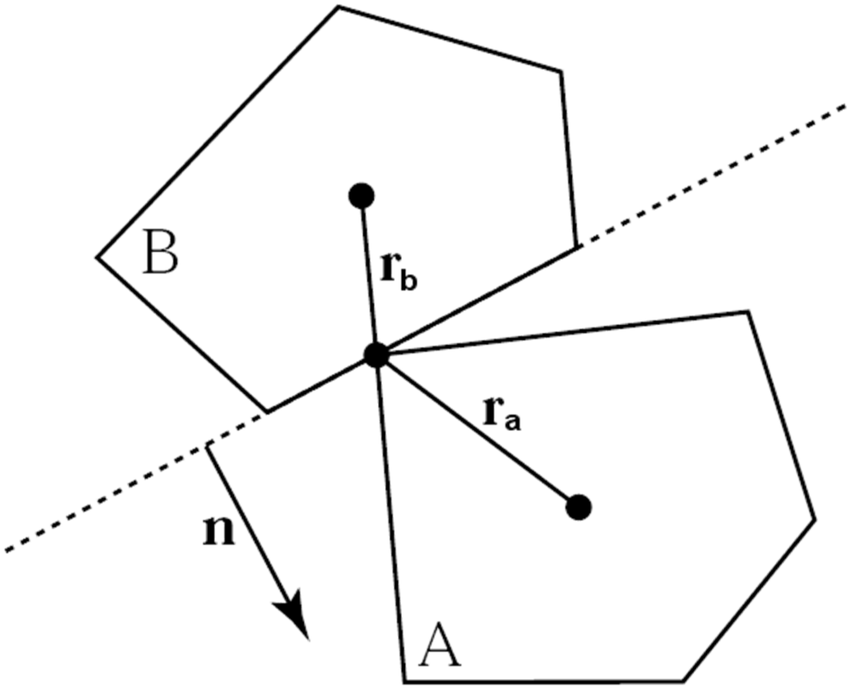
$$\Delta \boldsymbol{\omega} = \boldsymbol{I}^{-1}(\boldsymbol{r} \times \boldsymbol{J})$$
$$= \boldsymbol{I}^{-1}((\boldsymbol{p} - \boldsymbol{x}) \times \boldsymbol{J})$$

But what is the value of $\boldsymbol{J}$*?*

RIGID BODY COLLISION

$$v_{rel}^{+} = -\epsilon v_{rel}^{-}$$

$$v_{rel} = \hat{\boldsymbol{n}}(\dot{\boldsymbol{p}}_A - \dot{\boldsymbol{p}}_B)$$



Relative velocity of **colliding points** on two object is used to calculate the magnitude of the impulse.

How do we get this?

N.B $\boldsymbol{p} \approx \boldsymbol{p_A} \approx \boldsymbol{p_B}$

Point of collision

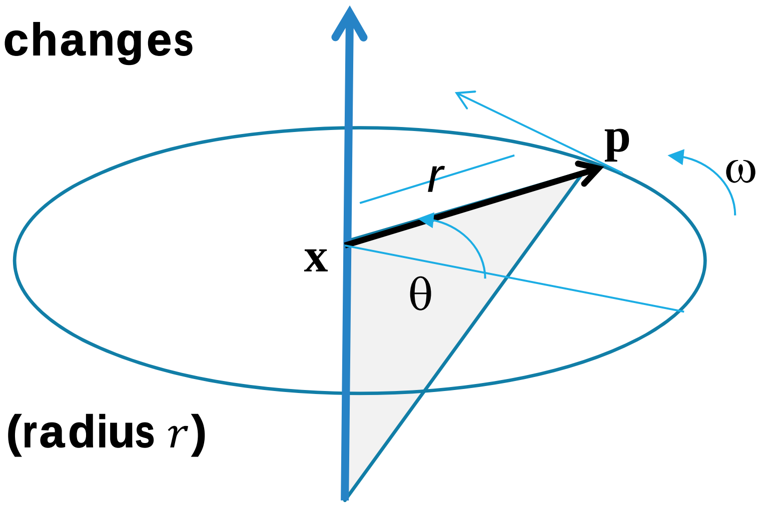
$\boldsymbol{p}_A$ and $\boldsymbol{p}_B$ may be collocated but are separate points on respective objects **with different velocities**

ASIDE: VELOCITY OF A POINT

Angular velocity: rate at which orientation changes

- magnitude ω
- axis $\hat{\boldsymbol{\omega}}$
  - where $\boldsymbol{\omega}(t) = \dot{\theta}(t)$

RATE AT WHICH θ CHANGES



Instantaneous speed of a point p on circle (radius r) due to its rotation is:

- $\dot{p}(t) = |\boldsymbol{\omega}(t) r|$

Linear Velocity of a point on an object due to its rotation:

- $\dot{\mathbf{p}}(t) = |\boldsymbol{\omega}(t) \times \mathbf{r}(t)|$
- $\quad = |\boldsymbol{\omega}(t) \times (\mathbf{p}(t) - \mathbf{x})|$

$\boldsymbol{r}$ is a vector representing the displacement of the point $\boldsymbol{p}$ from the center of mass $\boldsymbol{x}$

RIGID BODY COLLISION

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$$v_{rel} = \hat{\mathbf{n}}(\dot{\mathbf{p}}_A - \dot{\mathbf{p}}_B)$$

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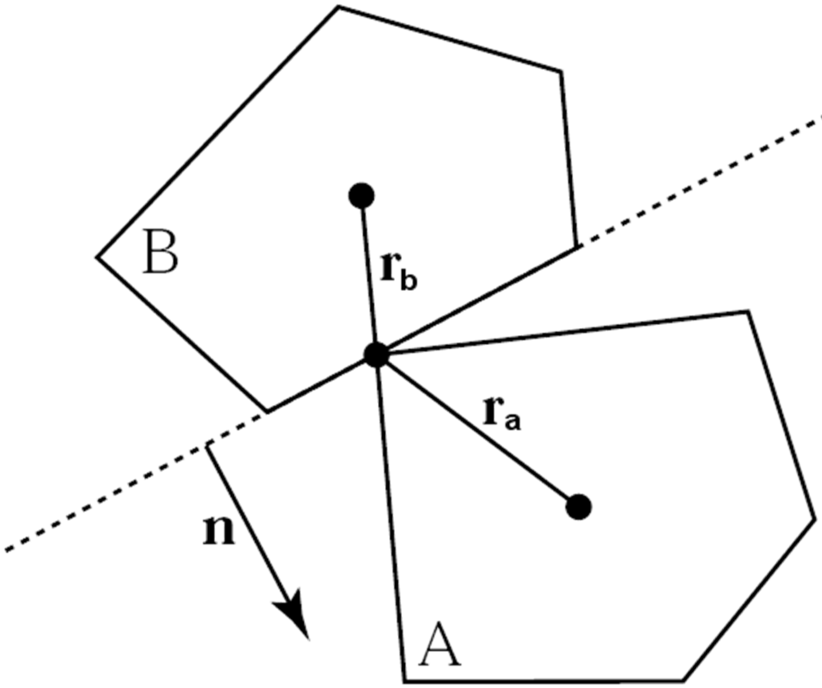

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RIGID BODY COLLISION

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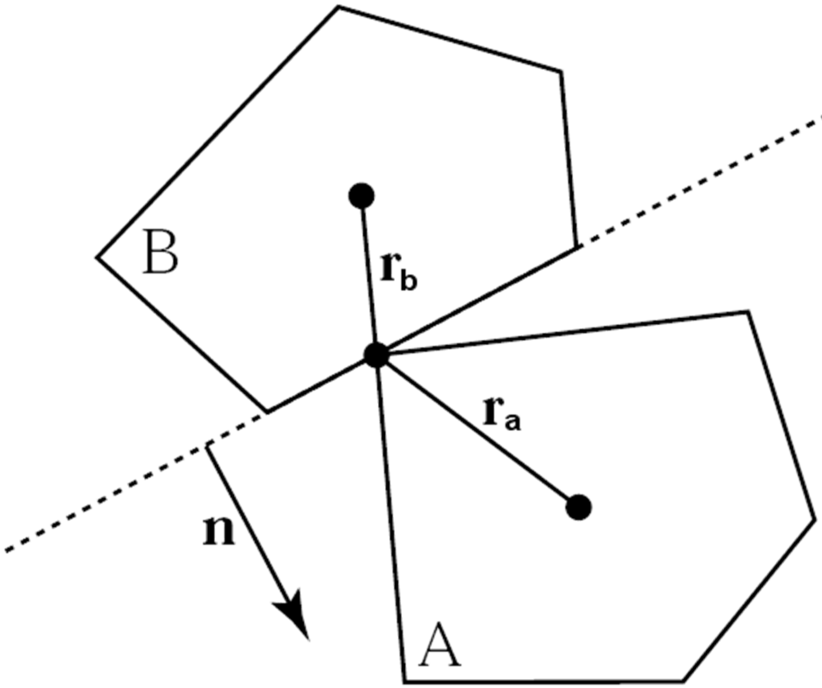

$$\begin{aligned}\dot{\boldsymbol{p}}_A &= \boldsymbol{v}_A + \boldsymbol{\omega}_A \times (\boldsymbol{p}_A - \boldsymbol{x}_A) \\ &= \boldsymbol{v}_A + \boldsymbol{\omega}_A \times \boldsymbol{r}_A\end{aligned}$$

$$\begin{aligned}\dot{\boldsymbol{p}}_A &= \boldsymbol{v}_B + \boldsymbol{\omega}_B \times (\boldsymbol{p}_B - \boldsymbol{x}_B) \\ &= \boldsymbol{v}_A + \boldsymbol{\omega}_A \times \boldsymbol{r}_A\end{aligned}$$

Linear velocity component

Angular component: Linear velocity of point p due to its rotation. See previous slide

COLLISION IMPULSE

Post-collision velocity should be

$$\dot{\boldsymbol{p}}_A^+ = \boldsymbol{v}_A^+ + \boldsymbol{\omega}_A^+ \times \boldsymbol{r}_A$$

However we need to express this in terms of the previous state of the object (pre-collision values)

$$\boldsymbol{v}_A^+ = \boldsymbol{v}_A^- + \Delta\boldsymbol{v} = \boldsymbol{v}_A^- + \frac{j\widehat{\boldsymbol{n}}}{m_A}$$

$$\boldsymbol{\omega}_A^+ = \boldsymbol{\omega}_A^- + \Delta\boldsymbol{\omega} = \boldsymbol{\omega}_A^- + \boldsymbol{I}_A^{-1}(\boldsymbol{r}_A \times j\widehat{\boldsymbol{n}})$$

SEE SLIDE 19

$$\dot{\boldsymbol{p}}_A^+ = \boldsymbol{v}_A^- + \frac{j\widehat{\boldsymbol{n}}}{m_A} + \left(\boldsymbol{\omega}_A^- + \boldsymbol{I}_A^{-1}(\boldsymbol{r}_A \times j\widehat{\boldsymbol{n}})\right) \times \boldsymbol{r}_A$$

$$= \boxed{\boldsymbol{v}_A^- + \boldsymbol{\omega}_A^- \times \boldsymbol{r}_A} + j\left(\frac{\widehat{\boldsymbol{n}}}{m_A} + \boldsymbol{I}_A^{-1}(\boldsymbol{r}_A \times \widehat{\boldsymbol{n}})\right) \times \boldsymbol{r}_A$$

PREVIOUS SLIDE

$$= \dot{\boldsymbol{p}}_A^- + j\left(\frac{\widehat{\boldsymbol{n}}}{m_A} + \boldsymbol{I}_A^{-1}(\boldsymbol{r}_A \times \widehat{\boldsymbol{n}})\right) \times \boldsymbol{r}_A$$

Similarly (by Newton's 3rd Law: every reaction has equal and opposite reaction):

$$\dot{\boldsymbol{p}}_B^+ = \dot{\boldsymbol{p}}_B^- + j\left(\frac{\widehat{\boldsymbol{n}}}{m_A} + \boldsymbol{I}_A^{-1}(\boldsymbol{r}_A \times \widehat{\boldsymbol{n}})\right) \times \boldsymbol{r}_A$$

COLLISION IMPULSE MAGNITUDE

Putting this in $v_{rel}^{+} = \hat{\boldsymbol{n}}(\dot{\boldsymbol{p}}_A^{+} - \dot{\boldsymbol{p}}_B^{+})$ *we get*

From previous slide

$$v_{rel}^{+} = \hat{\boldsymbol{n}}\left(\left(\boxed{\boldsymbol{p}_A^{-} + j\left(\frac{\hat{\boldsymbol{n}}}{m_A} + \boldsymbol{I}_A^{-1}(\boldsymbol{r}_A \times \hat{\boldsymbol{n}})\right) \times \boldsymbol{r}_A}\right) - \left(\boxed{\boldsymbol{p}_B^{-} + j\left(\frac{\hat{\boldsymbol{n}}}{m_B} + \boldsymbol{I}_B^{-1}(\boldsymbol{r}_B \times \hat{\boldsymbol{n}})\right) \times \boldsymbol{r}_B}\right)\right)$$

$$= \boxed{\hat{\boldsymbol{n}}(\dot{\boldsymbol{p}}_A^{-} - \dot{\boldsymbol{p}}_B^{-})} + j\left(\frac{1}{m_A} + \frac{1}{m_B} + \hat{\boldsymbol{n}}\left(\boldsymbol{I}_A^{-1}(\boldsymbol{r}_A \times \hat{\boldsymbol{n}})\right) \times \boldsymbol{r}_A + \hat{\boldsymbol{n}}\left(\boldsymbol{I}_B^{-1}(\boldsymbol{r}_B \times \hat{\boldsymbol{n}})\right) \times \boldsymbol{r}_B\right)$$

$$= v_{rel}^{-} + j\left(\frac{1}{m_A} + \frac{1}{m_B} + \hat{\boldsymbol{n}}\left(\boldsymbol{I}_A^{-1}(\boldsymbol{r}_A \times \hat{\boldsymbol{n}})\right) \times \boldsymbol{r}_A + \hat{\boldsymbol{n}}\left(\boldsymbol{I}_B^{-1}(\boldsymbol{r}_B \times \hat{\boldsymbol{n}})\right) \times \boldsymbol{r}_B\right)$$

And since $v_{rel}^{+} = -\epsilon v_{rel}^{-}$

$$-\epsilon v_{rel}^{-} = v_{rel}^{-} + j\left(\frac{1}{m_A} + \frac{1}{m_B} + \hat{\boldsymbol{n}}\left(\boldsymbol{I}_A^{-1}(\boldsymbol{r}_A \times \hat{\boldsymbol{n}})\right) \times \boldsymbol{r}_A + \hat{\boldsymbol{n}}\left(\boldsymbol{I}_B^{-1}(\boldsymbol{r}_B \times \hat{\boldsymbol{n}})\right) \times \boldsymbol{r}_B\right)$$

giving

$$j = \frac{-(1+\epsilon)\, v_{rel}^{-}}{m_A^{-1} + m_B^{-1} + \hat{\boldsymbol{n}}\left(\boldsymbol{I}_A^{-1}(\boldsymbol{r}_A \times \hat{\boldsymbol{n}})\right) \times \boldsymbol{r}_A + \hat{\boldsymbol{n}}\left(\boldsymbol{I}_B^{-1}(\boldsymbol{r}_B \times \hat{\boldsymbol{n}})\right) \times \boldsymbol{r}_B}$$

This gives us the impulse magnitude we were looking for

IMPULSE MAGNITUDE EQUATION

N.B. YOU WILL NEED TO IMPLEMENT THIS

$$j = \frac{-(1+\epsilon)\, v_{rel}^{-}}{m_A^{-1} + m_B^{-1} + \hat{\boldsymbol{n}}\left(\boldsymbol{I}_A^{-1}(\boldsymbol{r}_A \times \hat{\boldsymbol{n}})\right) \times \boldsymbol{r}_A + \hat{\boldsymbol{n}}\left(\boldsymbol{I}_B^{-1}(\boldsymbol{r}_B \times \hat{\boldsymbol{n}})\right) \times \boldsymbol{r}_B}$$

$\boldsymbol{I}_A^{-1} = \boldsymbol{R}_A \boldsymbol{I}_{bodyA}^{-1} \boldsymbol{R}_A^T$ and $\boldsymbol{I}_B^{-1} = \boldsymbol{R}_B \boldsymbol{I}_{bodyB}^{-1} \boldsymbol{R}_B^T$

$\boldsymbol{r}_A = \boldsymbol{p}_A - \boldsymbol{x}_A$ and $\boldsymbol{r}_B = \boldsymbol{p}_B - \boldsymbol{x}_B$

$$v_{rel}^{-} = \hat{\boldsymbol{n}}(\dot{\boldsymbol{p}}_A^{-} - \dot{\boldsymbol{p}}_B^{-})$$

$\dot{\boldsymbol{p}}_A = \boldsymbol{v}_A + \boldsymbol{\omega}_A \times (\boldsymbol{p}_A - \boldsymbol{x}_A)$ and $\dot{\boldsymbol{p}}_B = \boldsymbol{v}_B + \boldsymbol{\omega}_B \times (\boldsymbol{p}_B - \boldsymbol{x}_B)$

---

j: impulse magnitude

ϵ: coefficient of restitution

m_A, m_B: mass of object A and B

$\mathbf{R}_A, \mathbf{R}_B$: orientation of A and B

$\boldsymbol{\omega}_A, \boldsymbol{\omega}_B$: angular velocity of A and B

$\mathbf{x}_A, \mathbf{x}_B$: centre of mass position of A and B

$\mathbf{v}_A, \mathbf{v}_B$: velocity of A and B

v_{rel}^{-}: pre-collision relative velocity of contact points

$\mathbf{I}_A, \mathbf{I}_B$: world space inertial tensor

$\mathbf{I}_{bodyA}, \mathbf{I}_{bodyB}$: object space inertial tensor

$\hat{\mathbf{n}}$: contact plane normal

$\mathbf{p}_A, \mathbf{p}_B$: contact point on A and B

IMPULSE MAGNITUDE EQUATION

WHERE THE VARIABLES COME FROM

$$j = \frac{-(1+\epsilon)\, v_{rel}^{-}}{m_A^{-1} + m_B^{-1} + \hat{\boldsymbol{n}}\left(\boldsymbol{I}_A^{-1}(\boldsymbol{r}_A \times \hat{\boldsymbol{n}})\right) \times \boldsymbol{r}_A + \hat{\boldsymbol{n}}\left(\boldsymbol{I}_B^{-1}(\boldsymbol{r}_B \times \hat{\boldsymbol{n}})\right) \times \boldsymbol{r}_B}$$

$$\boxed{\boldsymbol{I}_A^{-1}} = \boldsymbol{R}_A \boldsymbol{I}_{bodyA}^{-1} \boldsymbol{R}_A^T \quad \text{and} \quad \boxed{\boldsymbol{I}_B^{-1}} = \boldsymbol{R}_B \boldsymbol{I}_{bodyB}^{-1} \boldsymbol{R}_B^T$$

$$\boxed{\boldsymbol{r}_A} = \boldsymbol{p}_A - \boldsymbol{x}_A \quad \text{and} \quad \boxed{\boldsymbol{r}_B} = \boldsymbol{p}_B - \boldsymbol{x}_B$$

$$\boxed{v_{rel}^{-}} = \hat{\boldsymbol{n}}(\dot{\boldsymbol{p}}_A^{-} - \dot{\boldsymbol{p}}_B^{-})$$

$$\boxed{\dot{\boldsymbol{p}}_A} = \boldsymbol{v}_A + \boldsymbol{\omega}_A \times (\boldsymbol{p}_A - \boldsymbol{x}_A) \quad \text{and} \quad \boxed{\dot{\boldsymbol{p}}_B} = \boldsymbol{v}_B + \boldsymbol{\omega}_B \times (\boldsymbol{p}_B - \boldsymbol{x}_B)$$

Constants

Intermediate Variables

Rigid Body State

Contact Model

j: impulse magnitude

ϵ: coefficient of restitution

m_A, m_B: mass of object A and B

$\mathbf{R}_A, \mathbf{R}_B$: orientation of A and B

$\boldsymbol{\omega}_A, \boldsymbol{\omega}_B$: angular velocity of A and B

$\mathbf{x}_A, \mathbf{x}_B$: centre of mass position of A and B

$\mathbf{v}_A, \mathbf{v}_B$: velocity of A and B

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$\mathbf{I}_{bodyA}, \mathbf{I}_{bodyB}$: object space inertial tensor

$\hat{\mathbf{n}}$: contact plane normal

$\mathbf{p}_A, \mathbf{p}_B$: contact point on A and B

APPLYING IMPULSE

Impulse magnitude is given by:

$$j = \frac{-(1+\epsilon)\ v_{rel}^{-}}{m_A^{-1} + m_B^{-1} + \widehat{\boldsymbol{n}}\left(\boldsymbol{I}_A^{-1}(\boldsymbol{r}_A \times \widehat{\boldsymbol{n}})\right) \times \boldsymbol{r}_A + \widehat{\boldsymbol{n}}\left(\boldsymbol{I}_B^{-1}(\boldsymbol{r}_B \times \widehat{\boldsymbol{n}})\right) \times \boldsymbol{r}_B}$$

The actual impulse vector is simply $\boldsymbol{J} = j\widehat{\boldsymbol{n}}$

This is applied to the objects as follows:

- Change in Linear momentum is directly equal to the impulse:

$$\Delta\boldsymbol{P} = \boldsymbol{J} \quad \Leftrightarrow \quad \Delta\boldsymbol{v} = \boldsymbol{J}m^{-1}$$

- Change in Angular momentum is equal to the imulsive torque ($\boldsymbol{\tau}_{\text{IMPULSE}}$):

$$\Delta\boldsymbol{L} = (\boldsymbol{r} \times \boldsymbol{J}) \quad \Leftrightarrow \quad \Delta\boldsymbol{\omega} = \boldsymbol{I}^{-1}(\boldsymbol{r} \times \boldsymbol{J})$$

COLLISION RESPONSE

Given this relatively simple equation we can calculate the instantaneous change in Rigid Body state quite straightforwardly

We now have a method for 3D Collision response using impulses given a contact model

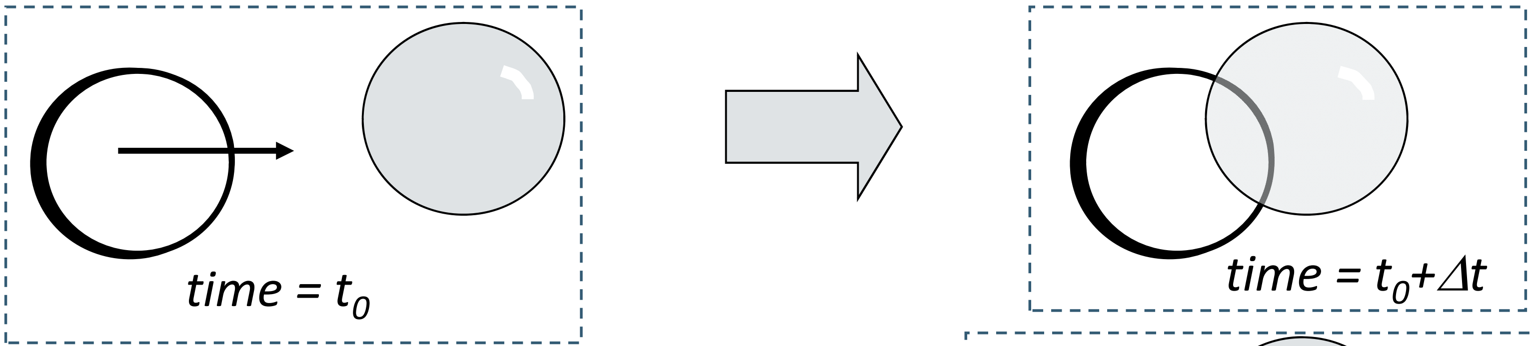
Thus quantitatively, response is easy but there are some issues to deal with

- Contact Modelling – getting accurate collision points+normals, manifolds etc is still expensive
- Interpenetrations often occur and can cause lack of robustness
- The issue is mainly related to accurate Time of Impact (TOI) determination

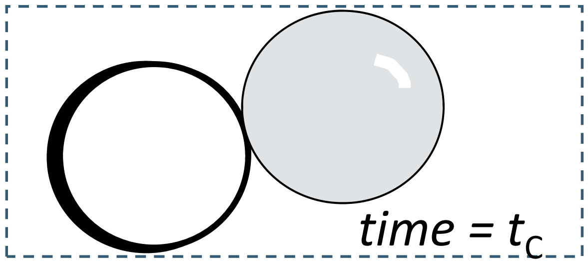
NON-INTERPENETRATION

Due to discrete timesteps the instant of collision (or point of discontinuity) is not obvious.

In most cases collision detection returns the fact that objects have interpenetrated at time $t_0 + \Delta t$



For accurate calculations we need the exact collision time – or at least correct within a certain threshold ε



RETROACTIVE DETECTION

A.K.A. BACKTRACKING

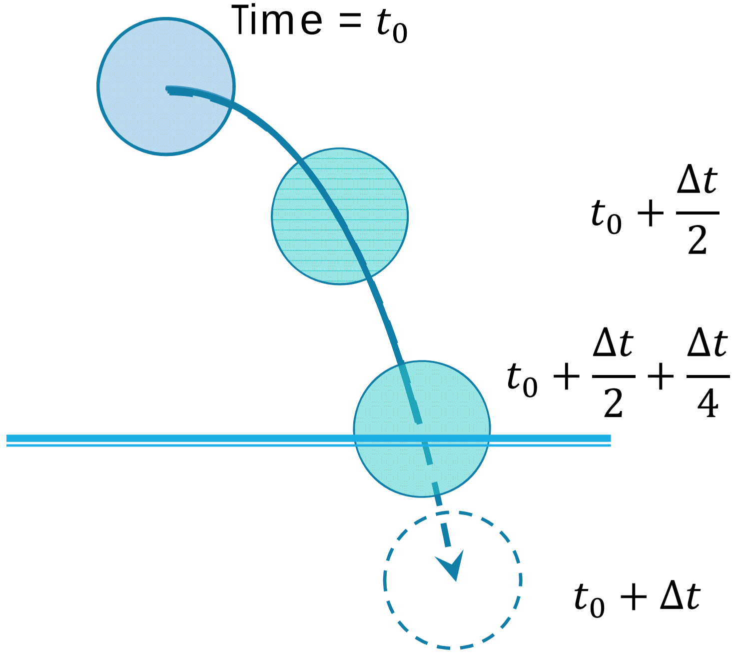
Wait for an interpenetration then step backwards in simulation time and redo collision detection to find TOI

- e.g. **Timestep Bisection**:
  - a useful (if not reasonable) guess might be $t_0 + \frac{\Delta t}{2}$
  - This process is repeated until we achieve a result under some threshold accuracy ε

We could try to analytically calculate t_C (continuous collision detection)

- this can be quite expensive

Problems with tunneling



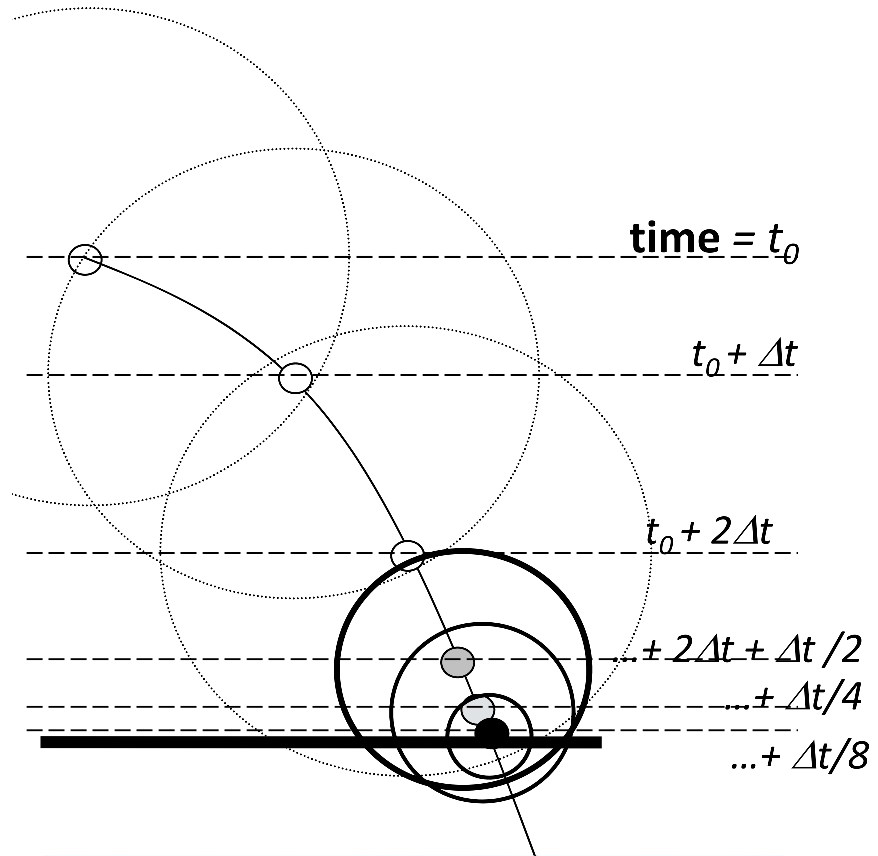
CONSERVATIVE ADVANCEMENT

A.K.A. ADAPTIVE TIMESTEPS

Reduces wasted computation encountered in bisection by pre-emptively reducing timesteps

A conservative approximation is taken as to when discontinuity occurs (e.g. space-time bound) and simulation takes place to this point

As discontinuity gets closer increasingly small steps are taken

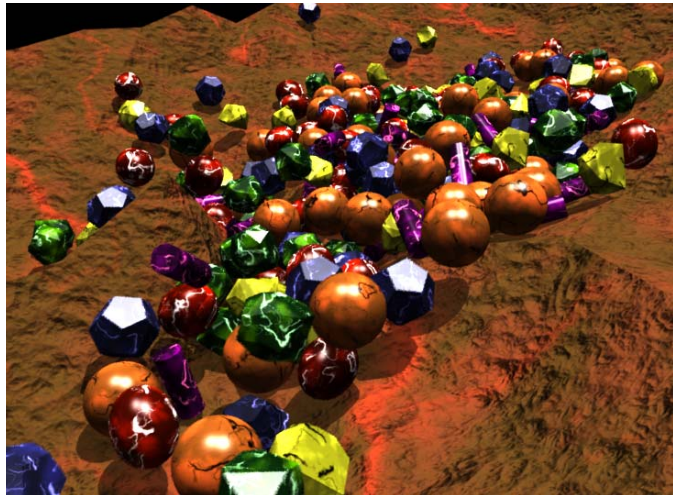


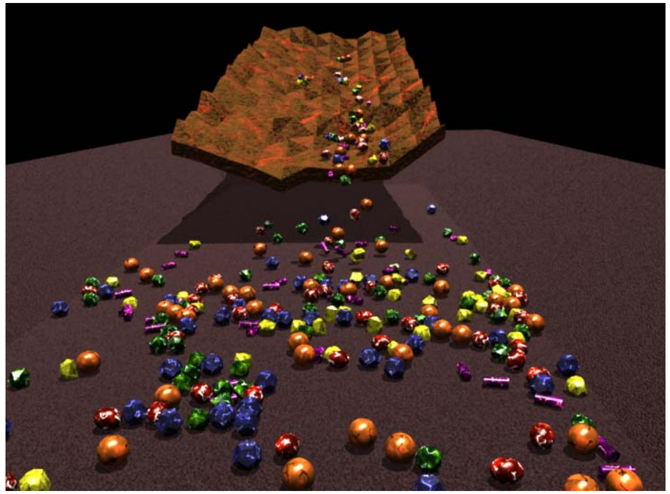
Timestep is halved when the spacetime-bound suggests that the object will interpenetrate in the next step

TIMEWARP

Main problem with previous methods are that they deal with the synchronous evolution of the entire scene

- Possible solution: asynchronous processing of individual interactions [Mirtich 2000]
- Basic idea: rollback to time of discontinuity but only for objects that have penetrated and for objects dependent on these
- Similar to having several different simulation processes communicating at key points (collisions)





Brian Mirtich - **Timewarp Rigid Body Simulation**, ACM Siggraph 2000

TIMEWARP

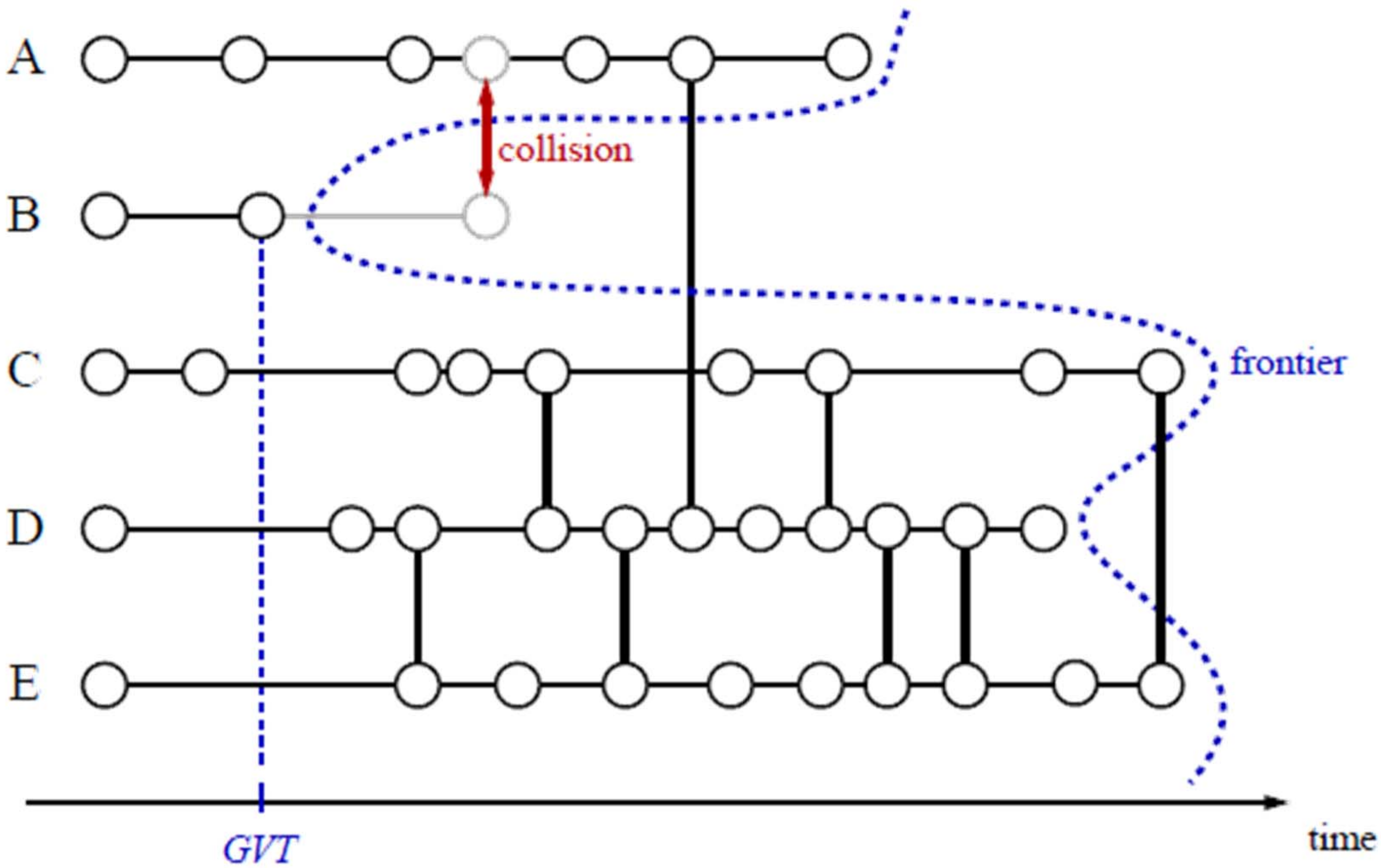


Figure © 2000 B. Mirtich

TIMEWARP

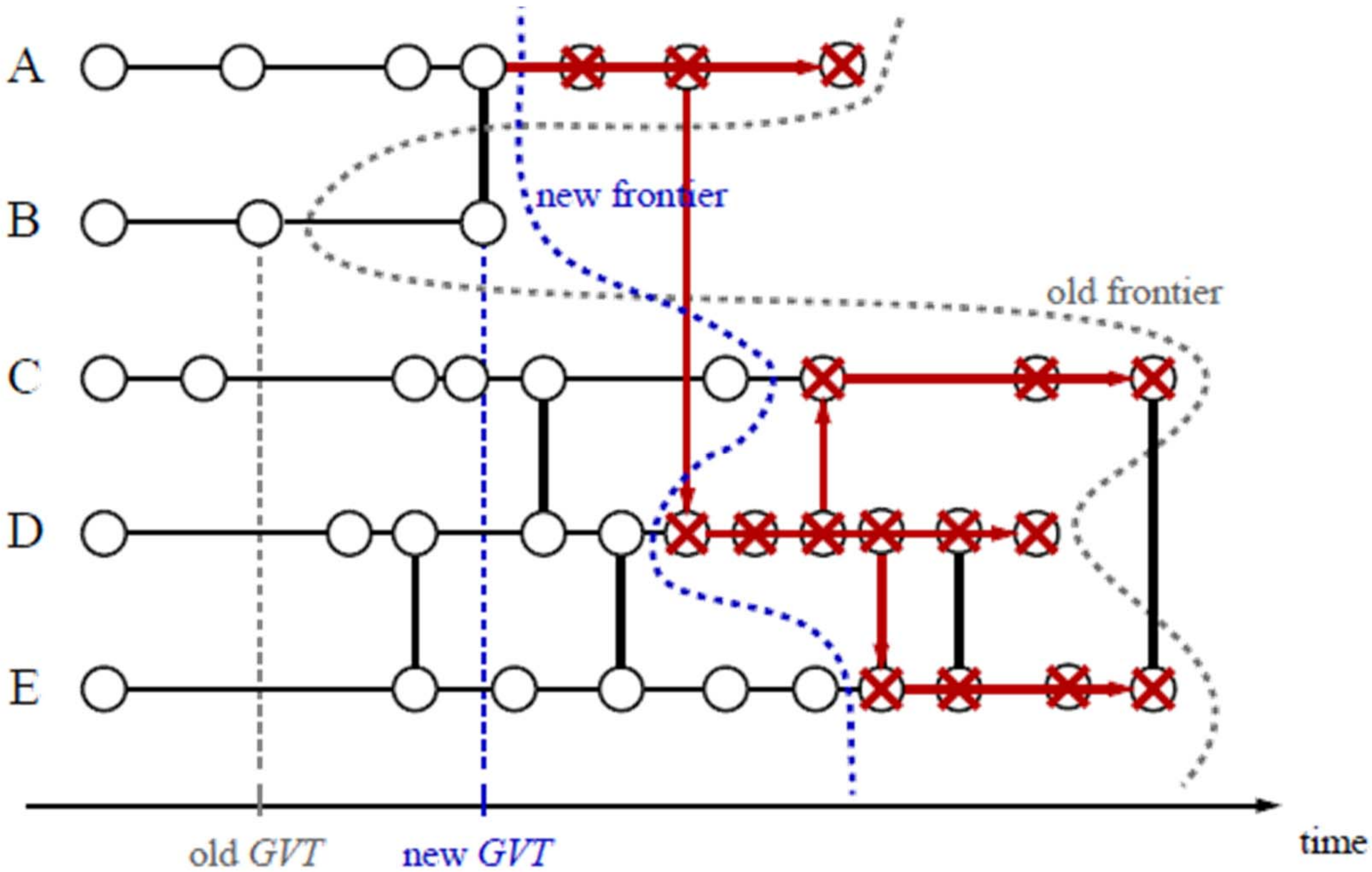


Figure © 2000 B. Mirtich

REQUIRED READING

Baraff, Witkin and Kass Siggraph 2001 Course on Physically Based Modelling

- Rigid Body Simulation Part II Nonpenetration Constraints: Sec 8 (Colliding Contact)

OR

Chris Hecker has a few Gamasutra Articles on the topic that are relatively easy reading:

- In particular, "Physics Part 4 – The 3rd Dimension"
- Available http://chrishecker.com/Rigid_Body_Dynamics

OTHER REFERENCES

2D Collision Response Demo

- Erik Neumann’s MyPhysics Lab: http://www.myphysicslab.com/collision.html

Time of Impact & Continuous Collision Detection

- Erwin Coumans: http://gamedevs.org/uploads/continuous-collision-detection-and-physics.pdf
- Gino van den Bergen: http://www.continuousphysics.com/ftp/pub/test/files/physics/papers/jgt04raycast.pdf

Papers Referenced in this lecture

- [Moore & Wilhelms88] M. Moore and J. Wilhelms. Collision Detection and Response for Computer Animation. Siggraph 1988
- [Mirtich00] B. Mirtich. Timewarp Rigid Body Simulation. Siggraph 2000. (http://www.merl.com/papers/docs/TR2000-17.pdf)
- [Mirtich96] B. Mirtich. Impulse based Rigid Body Simulationof Rigid Body Systems. PhD Thesis. Univ. of California Berkley. 1996.