Internet Appendix for “Volatility-Managed Portfolios”

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This document has two main sections. In section I, we extend our empirical analysis in several fronts. In section II, we provide proofs and a few extensions to the theoretical framework presented in Section III of the main text.

I. Additional Empirical Results

This subsection performs various robustness checks of our main result. Specifically, we use a measure of conditional variance instead of realized variance in our portfolio and find similar results, extend the analysis to international data, evaluate potential risk-based explanations for our findings, study how our strategy performs with respect to alternative performance metrics, evaluate to what extent our strategy generates option-like payoffs, and finally alternative sub-samples.

A. Additional Evidence: Credit and Term Premia

Table IA. I uses monthly data from Asvanunt and Richardson (2016) on credit risk premia and term premia for the United States from 1926-2014. The credit risk factor is formed from corporate bond returns which are duration-matched to government bonds.

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The credit risk factor thus captures only credit risk, but no term premium. The term premium factor is formed from long versus short maturity government bonds. We refer the reader to Asvanunt and Richardson (2016) for specific details. We only have monthly data for each series, hence we form our volatility-managed portfolios for these factors using the variance of the past 18 months of monthly returns to scale the factors. This is in contrast to the daily returns used in the main text. The volatility-managed credit factor has a statistically significant alpha of 1.5% per annum (for reference, the average return of the credit risk factor is 2.2% per annum) and an annualized appraisal ratio of 0.4. The volatility-managed term premium has an alpha of 1.25% which is not significant yet still economically fairly large. Thus, we conclude that there is strong evidence that volatility timing improves the Sharpe ratio for credit, but only weak evidence that it does so for the term premium.

B. Using Expected Variance Instead of Realized Variance

Table IA. II shows the results when, instead of scaling by past realized variance, we scale by the expected variance from our forecasting regressions where we use three lags of realized log variance to form our forecast. This offers more precision but comes at the cost of assuming that an investor could forecast volatility using the forecasting relationship in real time. As expected, the increased precision generally increases significance of alphas and increases appraisal ratios. We favor using the realized variance approach because it does not require a first stage estimation and has a clear appeal from the perspective of practical implementation. Other variance forecasting methods behave similarly, see, for example Andersen and Bollerslev (1998).

C. International Data

As an additional robustness check, we show that our results hold for the stock market indices of 20 OECD countries. On average, the volatility-managed version of the index has an annualized Sharpe ratio that is 0.15 higher than a passive buy-and-hold strategy. Moreover, the volatility-managed index has a higher Sharpe ratio than the passive strategy in 80% of cases. These results are detailed in Figure IA. 1. Note that this is a strong condition – a portfolio can have positive alpha even when its Sharpe ratio is below that of the nonmanaged factor.

D. Other Risk Based Explanations

Variance risk premia: Because our strategy aggressively times volatility, a reasonable concern is that our strategy's high Sharpe ratio is due to a large exposure to variance shocks, which would require a high risk premium (Ang et al. (2006b), Carr and Wu (2009)). However, it turns out that our strategy is much less exposed to volatility shocks than the buy-and-hold strategy. This follows from the fact that the volatility of volatility is higher when volatility is high. Because our strategy takes less risk when volatility is high, it also less sensitive to volatility shocks.

Downside risk: In unreported results, we find that the downside betas of our strategy following the methodology in Lettau, Maggiori, and Weber (2014) are always substantially lower than unconditional betas. For example, for the volatility-managed market return, the downside beta we estimate is 0.11 and is not significantly different from zero. Thus, alphas would be even larger if we evaluated them relative to the downside risk CAPM (Ang, Chen, and Xing (2006a), Lettau, Maggiori, and Weber (2014)). Intuitively, periods of very low market returns are typically preceded by periods of high volatility when our strategy has a low risk exposure.

Disaster risk: For disaster risk to explain our findings, our volatility-managed portfolio would have to be more exposed to disaster risk than the static portfolio. Because empirically, macro-economic disasters unfold over many periods (Nakamura et al. (2013)) and feature above-average financial market volatility (Manela and Moreira (2016)), the volatility timing strategy tends to perform better during disaster events than the static counterpart. This is further supported by the fact that our strategy takes less risk during the Great Depression and the recent financial crisis (see Figure 3 of the main text), the two largest consumption declines in our sample.

Jump risk: Jump risk is the exposure to sudden market crashes. To the extent that crashes after low-volatility periods happen frequently, our strategy should exhibit much fatter tails than the static strategy, yet we do not see this when analyzing the unconditional distribution of the volatility-managed portfolios. Overall, crashes during low-volatility times are not frequent enough (relative to high-volatility times) to make our volatility-managed portfolio more exposed to jump risk than the static buy-and-hold. If anything, jumps seem to be much more likely when volatility is high (Bollerslev and Todorov (2011)), suggesting that our strategy is less exposed to jump risk than the buy-and-hold portfolio.

Betting against beta controls: Table IA. III gives the alphas of our volatility-managed factors when we include the BAB factor of Frazzini and Pedersen (2014). As we can see from Table IA. III, the results are identical to those in the main text. Moreover, the BAB factor does not appear significant, which means that it is not strongly correlated with our volatility-managed portfolios. This again reinforces the point that our strategy is quite different from this cross-sectional low-risk anomaly.

Multivariate analysis: We study whether some of the single-factor volatility timing strategies are priced by other aggregate factors. Consistent with Table II, Tables IA. IV and IA. V show that the scaled factors expand the mean-variance frontier of the existing factors because the appraisal ratio of HML, RMW, and Mom are positive and large when including all factors. Notably, the alpha for the scaled market portfolio is lower when we include all factors. Thus, the other asset pricing factors, specifically momentum, contain some of the pricing information of the scaled market portfolio. For an investor who only has the market portfolio available, the univariate results are the appropriate benchmark; in this case, the volatility-managed market portfolio does have a large alpha. For the multivariate results (i.e., for an investor who has access to all factors) the relevant benchmark is the MVE portfolio, or "tangency portfolio," since this is the portfolio investors with access to these factors will hold (within the set of static portfolios). We find that the volatility-managed version of each of the different MVE portfolios has a substantially higher Sharpe ratio and a large positive alpha with respect to the static factors.

E. An Alternative Performance Measure and Simulation Exercises

So far, we have focused on time-series alphas, Sharpe ratios, and appraisal ratios as our benchmark for performance evaluation. This section considers alternative measures and discusses some statistical concerns. We also conduct simulations to better evaluate our results.

In our simulations, we consider a world in which the price of risk is constant, $E_t[R_{t+1}] = \gamma Var_t[R_{t+1}]$, and choose parameters to match the average equity premium, average market standard deviation, and volatility of the market standard deviation. We model volatility as lognormal and returns as conditionally lognormal. Using these simulations we can ask, if the null that the risk-return trade-off is strong were true, what is the probability we would see the empirical patterns we document in the data (alphas, Sharpe ratios, etc.).

First, we study the manipulation-proof performance measure (henceforth MPPM) of Goetzmann et al. (2007). This measure is useful because, unlike alphas and Sharpe ratios, it can not be manipulated to produce artificially high performance. This manipulation could be done intentionally by a manager, say by decreasing risk exposure after experiencing a string of lucky returns, or through a type of strategy that uses highly non-linear payoffs. Essentially, the measure is based on the certainty equivalent for a power utility agent with risk-aversion ranging from 2 to 4 and evaluates their utility directly. We choose risk aversion of 3, although our results are not sensitive to this value. We find the market MPPM to be 2.48% and the volatility-managed market portfolio MPPM to be 4.33%, with the difference between the two 1.85% per year. This demonstrates that even under this alternative test that overcomes many of the potential shortcomings of traditional performance measures, we find that our volatility-managed strategy beats the buy-and-hold portfolio.

It is useful to consider the likelihood of this finding in relation to the null hypothesis that the price of risk is constant. In our simulations, we can compute the MPPM measure of the scaled market portfolio and compare it to the market portfolio MPPM. We find that the volatility-managed MPPM beats the market MPPM measure only 0.2% of the time. Hence, if the price of risk is not moving with volatility, it is highly unlikely that the MPPM measure would favor the volatility-managed portfolio. Using these simulations, we can also ask what is the likelihood that we would observe an alpha as high as we see in the data. The median alpha in our simulations with a constant price of risk is about 10bps and the chance of seeing an alpha as high as we see empirically (4.86%) is essentially zero.

F. Are Volatility-Managed Portfolios Option Like?

At least since Black and Scholes (1973), it has been well known that under some conditions option payoffs can be replicated by dynamically trading the reference asset. Since our strategy is dynamic, a plausible concern is that our strategy might be replicating option payoffs. A large literature discusses potential issues with evaluating strategies that have a strong option-like return profile.

We discuss each of the potential concerns and explain why it does not apply to our volatility-managed portfolios. First, a linear asset pricing factor model where a return is a factor implies a stochastic discount factor that can be negative for sufficiently high factor return realizations (Dybvig and Ingersoll (1982)). Thus, there are states with a negative state price, which implies an arbitrage opportunity. A concern is that our strategy may be generating alpha by implicitly selling these negative state-price states. However, empirically this cannot be the source of our strategy alpha, as the implied stochastic discount factor is always positive in our sample.[1]

Second, the nonlinearity of option-like payoffs can make the estimation of our strategy's beta challenging. Because some events happen only with very low probability, sample moments are potentially very different from population moments. This concern is much more pronounced for short samples. For example, most option and hedge fund strategies for which such biases are shown to be important have no more than 20 years of data. However, we have 90 years of data for the market portfolio. In Figure IA. 2 we also look at kernel estimates of the buy-and-hold and volatility-managed factor return distributions. No clear pattern emerges; if anything, the volatility-managed portfolio appears to have less mass on the left tail for some portfolios.

Third, another concern is that our strategy loads on high-price-of-risk states. For example, strategies that implicitly or explicitly sell deep out-of-the-money puts can capture the expected return resulting from the strong smirk in the implied volatility curve. Note that our strategy reduces risk exposure after a volatility spike, which is typically associated with low return realizations, while one would need to increase exposure following a low return realization to replicate the sale of a put option. Mechanically our strategy does exactly the opposite of what a put-selling strategy would call for. This also implies that our strategy will typically have less severe drawdowns than the static portfolio, which

[1]For example, for the market factor the implied SDF can be written as $\approx 1/R_t^f - b(R_{t+1}^m - R_t^f)$, where empirically $b = E[R_{t+1}^m - R_t^f]/Var(R_t^e) \approx 2$. In our sample the highest return realization is 38%, so the SDF is never negative.

accords with our Figure 3.

A more general way of addressing the concern that our strategy alpha is due to its option-like returns is to use the manipulation proof performance measure (MPPM) proposed by Goetzmann et al. (2007). We find that the volatility-managed MPPM is 75% higher than the market MPPM. Using simulations we show that a volatility-managed portfolio would beat the market (as measured by MPPM) only 0.2% of the time if the risk-return trade-off was constant. This again shows that our strategy increases Sharpe ratios simply by avoiding high risk times and does not load on other unwanted risks.

Overall, there is no evidence that our volatility-managed portfolios generate option-like returns.

G. *Additional Results on Subsamples*

In addition to showing results for discreet subsamples in the main text, we provide rolling 30-year window alphas for our strategy in Figure IA. 3. We do this only for the market return. Consistent with our formula for alphas in the main text, we find that our strategy is most profitable during relatively high-volatility periods. This is due in large part to volatility varying the most during these subsamples. In subsamples where volatility does not vary much, the strategy weight is very close to a static buy-and-hold weight, and hence it is not able to increase Sharpe ratios relative to the buy-and-hold portfolio or generate alpha.

II. Theoretical Framework: Proofs and Extensions

A. *Conditional Risk-Return Trade-off*

We decompose variation in expected returns in a component due to volatility and an orthogonal component, $\mu_t = b\sigma_t^2 + \zeta_t$, for a constant b. We assume that the process ζ_t satisfies $E[\zeta_t|\sigma_t] = E[\zeta_t]$. The coefficient b represents the conditional risk-return trade-off.

We then have

$$\alpha = c(E[\sigma_t^2]E[1/\sigma_t^2] - 1)\,(\gamma - b)\,, \tag{IA. 1}$$

where alpha is positive if and only if $b < \gamma$, which means that the conditional risk-return trade-off is weaker than the unconditional risk-return trade-off. Moreover, the weaker the conditional risk-return trade-off, b, the higher the alpha.

B. *Individual Stocks*

Consider a simple example in which the CAPM holds, and the market portfolio return given by dF_t has constant expected returns and variance. Consider a individual stock with returns $dR_t = (r_t dt + \mu_{R,t})dt + \beta_R(dF_t - E_t[dF_t]) + \sigma_{R,t}dB_{R,t}$, where $dB_{R,t}$ shocks are not priced. We have that the volatility-managed alpha is

$$\alpha_R \propto -cov\left(\sigma_{R,t}^2, \frac{1}{\beta_R^2\sigma_F^2 + \sigma_{R,t}^2}\right), \tag{IA. 2}$$

which is positive (negative) if $\beta_R > 0$ ($\beta_R < 0$), but CAPM alphas are always zero.

While volatility timing can "work" for any asset with positive expected returns for which volatility is forecastable but does not predict returns, the alphas become economically interesting when studying systematic factors.

C. *Proof of Implication 1*

The fact that $\Pi(\gamma^u)$ must price factors F unconditionally immediately implies that $\gamma_i^u = E[\mu_{i,t}]/E[\sigma_{i,t}]$. Similarly, the fact that $\Pi(\gamma_t^*)$ must price factors F conditionally implies that $\gamma_{i,t}^* = \mu_{i,t}/\sigma_{i,t}$. We can thus write $\gamma_{i,t}^* = b + \zeta_t/\sigma_{i,t}$, with conditional expectation given by $E[\gamma_{i,t}^*|\Sigma_t] = b + E[\zeta_t]/\sigma_{i,t}$.

We now use result (IA. 1) to substitute out b and $E[\zeta_t]$. Specifically, we use that

$$b = \gamma_i^u - \alpha_i(c_i J_{\sigma,i})^{-1} \tag{IA. 3}$$

$$E[\zeta_t] = E[\mu_{i,t}] - bE[\sigma_{i,t}^2] \tag{IA. 4}$$

to obtain equation (10).

Now we show that the SDF $\Pi(\gamma_t^\sigma)$ prices all volatility-based strategies. We need to show that $E\left[d\left(\Pi_t(\gamma_t^\sigma)w(\Sigma_t)\widetilde{R}_t\right)\right] = 0$, which is equivalent to

$$E\left[d\left(\Pi_t(\gamma_t^\sigma)w(\Sigma_t)\widetilde{R}_t\right)\right] = E[w(\Sigma_t)\mu_t] - E[\gamma_t^\sigma(dF_t - E_t[])w(\Sigma_t)\widetilde{R}_t] \qquad \text{(IA. 5)}$$

Using the fact that factors are on the conditional mean-variance frontier, it is sufficient to show that the expression holds for the factors themselves. Furthermore, it is sufficient to show that the pricing equation holds for each portfolio conditional on Σ_t information.

This yields

$$\begin{aligned}
E\left[d\left(\Pi_t(\gamma_t^\sigma)F_t\right)|\Sigma_t\right] &= E[\mu_t|\Sigma_t] - E[\gamma_t^\sigma(dF_t - E_t[dF_t])dF_t|\Sigma_t] && \text{(IA. 6)}\\
&= b\Sigma_t + E[\zeta_t] - \gamma_t^\sigma\Sigma_t && \text{(IA. 7)}\\
&= b\Sigma_t + E[\zeta_t] - (b + E[\zeta_t]\Sigma_t^{-1})\Sigma_t && \text{(IA. 8)}\\
&= 0, && \text{(IA. 9)}
\end{aligned}$$

where in the last line we use the fact that $\gamma_{i,t}^\sigma = E[\gamma_{i,t}^*|\Sigma_t] = b + E[\zeta_t]/\sigma_{i,t}$. This proves Implication 1.

D. *Spanning the Unconditional Mean-Variance Frontier with Volatility-Managed Portfolios*

The price of risk γ_t^σ is also the unconditional MVE portfolio from the perspective of an investor who can measure time-variation in volatility but not variation in ζ_t. The price of risk process can be represented with constant positions on the buy-and-hold factors and the volatility-managed factors.

IMPLICATION 1: *The unconditional MVE portfolio spanned by conditional information on volatility can be replicated by a constant position of the factors and the volatility-managed factors* $[dF; dF^{\sigma}]$,

$$E[w_i^*(\mu_t, \Sigma_t)|\Sigma_t] \propto \left[\gamma_i^u - \frac{\alpha_i}{c_i} J_{\sigma,i}^{-1}; \frac{\alpha_i}{c_i} J_{\sigma,i}^{-1} \frac{E[\sigma_{i,t}^2]}{c_i}\right]. \tag{IA. 10}$$

These weights are simple functions of our strategy alpha. Assuming the market portfolio is on the conditional mean-variance frontier, we can plug in numbers for the market portfolio to get a sense of magnitudes. We get $[0.14; 0.86]$ for the weights on the market and the volatility-managed market portfolio. Our volatility-managed portfolio gets close to being unconditionally MVE because the relationship between return and volatility is so weak.

E. Correlated Factors

Our approach can be easily extended to the case in which factors are correlated. Let the factors' variance-covariance matrix be block-diagonal: it can be decomposed into N blocks as follows $\Sigma_t \Sigma_t' = diag\left(\left[H_1 \sigma_{1,t}^2 ..., H_N \sigma_{N,t}^2\right]\right)$, where $\sigma_{n,t}^2$ are scalars, and H_n are constant full-rank matrixes.

Given this factor structure in factor variances (see Section II.E of the main text to see that this is a good description of the data), we can apply our analysis to "block-specific" MVE portfolios constructed as follows. For block n, let $df_{n,t}$ be the vector of factor returns and $\mu_{n,t}$ be the vector of expected excess returns. Then the MVE portfolios are given by $df_{n,t}^{MVE} \equiv rdt + \mu_{n,t}' H_n^{-1} \left(df_{n,t} - rdt\right)$, which is exactly the procedure we follow in Section I.E of the main text.

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Table IA. I. Volatility-Managed Factor Alphas.

We run time-series regressions of each volatility-managed factor on the nonmanaged factor $f_t^{\sigma} = \alpha + \beta f_t + \varepsilon_t$. The managed factor, f^{σ}, scales by the factor's inverse realized variance computed using the preceding 18 months of data $f_t^{\sigma} = \frac{c}{RV_{t-1}^2} f_t$. The data are monthly and the sample period is 1926 to 2014 for Credit and Term. Standard errors are in parentheses and adjust for heteroskedasticity. All factors are annualized in percent per year by multiplying monthly factors by 12.

Univariate Regressions		
	(1)	(2)
	Credit$^{\sigma}$	Term$^{\sigma}$
Credit	0.44	
	(0.05)	
Term		0.18
		(0.02)
Alpha (α)	1.48	1.25
	(0.40)	(0.86)
N	1,050	1,050
R^2	0.20	0.03
RMSE	12.75	28.45

Table IA. II. Alphas Using Expected Variance. We run time-series regressions of each managed factor on the nonmanaged factor. Here our managed portfolios make use of the full forecasting regression for log variances rather than simply scaling by lagged realized variances. The managed factor scales by the factors' inverse conditional variance. The data are monthly and the sample is 1926 to 2015, except for the factors RMW and CMA, which start in 1963, and the FX carry factor, which starts in 1983. Standard errors are in parentheses and adjust for heteroskedasticity. All factors are annualized in percent per year by multiplying monthly factors by 12.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Mkt^{σ}	SMB^{σ}	HML^{σ}	Mom^{σ}	RMW^{σ}	CMA^{σ}	MVE^{σ}	FX^{σ}
MktRF	0.73							
	(0.06)							
SMB		0.71						
		(0.09)						
HML			0.65					
			(0.08)					
Mom				0.59				
				(0.08)				
RMW					0.70			
					(0.08)			
CMA						0.78		
						(0.05)		
MVE							0.74	
							(0.03)	
Carry								0.89
								(0.05)
Constant	3.85	-0.60	2.09	12.54	1.95	0.41	3.83	1.77
	(1.36)	(0.78)	(0.92)	(1.67)	(0.75)	(0.57)	(0.67)	(0.90)
Observations	1,063	1,063	1,063	1,059	619	619	1,059	358
R^2	0.53	0.51	0.43	0.35	0.49	0.61	0.54	0.81
RMSE	44.33	27.02	32.06	46.01	18.31	14.96	20.97	13.66

Table IA. III. Time-series Alphas Controlling for Betting Against Beta Factor. We run time-series regressions of each managed factor on the nonmanaged factor plus the betting against beta (BAB) factor from Frazzini and Pedersen (2014). The managed factor, f^{σ}, scales by the factors' inverse realized variance in the preceding month, $f_t^{\sigma} = \frac{c}{RV_{t-1}^2} f_t$. The data are monthly and the sample is 1929 to 2012 based on availability of the BAB factor. Standard errors are in parentheses and adjust for heteroskedasticity. All factors are annualized in percent per year by multiplying monthly factors by 12.

	(1) Mkt^{σ}	(2) SMB^{σ}	(3) HML^{σ}	(4) Mom^{σ}	(5) RMW^{σ}	(6) CMA^{σ}	(7) MVE^{σ}
MktRF	0.60						
	(0.05)						
BAB	0.09	0.01	0.02	-0.07	-0.13	-0.06	0.04
	(0.06)	(0.05)	(0.05)	(0.04)	(0.02)	(0.02)	(0.02)
SMB		0.61					
		(0.09)					
HML			0.56				
			(0.07)				
Mom				0.47			
				(0.06)			
RMW					0.65		
					(0.08)		
CMA						0.69	
						(0.04)	
MVE							0.57
							(0.04)
Constant	3.83	-0.77	2.05	13.52	3.97	0.94	4.10
	(1.80)	(1.10)	(1.15)	(1.86)	(0.89)	(0.71)	(0.85)
Observations	996	996	996	996	584	584	996
R^2	0.37	0.37	0.31	0.21	0.40	0.46	0.33
rmse	52.03	31.36	35.92	51.73	19.95	17.69	26.01

Table IA. IV. Alphas of Volatility-managed Factors Controlling for Basic Risk Factors. We run time-series regressions of each managed factor on the four Fama-French Carhart factors. The managed factor, f^{σ}, scales by the factors' inverse realized variance in the preceding month, $f_t^{\sigma} = \frac{c}{RV_{t-1}^2} f_t$. The data are monthly and the sample is 1926 to 2015. Standard errors are in parentheses and adjust for heteroskedasticity. All factors are annualized in percent per year by multiplying monthly factors by 12.

	(1)	(2)	(3)	(4)	(5)
	Mkt^{σ}	SMB^{σ}	HML^{σ}	Mom^{σ}	MVE^{σ}
MktRF	0.70	-0.02	-0.10	0.16	0.23
	(0.05)	(0.01)	(0.02)	(0.03)	(0.02)
HML	-0.03	-0.02	0.63	0.09	0.08
	(0.05)	(0.04)	(0.05)	(0.05)	(0.02)
SMB	-0.05	0.63	-0.00	-0.10	-0.15
	(0.06)	(0.08)	(0.05)	(0.04)	(0.02)
Mom	0.25	0.01	0.06	0.54	0.30
	(0.03)	(0.03)	(0.04)	(0.05)	(0.02)
Constant	2.43	-0.42	1.96	10.52	4.47
	(1.60)	(0.94)	(1.06)	(1.60)	(0.77)
Observations	1,060	1,060	1,060	1,060	1,060
R^2	0.42	0.38	0.35	0.25	0.35
RMSE	49.56	30.50	34.21	49.41	25.13

Table IA. V. Alphas of Volatility-managed Factors Controlling for Additional Risk Factors. We run time-series regressions of each managed factor on the 6 Fama-French Carhart factors. The managed factor, f^{σ}, scales by the factors' inverse realized variance in the preceding month, $f_t^{\sigma} = \frac{c}{RV_{t-1}^2} f_t$. The data are monthly and the sample is 1963 to 2015. Standard errors are in parentheses and adjust for heteroskedasticity. All factors are annualized in percent per year by multiplying monthly factors by 12.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Mkt^{σ}	SMB^{σ}	HML^{σ}	Mom^{σ}	RMW^{σ}	CMA^{σ}	MVE^{σ}	MVE2^{σ}
MktRF	0.79	0.03	-0.06	0.12	0.02	0.02	0.26	0.23
	(0.05)	(0.03)	(0.03)	(0.04)	(0.02)	(0.01)	(0.03)	(0.02)
HML	0.11	0.09	1.03	0.15	-0.21	0.03	0.16	0.05
	(0.09)	(0.06)	(0.08)	(0.09)	(0.04)	(0.03)	(0.04)	(0.03)
SMB	0.02	0.75	-0.05	-0.12	-0.02	-0.03	-0.15	-0.09
	(0.05)	(0.05)	(0.04)	(0.07)	(0.03)	(0.02)	(0.03)	(0.02)
Mom	0.15	-0.01	0.05	0.64	-0.00	-0.02	0.32	0.23
	(0.03)	(0.03)	(0.03)	(0.08)	(0.02)	(0.02)	(0.03)	(0.02)
RMW	0.15	0.23	-0.56	-0.04	0.64	-0.18	0.01	0.04
	(0.06)	(0.07)	(0.08)	(0.08)	(0.06)	(0.04)	(0.04)	(0.03)
CMA	0.04	0.00	-0.28	-0.25	-0.00	0.63	-0.04	0.14
	(0.12)	(0.07)	(0.10)	(0.11)	(0.06)	(0.05)	(0.06)	(0.04)
Constant	0.18	-1.68	4.16	12.91	3.21	1.07	4.00	3.03
	(1.87)	(1.25)	(1.44)	(2.17)	(0.81)	(0.72)	(1.02)	(0.77)
Observations	622	622	622	622	621	621	622	621
R^2	0.47	0.49	0.51	0.31	0.46	0.50	0.40	0.43
RMSE	42.70	26.82	32.82	48.10	18.85	17.01	23.26	16.96

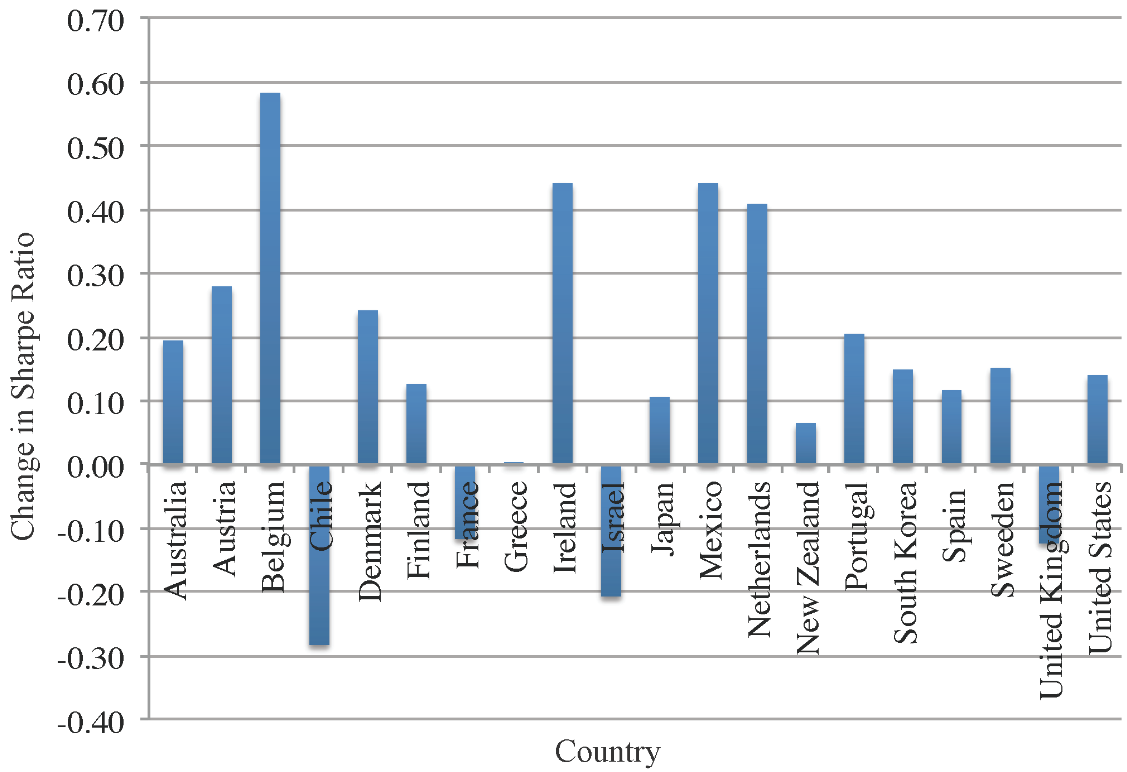

Figure IA. 1. Increase in Volatility-Managed Sharpe Ratios by Country. The figure plots the change in Sharpe ratio for managed versus nonmanaged portfolios across 20 OECD countries. The change is computed as the Sharpe ratio of the volatility-managed country index minus the Sharpe ratio of the buy-and-hold country index. All indices are from Global Financial Data. For many series, the index only contains daily price data and not dividend data, and thus our results are not intended to accurately capture the level of Sharpe ratios but should still capture their difference well to the extent that most of the fluctuations in monthly volatility are driven by daily price changes. All indices are converted to USD and are taken over the U.S. risk-free rate from Ken French. The average change in Sharpe ratio is 0.15 and the value is positive in 80% of cases.

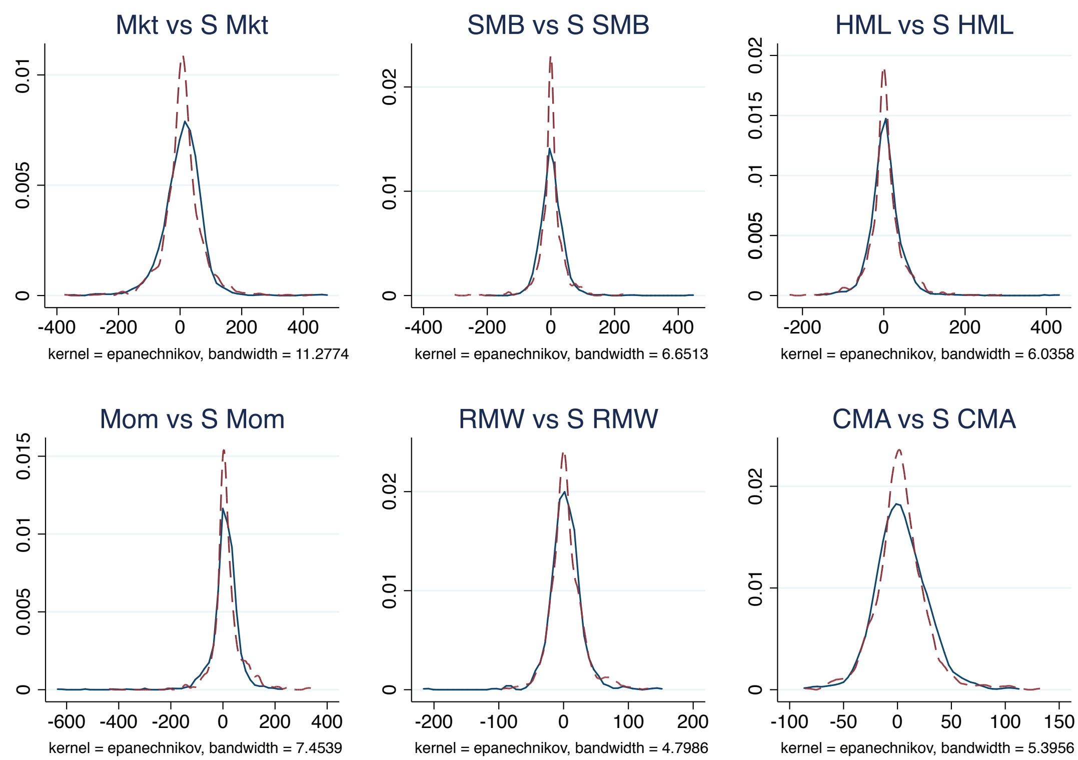

Figure IA. 2. Distribution of Volatility-Managed Factors. The figure plots the full distribution of managed factors (S) versus nonmanaged factors estimated using kernel density estimation. The managed factor, f^{σ}, scales by the factors' inverse realized variance in the preceding month, $f_t^{\sigma} = \frac{c}{RV_{t-1}^2} f_t$. In particular, for each panel we plot the distribution of f_t (solid line) along with the distribution of $\frac{c}{RV_{t-1}^2} f_t$ (dashed line).

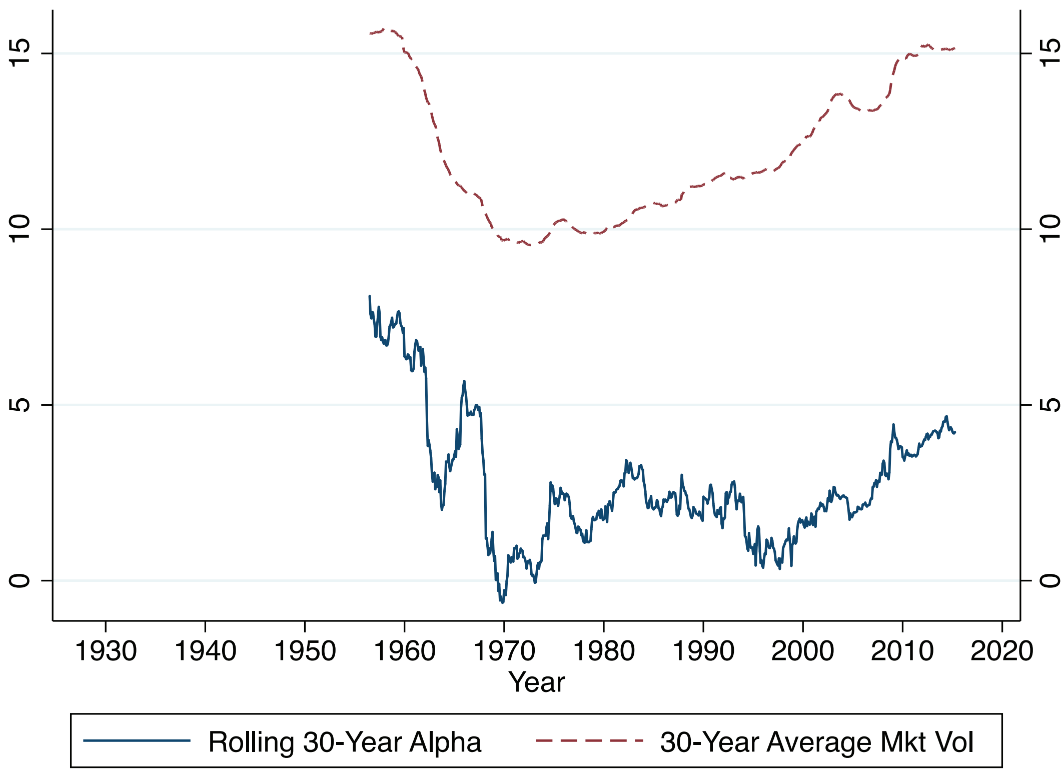

Figure IA. 3. Rolling Window Alphas. We plot the alpha of our strategy for rolling 30-year windows. We only show results for the market return here for brevity. Each point corresponds to using the 30 years of data up until that time period. We also plot the average experienced volatility for the same window. We see that the alpha of our strategy is highest during subsamples when volatility is relatively high. This is consistent with the strategy being more profitable in times when volatility varies substantially.