VFI Toolkit Workshop, pt3A: Alternative Preferences

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VFI Toolkit

- We saw how to setup and solve basic Life-Cycle models.
- *d* (decision varialbe), *a* (endogenous state) and *z* (exogenous markov state).
- We saw other shocks, *e* and *semiz*
- We saw permanent types (by number and by name).
- We saw basic analysis: AllStats and AgeConditionalStats.
- We saw more model analysis (conditional stats, simulate panel data).
- Now we are starting on 'advanced' features.

Life-Cycle Model

- Now, let's talk about..
- Alternative preferences:

  Epstein-Zin, Quasi-Hyperbolic Discounting, Gul-Pesendorfer, Loss-Aversion, Ambiguity Aversion.
- Largely, just add a few lines of code and tell *vfoptions*.
- Examples all build from Workshop Model 3.
- See also: Intro to Life-Cycle Models, Appendix B.

Life-Cycle Model: Alternative preferences

- Concepts these preference aim to capture:
- Epstein-Zin: separate risk-aversion from elasticity of intertemporal substitution
- Quasi-Hyperbolic Discounting: impatience
- Gul-Pesendorfer: temptation and self-control.
- Loss-Aversion: people dislike loses more than they like gains.
- Ambiguity Aversion: risk vs ambiguity (nests Knightian Uncertainty).

Life-Cycle Model: Epstein-Zin preferences

- Standard vonNeumann-Morgenstern preferences have a single parameter that determines both
  - Elasticity of intertemporal substitution.
  - Risk aversion.
- Epstein-Zin preferences use two separate parameters to determine these distinct concepts.

Life-Cycle Model: Epstein-Zin preferences

- Household problem

$$
\begin{aligned}
V(a,z,j) \quad = \quad & \max_{h,c,aprime} \frac{c^{1-\sigma}}{1-\sigma} - \psi \frac{h^{1+\eta}}{1+\eta} \\
& -\beta E[-s_j V(aprime, zprime, j+1)^{1+\phi} | z]^{\frac{1}{1+\phi}} \\
& \text{if } j < Jr: \; c + aprime = (1+r)a + w\kappa_j h \, exp(z) \\
& \text{if } j >= Jr: \; c + aprime = (1+r)a + pension \\
& 0 \leq h \leq 1, aprime \geq 0 \\
& z' = \rho_z z + \epsilon', \quad \epsilon \sim N(0, \sigma_{z,\epsilon})
\end{aligned}
$$

- $\phi > 0$ increases risk aversion (relative to vonNeumann-Morgenstern).

The 'extra minuses' are needed because the utility function is negative valued.
Can think of ϕ as adding 'extra risk aversion'.

Life-Cycle Model: Epstein-Zin preferences

- Code: *WorkshopModel8.m* (still uses *WorkshopModel3_ReturnFn.m*)
- Three changes to code:
  - *vfoptions.exoticpreferences='EpsteinZin'*
    Tell toolkit we are using Epstein-Zin preferences
  - *vfoptions.EZriskaversion='phi'*
    Name of the 'additional relative risk aversion' parameter, and add it to parameter structure.
  - *vfoptions.survivalprobabilites='sj'*
    Name of conditional survival probabilites so they will be treated differently to discount factor (and remove it from DiscountFactorParamNames).
- Note: Once you have Policy, the preferences are no longer (directly) relevant to anything. Hence we don't put anything about them into simoptions, etc.

Life-Cycle Model: Epstein-Zin preferences

- There are two versions of Epstein-Zin preferences: consumption units, utility units (utils).
  $vfoptions.EZutils = 0$ gives consumption units.
  consumption-units is the 'classical' form, but utility-units is much easier to extend
- Warm-glow of bequests has to be done specially with EZ prefs.
  Use $vfoptions.WarmGlowBequestsFn$
- Appendix B of Intro to Life-Cycle Models explains EZ prefs.

Life-Cycle Model: Quasi-hyperbolic discounting

- Standard exponential discounting (β^t) uses same discount factor between any/every two periods.
- Question: would you prefer \$10 today or \$11 tomorrow?
- Question: would you prefer \$10 one year from now or \$11 one year and one day from now?
- If you are a regular human being, you prefer \$10 today, but \$11 one year and one day from now.
  Studies find almost everyone does.
- Violates exponential discounting.
- Hyperbolic discounting permits this very human behaviour.
- Quasi-hyperbolic discounting ($\beta_0 \beta^t$) is a tractable form.
  Also gets called 'alpha-beta discounting' or 'delta-beta discounting' (I used 'beta-0 beta').
- Captures concept of impatience.

Life-Cycle Model: Quasi-hyperbolic discounting

- Quasi-hyperbolic discounting ($\beta_0\beta^t$).
- Use β to discount between any two periods.
- Use 'additional' β_0 to discount between now and next period.
  So discount factor between today and tomorrow is $\beta_0\beta$.
  (Is equivalent but just easier/cleaner to consider β_0 as additional discount, rather than focus on $\beta_0\beta$ as the discount factor).
- Naive vs Sophisticated: you are impatient, but do you think your future self will be patient or impatient?
  Naive: beleive future self behaves as (patient) exponential discounter.
  Sopisticated: beleive future self behaves as (impatient) quasi-hyperbolic discounter.

Life-Cycle Model: Quasi-Hyperbolic discounting

- Household problem

$$
\begin{aligned}
V(a,z,j) \quad = \quad & \max_{h,c,aprime} \frac{c^{1-\sigma}}{1-\sigma} - \psi \frac{h^{1+\eta}}{1+\eta} \\
& \quad + s_j \beta_0 \beta E[\tilde{V}(aprime, zprime, j+1)|z] \\
& \text{if } j < Jr: \; c + aprime = (1+r)a + w\kappa_j h \, exp(z) \\
& \text{if } j >= Jr: \; c + aprime = (1+r)a + pension \\
& 0 \leq h \leq 1, aprime \geq 0 \\
& z' = \rho_z z + \epsilon', \quad \epsilon \sim N(0, \sigma_{z,\epsilon})
\end{aligned}
$$

- $\tilde{V}$ is the 'continuation value' and definition depends if Naive or Sophisticated.
Roughly, in both cases you discount with β, if Naive is value of using policy function of exponential discounter, if Sophisticated is value of using policy function of quasi-hyperbolic discounter.
See Intro to Life-Cycle Models, Appendix B, for exact formulation.

Life-Cycle Model: Quasi-Hyperbolic discounting

- Code: *WorkshopModel9.m* (still uses *WorkshopModel3_ReturnFn.m*)
- Three changes to code:
  - *vfoptions.exoticpreferences='QuasiHyperbolic'*
    Tell toolkit we are usingQuasi-Hyperbolic discounting
  - *vfoptions.quasi_hyperbolic='Sophisticated'*
    Or 'Naive'.
  - *vfoptions.QHadditionaldiscount='beta0';*
    And put this in parameter structure, e.g. *Params.beta0=0.85;*
- Will output both V and the continuation value (interpretation depends on Naive vs Sophisticated).
- Note: Once you have Policy, the preferences are no longer (directly) relevant to anything. Hence we don't put anything about them into simoptions, etc.

Life-Cycle Model: Gul-Pesendorfer preferences

- Temptation: if you have money, you will spend it.
- Odysseus had himself tied to the mast to avoid temptation.
  Listening to the Sirens without jumping into the water and drowning.
- Homo-Economicus buys a house to force him/herself to save money.
  No more Avo toast. Goodbye holidays to the islands of Greece, the beaches of Spain, and the glaciers of Alaska!
- Gul-Pesendorfer adds a 'temptation function' to standard setup.
- Decisions today that reduce 'temptation' in the future help create self-control.
  Buying a house is a form of tying yourself to the mast.

Historical note: I feel like Odyesseus got a better deal than us poor Homo-Economi.

Life-Cycle Model: Gul-Pesendorfer preferences

- Household problem

$$
\begin{aligned}
V(a,z,j) \quad = \quad & \max_{h,c,aprime} \frac{c^{1-\sigma}}{1-\sigma} - \psi \frac{h^{1+\eta}}{1+\eta} \\
& \quad + v(c) - \max_{\hat{c}} v(\hat{c}) \\
& \quad + s_j \beta E[V(aprime, zprime, j+1)|z] \\
& \text{if } j < Jr: \ c + aprime = (1+r)a + w\kappa_j h\, exp(z) \\
& \text{if } j >= Jr: \ c + aprime = (1+r)a + pension \\
& 0 \leq h \leq 1, aprime \geq 0 \\
& z' = \rho_z z + \epsilon', \quad \epsilon \sim N(0, \sigma_{z,\epsilon})
\end{aligned}
$$

- Idea: $v(c)$ is the tempation function.
- So $v(c) - \max_{\hat{c}} v(\hat{c})$ is the cost of resisting temptation (of the most tempting alternative).

Life-Cycle Model: Gul-Pesendorfer preferences

- Code: *WorkshopModel10.m* (still uses *WorkshopModel3_ReturnFn.m*)
- Two changes to code:
  - *vfoptions.exoticpreferences='GulPesendorfer'*
    Tell toolkit we are using Gul-Pesendorfer preferences
  - *vfoptions.temptationFn*
    Set up the temptation function, $v(c)$.
- Notice that we do not change the ReturnFn.
- Note: Once you have Policy, the preferences are no longer (directly) relevant to anything. Hence we don't put anything about them into simoptions, etc.

Life-Cycle Model: Loss Aversion

- Loss Aversion (a.k.a. Prospect Theory).
- People appear to dislike losses more than they like gains.
- To implement, we need: a reference point (higher than reference is gain, lower than reference is loss).
  We will assume that 'lag of consumption' is the reference point.
- Then just make utility 'steeper' below reference point than above it.
- Loss Aversion implements this concept.

Life-Cycle Model: Loss Aversion

- Household problem

$$
\begin{aligned}
V(a,z,j) \quad = \quad & \max_{h,c,aprime} U(c,c_{lag}) - \psi \frac{h^{1+\eta}}{1+\eta} \\
& \quad + s_j \beta E[V(aprime, zprime, j+1)|z] \\
& \text{if } j < Jr: \;\; c + aprime = (1+r)a + w\kappa_j h \, exp(z) \\
& \text{if } j >= Jr: \;\; c + aprime = (1+r)a + pension \\
& 0 \leq h \leq 1, aprime \geq 0 \\
& z' = \rho_z z + \epsilon', \quad \epsilon \sim N(0, \sigma_{z,\epsilon})
\end{aligned}
$$

where $U(c, c_{lag}) = \theta u(c) + (1-\theta) v(u(c) - u(c_{lag}))$.
$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. $\Delta = u(c) - u(c_{lag})$
$v(\Delta) = (1 - exp(-\mu)\Delta))/\mu$ for $\Delta > 0$, and
$v(\Delta) = -\lambda(1 - exp(-\upsilon/\lambda)\Delta))/\mu$ for $\Delta > 0$
$v(\Delta)$ is 'creating' the loss aversion. Notice how it differs for gains, $\Delta > 0$ and losses, $\Delta < 0$.

- Assumes that reference point is last period consumption.

Life-Cycle Model: Loss Aversion

- Code: *WorkshopModel11.m* (uses *WorkshopModel11_ReturnFn.m*)
- Two changes to code:
  - Change ReturnFn to include the loss-aversion.
  - Add 'lag of consumption' to the state space.
    The 'reference point' for the loss aversion.
- Note: We do have to tell simoptions, because we added a 'residualasset' (which changes state-space and action-space).
- Notice that there is no 'vfoptions.lossaversion', because loss-aversion actually ends up as just an unusual ReturnFn (and a change to the action and state spaces), and so can be solved with entirely standard algorithms.

Life-Cycle Model: Ambiguity Aversion

- Known-knowns, known-unknowns, unknown-knowns, and unknown-unknowns.
- Uncertainty vs Amgibuity
- Uncertainty (vonNeumann-Morgenstern expected utility) is 'averaging across the utility in each of the possible futures based on their probabilities'.
- Knightian Uncertainty is 'worst possible utility across the possible futures'
- Ambiguity is 'I have *p* priors about future probabilties, first for each one of my priors I do average utility across each possible future based on the probabilities of that prior, then I take the worst possible one of these average-utility-under-a-prior'. Note: Knightian Uncertainty is nested as Ambiguity where the priors include a prior that puts probability one on a single possible future and zero on all others, with one such prior for each possible future. (Typically Ambiguity would not include all such priors, so is not as 'pessimistic' as Knightian Uncertainty.)
- Define, SET OF PRIORS

- Household problem

$$\begin{aligned}
V(a,z,j) \quad = \quad & \max_{h,c,aprime} \frac{c^{1-\sigma}}{1-\sigma} - \psi\frac{h^{1+\eta}}{1+\eta} \\
& \quad +s_j\beta E[V(aprime, zprime, j+1)|z] \\
& \text{if } j < Jr: \;\; c + aprime = (1+r)a + w\kappa_j h\, exp(z) \\
& \text{if } j >= Jr: \;\; c + aprime = (1+r)a + pension \\
& 0 \leq h \leq 1, aprime \geq 0 \\
& z' = \rho_z z + \epsilon', \quad \epsilon \sim N(0, \sigma_{z,\epsilon})
\end{aligned}$$

- Let's write the code to solve this.
  Code: *WorkshopModel12.m* (still uses *WorkshopModel3_ReturnFn.m*)

Life-Cycle Model: Ambiguity Aversion

- You can implement Knightian-Uncertainty via Ambiguity Aversion by specific choice of priors.
- Ambiguity Aversion works with *riskyasset* (which we will see in Part 2C, for portfolio-choice models).

If you just want people to, e.g., overweight the risk of large disaster, you can do this by using different *pi_z* for solving value fn (what people believe) from the *pi_z* for agent dist/panel data simulation (what actually happens).

Life-Cycle Model: Alternative preferences

- Summary. We saw how to implement...
- Epstein-Zin: separate risk-aversion from elasticity of intertemporal substitution
- Quasi-Hyperbolic Discounting: impatience
- Gul-Pesendorfer: temptation and self-control
- Loss-Aversion: people dislike loses more than they like gains.
- Ambiguity Aversion: risk vs ambiguity (nests Knightian Uncertainty).
- All take just a few lines of code.
- All are explained in more depth in Intro to Life-Cycle Models, Appendix B.
- All are demoed in Intro to Life-Cycle Models, Life-Cycle Models 12, 36, 37, 38, 39.

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