VFI Toolkit: Workshop, Part 2

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VFI Toolkit

- We saw how to setup and solve basic Life-Cycle models.
- d (decision varialbe), a (endogenous state) and z (exogenous markov state).
- And basic analysis, StationaryDist, AllStats, and AgeConditionalStats.
- VFI Toolkit does much more: i.i.d shocks, panel data, human capital, portfolio-choice, conditional statistics, calibration, GMM estimation.
- Let's take a whirlwind tour!

Life-Cycle Model

- All of this is covered in the examples of the Intro to Life-Cycle Models.
- Intro to Life-Cycle Models: pdf of 50 example Life-Cycle models, adding features one at a time. Covers everything we did here, plus much more.

Life-Cycle Model

- Rough outline:
    1. Exogenous shocks: i.i.d, correlated shocks (e.g., VAR), semi-exogenous state.
    2. Permanent Types
    3. Analyse model: simulate panel data, conditional stats, faster stats.
    4. Calibrate model
    5. Alternative preferences.
    6. Alternative endogenous states: human capital, riskyasset (portfolio choice).
    7. GMM estimation.

Life-Cycle Models: Exogenous shocks

- We saw markov shocks z.
- Can also add i.i.d. exogenous shocks e.
- Beyond the basics, we set things up using *vfoptions* and *simoptions* to explain the model to VFI Toolkit.
- So we would create *vfoptions.n_e*, *vfoptions.e_grid* and *vfoptions.pi_e*, put copies in simoptions.
  pi_e will be a column vector.
- Action space: $(d, aprime, a, z, e, \ldots)$ [e always comes just after z]
- Explain e vars: e vars concept

See Intro to Life-Cycle Models: Life-Cycle Model 11.

Life-Cycle Models: Exogenous shocks

- Can have up to four of each of z and e.
  And *semiz* which are explained next slide.
- Can have grids and probabilites depend on age.
  Can also be created by parameterized functions.
- Can use joint-grids, and so can have correlated shocks.
- VFI Toolkit contains many discretization methods: AR(1), AR(1) with gaussian mixture, VAR(1), and versions for 'non-stationary life-cycle' processes.

See Intro to Life-Cycle Models: Appendix A.

Life-Cycle Models: Exogenous shocks

- Semi-exogenous states.
- *semiz* probabilities depend on a decision variable.
- Create *vfoptions.n_semiz*, *vfoptions.semiz_grid*.
- Create *vfoptions.SemiExoStateFn* for the transition probabilities, will be a function of $(semiz, semizprime, d, ...)$.
  Put them in *simoptions* as well.
- Uses: endogenous fertility, search labor, years owned house.
- Action space: $(d, aprime, a, semiz, z, e, ...)$ [*semiz* always comes after a, before z]
- Explain *semiz* vars: semiz vars concept

See Intro to Life-Cycle Models: Life-Cycle Model 29, and Appendix A.

Life-Cycle Models: Permanent Types

- Permanent Types: agents with permanent differences.
- Examples
  - Different values of a parameter.
  - Different exogenous shock processes.
  - Different utility functions.
  - Even that one is finite-horizon and one infinite-horizon.
  - Essentially, anything in toolkit can differ between the permanent types.
- Two ways to set up:
  - Number of permanent types: $N\_i = 2$, $Params.beta = [0.8, 0.95]$
  - Names of permanent types: $Names\_i = 'patient', 'impatient'$ $Params.beta.impatient = 0.8; Params.beta.patient = 0.95$
- VFI Toolkit commands are same in both cases. And all output uses the names. E.g., $V.patient$ and $V.impatient$.
  If N_i, names are allocated as $ptype001$, $ptype002$.
- All model stats reported as aggregate and conditional on ptype.
  E.g, $AllStats.earnings.Mean$, $AllStats.earnings.patient.Mean$ and $AllStats.earnings.impatient.Mean$.

See Intro to Life-Cycle Models: Life-Cycle Models 24, 25, 27.

Life-Cycle Models: Analyse Model

- We saw AllStats and AgeConditionalStats.
- Can simulate panel data: same setup (FnsToEvaluate), different command $SimPanelValues = SimPanelValues\_FHorz\_Case1()$.
- Conditional stats: $simoptions.conditionalrestrictions$ can setup a function in same way we do FnsToEvaluate, then when you call AllStats or LifeCycleProfiles, it will also report moments conditional on this function (function must be binary, e.g. $a > 0$)
  See: explain conditionalrestrictions.
- Faster stats: $simoptions.whichstats = ones(7, 1)$ is default. By setting some zeros, you can e.g., skip lorenz curve and quantiles which take most of the time.
  See: explain whichstats.
  Really this is for when you want write a custom code that will need to use AllStats or LifeCycleProfiles a large number of times. If you are using them once you probably don't care about the runtime to calculate a few unnecessary stats.

See Intro to Life-Cycle Models: Life-Cycle Models 12.

Life-Cycle Models: Calibrate model

- You can easily choose parameters (*CalibParamNames*) to target moments (*TargetMoments*).
- Setup targets, then just *CalibrateLifeCycleModel*().
- Targets can be anything in AllStats and AgeConditionalStats.
  Includes conditional restrictions, and both aggregating across or conditional on permanent types.
- Can include, e.g., parametrizing initial agent distribution and/or exogenous shocks with parameters that will be calibrated.

See Intro to Life-Cycle Models: No examples of calibration yet, but if you read Life-Cycle Models 45-50 about GMM estimation, is essentially the same but without Variance-Covariance matrix. Is used in Huggett, Ventura & Yaron (2006) example.

Life-Cycle Models: Alternative Preferences

- Can solve Epstein-Zin preferences, Quasi-Hyperbolic discounting, Gul-Pesendorfer preferences, Prospect Theory (loss aversion), and Ambiguity Aversion.
- Most just require a few lines to tell *vfoptions* you want to use them.
- See Life-Cycle Model 12 for Epstein-Zin preferences (and 32 to 35 for EZ prefs in portfolio-choice models).
- See Life-Cycle Models 36-39 for the rest.
- Intro to Life-Cycle Models, Appendix B explains how each of these works.

Once you solve for Policy, alternative preferences are actually irrelevant to StationaryDist and model statistics, so we don't need to tell *simoptions*.

Life-Cycle Models: Alternative Endogenous States

- We can have two endogenous states.
- Set $n\_a = [201, 51]$, $a\_grid = [a1\_grid; a2\_grid]$, then action space is $(.., a1prime, a2prime, a1, a2, ...)$.
- In practice, only going to work nicely as of the year 2025 if using $vfoptions.divideandconquer = 1$ and second state is say 51 points.
- E.g., for 'house', 51 points is probably plenty.

See Intro to Life-Cycle Models: Life-Cycle Model 42

Life-Cycle Models: Alternative Endogenous States

- Instead of standard endogenous state, where *a* is in today's state-space, and we can choose *aprime* directly (so *aprime* is also in the action-space)...
- ... risky asset, 'riskyasset', is $aprime(d, u)$, useful for portfolio-choice models.
- ... experience asset, 'expasset' is $aprime(d, a)$, useful for human-capital models.
- ... experience asset u, 'expassetu' is $aprime(d, a, u)$, useful for uncertain human-capital models.
- All require setting up *vfoptions.aprimeFn* (inputs follow from which we are using)
- All of these mean *aprime* is no longer in the decision space, so e.g., for expasset, we likely have action-space $(d, a, z, ...)$ [essentially, write out action-space as usual, but remove *aprime* for the relevant endogenous state that is of this nature]

E.g., $aprime(d, a)$, means you cannot choose *aprime*, but you can choose d, and together with between period i.i.d. shocks u, you end up in $aprime(d, u)$. These alternative assets all evaluate $aprime(x)$ on the grids for x, and then linearly interpolate $aprime(x)$ onto the grid on *aprime* (which in practice is the same grid as a, so *a_grid*.

Life-Cycle Models: Alternative Endogenous States

- You can have two endogenous states where the first state is a standard endogenous state and the second state is a riskyasset/expasset/expassetu.
- Action space: $(d, a1prime, a1, a2, ...)$
  Notice that we put $a1prime$ for the standard endogenous state, but not for the riskyasset/expasset/expassetu.
- See Intro to Life-Cycle Models: Life-Cycle Model 35 for riskyasset (with housing), Life-Cycle Model 43 for expasset (with savings), Life-Cycle Model 44 for expassetu (with savings).

You can do without a standard endogenous state. See Intro to Life-Cycle Models, Life-Cycle Models 31-34 for 'riskyasset' in portfolio-choice problems. See Huggett, Ventura & Yaron (2006) example for 'expasset'.

Life-Cycle Models: GMM estimation

- You can easily choose parameters (*EstimParamNames*) to target moments (*TargetMoments*).
  This part is identical to calibration.
- Set *WeightingMatrix* and *CovarMatrixDataMoments*.
- Then *EstimateLifeCycleModel_MethodOfMoments*().
- Outputs: [*EstimParams*, *EstimParamsConfInts*, *estsummary*]
- *estsummary* contains information on identification, sensitivity and more.

See Intro to Life-Cycle Models: Life-Cycle Models 45-50 about GMM estimation. Appendix C explains concepts around GMM estimation of Life-Cycle Models.
Note, permanent types mean you can handle unobserved heterogeneity.

Life-Cycle Models: Combinations?

- Some combinations are coded: e.g., riskyasset with Epstein-Zin preferences.
- Some combination have not been coded yet.
- If you want a combination that is not yet coded, or just want to know if some combo is coded, please ask via discourse.vfitoolkit.com
  First time someone asks a combo, decent odds I ignore, but if two or three people are asking then I make it a priority and then it happens :D

Closing

- VFI Toolkit can do a lot of models, but equally there are many models it cannot do (yet :P).
- Intro to Life-Cycle Models explains many of these features by examples.
- Examples implementing models from literature: Life-Cycle OLG Reading List
- If you want a combination that is not yet coded, or just want to know if some combo is possible, please ask via discourse.vfitoolkit.com

References I

Robert Kirkby. VFI toolkit, v2. *Zenodo*, 2022. doi: https://doi.org/10.5281/zenodo.8136790.

- Can write Bellman equation like
$$V(a,z,e) = \max_{d,a'} F(d,a',a,z,e) + s_j \beta E[V(a',z',e')|z]$$
- ReturnFn will be a function that plays role of $F(d,a',a,z,e)$
- e is an i.i.d exogenous state (so will have probability distriubtion pi_e).

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Life-Cycle Model

- Can write Bellman equation like

$$V(a, semiz, z) = \max_{d,a'} F(d, a', a, semiz, z) + s_j \beta E[V(a', semiz', z')|semiz, z, d]$$

- ReturnFn will be a function that plays role of $F(d, a', a, semiz, z)$
- *semi* is an semi-exogenous markov state (so will have probability transitions that depend on $(semiz, semizprime, d..)$ and a set using $vfoptions.SemiExoStateFn$.
- Roughly, *semiz* is a markov state, but where depending on value of the decision variable, you get different transition matrix.
- I here simplify and have d in everything, in practice often you can split d in $d1$ and $d2$, with both in ReturnFn, but only $d2$ determine semi-exo state.

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