VFI Toolkit: Workshop, Part 4

vfitoolkit.com
Robert Kirkby

robertdkirkby.com
Victoria University of Wellington

March 11, 2025

VFI Toolkit

- We have seen Life-Cycle Models, which were partial equilibrium.
- We have seen OLG models, which add (stationary) general equilibrium.
- Now for OLG general eqm transition paths.
- Note: end point of transition must be a stationary general equilibium.
  Initial point can be, but does not need to be.
  Intuitively, if end point is not a stationary general equilibium why might we expect to actually end up converging to it?

OLG Model Transition

- Code: first just solve for a final stationary general equilibium.
  Is just the same code we did to solve OLG models.
- In many cases, also solve for an initial stationary general equilibrium.
- Set $T = 100$, the number of time periods for our transition.
- So we identify initial with period 0 and final with period T.

OLG Model Transition

- In period 1 a path on exogenous parameters will be revealed.
- Call this *ParamPath*
  E.g., we might set $ParamPath.tau = [Params.tau\_initial * ones(1,9), Params.tau\_final.r * ones(1, T-9)]$;
  (announce today that tax *tau* will be increased in 10 periods time)
- We want to find the path on endogenous parameters
  E.g., we might set initial guess
  $PricePath0.r = [linspace(p\_eqm\_initial.r, p\_eqm\_final.r, ceil(T/2)), p\_eqm\_final.r * ones(1, T - ceil(T/2))]$;
- That satisfies some general eqm conditions *GeneralEqmEqns_Transition*, set up in same way we would stationary general eqm eqns.
  E.g., *GeneralEqmEqns_Transition.capitalmarket* set up to evaluate to zero in general eqm.
  Can depend on lag/leads of parameters on the path. E.g., bequests received today must equal bequests left yesterday.
- Tell VFI Toolkit how to update price path based on general eqm eqns using *transpathoptions.GEnewprice3.howtoupdate*.
  E.g. we might say that new interest rate r is equal to 0.9 times old interest rate minus 0.1 times the general eqm eqn $r - \alpha K^{\alpha-1} L^{1-\alpha}$. Note, this general eqm eqn is positive if r is 'too big', hence 'minus'.

OLG Model Transition

- Solve the general equilibrium transition path

$$PricePath = TransitionPath\_Case1\_FHorz(PricePath0, ParamPath, T, V\_final, StationaryDist\_init, ...)$$

- Done!

Currently, all transition paths are solved using 'shooting' algorithms.

OLG Model Transition

- Once you solve, there are commands to calculate V, Policy, AgentDist, and AggVars across the transition path.
- Transition paths do not work for more advanced life-cycle model setups, currently they handle d, a, z and e.
  More features hopefully coming soon :)
- Most of the time, you want to set $transpathoptions.fastOLG = 1$ (which parallizes across age j, faster but requires more gpu memory) and $vfoptions.divideandconquer = 1$ (exploit monotonicity, faster and also uses less gpu memory making fastOLG easier to use).

Basic OLG Model

- Intro to OLG Transition Paths has some examples.
- This document is not yet online, but if you email I am happy to send a copy (contains about 7 examples).
- See Conesa & Krueger (1999) example.
- For infinite horzion transition paths, there is an example with Aiyagari (1994) model, and example based on Guerrieri & Lorenzoni (2017) model.

References I

Robert Kirkby. VFI toolkit, v2. *Zenodo*, 2022. doi: https://doi.org/10.5281/zenodo.8136790.