VFI Toolkit Workshop, pt5: OLG Transition Paths

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VFI Toolkit

- We have seen Life-Cycle Models, which were partial equilibrium.
- We have seen OLG models, which add stationary general equilibrium.
- Now for OLG general eqm transition paths.

OLG Model Transition

- Why we solve transition paths?
- Say you want to think about impact of cutting pensions by 20%.
- We could solve an OLG model twice, before and after the pension cut.
- This would involve comparing stationary general eqms.
- One of these does represent 'before'.
- But the other represents the 'infinite future after'.
- Infinite future is very different to 'what will happen in the years after the reform'.
- So we solve transition paths, to see how things evolve between the 'before' and the 'infinite future after'.

Numerous papers in the literature show that the transition paths give meaningfully different answers to just comparing stationary general eqm. So while it requires much more work to solve the transition path, we cannot skip this on the grounds it is unimportant.

OLG Model Transition

- Furthermore, many policy questions only make sense with a transition.
- Example application: Announce today that the pension will be cut by 20% in 10 years time.
  No change for periods 1 (today) to 9, then 20% lower for periods 10 to T.
- Because the reform here is 'preannounced', comparing stationary general eqm is not even a possible alternative.
- Example application: how will societal ageing change interest rates over the next 30 years?
- We are interested in the whole path for the interest rate over the next 30 years.

OLG Model Transition

- Concept Intuition: Transition Path
- In period 1, wake up and learn about an exogenous path for parameters from now until period T.
- Need to solve for the general eqm (e.g., find path on r) for all T periods.
- Assume: in period T we reach the final stationary general equilibium.
If the final is not a stationary general eqm, why might we ever be expected to converge to there?
Note: no need to start in a stationary general eqm, only need to finish in one.

OLG Model Transition

- Concept for Code: Transition Path
- In period 1, we have an initial distribution of agents.
  No reason this should be associated with a stationary general eqm, but in many applications it will be.
- Wake up and learn 'ParamPath', an exogenous path on (some) model parameters.
  Conceptually, there is a path on every model parameter, but since in applications most will be constant we only define those which are not constant.
- Period T must be a stationary general eqm, so we need to solve for V_T (the value function associated with the final general eqm) as this is needed for computing 'PricePath'.
  Solve the stationary eqm, then calculate V.
- Solve for 'PricePath' an endogenous path of general eqm parameters, based on satisfying some GeneralEqmEqns.
  TransitionPath_Case1_FHorz()

OLG Model Transition: Example 1

- Let's do an example.
- Code: *WorkshopOLGTPath1.m* (and *WorkshopOLGTPath1_ReturnFn*)
- We will reuse Workshop OLG Model 1, with minor change that we add a tax on earnings and use it to pay for 'G'.
- Transition path: in period 1 we find out that 'tax' has been cut to 0.05 (it was previously 0.1), and will stay there.
- Model has three general equilibium conditions: r for capital market, w for labor market, and G for government budget.
- Solve initial and final stationary general eqm to get $AgentDist_1$ and V_T.
- So we want to solve the general eqm transition path on $\{r_t, w_t, G_t\}_{t=1}^T$ that satifies the three general equilibrium conditions in every period, given the exogenous parameter path on $\{\tau_t\}_{t=1}^T$ (and given $AgentDist_1$ and V_T).

OLG Model Transition: Example 1

- Household problem

$$\begin{aligned} V(a,z,j) &= \max_{h,c,aprime} \frac{c^{1-\sigma}}{1-\sigma} - \psi \frac{h^{1+\eta}}{1+\eta} + \beta E[V(aprime, zprime, j+1)|z] \\ & \text{if } j < Jr: \; c + aprime = (1+r)a + (1-\tau)w\kappa_j hexp(z) \\ & \text{if } j >= Jr: \; c + aprime = (1+r)a + pension \\ & 0 \leq h \leq 1, aprime \geq 0 \\ & z' = \rho_z z + \epsilon', \quad \epsilon \sim N(0, \sigma_{z,\epsilon}) \end{aligned}$$

- Workshop OLG Model 1, but add tax (τ) and remove sj (and don't use trick for w).

OLG Model Transition: Example 1

- First, we want to solve the initial and final stationary general equilibrium.
- Use GEPriceParamNames=$\{$'r','w','tau'$\}$ and three GeneralEqmEqns.
- For initial, solve stationary general eqm with $\tau = 0.1$.
- For final, solve stationary general eqm with $\tau = 0.05$.
- For transition path, really we just need $AgentDist_0$ and V_T.
- We already know how to do this part.

OLG Model Transition: Example 1

- Set T, the number of periods for the transition, e.g., $T = 100$.
  T must be large enough that we have time to converge to the final stationary general eqm.
  If you set T too small, it will fail (as cannot converge).
  If you set T too large, it just wastes time (as compute lots of periods after converge is already reached).
- Advice: Good choice of T varies widely by model. Err on the larger side the first time you solve (so it does converge), then looking at solution path shorten T to be not long after convergence is reached, so that codes are faster from now on.
  Computing periods after the convergence has occured is wasteful. That said, best to still include a few of them as otherwise is a bit knife-edge whether or not things solve.
- Alway make sure to check the solution to ensure it has converged! (Compare prices in the last few periods with those of the final stationary general eqm).

OLG Model Transition: Example 1

- In period 1 a path on exogenous parameters will be revealed.
- Call this *ParamPath*
- In our example, there is just one path on tax
- $ParamPath.tau = 0.05 * ones(1, T);$
  Values of 'tau' for periods 1 to T.

Implicitly, all other exogenous parameters are assumed to be constant over periods 1 to T. So there is a path on all parameters, but we only need to write the path for parameters that change value.

OLG Model Transition: Example 1

- We want to find the path on endogenous parameters.
- Set initial guess for the general eqm parameters,
- E.g.,

  $PricePath0.r = [linspace(p\_eqm\_initial.r,\ p\_eqm\_final.r, ceil(T/2)),$ $p\_eqm\_final.r * ones(1, T - ceil(T/2))];$

  $PricePath0.w = [linspace(p\_eqm\_initial.w,$ $p\_eqm\_final.w, ceil(T/2)), p\_eqm\_final.w * ones(1, T - ceil(T/2))];$

  $PricePath0.G = p\_eqm\_final.r * ones(1, T);$

  No need to set a cell with the names, toolkit reads the names from the 'fields' of the 'PricePath0' structure.

- That satisfy some general eqm conditions (in every period 1 to T)
- *GeneralEqmEqns_Transition*, set up in same way we would stationary general eqm eqns.

  So, e.g., *GeneralEqmEqns_Transition.capitalmarket* set up to evaluate to zero in general eqm.
- So our example will have *GeneralEqmEqns_Transition.capitalmarket*, *GeneralEqmEqns_Transition.labormarket*, and *GeneralEqmEqns_Transition.govbudget*.

OLG Model Transition

- We want to find the path on endogenous parameters, *PricePath* that satisfies some general eqm conditions *GeneralEqmEqns_Transition*.
- Main algorithm: shooting.
- Set $p_{new} = p_{old} + \theta GEcondn(p_{old})$
- Tell VFI Toolkit how to update price path based on general eqm eqns using *transpathoptions.GEnewprice3.howtoupdate*.
  E.g. we might say that new interest rate r is equal to 0.9 times old interest rate minus 0.1 times the general eqm eqn $r - \alpha K^{\alpha-1} L^{1-\alpha}$. Note, this general eqm eqn is positive if r is 'too big', hence 'minus'.

- Solve the general equilibrium transition path

  $PricePath = TransitionPath\_Case1\_FHorz(PricePath0, ParamPath, T, V\_final, StationaryDist\_init, ...)$
- Done!

OLG Model Transition

- Once you solve, there are commands to calculate V, Policy, AgentDist, and AggVars across the transition path.
- Transition paths do not work for more advanced life-cycle model setups, currently they handle d, a, z and e.
  More features hopefully coming soon :)
- You essentially always will want to set $transpathoptions.fastOLG = 1$ (which parallizes across age j, faster but requires more gpu memory) and $vfoptions.divideandconquer = 1$ (exploit monotonicity, faster and also uses less gpu memory making fastOLG easier to use).

OLG Model Transition: Example 1

- Code: *WorkshopOLGTPath1.m* (and *WorkshopOLGTPath1_ReturnFn*)
- Lines 129-175ish are initial and final stationary general eqm.
- Transition path starts lines 178ish.
- Get results like,
  Notice how r jumps in period 1 (red is periods -3 to 0, the initial stationary eqm), but K does not (because agent dist cannot change in period 1, but prices can).

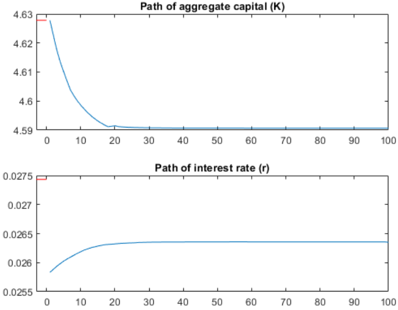

Figure: Example 1

OLG Model Transition

- Note that having period 0 in graph was handy.
  Makes jumps clearer.
- Also always good to double-check that period T is reaching the final stationary general eqm.

OLG Model Transition

- We will do a few more examples:
- Preannounced policy changes.
- Multiple reveals.
- Bequests ('tminus1' in general eqm conditions)
- Demographic transition.
- Permanent types

OLG Model Transition: Example 2

- Example 1: in period 1 we find out that 'tax' has been cut to 0.05 (it was previously 0.1), and will stay there.
- Example 2: in period 1 we find out that 'tax' has been cut to 0.05 in period 10 (remains 0.1 until then), and will stay there.
- Code: $WorkshopOLGTPath2.m$ (and reuse $WorkshopOLGTPath1\_ReturnFn$)
- Only change is trivial,

  $ParamPath.tau = [0.1 * ones(1,9), 0.05 * ones(T-9)];$

Trivial example to emphasize that you put a whole path.

OLG Model Transition: Example 2

- For Example 2, get results like

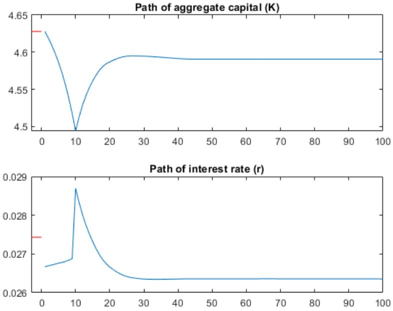

Figure: Example 2

- Can clearly see the spike at period 10, and equally how everyone begins reacting to the knowledge it is coming.
- Looking carefully can see that initial (red) and final eqm are same as in Example 1.

The y-axes change.

OLG Model Transition: Example 3

- Multiple-Reveals.
- Reveal 1: In period 1 wake up to announcement that tax will be cut in period 10.
- Reveal 2: In period 8 wake up to new announcement that tax won't be cut until period 15.
- Code: *WorkshopOLGTPath3.m* (and reuse *WorkshopOLGTPath1_ReturnFn*)
- Is just solving transition path for 'reveal 1', and then using the agent dist in period 8 of that path as the starting point to solve the transition path for 'reveal 2'.
- VFI Toolkit has command that automates this:

Here there are just two, so would be easy to do 'by hand'. But, e.g., you might want to solve 'repeated transition paths', where a new path is revealed every period.
This command is also useful for 'Repeated Transition Paths', where a new path is revealed every period.

OLG Model Transition: Example 3

- For Example 3, get results like

./Material/tpath3.png

Figure: Example 2

- Solid line is the realized path. Dotted lines are the two reveals, notice how realization follows first reveal until period 8, then switch to

OLG Model Transition: Example 4

- Add conditional survival probabilities, *sj*, and a general equilibium condition for accidental bequests.
- Challenge: bequests left are in period $t-1$ and bequests received are in period t.
- VFI Toolkit allows you to use '_tminus1' in general eqm eqns.
- Code: *WorkshopOLGTPath4.m* (and *WorkshopOLGTPath4_ReturnFn*)
- Look carefully at the inputs to the general eqm eqns for transition path.

There is also '_tplus1'. '_tminus1' and '_tplus1' can be used for parameters and for AggVars.

OLG Model Transition: Example 5

- Demographic transition:
- People are living longer.
- Population is getting older.
- So changes in *sj* (conditional surivival probabilities) and *mewj* (population age masses).
- Easy, just put *ParamPath.sj* and *ParamPath.mewj*.
- Both *sj* and *mewj* are age-dependent, so these paths will each be $N\_j - by - T$.
- Code: *WorkshopOLGTPath*5*.m* (and resuse *WorkshopOLGTPath*4*_ReturnFn*)

As usual, VFI Toolkit handles the age-dependence of parameters for you. Note we need to 'fix' the parameters at a final value long enough (large enough T) to allow for convergence to final stationary general eqm.
In practice, you probably want to derive the *mewj* from *sj* and population growth (of age $j = 1$), possibly also net immigration. Here I just make up numbers for them.
No 'reform' is modelled. The demographic transition alone already creates a transition path.

OLG Model Transition: Example 6

- Permanent types: Two agents, 'scaredycat' and 'curiouscat', differ by risk aversion σ.
- Redo same reform as Example 1.
- Code: *WorkshopOLGTPath6.m* (and resuse *WorkshopOLGTPath1_ReturnFn*)

As usual, use *_PType* versions of all the commands.

OLG Model Transition: Permanent Types

- Permanent Types.
- Both *N_i* and *Names_i* can be used.
- Transition path general eqm conditions can be solved 'conditional on ptype', same as for stationary general eqm.

OLG Model Transition

- Random advice: when solving a transition paths in a new model for the first time, trying to solve 'no change' is often a good way to test your code.
  Solving 'no change' does not guarantee your code will work for a reform, but if you cannot solve 'no change' you cannot solve a reform, and debugging 'no change' tends to be much easier as you know exactly what should happen, namely nothing.

OLG Model Transition

- Advanced features of life-cycle models have not yet been implemented for transition paths.
- If you want to use one, ask on forum: discourse.vfitoolkit.com

OLG Model Transition

- Intro to OLG Transition Paths has some examples.
- This document is not yet online, but if you email I am happy to send a copy (contains about 7 examples).
- See also the Conesa & Krueger (1999) example.

References I

Robert Kirkby. VFI toolkit, v2. *Zenodo*, 2022. doi: https://doi.org/10.5281/zenodo.8136790.