ФЕДЕРАЛЬНОЕ ГОСУДАРСТВЕННОЕ АВТОНОМНОЕ ОБРАЗОВАТЕЛЬНОЕ УЧЕРЕЖДЕНИЕ ВЫСШЕГО ОБРАЗОВАНИЯ
«Санкт-Петербургский политехнический университет Петра Великого»

ИНСТИТУТ КОМПЬЮТЕРНЫХ НАУК И ТЕХНОЛОГИЙ
Высшая школа программной инженерии

Отчет по курсовой работе
по дисциплине «Математические модели»

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1 Задание

Для **МОДЕЛИ 3** в плоскости параметров (**p4, p6**) построить бифуркационные диаграммы точек поворота (**p4, p6** > 0). При построении диаграммы целесообразно использовать логарифмический масштаб $(\log(\mathbf{p4}, \mathbf{p6}))$ по обеим осям. Убедиться, что это точки поворота, а не ветвления. Проиллюстрирова[...] количество решений в каждой области.
(p1 = 8.4e-6, p2 = 6.6667e-4, p3 = 1.7778e-5, p5 = 2).

$\frac{dx_1}{dt} = (p_1 x_2 - x_1 x_2 + x_1 - x_1^2)/p_2 - p_4 x_1;$
$\frac{dx_2}{dt} = (-p_1 x_2 - x_1 x_2 + p_5 x_3)/p_3 + p_4(p_6 - x_2);$
$\frac{dx_1}{dt} = x_1 - x_3 - p_4 x_3;$

2 Блок аналитических преобразований

Будем решать систему, в качестве ε варьируя p_4

$$\begin{cases} det(J_N) = 0 \\ f(x, \varepsilon) = 0 \end{cases} \tag{1}$$

Посчитаем матрицу Якоби:

$$J_N = \begin{pmatrix} -\frac{x_2}{p_2} + \frac{1}{p_2} - \frac{2x_1}{p_2} - p_4 & \frac{p_1}{p_2} - \frac{x_1}{p_2} & 0 \\ -\frac{x_2}{p_3} & -\frac{p_1}{p_3} - \frac{x_1}{p_3} - p_4 & \frac{p_5}{p_3} \\ 1 & 0 & -1 - p_4 \end{pmatrix}$$

Запишем систему $f(x, p_4) = 0$:

$$\begin{cases} (p_1 x_2 - x_1 x_2 + x_1 - x_1^2)/p_2 - p_4 x_1 = 0; \\ (-p_1 x_2 - x_1 x_2 + p_5 x_3)/p_3 + p_4(p_6 - x_2) = 0; \\ x_1 - x_3 - p_4 x_3 = 0; \end{cases} \tag{2}$$

Из первого уравнения выразим x_2 через x_3, из третьего уравнения выразим x_3 через x_1:

$x_2 = \frac{x_1^2 + p_2 p_4 x_1 - x_1}{p_1 - x_1}$
$x_3 = \frac{x_1}{1 + p_4}$

Подставим их в уравнение $det(J_N) = 0$ и получим кубическое уравнение относительно x_1. Находим корни, подставляем их в выражения для x_2 и x_3.

p_6 можем найти из второго уравнения:

$p_6 = \frac{p_1 x_2 + x_1 x_2 - p_5 x_3}{p_3 p_4} + x_2$

3 Блок проверки

Проверку реализуем несколькими способами. При подстановке полученных значений в исходные данные:

1. Равенства в системе должны соблюдаться:
   $f(x, p_4, p_6) = 0$

2. Определитель матрицы Якоби должен быть близок к нулю

3. Одно из собственных значений матрицы Якоби должно быть близко нулю

Также необходимо проверить, что полученные точки являются точками поворота, а не ветвления: в матрице Якоби вычеркиваем один столбец и заменяем его на частные производные по параметру **p4**, подставляем все значения и проверяем, что определитель полученной матрицы не равен нулю.

В приложении приведен код программы с реализацией данных проверок.

4 Результаты работы программы

```
p4 = 1.000000e+00
x1 = 2.296225e-04  x1 = 2.296225e-04  x1 = -4.611574e-04

p4 = 1.100000e+00
x1 = -5.538053e-05  x1 = 7.200466e-05  x1 = 2.378996e-02
x2 = 1.131154e+00  x3 = 3.428793e-05  p6 = 2.275275e+00
x2 = 9.758213e-01  x3 = 1.132855e-02  p6 = 2.991050e+01

p4 = 1.200000e+00
x1 = -3.832125e-05  x1 = 5.496644e-05  x1 = 4.543393e-02
x2 = 1.179378e+00  x3 = 2.498474e-05  p6 = 2.340153e+00
x2 = 9.539424e-01  x3 = 2.065178e-02  p6 = 9.684935e+01

p4 = 1.300000e+00
x1 = -3.106441e-05  x1 = 4.773154e-05  x1 = 6.519572e-02
x2 = 1.212459e+00  x3 = 2.075284e-05  p6 = 2.361309e+00
x2 = 9.340580e-01  x3 = 2.834596e-02  p6 = 1.832033e+02

p4 = 1.400000e+00
x1 = -2.691505e-05  x1 = 4.359485e-05  x1 = 8.331062e-02
x2 = 1.237461e+00  x3 = 1.816452e-05  p6 = 2.362952e+00
x2 = 9.158484e-01  x3 = 3.471276e-02  p6 = 2.774243e+02

p4 = 1.500000e+00
x1 = -2.420245e-05  x1 = 4.088966e-05  x1 = 9.997624e-02
x2 = 1.257234e+00  x3 = 1.635586e-05  p6 = 2.354354e+00
x2 = 8.990993e-01  x3 = 3.999050e-02  p6 = 3.727117e+02

p4 = 1.600000e+00
x1 = -2.228721e-05  x1 = 3.897874e-05  x1 = 1.153598e-01
x2 = 1.273291e+00  x3 = 1.499182e-05  p6 = 2.340034e+00
x2 = 8.836378e-01  x3 = 4.436916e-02  p6 = 4.651303e+02

p4 = 1.700000e+00
x1 = -2.086596e-05  x1 = 3.755986e-05  x1 = 1.296038e-01
x2 = 1.286559e+00  x3 = 1.391106e-05  p6 = 2.322471e+00
x2 = 8.693192e-01  x3 = 4.800141e-02  p6 = 5.524949e+02

p4 = 1.800000e+00
x1 = -1.977426e-05  x1 = 3.646923e-05  x1 = 1.428303e-01
x2 = 1.297654e+00  x3 = 1.302473e-05  p6 = 2.303118e+00
x2 = 8.560201e-01  x3 = 5.101082e-02  p6 = 6.336964e+02

p4 = 1.900000e+00
x1 = -1.891450e-05  x1 = 3.560960e-05  x1 = 1.551445e-01
x2 = 1.307010e+00  x3 = 1.227917e-05  p6 = 2.282866e+00
x2 = 8.436345e-01  x3 = 5.349811e-02  p6 = 7.082861e+02

p4 = 2.000000e+00
x1 = -1.822469e-05  x1 = 3.491923e-05  x1 = 1.666377e-01
x2 = 1.314950e+00  x3 = 1.163974e-05  p6 = 2.262276e+00
x2 = 8.320709e-01  x3 = 5.554592e-02  p6 = 7.762166e+02

p4 = 2.000000e+00
```

```
x1 = -1.822469e-05 x1 = 3.491923e-05 x1 = 1.666377e-01
x2 = 1.314950e+00 x3 = 1.163974e-05 p6 = 2.262276e+00
x2 = 8.320709e-01 x3 = 5.554592e-02 p6 = 7.762166e+02


p4 = 3.000000e+00
x1 = -1.542839e-05 x1 = 3.209895e-05 x1 = 2.499608e-01
x2 = 1.351694e+00 x3 = 8.024738e-06 p6 = 2.077173e+00
x2 = 7.480643e-01 x3 = 6.249020e-02 p6 = 1.163471e+03


p4 = 4.000000e+00
x1 = -1.511175e-05 x1 = 3.174281e-05 x1 = 2.999505e-01
x2 = 1.356184e+00 x3 = 6.348561e-06 p6 = 1.943201e+00
x2 = 6.974023e-01 x3 = 5.999010e-02 p6 = 1.255223e+03


p4 = 5.000000e+00
x1 = -1.546367e-05 x1 = 3.204575e-05 x1 = 3.332736e-01
x2 = 1.350683e+00 x3 = 5.340958e-06 p6 = 1.845085e+00
x2 = 6.634098e-01 x3 = 5.554560e-02 p6 = 1.238275e+03


p4 = 6.000000e+00
x1 = -1.607331e-05 x1 = 3.259820e-05 x1 = 3.570729e-01
x2 = 1.341701e+00 x3 = 4.656886e-06 p6 = 1.770072e+00
x2 = 6.389422e-01 x3 = 5.101041e-02 p6 = 1.183125e+03


p4 = 7.000000e+00
x1 = -1.679783e-05 x1 = 3.325775e-05 x1 = 3.749198e-01
x2 = 1.331635e+00 x3 = 4.157218e-06 p6 = 1.710582e+00
x2 = 6.204275e-01 x3 = 4.686497e-02 p6 = 1.116653e+03


p4 = 8.000000e+00
x1 = -1.757607e-05 x1 = 3.396343e-05 x1 = 3.887984e-01
x2 = 1.321463e+00 x3 = 3.773715e-06 p6 = 1.662013e+00
x2 = 6.058813e-01 x3 = 4.319982e-02 p6 = 1.049452e+03


p4 = 9.000000e+00
x1 = -1.837849e-05 x1 = 3.468581e-05 x1 = 3.998993e-01
x2 = 1.311601e+00 x3 = 3.468581e-06 p6 = 1.621436e+00
x2 = 5.941132e-01 x3 = 3.998993e-02 p6 = 9.856478e+02


p4 = 1.000000e+01
x1 = -1.918969e-05 x1 = 3.540951e-05 x1 = 4.089800e-01
x2 = 1.302215e+00 x3 = 3.219047e-06 p6 = 1.586900e+00
x2 = 5.843653e-01 x3 = 3.718000e-02 p6 = 9.266651e+02


p4 = 1.000000e+01
x1 = -1.918969e-05 x1 = 3.540951e-05 x1 = 4.089800e-01
x2 = 1.302215e+00 x3 = 3.219047e-06 p6 = 1.586900e+00
x2 = 5.843653e-01 x3 = 3.718000e-02 p6 = 9.266651e+02


p4 = 2.000000e+01
x1 = -2.696682e-05 x1 = 4.190594e-05 x1 = 4.521682e-01
x2 = 1.233973e+00 x3 = 1.995521e-06 p6 = 1.397336e+00
x2 = 5.345084e-01 x3 = 2.153182e-02 p6 = 5.591702e+02


p4 = 3.000000e+01
x1 = -3.400617e-05 x1 = 4.692934e-05 x1 = 4.674280e-01
x2 = 1.193598e+00 x3 = 1.513850e-06 p6 = 1.311747e+00
x2 = 5.125811e-01 x3 = 1.507832e-02 p6 = 3.932122e+02


p4 = 4.000000e+01
x1 = -4.064265e-05 x1 = 5.081503e-05 x1 = 4.751955e-01
x2 = 1.166034e+00 x3 = 1.239391e-06 p6 = 1.259644e+00
x2 = 4.981465e-01 x3 = 1.159013e-02 p6 = 3.007863e+02
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3

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p4 = 5.000000e+01
x1 = -4.712664e-05 x1 = 5.381312e-05 x1 = 4.798782e-01
x2 = 1.145406e+00 x3 = 1.055159e-06 p6 = 1.223197e+00
x2 = 4.867968e-01 x3 = 9.409377e-03 p6 = 2.421209e+02


p4 = 6.000000e+01
x1 = -5.362214e-05 x1 = 5.608721e-05 x1 = 4.829937e-01
x2 = 1.129036e+00 x3 = 9.194625e-07 p6 = 1.195569e+00
x2 = 4.770143e-01 x3 = 7.917930e-03 p6 = 2.016274e+02


p4 = 7.000000e+01
x1 = 5.775365e-05 x1 = -6.024587e-05 x1 = 4.852045e-01
x2 = 1.115523e+00 x3 = 8.134317e-07 p6 = 1.173515e+00
x2 = 4.681367e-01 x3 = 6.833866e-03 p6 = 1.720110e+02


p4 = 8.000000e+01
x1 = 5.890190e-05 x1 = -6.708770e-05 x1 = 4.868459e-01
x2 = 1.104057e+00 x3 = 7.271840e-07 p6 = 1.155280e+00
x2 = 4.598284e-01 x3 = 6.010443e-03 p6 = 1.494140e+02


p4 = 9.000000e+01
x1 = 5.960510e-05 x1 = -7.422117e-05 x1 = 4.881059e-01
x2 = 1.094134e+00 x3 = 6.550011e-07 p6 = 1.139819e+00
x2 = 4.519016e-01 x3 = 5.363801e-03 p6 = 1.316078e+02


p4 = 1.000000e+02
x1 = 5.992564e-05 x1 = -8.170907e-05 x1 = 4.890980e-01
x2 = 1.085421e+00 x3 = 5.933232e-07 p6 = 1.126469e+00
x2 = 4.442427e-01 x3 = 4.842554e-03 p6 = 1.172160e+02


p4 = 1.000000e+02
x1 = 5.992564e-05 x1 = -8.170907e-05 x1 = 4.890980e-01
x2 = 1.085421e+00 x3 = 5.933232e-07 p6 = 1.126469e+00
x2 = 4.442427e-01 x3 = 4.842554e-03 p6 = 1.172160e+02


p4 = 2.000000e+02
x1 = 5.178628e-05 x1 = -1.858662e-04 x1 = 4.931049e-01
x2 = 1.034399e+00 x3 = 2.576432e-07 p6 = 1.051764e+00
x2 = 3.735675e-01 x3 = 2.453258e-03 p6 = 5.080235e+01


p4 = 3.000000e+02
x1 = 3.984376e-05 x1 = -3.603206e-04 x1 = 4.939129e-01
x2 = 1.013663e+00 x3 = 1.323713e-07 p6 = 1.022783e+00
x2 = 3.060913e-01 x3 = 1.640907e-03 p6 = 2.803760e+01


p4 = 4.000000e+02
x1 = 3.116581e-05 x1 = -6.124341e-04 x1 = 4.939709e-01
x2 = 1.003870e+00 x3 = 7.772023e-08 p6 = 1.009434e+00
x2 = 2.393651e-01 x3 = 1.231848e-03 p6 = 1.652041e+01


p4 = 5.000000e+02
x1 = 2.528165e-05 x1 = -9.419647e-04 x1 = 4.937729e-01
x2 = 9.983474e-01 x3 = 5.046238e-08 p6 = 1.002119e+00
x2 = 1.728951e-01 x3 = 9.855746e-04 p6 = 9.555413e+00


p4 = 6.000000e+02
x1 = 2.116873e-05 x1 = -1.348052e-03 x1 = 4.934839e-01
x2 = 9.946758e-01 x3 = 3.522252e-08 p6 = 9.974264e-01
x2 = 1.065159e-01 x3 = 8.211047e-04 p6 = 4.880446e+00


p4 = 7.000000e+02
x1 = 1.816874e-05 x1 = -1.830133e-03 x1 = 4.931751e-01
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4

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x2 = 9.919011e-01 x3 = 2.591831e-08 p6 = 9.940146e-01
x2 = 4.015655e-02 x3 = 7.035309e-04 p6 = 1.518508e+00

p4 = 8.000000e+02
x1 = 1.589556e-05 x1 = -2.387848e-03 x1 = 4.928820e-01
x2 = 9.896037e-01 x3 = 1.984464e-08 p6 = 9.912914e-01
x2 = -2.621842e-02 x3 = 6.153333e-04 p6 = -1.021372e+00

p4 = 9.000000e+02
x1 = 1.411796e-05 x1 = -3.020925e-03 x1 = 4.926241e-01
x2 = 9.875799e-01 x3 = 1.566921e-08 p6 = 9.889678e-01
x2 = -9.262872e-02 x3 = 5.467527e-04 p6 = -3.012932e+00

p4 = 1.000000e+03
x1 = 1.269161e-05 x1 = -3.729114e-03 x1 = 4.924134e-01
x2 = 9.857224e-01 x3 = 1.267893e-08 p6 = 9.868905e-01
x2 = -1.590861e-01 x3 = 4.919215e-04 p6 = -4.620854e+00

p4 = 1.100000e+03
x1 = 1.152256e-05 x1 = -4.512143e-03 x1 = 4.922571e-01
x2 = 9.839711e-01 x3 = 1.046554e-08 p6 = 9.849725e-01
x2 = -2.255979e-01 x3 = 4.471000e-04 p6 = -5.950154e+00

p4 = 1.200000e+03
x1 = 1.054734e-05 x1 = -5.369709e-03 x1 = 4.921600e-01
x2 = 9.822901e-01 x3 = 8.782136e-09 p6 = 9.831617e-01
x2 = -2.921690e-01 x3 = 4.097918e-04 p6 = -7.070956e+00

p4 = 1.300000e+03
x1 = 9.721647e-06 x1 = -6.301460e-03 x1 = 4.921253e-01
x2 = 9.806572e-01 x3 = 7.472442e-09 p6 = 9.814255e-01
x2 = -3.588025e-01 x3 = 3.782670e-04 p6 = -8.031870e+00

p4 = 1.400000e+03
x1 = 9.013637e-06 x1 = -7.306993e-03 x1 = 4.921552e-01
x2 = 9.790581e-01 x3 = 6.433716e-09 p6 = 9.797426e-01
x2 = -4.255005e-01 x3 = 3.512885e-04 p6 = -8.867653e+00

p4 = 1.500000e+03
x1 = 8.399885e-06 x1 = -8.385854e-03 x1 = 4.922510e-01
x2 = 9.774833e-01 x3 = 5.596192e-09 p6 = 9.780987e-01
x2 = -4.922644e-01 x3 = 3.279487e-04 p6 = -9.603813e+00
```
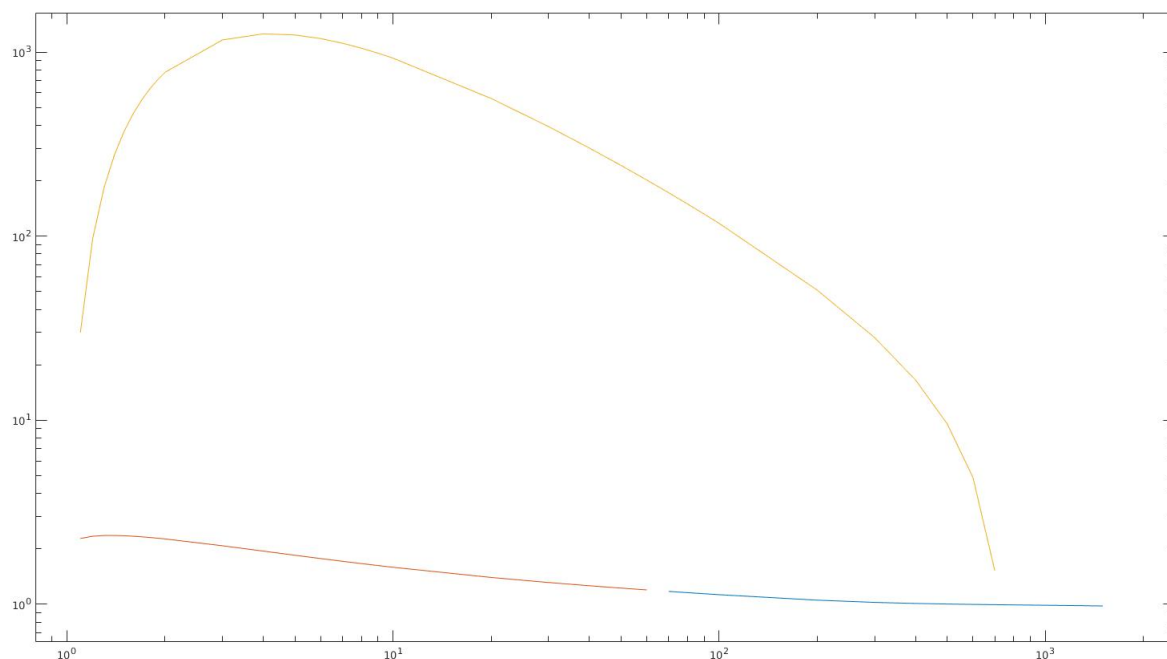
5 Бифуркационная диаграмма точек поворота

p4 по оси абсцисс, p6 по оси ординат.

6 Результаты работы блока проверки

```
p4 = 1.100000
1:
 det = 0.051595
eigenvalues: 79.035395 -8850.874123 -0.000000
System: 0.000000, 0.000000, , 0.000000
detcheck: -32.328115

2:
 det = -4.753448
eigenvalues: 13740.356714 -19535.618289 0.000000
System: -0.000002, -0.000326, 0.000000
detcheck: -8887.142126

p4 = 1.200000
1:
 det = 0.150740
eigenvalues: 39.544679 -12021.331986 -0.000000
System: 0.000000, 0.000000, , 0.000000
detcheck: -27.423629

2:
 det = -10.985002
eigenvalues: 17269.420917 -28340.123947 0.000000
System: 0.000002, -0.000623, 0.000000
detcheck: -17775.862340

p4 = 1.300000
1:
 det = 0.243875
```

```
eigenvalues: 26.227793 -14211.475248 -0.000001
System: 0.000000, 0.000001, , 0.000000
detcheck: -25.941850


2:
 det = -7.068510
eigenvalues: 19163.083596 -35056.622866 0.000000
System: -0.000004, -0.000706, 0.000000
detcheck: -26664.563443


p4 = 1.400000
1:
 det = 0.120861
eigenvalues: 19.506086 -15870.574892 -0.000000
System: 0.000000, 0.000001, , 0.000000
detcheck: -25.544747


2:
 det = -18.321732
eigenvalues: 20270.530901 -40586.189767 0.000000
System: -0.000002, 0.000169, -0.000000
detcheck: -35553.252901


p4 = 1.500000
1:
 det = -0.108941
eigenvalues: 15.429315 -17184.163091 0.000000
System: -0.000000, -0.000001, , 0.000000
detcheck: -25.656789


2:
 det = 24.590462
eigenvalues: 20920.511382 -45304.765699 -0.000000
System: -0.000000, 0.000382, -0.000000
detcheck: -44441.923248


p4 = 1.600000
1:
 det = 0.216833
eigenvalues: 12.676766 -18251.588045 -0.000001
System: 0.000000, 0.000000, , 0.000000
detcheck: -26.059780


2:
 det = 170.461503
eigenvalues: 21278.983945 -49418.807326 -0.000000
System: 0.000012, 0.001182, -0.000000
detcheck: -53330.572350


p4 = 1.700000
1:
 det = -0.009814
eigenvalues: 10.681390 -19133.988994 0.000000
System: 0.000000, 0.000000, , -0.000000
detcheck: -26.651009


2:
 det = 38.130305
eigenvalues: 21441.728099 -53058.781190 -0.000000
System: 0.000001, 0.000296, -0.000000
detcheck: -62219.223673


p4 = 1.800000
```

7

```
1:
 det = -0.408093
eigenvalues: 9.159510 -19872.098171 0.000002
System: -0.000000, -0.000001, , -0.000000
detcheck: -27.375185


2:
 det = -137.328879
eigenvalues: 21468.713662 -56314.404975 0.000000
System: -0.000013, -0.000729, 0.000000
detcheck: -71107.859441

p4 = 1.900000
1:
 det = 0.272557
eigenvalues: 7.953478 -20494.661144 -0.000002
System: 0.000000, 0.000001, , 0.000000
detcheck: -28.199677


2:
 det = 267.482603
eigenvalues: 21399.591879 -59250.992415 -0.000000
System: 0.000001, 0.000780, -0.000000
detcheck: -79996.449430

p4 = 2.000000
1:
 det = -0.283135
eigenvalues: 6.968469 -21023.097334 0.000002
System: -0.000000, -0.000001, , 0.000000
detcheck: -29.103878


2:
 det = 560.181153
eigenvalues: 21261.610655 -61918.096095 -0.000000
System: 0.000001, 0.002357, -0.000000
detcheck: -88885.030974

p4 = 2.000000
1:
 det = -0.283135
eigenvalues: 6.968469 -21023.097334 0.000002
System: -0.000000, -0.000001, , 0.000000
detcheck: -29.103878


2:
 det = 560.181153
eigenvalues: 21261.610655 -61918.096095 -0.000000
System: 0.000001, 0.002357, -0.000000
detcheck: -88885.030974

p4 = 3.000000
1:
 det = -0.537817
eigenvalues: 1.984879 -23469.848574 0.000012
System: -0.000000, -0.000001, , -0.000000
detcheck: -40.799613


2:
 det = -185.927167
eigenvalues: 18607.701043 -79589.701414 0.000000
System: 0.000011, 0.001712, 0.000000
detcheck: -177770.097504
```

```
p4 = 4.000000
1:
 det = 1.158111
eigenvalues: 0.000142 -23769.744374 -0.343480
System: 0.000000, 0.000002, , 0.000000
detcheck: -55.682944


2:
 det = 683.735601
eigenvalues: 15952.877156 -89112.889499 -0.000000
System: 0.000023, -0.000138, 0.000000
detcheck: -266653.466210

p4 = 5.000000
1:
 det = -1.044646
eigenvalues: -0.000022 -23404.439230 -2.007542
System: -0.000000, -0.000004, , 0.000000
detcheck: -73.142673


2:
 det = -515.850727
eigenvalues: 13839.060814 -95103.178093 0.000000
System: -0.000015, -0.002852, -0.000000
detcheck: -355535.243760

p4 = 6.000000
1:
 det = -1.109873
eigenvalues: -0.000014 -22807.409570 -3.406091
System: -0.000000, -0.000004, , -0.000000
detcheck: -93.020620


2:
 det = -2507.079460
eigenvalues: 12180.147701 -99220.378913 0.000002
System: -0.000048, -0.003858, 0.000000
detcheck: -444415.394598

p4 = 7.000000
1:
 det = -1.767959
eigenvalues: -0.000017 -22138.279034 -4.671798
System: -0.000000, -0.000002, , 0.000000
detcheck: -115.247318


2:
 det = -3335.834166
eigenvalues: 10860.116035 -102221.448190 0.000003
System: -0.000050, -0.002041, 0.000000
detcheck: -533293.883168

p4 = 8.000000
1:
 det = 1.858326
eigenvalues: 0.000015 -21462.172086 -5.861601
System: 0.000000, 0.000006, , -0.000000
detcheck: -139.782335


2:
 det = 174.586882
eigenvalues: 9790.531113 -104502.938510 -0.000000
```

```
System: 0.000018, -0.002196, 0.000000
detcheck: -622170.656136


p4 = 9.000000
1:
 det = -0.459851
eigenvalues: -0.000003 -20806.784507 -7.003950
System: -0.000000, -0.000001, , 0.000000
detcheck: -166.597953


2:
 det = -1816.968670
eigenvalues: 8908.681701 -106293.190580 0.000002
System: -0.000030, -0.001314, 0.000000
detcheck: -711045.909060


p4 = 10.000000
1:
 det = 0.251571
eigenvalues: 0.000002 -20183.162482 -8.114671
System: 0.000000, -0.000000, , -0.000000
detcheck: -195.673002


2:
 det = -3733.112529
eigenvalues: 8170.248615 -107733.027147 0.000004
System: 0.000001, 0.000222, 0.000000
detcheck: -799919.535272


p4 = 10.000000
1:
 det = 0.251571
eigenvalues: 0.000002 -20183.162482 -8.114671
System: 0.000000, -0.000000, , -0.000000
detcheck: -195.673002


2:
 det = -3733.112529
eigenvalues: 8170.248615 -107733.027147 0.000004
System: 0.000001, 0.000222, 0.000000
detcheck: -799919.535272


p4 = 20.000000
1:
 det = 8.000844
eigenvalues: 0.000028 -15655.227970 -18.580925
System: 0.000000, 0.000012, , -0.000000
detcheck: -607.811190


2:
 det = -7944.330932
eigenvalues: 4450.741337 -114156.163582 0.000016
System: -0.000048, -0.002502, -0.000000
detcheck: -1688567.074908


p4 = 30.000000
1:
 det = 3.583020
eigenvalues: 0.000010 -12984.965918 -28.725684
System: 0.000000, 0.000017, , -0.000000
detcheck: -1234.929952


2:
```

```
 det = 13565.670190
eigenvalues: 3049.099370 -116070.231670 -0.000038
System: 0.000008, -0.000526, 0.000000
detcheck: -2577056.138952


p4 = 40.000000
1:
 det = 18.916708
eigenvalues: 0.000044 -11168.495089 -38.796252
System: 0.000000, -0.000001, , -0.000000
detcheck: -2071.617169


2:
 det = -6176.577625
eigenvalues: 2311.614176 -116817.013054 0.000023
System: 0.000004, 0.000683, 0.000000
detcheck: -3465391.723022


p4 = 50.000000
1:
 det = -55.888667
eigenvalues: -0.000117 -9814.178998 -48.838055
System: -0.000000, -0.000031, , 0.000000
detcheck: -3114.072672


2:
 det = 28444.601194
eigenvalues: 1854.665078 -117090.266880 -0.000131
System: 0.000015, 0.002649, -0.000000
detcheck: -4353576.227148


p4 = 60.000000
1:
 det = -65.509040
eigenvalues: -0.000127 -8743.520031 -58.865722
System: -0.000000, -0.000002, , -0.000000
detcheck: -4359.336708


2:
 det = 32616.044839
eigenvalues: 1542.267200 -117124.974691 -0.000181
System: 0.000070, 0.002519, -0.000000
detcheck: -5241614.913618


p4 = 70.000000
1:
 det = -4.745887
eigenvalues: -0.000009 -7863.147750 -68.885403
System: -0.000000, -0.000019, , -0.000000
detcheck: -5805.016509


2:
 det = 12465.796329
eigenvalues: 1314.085217 -117022.760680 -0.000081
System: 0.000003, -0.000120, 0.000000
detcheck: -6129511.604702


p4 = 80.000000
1:
 det = 24.313606
eigenvalues: 0.000043 -7119.086486 -78.900134
System: 0.000000, 0.000036, , -0.000000
detcheck: -7449.159474
```

```
2:
 det = 11793.687375
eigenvalues: 1139.260640 -116834.709178 -0.000089
System: 0.000025, 0.001868, 0.000000
detcheck: -7017269.553774


p4 = 90.000000
1:
 det = -181.308442
eigenvalues: -0.000315 -6477.756811 -88.911585
System: -0.000000, 0.000013, , -0.000000
detcheck: -9290.185148


2:
 det = -14100.206055
eigenvalues: 1000.361477 -116589.357890 0.000121
System: -0.000017, 0.001654, 0.000000
detcheck: -7904893.383114


p4 = 100.000000
1:
 det = -337.563085
eigenvalues: -0.000577 -5916.977532 -98.920755
System: -0.000000, -0.000017, , -0.000000
detcheck: -11326.840160


2:
 det = -57903.475184
eigenvalues: 886.805913 -116303.863079 0.000561
System: -0.000052, -0.000540, 0.000000
detcheck: -8792386.914992


p4 = 100.000000
1:
 det = -337.563085
eigenvalues: -0.000577 -5916.977532 -98.920755
System: -0.000000, -0.000017, , -0.000000
detcheck: -11326.840160


2:
 det = -57903.475184
eigenvalues: 886.805913 -116303.863079 0.000561
System: -0.000052, -0.000540, 0.000000
detcheck: -8792386.914992


p4 = 200.000000
1:
 det = 73.981504
eigenvalues: 0.000137 -2712.919979 -198.963080
System: 0.000000, 0.000093, , -0.000000
detcheck: -42350.683289


2:
 det = -70485.547157
eigenvalues: 316.505324 -112568.463036 0.001978
System: -0.000027, -0.000430, 0.000000
detcheck: -17661002.177274


p4 = 300.000000
1:
 det = 857.198720
eigenvalues: 0.001878 -1526.782515 -298.979887
```

```
System: 0.000000, 0.000137, , -0.000000
detcheck: -92881.609471

2:
 det = -37026.463548
eigenvalues: 58.927955 -108363.833773 0.005798
System: 0.000004, 0.001670, -0.000000
detcheck: -26520907.942914

p4 = 400.000000
1:
 det = -49.256467
eigenvalues: -0.000115 -1071.198625 -398.992021
System: 0.000000, 0.000182, , -0.000000
detcheck: -163220.637890

2:
 det = 150272.358530
eigenvalues: 0.012104 -104055.212407 -119.308753
System: 0.000052, 0.003260, -0.000000
detcheck: -35376075.621554

p4 = 500.000000
1:
 det = -7.543190
eigenvalues: -0.000017 -901.180853 -499.004073
System: 0.000000, 0.000039, , -0.000000
detcheck: -253499.949792

2:
 det = -227686.063717
eigenvalues: -0.008599 -99716.014621 -265.529774
System: -0.000043, -0.000924, 0.000000
detcheck: -44230504.196514

p4 = 600.000000
1:
 det = -636.494239
eigenvalues: -0.001243 -855.111420 -599.018136
System: 0.000000, -0.000042, , -0.000000
detcheck: -363759.294020

2:
 det = 235171.866099
eigenvalues: 0.006233 -95370.716972 -395.628183
System: 0.000010, -0.000534, -0.000000
detcheck: -53088169.717494

p4 = 700.000000
1:
 det = -735.804352
eigenvalues: -0.001211 -869.179463 -699.035861
System: -0.000000, -0.000004, , 0.000000
detcheck: -494011.096809

2:
 det = 257056.713131
eigenvalues: 0.005469 -91029.909619 -516.383982
System: 0.000025, 0.000359, -0.000000
detcheck: -61953092.003630

p4 = 800.000000
1:
```

```
 det = -1550.862386
eigenvalues: -0.002120 -915.287310 -799.059121
System: -0.000000, -0.000008, , 0.000000
detcheck: -644259.730990


2:
 det = -276795.295653
eigenvalues: -0.005058 -86698.879137 -631.196299
System: -0.000020, 0.000104, 0.000000
detcheck: -70829270.304912


p4 = 900.000000
1:
 det = -2432.708083
eigenvalues: -0.002762 -979.781566 -899.090512
System: -0.000000, -0.000015, , 0.000000
detcheck: -814506.921879


2:
 det = 477780.833944
eigenvalues: 0.007817 -82380.534337 -741.942203
System: 0.000030, -0.000744, -0.000000
detcheck: -79720665.470850


p4 = 1000.000000
1:
 det = 2193.951586
eigenvalues: 0.002080 -1055.465120 -999.134090
System: -0.000000, 0.000077, , 0.000000
detcheck: -1004753.416092


2:
 det = -75377.927352
eigenvalues: -0.001136 -78076.686509 -849.715206
System: -0.000010, -0.000169, -0.000000
detcheck: -88631290.351604


p4 = 1100.000000
1:
 det = 252.225646
eigenvalues: 0.000202 -1138.282881 -1099.196826
System: -0.000000, 0.000050, , 0.000000
detcheck: -1214999.557821


2:
 det = -195352.362110
eigenvalues: -0.002772 -73788.432573 -955.198218
System: -0.000037, -0.000627, -0.000000
detcheck: -97565071.609590


p4 = 1200.000000
1:
 det = -1281.755213
eigenvalues: -0.000872 -1225.815863 -1199.292139
System: 0.000000, 0.000008, , -0.000000
detcheck: -1445245.518184


2:
 det = 131752.849986
eigenvalues: 0.001790 -69516.517594 -1058.813065
System: 0.000010, 0.000142, 0.000000
detcheck: -106525935.480552
```

```
p4 = 1300.000000
1:
 det = -1902.451648
eigenvalues: -0.001112 -1316.539912 -1299.450421
System: 0.000000, 0.000017, , -0.000000
detcheck: -1695491.387991

2:
 det = 670809.135131
eigenvalues: 0.008855 -65261.471084 -1160.815440
System: 0.000056, 0.000207, -0.000000
detcheck: -115517779.470986

p4 = 1400.000000
1:
 det = -1352.888899
eigenvalues: -0.000686 -1409.399378 -1399.763909
System: -0.000000, 0.000032, , 0.000000
detcheck: -1965737.210737

2:
 det = 99318.101355
eigenvalues: 0.001290 -61023.684536 -1261.341165
System: 0.000028, 0.000724, 0.000000
detcheck: -124544479.576008

p4 = 1500.000000
1:
 det = -82.549050
eigenvalues: -0.000037 -1502.984883 -1501.000915
System: -0.000000, 0.000026, , 0.000000
detcheck: -2255983.012017

2:
 det = -175005.788930
eigenvalues: -0.002265 -56803.443416 -1360.434530
System: -0.000000, 0.000510, 0.000000
detcheck: -133609828.074132
```

7 Выводы

Все проверки прошли успешно, следовательно можно утверждать, что полученные значения верны.

8 Приложение

8.1 Исходный код

```
1  clear all
2  clear global variables
3  syms x1 x2 x3 p4 p6
4
5  global eq x2val x3val
6
7  global det11 det12 det13 det21 det22 det23 det31 det32 det33
8
9  global p1 p2 p3 p5
10
11 global x1 x2 x3 p4 p6 p6eq
```

```matlab
p1 = double(8.4e-6);
p2 = 6.6667e-4;
p2opp = 66667;
p3 = 1.7778e-5;
p3opp = 177780;
p5 = 2;

x2val = (x1 * x1 + p2 * p4 * x1 - x1) / (p1 - x1);
x3val = x1 / (1 + p4);
p6 = (p1 * x2 + x1 * x2 - p5 * x3) / (p3 * p4) + x2;

det11 = -x2*p2opp + 1*p2opp - 2*x1*p2opp - p4;
det12 = p1 * p2opp - x1 * p2opp;
det13 = 0;
det21 = -x2*p3opp;
det22 = - p1 * p3opp - x1 * p3opp - p4;
det23 = p5*p3opp;
det31 = 1;
det32 = 0;
det33 = - 1 - p4;

det11 = subs(det11, x2, x2val);
det21 = subs(det21, x2, x2val);

res = det ( [det11 det12 det13; det21 det22 det23; det31 det32 det33] );
%res = det11*det22*det33 + det12*det23*det31 + det21*det32*det13 - det13*det22*det31
    - det12*det21*det33 - det23*det32*det11;

p6eq = (-p1*x2 + x1*x2 - p5*x3 + p4 * p3 * (p6 - x2))*p3opp == 0;

eq = res == 0;

global fileID fileIDcheck fileIDplot1 fileIDplot2 fileIDplot3;

fileIDplot3 = fopen('forplot3.txt', 'w');
fileID = fopen('resulst.txt', 'w');
fileIDcheck = fopen('check.txt', 'w');
fileIDplot1 = fopen('forplot1.txt', 'w');
fileIDplot2 = fopen('forplot2.txt', 'w');

forrange(1.0, 2.0, 0.1);
forrange(2.0, 10.0, 1.0);
forrange(10.0, 100.0, 10.0);
forrange(100.0, 1500.0, 100.0);

fclose(fileID);
fclose(fileIDcheck);
fclose(fileIDplot1);
fclose(fileIDplot2);
fclose(fileIDplot3);

A = load('forplot1.txt');
B = load('forplot2.txt');
C = load('forplot3.txt');

loglog(A(:,1), A(:,2));
hold on;
loglog(B(:,1),B(:,2));
loglog(C(:,1),C(:,2));
hold off;
```

```matlab
74
75 function[x11, x12, x13, x2tmp, x3tmp] = subsp4(i)
76 global p4 x1 eq x2val x3val;
77 inner = subs(eq, p4, i);
78 solution = solve(inner, x1);
79 x11 = solution(1);
80 x12 = solution(2);
81 x13 = solution(3);
82
83 x2tmp = subs(x2val, p4, i);
84 x3tmp = subs(x3val, p4, i);
85 %p6tmp = subs(p6, p4, i);
86 end
87
88 function[p6res] = subsx1(x1in, x2tmp, x3tmp, p6tmp)
89 global fileID fileIDcheck x1 x2 x3 ;
90 if (isAlways(x1in > 0) == true)
91         x2in = subs(x2tmp, x1, x1in);
92         x3in = subs(x3tmp, x1, x1in);
93         p6in = subs(p6tmp, x1, x1in);
94         p6in = subs(p6in, x2, x2in);
95         p6in = subs(p6in, x3, x3in);
96
97         %p6res = solve(p6in, p6);
98         p6res = p6in;
99
100        fprintf(fileID, 'x2 = %e x3 = %e p6 = %e\n', x2in, x3in, p6res);
101        fprintf(fileIDcheck, '%e %e %e %e\n', x1in, x2in, x3in, p6res);
102
103 else
104     p6res = -1;
105 end
106 end
107
108 function forrange(left, right, step)
109
110 global fileIDcheck fileID fileIDplot1 fileIDplot2 fileIDplot3 p6 p4;
111 for i = left: step: right
112
113    fprintf(fileID, 'p4 = %e \n', i);
114    fprintf(fileIDcheck, '%e\n', i);
115
116    %p6eqp4 = subs(p6eq, p4, i);
117    p6eqp4 = subs(p6, p4, i);
118
119    [x11, x12, x13, x2tmp, x3tmp] = subsp4(i);
120
121    fprintf(fileID, 'x1 = %e x1 = %e x1 = %e\n', x11, x12, x13);
122
123    p6res = subsx1(x11, x2tmp, x3tmp, p6eqp4);
124
125    fprintf(fileIDplot1, '%e %e \n', i, p6res);
126
127    p6res = subsx1(x12, x2tmp, x3tmp, p6eqp4);
128    fprintf(fileIDplot2, '%e %e \n', i, p6res);
129
130    p6res = subsx1(x13, x2tmp, x3tmp, p6eqp4);
131    fprintf(fileIDplot3, '%e %e \n', i, p6res);
132
133    fprintf(fileID, '\n');
134 end
135 end
```

8.2 Код блока проверки

```matlab
clear all
syms x1 x2 x3 p1 p2 p3 p4 p5 p6

p1 = double(8.4e-6);
p2 = 6.6667e-4;
p2opp = 66667;
p3 = 1.7778e-5;
p3opp = 177780;
p5 = 2;

det11 = -x2*p2opp + 1*p2opp - 2*x1*p2opp - p4;
det12 = p1 * p2opp - x1 * p2opp;
det13 = 0;
det21 = -x2*p3opp;
det22 = - p1 * p3opp - x1 * p3opp - p4;
det23 = p5*p3opp;
det31 = 1;
det32 = 0;
det33 = - 1 - p4;

res =  ( [det11 det12 det13; det21 det22 det23; det31 det32 det33] );
matrix = [det11 det12 det13; det21 det22 det23; det31 det32 det33];
detcheck = det ( [-1 det12 det13; 0 det22 det23; 0 det32 det33] );

eq1 = (p1*x2 - x1*x2 + x1 - x1^2) / p2 - p4*x1;
eq2 = (-p1*x2 - x1*x2 + p5*x3)/p3 + p4 * (p6 - x2);
eq3 = x1 - x3 - p4*x3;


fid = fopen('check.txt', 'r');
fres = fopen('check_res.txt', 'w');

while ~feof(fid)
    i = fscanf(fid, "%e", 1);

    x11 = fscanf(fid, "%e", 1);
    x21 = fscanf(fid, "%e", 1);
    x31 = fscanf(fid, "%e", 1);
    p61 = fscanf(fid, "%e", 1);

    x12 = fscanf(fid, "%e", 1);
    x22 = fscanf(fid, "%e", 1);
    x32 = fscanf(fid, "%e", 1);
    p62 = fscanf(fid, "%e", 1);
    %fscanf(fid, '%f %f %f %f\n', x11, x21, x31, p61);
    %fscanf(fid, '%f %f %f %f\n', x12, x22, x32, p62);
    check1 = subs(res, p4, i);
    check1 = subs(check1, x1, x11);
    check1 = subs(check1, x2, x21);
    check1 = subs(check1, x3, x31);
    check1 = subs(check1, p6, p61);
    check1tmp = det(check1);
    fprintf(fres, 'p4 = %f \n', i);
    fprintf(fres, '1:\n det = %f\n', check1tmp);
    eigenvalues = eig(check1);
    fprintf(fres, 'eigenvalues: %f % f %f\n', eigenvalues(1), eigenvalues(2),
    eigenvalues(3));

    checkeq1 = subs(eq1, p4, i);
    checkeq1 = subs(checkeq1, x1, x11);
    checkeq1 = subs(checkeq1, x2, x21);
```

```matlab
61        checkeq1 = subs(checkeq1, x3, x31);
62        checkeq1 = subs(checkeq1, p6, p61);
63        fprintf(fres, 'System: %f, ', checkeq1);
64
65        checkeq1 = subs(eq2, p4, i);
66        checkeq1 = subs(checkeq1, x1, x11);
67        checkeq1 = subs(checkeq1, x2, x21);
68        checkeq1 = subs(checkeq1, x3, x31);
69        checkeq1 = subs(checkeq1, p6, p61);
70        fprintf(fres, '%f, ', checkeq1);
71
72        checkeq1 = subs(eq3, p4, i);
73        checkeq1 = subs(checkeq1, x1, x11);
74        checkeq1 = subs(checkeq1, x2, x21);
75        checkeq1 = subs(checkeq1, x3, x31);
76        checkeq1 = subs(checkeq1, p6, p61);
77        fprintf(fres, ', %f\n', checkeq1);
78
79        detcheck1 = subs(detcheck, p4, i);
80        detcheck1 = subs(detcheck1, x1, x11);
81        detcheck1 = subs(detcheck1, x2, x21);
82        detcheck1 = subs(detcheck1, x3, x31);
83        detcheck1 = subs(detcheck1, p6, p61);
84        fprintf(fres, 'detcheck: %f\n\n', detcheck1);
85
86        check2 = subs(res, p4, i);
87        check2 = subs(check2, x1, x12);
88        check2 = subs(check2, x2, x22);
89        check2 = subs(check2, x3, x32);
90        check2 = subs(check2, p6, p62);
91        check2tmp = det(check2);
92        fprintf(fres, '2:\n det = %f \n', check2tmp);
93        eigenvalues = eig(check2);
94        fprintf(fres, 'eigenvalues: %f % f %f\n', eigenvalues(1), eigenvalues(2),
          eigenvalues(3));
95
96        checkeq2 = subs(eq1, p4, i);
97        checkeq2 = subs(checkeq2, x1, x12);
98        checkeq2 = subs(checkeq2, x2, x22);
99        checkeq2 = subs(checkeq2, x3, x32);
100       checkeq2 = subs(checkeq2, p6, p62);
101       fprintf(fres, 'System: %f, ', checkeq2);
102
103       checkeq2 = subs(eq2, p4, i);
104       checkeq2 = subs(checkeq2, x1, x12);
105       checkeq2 = subs(checkeq2, x2, x22);
106       checkeq2 = subs(checkeq2, x3, x32);
107       checkeq2 = subs(checkeq2, p6, p62);
108       fprintf(fres, '%f, ', checkeq2);
109
110       checkeq2 = subs(eq3, p4, i);
111       checkeq2 = subs(checkeq2, x1, x12);
112       checkeq2 = subs(checkeq2, x2, x22);
113       checkeq2 = subs(checkeq2, x3, x32);
114       checkeq2 = subs(checkeq2, p6, p62);
115       fprintf(fres, '%f \n', checkeq2);
116
117       detcheck2 = subs(detcheck, p4, i);
118       detcheck2 = subs(detcheck2, x1, x12);
119       detcheck2 = subs(detcheck2, x2, x22);
120       detcheck2 = subs(detcheck2, x3, x32);
121       detcheck2 = subs(detcheck2, p6, p62);
122       fprintf(fres, 'detcheck: %f\n\n', detcheck2);
```

```matlab
123
124
125
126 end
127
128
129 fclose(fid);
130 fclose(fres);
```