ФЕДЕРАЛЬНОЕ ГОСУДАРСТВЕННОЕ АВТОНОМНОЕ ОБРАЗОВАТЕЛЬНОЕ
УЧЕРЕЖДЕНИЕ ВЫСШЕГО ОБРАЗОВАНИЯ
«Санкт-Петербургский политехнический университет Петра Великого»

Институт компьютерных наук и технологий
Высшая школа программной инженерии

Отчет по лабораторной работе по дисциплине «Вычислительная математика»

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1 Задание

Решить систему дифференциальных уравнений:
$\frac{dx_1}{dt} = -40x_1 + 260x_2 + 1/(10t^2 + 1)$;
$\frac{dx_2}{dt} = 30x_1 - 270x_2 + e^{-2t}$;
$x_1(0) = 0, x_2(0) = 1; t \in [0, 0.4]$
Следующими способами с одним и тем же шагом печати $h_{print} = 0.02$:

1. по программе **RKF45** с EPS = 0.0001
2. методом Адамса 3-й степени точности
   $z_{n+1} = z_n + h(23f_n - 16f_{n-1} + 5f_{n-2})/12$
   с двумя постоянными шагами интегрирования:
   - $h_{int} = 0.002$
   - любой другой, позволяющий получить качественно верное решение

Сравнить результаты. Дополнительные начальные условия для метода Адамса получить с помощью **RKF45**.

2 Критический шаг интегрирования для метода Адамса 3 степени

Известное соотношение для метода Адамса 3 степени:
$h|\lambda_k|_{max} < \frac{6}{11}$
Собственные значения матрицы: $\lambda_1 = -10, \lambda_2 = -300$
Следовательно, $h < 0.00181$

3 Результаты работы программы

При шаге интегрирования 0.02 метод Адамса не сходится:

```
t = 0.02; x1 = 0.761431; x2 = 0.0931585; flag = 2
t = 0.04; x1 = 0.656251; x2 = 0.0784447; flag = 2
t = 0.06; x1 = 0.567499; x2 = 0.0680847; flag = 2
t = 0.08; x1 = 0.493912; x2 = 0.0594781; flag = 2
t = 0.1; x1 = 0.432642; x2 = 0.0523225; flag = 2
t = 0.12; x1 = 0.381437; x2 = 0.0463137; flag = 2
t = 0.14; x1 = 0.338422; x2 = 0.0412521; flag = 2
t = 0.16; x1 = 0.302068; x2 = 0.0369877; flag = 2
t = 0.18; x1 = 0.271199; x2 = 0.0333408; flag = 2
t = 0.2; x1 = 0.244819; x2 = 0.0302131; flag = 2
t = 0.22; x1 = 0.222113; x2 = 0.0275347; flag = 2
t = 0.24; x1 = 0.202482; x2 = 0.0251938; flag = 2
t = 0.26; x1 = 0.185391; x2 = 0.0231465; flag = 2
t = 0.28; x1 = 0.170401; x2 = 0.0213667; flag = 2
t = 0.3; x1 = 0.15721; x2 = 0.0197772; flag = 2
t = 0.32; x1 = 0.145524; x2 = 0.0183616; flag = 2
t = 0.34; x1 = 0.135096; x2 = 0.0171146; flag = 2
t = 0.36; x1 = 0.125778; x2 = 0.0159776; flag = 2
t = 0.38; x1 = 0.1174; x2 = 0.0149488; flag = 2
t = 0.4; x1 = 0.109811; x2 = 0.0140351; flag = 2
Adams:
t = 0.0002; x1 = 0.0467674; x2 = 0.95176; flag = 2
t = 0.0004; x1 = 0.0903754; x2 = 0.906701; flag = 2
t = 0.0006; x1 = 0.131011; x2 = 0.864639; flag = 2
t = 0.0026; x1 = 0.560329; x2 = 0.419419
t = 0.0226; x1 = 1.00269; x2 = -0.164603
t = 0.0426; x1 = 1.25758; x2 = -0.536418
```

```
t = 0.0626; x1 = 2.03833; x2 = -1.41386
t = 0.0826; x1 = 4.06013; x2 = -3.51588
t = 0.1026; x1 = 9.05404; x2 = -8.57659
t = 0.1226; x1 = 21.203; x2 = -20.7814
t = 0.1426; x1 = 50.6074; x2 = -50.2328
t = 0.1626; x1 = 121.651; x2 = -121.317
t = 0.1826; x1 = 293.197; x2 = -292.896
t = 0.2026; x1 = 707.331; x2 = -707.059
t = 0.2226; x1 = 1707.03; x2 = -1706.78
t = 0.2426; x1 = 4120.2; x2 = -4119.98
t = 0.2626; x1 = 9945.28; x2 = -9945.07
t = 0.2826; x1 = 24006.2; x2 = -24006
t = 0.3026; x1 = 57947.3; x2 = -57947.1
t = 0.3226; x1 = 139876; x2 = -139876
t = 0.3426; x1 = 337641; x2 = -337641
t = 0.3626; x1 = 815017; x2 = -815017
t = 0.3826; x1 = 1.96733e+06; x2 = -1.96733e+06
```

При шаге меньше критического полученные значения близки к результатам работы RKF45:

```
t = 0.02; x1 = 0.0296083; x2 = 0.00625063; flag = 2
t = 0.04; x1 = 0.0552798; x2 = 0.00909604; flag = 2
t = 0.06; x1 = 0.0754585; x2 = 0.0113139; flag = 2
t = 0.08; x1 = 0.0910685; x2 = 0.0129921; flag = 2
t = 0.1; x1 = 0.102826; x2 = 0.0142591; flag = 2
t = 0.12; x1 = 0.111402; x2 = 0.0151551; flag = 2
t = 0.14; x1 = 0.117328; x2 = 0.0157491; flag = 2
t = 0.16; x1 = 0.121056; x2 = 0.0161037; flag = 2
t = 0.18; x1 = 0.123006; x2 = 0.0162352; flag = 2
t = 0.2; x1 = 0.123478; x2 = 0.0162189; flag = 2
t = 0.22; x1 = 0.122774; x2 = 0.0160704; flag = 2
t = 0.24; x1 = 0.121158; x2 = 0.0157999; flag = 2
t = 0.26; x1 = 0.118795; x2 = 0.0154694; flag = 2
t = 0.28; x1 = 0.115879; x2 = 0.0150785; flag = 2
t = 0.3; x1 = 0.112578; x2 = 0.0146217; flag = 2
t = 0.32; x1 = 0.108973; x2 = 0.0141507; flag = 2
t = 0.34; x1 = 0.105178; x2 = 0.0136601; flag = 2
t = 0.36; x1 = 0.101291; x2 = 0.0131413; flag = 2
t = 0.38; x1 = 0.0973371; x2 = 0.0126406; flag = 2
t = 0.4; x1 = 0.0933874; x2 = 0.0121435; flag = 2
Adams:
t = 0.0002; x1 = 0.0467674; x2 = 0.95176; flag = 2
t = 0.0004; x1 = 0.0903754; x2 = 0.906701; flag = 2
t = 0.0006; x1 = 0.131011; x2 = 0.864639; flag = 2
t = 0.0008; x1 = 0.173943; x2 = 0.820117
t = 0.0028; x1 = 0.482229; x2 = 0.496101
t = 0.0048; x1 = 0.645398; x2 = 0.317505
t = 0.0068; x1 = 0.728974; x2 = 0.2188
t = 0.0088; x1 = 0.768985; x2 = 0.163951
t = 0.0108; x1 = 0.785201; x2 = 0.133182
t = 0.0128; x1 = 0.788469; x2 = 0.115642
t = 0.0148; x1 = 0.784739; x2 = 0.105372
t = 0.0168; x1 = 0.777275; x2 = 0.099106
t = 0.0188; x1 = 0.767865; x2 = 0.0950483
t = 0.0208; x1 = 0.757488; x2 = 0.0922144
t = 0.0228; x1 = 0.746681; x2 = 0.0900637
t = 0.0248; x1 = 0.735735; x2 = 0.0882991
t = 0.0268; x1 = 0.724808; x2 = 0.0867575
t = 0.0288; x1 = 0.713985; x2 = 0.0853491
t = 0.0308; x1 = 0.703311; x2 = 0.0840244
t = 0.0328; x1 = 0.692809; x2 = 0.082756
t = 0.0348; x1 = 0.682488; x2 = 0.0815288
t = 0.0368; x1 = 0.672354; x2 = 0.080334
t = 0.0388; x1 = 0.662406; x2 = 0.0791669
```

```
t = 0.0408; x1 = 0.652643; x2 = 0.0780245
t = 0.0428; x1 = 0.643063; x2 = 0.0769051
t = 0.0448; x1 = 0.633662; x2 = 0.0758075
t = 0.0468; x1 = 0.624438; x2 = 0.0747309
t = 0.0488; x1 = 0.615386; x2 = 0.0736745
t = 0.0508; x1 = 0.606504; x2 = 0.072638
t = 0.0528; x1 = 0.597788; x2 = 0.0716208
t = 0.0548; x1 = 0.589235; x2 = 0.0706225
t = 0.0568; x1 = 0.580841; x2 = 0.0696427
t = 0.0588; x1 = 0.572603; x2 = 0.068681
t = 0.0608; x1 = 0.564519; x2 = 0.0677371
t = 0.0628; x1 = 0.556584; x2 = 0.0668105
t = 0.0648; x1 = 0.548796; x2 = 0.065901
t = 0.0668; x1 = 0.541153; x2 = 0.0650081
t = 0.0688; x1 = 0.53365; x2 = 0.0641316
t = 0.0708; x1 = 0.526285; x2 = 0.063271
t = 0.0728; x1 = 0.519055; x2 = 0.0624262
t = 0.0748; x1 = 0.511958; x2 = 0.0615967
t = 0.0768; x1 = 0.504991; x2 = 0.0607822
t = 0.0788; x1 = 0.49815; x2 = 0.0599825
t = 0.0808; x1 = 0.491435; x2 = 0.0591972
t = 0.0828; x1 = 0.484841; x2 = 0.0584261
t = 0.0848; x1 = 0.478367; x2 = 0.0576688
t = 0.0868; x1 = 0.47201; x2 = 0.0569251
t = 0.0888; x1 = 0.465768; x2 = 0.0561947
t = 0.0908; x1 = 0.459638; x2 = 0.0554773
t = 0.0928; x1 = 0.453618; x2 = 0.0547727
t = 0.0948; x1 = 0.447706; x2 = 0.0540806
t = 0.0968; x1 = 0.441899; x2 = 0.0534007
t = 0.0988; x1 = 0.436197; x2 = 0.0527329
t = 0.1008; x1 = 0.430595; x2 = 0.0520768
t = 0.1028; x1 = 0.425093; x2 = 0.0514323
t = 0.1048; x1 = 0.419688; x2 = 0.050799
t = 0.1068; x1 = 0.414379; x2 = 0.0501768
t = 0.1088; x1 = 0.409163; x2 = 0.0495654
t = 0.1108; x1 = 0.404038; x2 = 0.0489647
t = 0.1128; x1 = 0.399003; x2 = 0.0483743
t = 0.1148; x1 = 0.394056; x2 = 0.0477941
t = 0.1168; x1 = 0.389195; x2 = 0.047224
t = 0.1188; x1 = 0.384419; x2 = 0.0466636
t = 0.1208; x1 = 0.379725; x2 = 0.0461128
t = 0.1228; x1 = 0.375111; x2 = 0.0455714
t = 0.1248; x1 = 0.370578; x2 = 0.0450392
t = 0.1268; x1 = 0.366122; x2 = 0.044516
t = 0.1288; x1 = 0.361742; x2 = 0.0440017
t = 0.1308; x1 = 0.357437; x2 = 0.043496
t = 0.1328; x1 = 0.353205; x2 = 0.0429988
t = 0.1348; x1 = 0.349044; x2 = 0.04251
t = 0.1368; x1 = 0.344954; x2 = 0.0420293
t = 0.1388; x1 = 0.340932; x2 = 0.0415565
t = 0.1408; x1 = 0.336978; x2 = 0.0410916
t = 0.1428; x1 = 0.333091; x2 = 0.0406344
t = 0.1448; x1 = 0.329267; x2 = 0.0401847
t = 0.1468; x1 = 0.325508; x2 = 0.0397424
t = 0.1488; x1 = 0.32181; x2 = 0.0393073
t = 0.1508; x1 = 0.318174; x2 = 0.0388793
t = 0.1528; x1 = 0.314597; x2 = 0.0384582
t = 0.1548; x1 = 0.311079; x2 = 0.0380439
t = 0.1568; x1 = 0.307618; x2 = 0.0376363
t = 0.1588; x1 = 0.304214; x2 = 0.0372352
t = 0.1608; x1 = 0.300864; x2 = 0.0368406
t = 0.1628; x1 = 0.297569; x2 = 0.0364522
t = 0.1648; x1 = 0.294327; x2 = 0.03607
```

```
t = 0.1668; x1 = 0.291136; x2 = 0.0356938
t = 0.1688; x1 = 0.287997; x2 = 0.0353235
t = 0.1708; x1 = 0.284908; x2 = 0.0349591
t = 0.1728; x1 = 0.281867; x2 = 0.0346003
t = 0.1748; x1 = 0.278875; x2 = 0.0342471
t = 0.1768; x1 = 0.275929; x2 = 0.0338994
t = 0.1788; x1 = 0.27303; x2 = 0.033557
t = 0.1808; x1 = 0.270176; x2 = 0.0332199
t = 0.1828; x1 = 0.267367; x2 = 0.032888
t = 0.1848; x1 = 0.264601; x2 = 0.0325611
t = 0.1868; x1 = 0.261877; x2 = 0.0322392
t = 0.1888; x1 = 0.259196; x2 = 0.0319221
t = 0.1908; x1 = 0.256555; x2 = 0.0316098
t = 0.1928; x1 = 0.253955; x2 = 0.0313022
t = 0.1948; x1 = 0.251394; x2 = 0.0309992
t = 0.1968; x1 = 0.248872; x2 = 0.0307007
t = 0.1988; x1 = 0.246388; x2 = 0.0304066
t = 0.2008; x1 = 0.243941; x2 = 0.0301169
t = 0.2028; x1 = 0.241531; x2 = 0.0298314
t = 0.2048; x1 = 0.239157; x2 = 0.0295501
t = 0.2068; x1 = 0.236817; x2 = 0.0292728
t = 0.2088; x1 = 0.234513; x2 = 0.0289996
t = 0.2108; x1 = 0.232242; x2 = 0.0287303
t = 0.2128; x1 = 0.230004; x2 = 0.0284649
t = 0.2148; x1 = 0.227799; x2 = 0.0282033
t = 0.2168; x1 = 0.225626; x2 = 0.0279454
t = 0.2188; x1 = 0.223485; x2 = 0.0276911
t = 0.2208; x1 = 0.221374; x2 = 0.0274405
t = 0.2228; x1 = 0.219293; x2 = 0.0271933
t = 0.2248; x1 = 0.217243; x2 = 0.0269497
t = 0.2268; x1 = 0.215221; x2 = 0.0267094
t = 0.2288; x1 = 0.213228; x2 = 0.0264724
t = 0.2308; x1 = 0.211263; x2 = 0.0262387
t = 0.2328; x1 = 0.209325; x2 = 0.0260082
t = 0.2348; x1 = 0.207415; x2 = 0.0257809
t = 0.2368; x1 = 0.205531; x2 = 0.0255566
t = 0.2388; x1 = 0.203673; x2 = 0.0253354
t = 0.2408; x1 = 0.201841; x2 = 0.0251172
t = 0.2428; x1 = 0.200034; x2 = 0.0249019
t = 0.2448; x1 = 0.198251; x2 = 0.0246895
t = 0.2468; x1 = 0.196493; x2 = 0.0244799
t = 0.2488; x1 = 0.194759; x2 = 0.024273
t = 0.2508; x1 = 0.193048; x2 = 0.0240689
t = 0.2528; x1 = 0.19136; x2 = 0.0238675
t = 0.2548; x1 = 0.189694; x2 = 0.0236687
t = 0.2568; x1 = 0.188051; x2 = 0.0234725
t = 0.2588; x1 = 0.186429; x2 = 0.0232788
t = 0.2608; x1 = 0.184829; x2 = 0.0230876
t = 0.2628; x1 = 0.18325; x2 = 0.0228989
t = 0.2648; x1 = 0.181691; x2 = 0.0227125
t = 0.2668; x1 = 0.180153; x2 = 0.0225286
t = 0.2688; x1 = 0.178634; x2 = 0.0223469
t = 0.2708; x1 = 0.177136; x2 = 0.0221675
t = 0.2728; x1 = 0.175656; x2 = 0.0219904
t = 0.2748; x1 = 0.174195; x2 = 0.0218155
t = 0.2768; x1 = 0.172753; x2 = 0.0216427
t = 0.2788; x1 = 0.171329; x2 = 0.0214721
t = 0.2808; x1 = 0.169923; x2 = 0.0213036
t = 0.2828; x1 = 0.168535; x2 = 0.0211371
t = 0.2848; x1 = 0.167164; x2 = 0.0209727
t = 0.2868; x1 = 0.16581; x2 = 0.0208102
t = 0.2888; x1 = 0.164473; x2 = 0.0206497
t = 0.2908; x1 = 0.163153; x2 = 0.0204912
```

```
t = 0.2928; x1 = 0.161848; x2 = 0.0203345
t = 0.2948; x1 = 0.16056; x2 = 0.0201797
t = 0.2968; x1 = 0.159287; x2 = 0.0200267
t = 0.2988; x1 = 0.158029; x2 = 0.0198755
t = 0.3008; x1 = 0.156787; x2 = 0.0197261
t = 0.3028; x1 = 0.15556; x2 = 0.0195784
t = 0.3048; x1 = 0.154347; x2 = 0.0194324
t = 0.3068; x1 = 0.153149; x2 = 0.0192881
t = 0.3088; x1 = 0.151965; x2 = 0.0191455
t = 0.3108; x1 = 0.150795; x2 = 0.0190045
t = 0.3128; x1 = 0.149638; x2 = 0.0188652
t = 0.3148; x1 = 0.148495; x2 = 0.0187274
t = 0.3168; x1 = 0.147366; x2 = 0.0185911
t = 0.3188; x1 = 0.146249; x2 = 0.0184564
t = 0.3208; x1 = 0.145145; x2 = 0.0183232
t = 0.3228; x1 = 0.144054; x2 = 0.0181914
t = 0.3248; x1 = 0.142976; x2 = 0.0180612
t = 0.3268; x1 = 0.141909; x2 = 0.0179323
t = 0.3288; x1 = 0.140855; x2 = 0.0178049
t = 0.3308; x1 = 0.139813; x2 = 0.0176789
t = 0.3328; x1 = 0.138782; x2 = 0.0175542
t = 0.3348; x1 = 0.137763; x2 = 0.0174309
t = 0.3368; x1 = 0.136755; x2 = 0.0173089
t = 0.3388; x1 = 0.135759; x2 = 0.0171882
t = 0.3408; x1 = 0.134773; x2 = 0.0170688
t = 0.3428; x1 = 0.133798; x2 = 0.0169507
t = 0.3448; x1 = 0.132834; x2 = 0.0168338
t = 0.3468; x1 = 0.131881; x2 = 0.0167182
t = 0.3488; x1 = 0.130937; x2 = 0.0166038
t = 0.3508; x1 = 0.130004; x2 = 0.0164905
t = 0.3528; x1 = 0.129081; x2 = 0.0163785
t = 0.3548; x1 = 0.128168; x2 = 0.0162676
t = 0.3568; x1 = 0.127264; x2 = 0.0161578
t = 0.3588; x1 = 0.12637; x2 = 0.0160492
t = 0.3608; x1 = 0.125486; x2 = 0.0159416
t = 0.3628; x1 = 0.124611; x2 = 0.0158352
t = 0.3648; x1 = 0.123745; x2 = 0.0157298
t = 0.3668; x1 = 0.122888; x2 = 0.0156255
t = 0.3688; x1 = 0.12204; x2 = 0.0155223
t = 0.3708; x1 = 0.121201; x2 = 0.0154201
t = 0.3728; x1 = 0.12037; x2 = 0.0153188
t = 0.3748; x1 = 0.119548; x2 = 0.0152186
t = 0.3768; x1 = 0.118734; x2 = 0.0151194
t = 0.3788; x1 = 0.117929; x2 = 0.0150212
t = 0.3808; x1 = 0.117131; x2 = 0.0149239
t = 0.3828; x1 = 0.116342; x2 = 0.0148275
t = 0.3848; x1 = 0.115561; x2 = 0.0147321
t = 0.3868; x1 = 0.114787; x2 = 0.0146376
t = 0.3888; x1 = 0.114021; x2 = 0.0145441
t = 0.3908; x1 = 0.113263; x2 = 0.0144514
t = 0.3928; x1 = 0.112512; x2 = 0.0143596
t = 0.3948; x1 = 0.111769; x2 = 0.0142686
t = 0.3968; x1 = 0.111033; x2 = 0.0141785
t = 0.3988; x1 = 0.110304; x2 = 0.0140893
```

4 Приложение

4.1 Код lab3.cpp

```cpp
#include <iostream>
#include <cmath>
```

```cpp
#include "../cmath.h"

int getSystem(int n, double t, double x[], double dx[])
{
  dx[0] = -40 * x[0] + 260 * x[1] + 1 / (10 * t * t + 1);
  dx[1] = 30 * x[0] - 270 * x[1] + std::exp(-2 * t);
  return 0;
}
int main()
{
  int fail = 0;
  double hprint = 0.02;

  const int n = 2;
  double x[2] = {0, 1};
  double dx[2];
  int flag = 1;
  double h;
  int nfe;
  int maxfe  = 5000;
  double relerr = 1.0e-4;
  double abserr = 1.0e-4;


  rkfinit(n, &fail);
  if (fail != 0)
  {
    std::cout << "Error occured in rkfinit.\n";
    return 1;
  }
  for (double t = 0; t < 0.4; /*t += hprint*/)
  {
    rkf45(getSystem, n, x, dx, &t, t + hprint, &relerr, abserr,
              &h, &nfe, maxfe, &flag);
    std::cout << "t = " << t << "; x1 = " << x[0] << "; x2 = " << x[1] << "; flag = "
     << flag << "\n";
  }
  rkfend();
  std::cout << "Adams: \n";

  x[0] = 0;
  x[1] = 1;
  double f[3][2];
  double t = 0;
  rkfinit(n, &fail);
  hprint = 0.0002;
  for (int i = 0; i < 3; ++i)
  {
    rkf45(getSystem, n, x, f[i], &t, t + hprint, &relerr, abserr,
              &h, &nfe, maxfe, &flag);
    std::cout << "t = " << t << "; x1 = " << x[0] << "; x2 = " << x[1] << "; flag = "
     << flag << "\n";
  }
  rkfend();
  double hint = 0.002;
  hprint = 0.02;
  int counter = 0;
  for (double t = 0.0006; t < 0.4; t += hint)
  {
    x[0] = x[0] +
        hint * (23 * f[2][0] - 16 * f[1][0] + 5 * f[0][0]) / 12;
    x[1] = x[1] +
        hint * (23 * f[2][1] - 16 * f[1][1] + 5 * f[0][1]) / 12;
```

```
64      if (counter++ % 10 == 0)
65      {
66        std::cout << "t = " << t + hint << "; x1 = " << x[0] << "; x2 = " << x[1] << "\
      n";
67      }
68      f[0][0] = f[1][0];
69      f[1][0] = f[2][0];
70      f[0][1] = f[1][1];
71      f[1][1] = f[2][1];
72
73      getSystem(n, t, x, f[2]);
74    }
75
76
77    return 0;
78  }
```