My Narrative Journey through the Beautiful Landscape of Mathematics

When I was in 8th grade, I got a 5 on the AP Calculus BC exam. In 9th grade, I got 5s on AP Physics C, both Mechanics and Electromagnetism. By the time I graduated, I had taken college courses in matrices (9th), PDEs (10th) and multivariable calculus (11th).

I did it all out of sheer love and passion for these beautiful areas of study, not to amass accomplishments, or because anyone forced me. These ideas are my oldest friends.

My dad taught me math outside of school starting before kindergarten at an accelerated pace that barely kept up with my intense curiosity for how and why these ideas really worked. No one trained me on how to mechanically solve math problems; rather, I learned to think through *why* the “right” approach actually made sense. Math was a beautiful, perfect game for me, and by 8th grade I was essentially teaching myself. I always deeply dwelt on concepts, however advanced they may seem at first, until they became crystal clear and intuitive.

One example comes to mind: in the summer before 8th grade, the moment that I had learned the definitions of derivative and integral and how to evaluate some of them, my first thought was: “wait a second here, *why should* slopes of tangent lines have anything to do with areas under curves??” I spent a long time thinking it through, visualizing telescoping dx’s, leveraging every analogy and intuition I had at my disposal until I synthesized a new one: I finally and viscerally understood *why* tangent lines and areas absolutely *must* be related!

I genuinely had no idea what a privilege learning this habit was until I discovered one day that all of my classmates had to memorize formulas. I didn’t get it; wasn’t it easier to understand the essential ideas behind the formulas and then derive them? How else can one learn??

To lay the groundwork for a running metaphor: there was something else I learned in my own unique way -- something I cannot imagine learning any other way, least of all the conventional approach. Before 10th grade, I didn’t play any instruments, but as with almost everything, I was curious about how music really *worked*. I plunged into wikipedia rabbit holes, immersing myself in the language of music theory. I probably became the most obsessed little amature music theorist in the world that didn’t even play any instrument! But sometimes, after looking up a concept, I would try to carefully and awkwardly play it on the family piano: the same piano I loathed during the attempted (and miserably failed) music lessons years earlier. “Oh ok, that’s what a major 7th chord sounds like! No wonder it resolves a V but not a dominant 7th IV chord”. Now I finally understood *why* each finger goes where it goes!

Even this early on, I implicitly understood a deep truth: that everything I love and everything that speaks to me -- whether it’s math or music, physics or literature -- I love them because they are beautiful and deep, and therefore profoundly meaningful to me. Even my hobbies, like origami, logic puzzles, and chess fit this paradigm; they allow endless creativity to flow from our spirits in a pristine and pure form, free from the messiness of the real world.

I couldn’t have known at the time which of my passions, if any, I would pursue as a career, but in retrospect, it was obvious the entire time, to everyone: I’m destined to be a mathematician.

When I first came to Vanderbilt, I took the famous Math 2500 and 2501: the year long honors class combining multivariable calculus with linear algebra, what I soon learned was ‘calculus on manifolds’. Not placing out of this class as I had done with 7th, 8th, 10th, and 11th grade math, even if I could have, would prove to be the single most impactful, life changing decision I’ve ever made. Proofs were easy, fun, and deeply insightful. Calculus in n dimensions made more sense than calculus in 3 dimensions, even visually. All my prior intuitions about what linear transformations “look like” fell into perfect place with my old intuitions for differentiation and integration, along with my new intuitions for differential forms. And the generalized Stokes theorem was by far the most beautiful, intuitive idea I had ever encountered. It grips me to this day like the love one has for a dear old friend. To skip ahead a bit, learning de Rham cohomology felt like running into this long lost friend again after we’ve both grown so much!

I had the rare and invaluable gift of *knowing*, even by sophomore year, what I was best at, and that there's an entire discipline for people with my natural mode of thought: pure math.

During the spring of freshman year, I read and deeply pondered over textbooks like “Visual Group Theory”, and “Visual Complex Analysis” that focused on intuition. Once I felt I knew what profound ideas groups were *meant* to capture, I started reading the graduate text “Algebra: Chapter 0” by Paolo Aluffi that summer. That book would become one of my greatest companions throughout undergrad -- from the very first chapter defining sets and categories to the very last on homological algebra. I had no money to buy it, but I found a free PDF and printed all 738 pages, 8 pages at a time, into a binder. Years of scribbles line the margins -- my early attempts to understand quotients right beside “cokernel of inclusion, ses”.

I spent freshman summer out-of-state and almost entirely alone, reading these texts and taking abstract algebra and complex analysis at UCLA. Some of my most formative memories are of pacing the palm tree studded streets, talking, explaining to myself as I often do, rewiring my neurons until universal properties seemed like second nature, visualizing infinite sums of infinitesimal dz’s spiraling through the night sky. Even then, algebra and analysis intertwined, dancing to the harmonies of some deeper math I was determined to uncover.

My sophomore and junior years were a blurred whirlwind of frantically obsessed learning.

The idea of universal properties in algebra especially opened my eyes. I had a very early-onset deep conviction that the category theory approach to math was “correct”, even before I fully knew what it was. I tore through every category theory book accessible to me, learning how to cast universal properties in terms of (co)limits of diagrams, adjunctions, and representable functors equally and ever more fluently over time. Learning category theory early paid dividends throughout my entire undergrad experience: it gave me access to guided studies from fixed point theorems for lattices and DCPOs (with Adam Prenosil) to research in classifying rank one fusion categories (with Cain Edie-Michelle), to a directed reading program in covering space theory and mapping class groups (with Ian Runnels), and others.

Every class I took, I couldn't help but to go beyond the curriculum, reading many full textbooks on my own and solving problems that caught my attention whether they were assigned or not. Taking graph theory and algorithms led to my self study of matroid theory. Taking game theory with an algebraic combinatorialist (Paul Edelman), I studied the representation theory of the symmetric group, and soon after, representation theory of finite groups, and then Lie algebras. When I took differential geometry, taught from the extrinsic point of view, in parallel I learned how to define a smooth structure in terms of sheaves. When I audited a graduate category theory course, I wrote up an expository intro to string diagrams for 2-categories -- an idea that was novel to my logician professor. When I audited a graduate harmonic analysis course, I presented on a representation theoretic version of the Fourier transform over finite, nonabelian groups, where Pontryagin duality generalizes to Tannaka-Krein duality as the noncommutativity shatters the dual group into a category. This last one led to my research in fusion categories with Cain -- abstractions of the category of representations of a finite group.

All the while, I was becoming a more natural piano player. Stiff fingers relaxed their posture out of sheer necessity over time. Music theory taught me what chord progressions should and shouldn't sound good; my technique developed organically as I attempted them over time. Just like with math, I was obsessed. I'd play a public piano for a bit anytime I passed one.

Yet throughout all of this, I genuinely struggled with focusing enough time and attention on what was assigned to maintain the grades that I knew matched my understanding in my classes. I wondered what was wrong with me, and I sought out counseling, eventually receiving the ADHD diagnosis that made so many of my behaviors finally make sense.

Things shifted dramatically when Covid swept the world. When we were all sent home, even though I was doing well in 4 math classes, I proactively switched all my classes to Pass/Fail, knowing I don't have a stable home life, much less a quiet working environment. The same father who taught me to love math so deeply almost disowned me for not pursuing something "useful" with it -- like CS or economics. And for a while, I did explore the idea of interesting industry jobs, but time and time again, I found theory to be the most interesting: game theory in economics, algorithms and complexity in computer science -- and of course music theory!

I took 2 years off after junior year, waiting for classes to resume in person, tutoring to support myself, buying myself time to figure it all out: my career, managing my ADHD -- basically, life. Still living near Vanderbilt, I decided to fully follow my true interests and passion, and to surrender to wherever my playful spirit took me. After all, that's how I became a musician -- and one who can increasingly fluently improvise melodies and songs! I continued my trend of auditing classes, which over the next 2 years would span graduate harmonic analysis, category theory, operator algebras, Riemannian geometry, and geometric group theory. I had no patience for things that seemed random or contrived, but fortunately I developed a good sense for when a little bit (or a lot) of further thinking should reveal a pattern that makes a concept seem more natural and less artificial. Just like with music theory, I learned to separate what was essential from what was notation -- what existed because it was *real* (as in, you could actually hear it) from what existed due to the limitations of human language, historical traditions, or even mere accident. The more I understood, the easier understanding things became. The math and music alike started to flow, imbued with emotion.

This approach, along with some deep philosophical pondering and a conscious decision to reorient my life, also helped manage my ADHD! As long as I followed my sense of “meaning”, my work ethic felt absolutely limitless. For example, it was much easier for me to follow the sequence “operator algebras, then real analysis, then operator algebras again” than just “real analysis then operator algebras”! Through my advanced audits, I discovered the *reasons* behind almost everything we learn in our first year graduate sequence, and then they became easy and fun again! The difference was night and day -- like rigid piano lessons vs free flowing improvisation -- like taking a language class in high school vs being a native speaker.

Leading Math Club throughout all this chaos centered me. The pivotal factor that led me to fully commit myself to math was really my ability to teach, and the meaning it brought me. Having learned so much math extremely naturally and organically, like a natural language, I found that when I convey my understanding to others, I so often see their eyes light up! My version and method of “understanding” is very intertwined with my ability to explain; I came to realize that my pursuit of beautiful ideas for their own sake wasn’t as “selfish” as I thought! I realized that I could be far more helpful, and “useful” inspiring others to see the beauty in math than I could ever hope to be as an engineer. I can't even help it -- love is contagious! I now had a stronger than ever reason to strive for math grad school: not only just to uncover the deep mysteries of math, but to share them. All externally imposed career goals fell away.

When I came back as a senior, I found all of the wide variety of concepts we learned in our graduate topology class completely well motivated and contextualized. Every problem that I understood “how to think about” I solved with ease, and almost every concept, named or unnamed, I recognized from my self study. I got to apply techniques I learned on my own, like filters, closure operators, and universal properties, to solve some problems in particularly elegant ways; my DRP in covering spaces, my homological algebra study, and everything else I had learned crystallized into a swelling crescendo, concluding the Ballad of Undergrad.

Besides the year-long graduate topology course, I achieved my goal of getting A’s in all of the hard math/CS classes I took that year, including numerical methods, linear optimization, and automata theory -- and I also graded for linear algebra. I still had engineering electives to finish, but I spent as much time on math as possible -- and I’m proud of the focus I was able to bring to my studies. It represented a notable shift away from my ADHD colored past. It was my sacrifice to the Math Grad School Gods: my proof that I *can* focus on what’s assigned!

But as I play back some old music I recorded -- not even five years after first googling music theory, already sounding classically trained, improvising nocturnes imbued with emotion I can’t possibly put into words -- I can't help but think to myself how grateful I am that I learn the way I do, and how excited I am to share what I learn and discover with the world.