John Baez recently said in a talk, “Mathematics in the 21st Century”, that there are five directions he sees math progressing toward over the next hundred years:

- **Embrace Homotopical / Higher-Categorical Thinking**
  - Lurie’s work with (infinity, 1)-category theory
  - Voevodsky’s work in homotopy type theory
  - Grothendieck’s full dream of infinity category theory
- **Understand quantum field theory and string theory**
  - According to Witten, coming to grips with QFT will be a central theme in math
- **Broadening our view of algebraic geometry**
  - We are realizing we need to generalize the duality between commutative algebra and geometry to account for phenomena like tropical algebra, etc.
  - What does Grothendieck’s work suggest commutative algebra is *really* about?
- **Make math more computer-friendly**
  - Ultimately, all results should be computer verified, easy to access world-wide, and annotated in many ways
- **Make math more human-friendly**
  - Math is harder to understand than it needs to be. Khan Academy, Wikipedia, nLab, etc are great, but we could do so much more

I was stunned when I heard these 5 themes all in one talk. I’ve read many “modern math” textbooks, so it might not be *too* surprising, but these directions almost completely align with my own intuitions for what’s important in math right now. I’ve actively engaged in all 5 of these areas throughout my undergraduate experience, both formally and informally.

Homotopical Thinking

Baez leads with a quote: “I am pretty strongly convinced that there is an ongoing reversal in the collective consciousness of mathematics: the right hemispherical and homotopical picture of the world becomes the basic intuition, and if you want to get a discrete set, then you pass to the set of connected components of a space defined only up to homotopy” -- Yuri Manin

My interest in category theory started early -- my study took an almost direct path from vector spaces to groups to categories, but I always felt the abstraction was powerful and well motivated. I was also interested in applied math, and sophomore year, I flew out to a conference on “Applied Category Theory”. I enjoyed talking about monads with a computer science friend (now at Yale) deeply interested in functional programming and type theory, and I tried to study homotopy type theory (using the HoTT/Univalent Foundations book) around junior year before realizing I’d like to see homotopy in action in the classical setting first. I shifted my interests toward algebraic topology, but at the back of my mind, I never forgot how universally applicable the ideas of category theory are, whether in algebraic topology or computer science. All along, I’ve tried listening to expositions of Lurie’s and Voevodsky’s groundbreaking work, periodically revisiting them as I learned more math. Maybe my intuition was “hijacked” early on by expositions like Baez’s, I can’t say for sure! But the idea of “categorification” always felt meaningful.

I've independently worked through the theory of string diagrams for 2-categories, which connected both to my study of fusion categories (see below) and to homotopy theory in general, easily showing how an adjoint pair of functors composes to a monad, for example. When I showed this idea to Dennis Sullivan, he pointed out a few ideas from classical topology that end up naturally having the structure of a 2-category. This style of math feels intrinsically interesting to me, and over the course of my career, I hope to play a prominent role in this "ongoing reversal in the collective consciousness of mathematicians"!

In addition, I've become especially interested in classical homotopy theory recently. I plan to revisit homotopy type theory, but for now, I'm interested in viewing topology from the point of view of fiber and cofiber sequences, classifying spaces, and characteristic classes, and I've studied bit of abstract frameworks that help clarify the structures at play, like model categories and simplicial sets. I'm influenced by discussions of the following sort, from tom Dieck's book:

Homotopy theory shows that the category of topological spaces has itself a kind of (hidden) algebraic structure [...] The notions of fibration and cofibration, which are at first sight of a technical nature, are used to indicate that an arbitrary continuous map has something like a kernel and a cokernel – the beginning of the internal algebraic structure of topology." Tammo tom Dieck, Algebraic Topology

My graduate course in topology, my directed reading program in covering space theory, and my audit of algebraic topology (homology and cohomology) prepares me well to go further in this direction, and I'm quite excited to do so, in either my thesis or postgrad work.

QFT and String Theory

"Only the loftiest peaks, which reach above the clouds, are seen in the mathematical theories of today, and these splendid peaks are studied in isolation [Seiberg-Witten theory, Chern-Simons theory, etc], because above the clouds they are isolated from one another. Still lost in the mist is the body of the range, with its quantum field theory bedrock and the great bulk of the mathematical treasures. So there is one rather safe, though perhaps seemingly provocative, prediction about twenty-first century mathematics: trying to come to grips with quantum field theory will be one of the main themes." -- Edward Witten

I have an old friend that just got his PhD in string theory, and we've talked at length about ideas I've seen attending math and physics seminars, like the AdS-CFT correspondence and supersymmetry. I've slowly become aware of just how valuable these physics ideas are to pure mathematics, like how a symmetry between different Calabi-Yau manifolds in string theory implies you can't actually "observe" the Euler characteristic of the manifold, only its absolute value. Even though more such manifolds with negative Euler characteristic than positive were known to mathematicians, the physicist Cumrun Vafa conjectured, via string theoretic intuition, they come in pairs. Soon enough, mathematicians formally proved this duality, mirror symmetry, and it's now an inseparable feature of the theory of 4-manifolds. I'm inspired by stories like these, and hope to mine quantum field theory for mathematical ideas myself. The following description of the Einstein Chair Mathematics Seminar at CUNY agrees:

“There are rigorous and famous discussions in mathematics that are separated in mathematics but are actually unified in the minds of theoretical physicists by these algorithms of quantum field theory [...] one can mention the celebrated invariants of differential topology (Donaldson and Vaughn Jones), of symplectic topology and algebraic geometry {Gromov and Witten), and of complex structures (Kodaira Spencer Griffiths...Kontsevich)” -- Dennis Sullivan

Along the lines of physics *and* homotopy, I’ve also seen concrete instances of categorification in quantum algebra, like fusion categories, with their monoidal product failing to be associative on the nose, but with natural associator transformations given very concretely by sets of equations. In this way, this categorification of representation theory seems to me not only natural, but extremely practical. Studying objects “up to associativity” allowed me to simplify my analysis, knowing these associators exist without having to carefully compute them, or on the other hand, posing new interesting concrete problems like computing them if their existence isn’t guaranteed. My study of string diagrams for 2-categories was also very helpful here, since the graphical calculus of fusion categories are just an enriched version of those.

In fact, several threads of my exploration through math seem to be deeply tied with quantum field theory, including the exceptional simple Lie algebras, the monster group and Moonshine, the Jones polynomial and representations of the Temperley-Lieb algebra, the Weil conjectures, the Hopf fibration, spinors, and gauge theory. I’m viscerally aware that I’ll likely never see satisfying resolutions to each of the deep mysteries these ideas pose, but I’m certain that over a lifetime of work, I’ll find a sense of fulfilment in the pursuit of many of them.

Algebraic Geometry

“The struggle to understand the “field with one element”, tropical algebra, etc, is pushing us to generalize the duality: COMMUTATIVE ALGEBRA^{OP} ≃ GEOMETRY that underlies AG.

What sort of “space” has Z as its ring of functions, really? Can understanding this help prove the Riemann hypothesis? You can answer “SpecZ”, but that answer doesn't seem adequate for understanding the relation between the Riemann hypothesis, which is about Z, to the modified versions of the Riemann hypothesis, like the Weil conjectures, which *have* successfully been proven.

What’s the subject of commutative algebra *really* about? Fields? Commutative rings? Commutative algebra includes Z. Sheaves of commutative rings? Commutative algebra must work “locally”. Commutative *rigs*? Commutative algebra must include N. Commutative monoids? Sheaves of commutative monoids? Commutative monoid objects in an arbitrary symmetric monoidal category? Commutative monoid objects “up to homotopy” [as in Lurie’s derived algebraic geometry, E-infinity objects]?

Items 1-3 are all related, and they interact in exciting ways! For example, you can think of string theory as a categorified version of particle physics, where particles are points and strings are paths, membranes are higher dimensional things -- all very akin to what you're doing when you go from sets to categories to 2-categories, etc.” -- John Baez

This idea is near and dear to me. The graduate algebra book I've been reading on my own starting freshman year, Algebra Chapter 0, put a heavy emphasis on universal properties, and culminated in homological algebra, preparing the reader to move into spectral sequences, triangulated categories, and derived functors. I got the impression that the framework Grothendieck set up is so powerful and natural that I immediately knew I wanted it to play a role in my research at some point, and at the least that it was a revolution in math at the scale of general relativity in physics. I've been learning bits and pieces of the theory of sheaves and schemes ever since, and I even audited several operator algebras courses knowing Grothendieck himself started there (and that he even named the spectrum of a ring after spectral theory). I've heard mathematicians talk about fields like noncommutative geometry and K-theory, and independently got the impression that much of recent math has been an attempt to broaden Grothendieck's techniques to problems beyond algebraic geometry.

Within algebraic geometry itself, through lots of pondering over time, I've come to know and appreciate the intuitions behind the scheme framework. The analogy between maximal/prime ideals and points/generic points in space makes sense, as does the way one computes the stalks and sections over a distinguished Zariskii open set purely algebraically with localization. It was a substantial learning curve because of how interrelated the algebra and geometry are -- sometimes it feels like they are mutual prerequisites, meaning you can never start! But in fact, I've always thrived in my self study of those areas of math where you have to lean on analogy and interconnected reasoning. The search to push the limits of these ideas is very appealing to me in the long run, and I'm happy to start researching in any area that remotely touches algebraic geometry and homological algebra (which seem to span much of math!).

For example, some heuristics I'm aware of are: "Sn should be thought of as GLn over the field with one element", and "stacks are to sheaves over schemes as representation theory is to modules over a ring". The idea of stacks as *local representation theory* especially excites me!

In fact, the interrelatedness of the first three directions, as Baez points out, I hope plays a huge role in my life's work. I've learned of these connections online from Edward Frenkel, and at Vanderbilt from the number theorist Larry Rolen, and many other sources, and whenever I do, I become extremely excited. Dennis Sullivan's advice to me was to keep these interests in mind as I progress through grad school and postdocs, to always look for the simplest essence of what a field of math is about, and to start in one place, picking up specialized skills that I can broaden over the years. I'm honestly elated to start my journey in any of these fields!

Computer-Friendly

For a long time, I kept computer science as a deep secondary passion (and my minor), and at times, theoretical computer science almost overtook my passion for math. Category theory is very close in spirit to functional programming -- so much so that the product-hom adjudication underlies how Haskell deals with functions in the first place! Functional programming has a concept called a monad, exactly the same as category theory (though they use different works, like "return" instead of "unit", and "bind" instead of "multiplication"). The cliche is that without monads, functional languages are only good for heating up your computer.

Over time, I've involved myself in a number of small reading groups at the intersection of math, logic, category theory, universal algebra, and type theory, with a number of professors. I had the sense that formalizing math in formal verification languages like Coq or Julia might be a part of my research. I want to stay true and honest to the history of math as a *human* discipline, by and for humans, and I mostly think about abstract ideas without the aid of a computer, but I'd also like to be open minded about the tools I use. I believe I'm well prepared for any project that formalizes math in these computer languages, especially if that's the direction cutting edge math research takes. Beyond being fluent in some programming languages (like Haskell, Java, C++, python, etc), and especially in algorithms, data structures, and automata theory, I also already have a network of potential collaborators on the computer verification side of these research directions, if they become relevant to my math research.

Beyond direct computer verification of math, the ideas of logic I've studied might be useful in various areas of pure math. Homotopy type theory might end up furnishing mathematicians with new tools. Adapting logic to quantum mechanics, as in von Neumann and Birkoff's attempt to view the lattice of closed subspaces of a Hilbert space as a nonclassical analog of logic suited to QM hasn't found great success in actually influencing physicists and quantum information theorists, but mathematicians continue to propose different logics that might do the trick, based on structures like partial Boolean algebras for example (Kochen, Specker). Even the structure of a topos, present throughout much of algebraic geometry, has its own internal logic, which is generally nonclassical ("intuitionistic") logic in which the law of excluded middle fails, related more to modal logic and Kripke semantics than classical two-valued logic. I feel confident that I would recognize any underlying non classical logic structures present, even as I study classical mathematics, given my background.

Human-Friendly

I'm glad Dr. Baez mentioned this. Math *is* harder to learn than it needs to be, for so many people. I have a unique perspective on this that prepares me to become an empathetic educator. My interest in math always originated outside the classroom. One on one with my dad, I would learn math at double the curriculum pace, eventually placing out of 7th, 8th, 10th, and 11th grade math when we finally convinced my rigid public school system to offer these placement tests. I would go on to take AP Calculus in 8th grade and AP Physics, both mechanics and E&M, in 9th, and exhausted most of the local 2 year college's math courses by 11th. Over time, it dawned on me how differently and organically I thought about math; starting early afforded me the freedom of childhood to focus on making creative connections between different ideas that math teachers have neither the time, nor in some cases, the expertise, to present. Self discovery and big picture thinking have always been a part of my identity, and I bring that viewpoint every time I'm explaining a piece of math to someone. I've been told I'm quite good at teaching, and over time I've honed these skills, organizing weekly math club meetings at Vanderbilt, often developing novel explanations and presentations aimed at making some terse, abstract idea in higher math accessible and beautiful for other undergrads. I've always followed my internal sense of beauty and simplicity, and I've been told my style of explanation inspires others to be as passionate about math as I am.

I've delighted in other opportunities I've had for honing this skill, always pouring more effort into these kinds of roles than required or expected, such as grading for proof based linear algebra, TA'ing a high school graph theory summer camp at the University of Michigan, hours and hours of experience tutoring a wide range of math subjects, and countless informal conversations held with students after class, in the halls, at lunch, etc. Making an impact on others, and viewing mathematics as a deeply human, communal endeavor, will always be a part of my life, and I'm grateful for the experience I've had in fostering these communities.

I've also been influenced by my self study of psychology, which has made me more empathetic and emotionally intelligent, and especially sensitive to just how differently people are able to learn and conceive of mathematical ideas. I initiated this self study in part to understand my students better, but also in large part to understand my own psychology and my own mechanisms of thought and discovery. For example, I'm diagnosed with ADHD, and a related diagnosis with many overlapping features is that of dyslexia. Thought by some to be strictly a disadvantage, with dyslexic students often struggling to master rote tasks (such as abstract symbol manipulations) as quickly as their peers, in truth, the dyslexic brain is simply organized to function optimally for different tasks than the non-dyslexic brain. Dyslexic people often excel at creative thought, far reaching lateral connections, quickly assimilating the gist of an idea, spotting analogies between ideas that on the surface seem unrelated, 3-dimensional spatial manipulation… the list goes on. The acronym MIND, standing for material, interconnected, narrative, and dynamic reasoning, concisely summarizes the different cognitive strengths of the dyslexic brain, and there's even recent neurological evidence providing possible explanations for these tradeoffs in cognitive function at the level of physiology, as laid out in The Dyslexic Advantage (by Eide and Eide).

This provides a snapshot of my commitment to constantly push myself to learn more about the human condition in service of making math accessible to all. I'm also inspired by recent developments in technology that have already made math far more accessible, such as animated videos, large language models than can act as personalized, customizable tutors, virtual reality lessons that can elucidate concepts in 3 dimensional geometry for students whose 3d spatial reasoning far exceeds their 2d reasoning (as in dyslexia), and others.

I've even begun working on a series of animated video, leveraging visualization and a narrative storyline to explain the concepts in covering space theory I've studied and visualized for years now, as well as their connections to other curiosity-inspiring areas of math, like Riemann surfaces, spinors, monodromy, fibrations, graph theory, and hyperbolic geometry. There's a veritable revolution in animated mathematical explanations of a wide range of pure math happening online at the moment, and I hope to contribute my part to this dynamic, exciting medium. I take huge inspiration from mathematicians like Edward Frenkel, who was able to explain nontrivial ideas from the Langlands correspondence in his bestselling book Love and Math. I see myself as carefully heading to a future where, as a professional research mathematician, I'm able to inspire the world with cutting edge ideas in pure math made accessible to everyone, as Frenkel has. I'm excited to begin my journey!