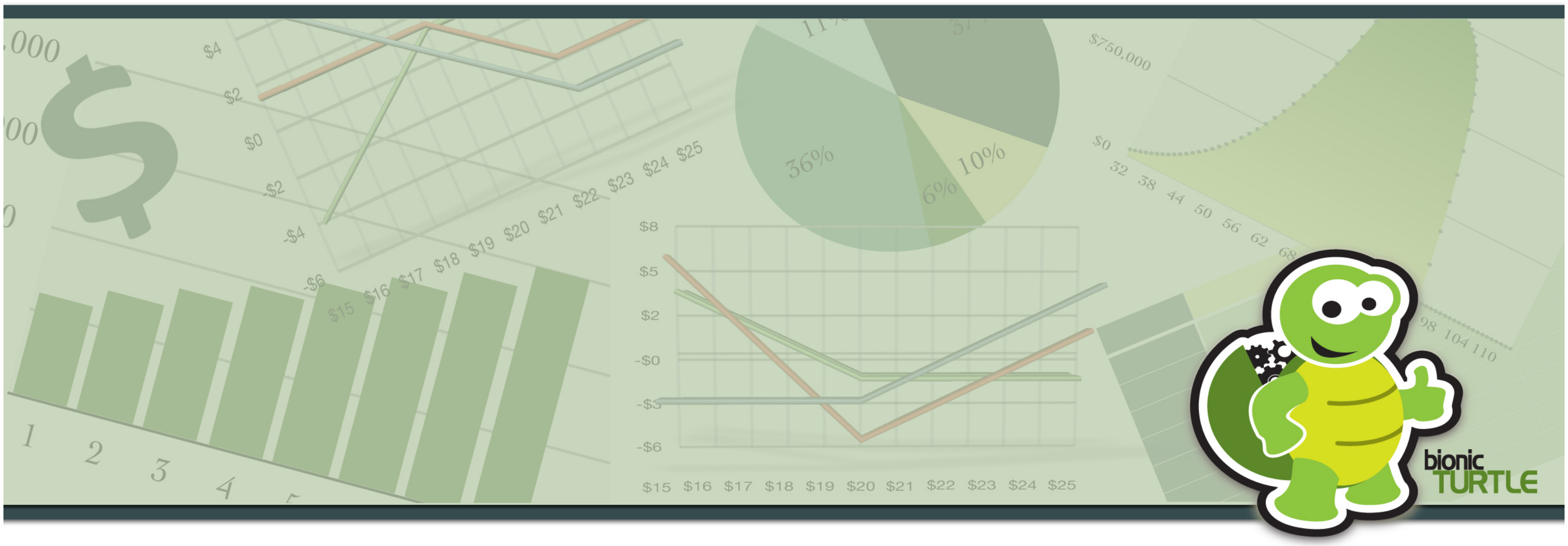


P1.T3. Financial Markets and Products

Bionic Turtle FRM 2013 Study Notes

Hull, Chapter 1, Introduction

Learning Outcomes:

Differentiate between an open outcry system and electronic trading.

Describe the over-the-counter market and how it differs from trading on an exchange, including advantages and disadvantages.

Differentiate between options, forwards, and Futures contracts.

Calculate and identify option and forward contract payoffs.

Describe, contrast, & calculate the payoffs from hedging strategies involving forward contracts and options.

Describe, contrast, and calculate the payoffs from speculative strategies involving Futures and options.

Calculate an arbitrage payoff and describe how arbitrage opportunities are ephemeral.

Describe some of the risks that can arise from the use of derivatives.

Differentiate between an open outcry system and electronic trading

Open outcry

Traders physically meet on exchange floor, shouting, using hand signals

Electronic trading

Electronic matching of trades has led to a growth in algorithmic trading (a.k.a., black-box trading, automated trading, high frequency trading or robo-trading).

"Traditionally derivatives exchanges have used what is known as the open outcry system. This involves traders physically meeting on the floor of the exchange, shouting, and using a complicated set of hand signals to indicate the trades they would like to carry out. Exchanges are increasingly replacing the open outcry system by electronic trading. This involves traders entering their desired trades at a keyboard and a computer being used to match buyers and sellers. The open outcry system has its advocates, but, as time passes, it is becoming less and less common." –Hull

Describe the over the counter market and how it differs from trading on an exchange, including advantages and disadvantages

Over-the-counter (OTC)
Network of dealers linked by recorded phone conversations and computers (If there is a dispute about what was agreed, the tapes are replayed to resolve the issue)
Trades between two counterparties. Trades in the over-the-counter market are typically much larger than trades in the exchange-traded market. And, in terms of total volume, the OTC market is "much larger."

Advantage of OTC

- Customization (a.k.a., "tailored" exposure): The terms of a contract do not have to be those specified by an exchange.
- Market participants are free to negotiate any mutually attractive deal.

Disadvantage of OTC

- Counterparty risk

Differentiate between options, forwards, and Futures contracts

A forward contract is an obligation (agreement) to buy or sell an asset at a certain future time for a certain price. For example, an oil producer promised to sell 10 million barrels of oil next December for the pre-agreed price of $110.00 per barrel
An option gives holder the right (but not the obligation) to buy/sell at a certain price.
For example, an executive has the right (but not the obligation) to buy 10,000 shares of her company's stock next December, at the pre-agreed (strike or exercise) price of $35 per share. Unlike a long forward position, she will not be obligated to purchase.

Options

A call (put) option is an option to buy (sell) a certain asset by a certain date for a certain price (the strike price)

Forwards

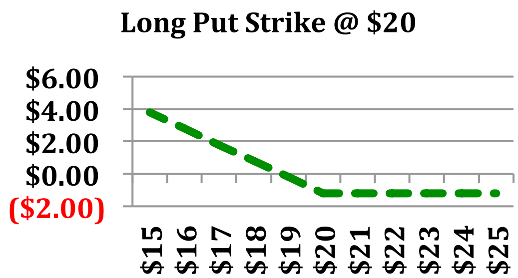
Agreement to buy/sell asset at future time for certain price. Traded in the over-the-counter (OTC) market

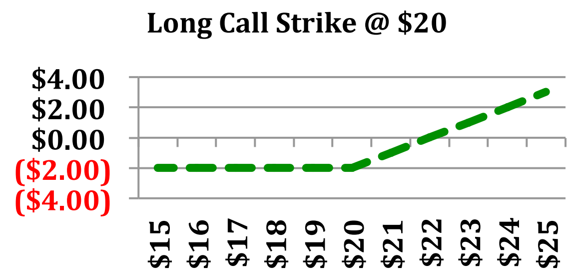
Futures

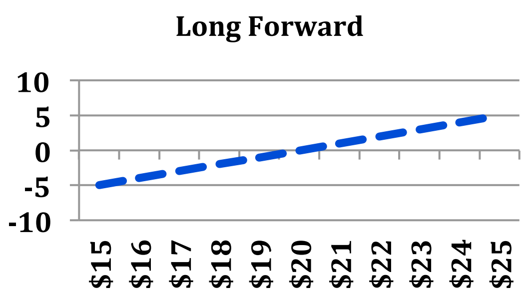
Like forward, agreement to buy/sell asset at certain price & time. But futures contract trades on an exchange

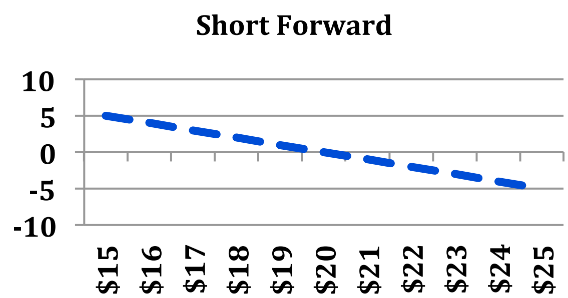
Calculate and identify option and forward contract payoffs

The call and put option charts plot the option payoff: payoff = payout (-) minus premium cost of option. The forward has no initial cost, so its payoff plot equals its profit plot.









In regard to stock options:

Premium = initial cost (or initial investment or up-front cost)
Payoff = gain on exercise (i.e., intrinsic value at exercise)
To the long position, who buys the option, (Net) Profit = Payoff – Premium
To the short position, who writes the option, (Net) Profit = Premium - Payoff
For Example:
Question: If the price (premium) is $4.00 for a call option with a strike (exercise) of $30.00, what are the payoff and profit on a long position (option buyer), if the option expires when the stock is $38.50?

Answer: Payoff on a long call = MAX[0, S(t) – K] = MAX[0, 38.50 – 30.00] = $8.50
Profit on the long call = payoff – premium = $8.50 – 4.00 = $4.50.
(does not account for the time value of money)
Question: If the price (premium) is $3.80 for a put option with a strike (exercise) of $20.00, what are the payoff and profit on a short position (option writer), if the option expires when the stock is $13.00?

Answer: Payoff on a short put = -MAX[0, K – S(t)] = -MAX[0, 20 - 13] = -$7.00
Profit on the short put = premium – payoff = 3.80 - $7.00 = -$3.20.

Describe, contrast, and calculate the payoffs from hedging strategies involving forward contracts and options. Describe, contrast, and calculate the payoffs from speculative strategies involving Futures and options.

Both forwards and options can be used to hedge but there is a key difference.

Forward contract:

A forward contract does not require up-front investment. This is the advantage of "synthetic" exposure: instead of funding a purchase, our exposure is leveraged with a forward position. This is the essential difference between cash and synthetic markets: the spot market requires fully funding the purchase, but a forward does not. However, the contract can produce a loss as well as a profit. There is no guarantee that the outcome in a scenario with hedging will produce a more favorable outcome than one without hedging.

Option:

- Requires an up-front premium when buying the option.
- The payoff structure is asymmetric.
- Options can provide insurance.

Unlike the forward contract, when going long an option, there is a limited downside. This is the essential difference between a forward hedge and an option hedge (e.g., buying a put option): the forward does not have a premium, while the option requires a premium. But the option is asymmetric: it does not need to be exercised, so the gain can be preserved.

Example: Illustrating option leverage by comparing outright shares to options
The following comparison illustrates how options bestow leverage. The investor has $2,000 to invest. He/she can employ two strategies:

1. Buy 100 shares @ $20, or
2. Purchase 2,000 call options.

Then consider the payoff and profit outcomes under two scenarios: the stock price drops to $15 or the stock price rises to $27. Both strategies invest the same $2,000. But the option profits have greater upside but also greater downside. Invest $2,000 in either of two strategies (purchase 100 shares or purchase 2,000 call options):

Share Price in October:		$20
Call option price, Strike @ $22.50		$1
Investor's Two Strategies:		
Buy 100 Shares, or	$2,000	
Buy 2,000 Call Options	$2,000	
	December Stock Price	
Payoff	$15	$27
Buy 100 Shares	$1,500	$2,700
Buy 2,000 Call Options	$0	$9,000
Profit		
Buy 100 Shares	($500)	$700
Buy 2,000 Call Options	($2,000)	$7,000

Calculate an arbitrage payoff & ephemeral arbitrage opportunities

Consider the following assumptions:

- The spot price of gold, $S_0$is $900.00
- The risk-free interest rate, r_f is 10.0%
- Assume no transaction costs[1]
- Assume zero storage cost, zero convenience yield, and no lease rate.

These add a sense of reality to our cost of carry model, however; our carry model is simple so we do not expect accuracy. Our cost of carry model returns a “model forward price” of $990; i.e., our model “predicts a forward price, $F_{0,1}$of $990. Then we “analyze” two scenarios:

1. The observed one-year forward price, $F_{0,1}$ is $1,000. In this case, the forward is “*trading rich*” as the observed [trading] price of $1,000 is greater than the model price of $990.

[1] The learning spreadsheet allows for transaction costs; if we enter a non-zero transaction cost the model forward price becomes, instead, a model forward interval with a lower and upper bound. Below, as we assume zero transaction costs, the lower and upper bound give the same value

2. The observed one-year forward price, $F_{0,1}$ is $980.00. In this case, the forward is "*trading cheap*" as the observed [trading] price of $1,000 is less than the model price of $990.

Futures trades rich: profit with cash and carry					
Spot price of gold:	$900.00				
Interest rate:	10%				
Transaction:	0%				
Model (carry) price:	$990.00	No lower/upper bound since transaction cost = 0			
Forward price:	$1,000.00	"Trades rich" as 1,000 > 990			
Cash & carry: Short forward, borrow to buy spot					
			T0	T1	Net
		Spot commodity market	($900)		
		Transaction	$0		
		Cash	$900	($990)	
		Futures contract		$1,000	
		Net Cash Flow	$0	$10	$10

In the first case, because the forward price "trades rich"—i.e., observed $F_{0,1}$ price exceeds the model's predicted $F_{0,1}$ price—the correct arbitrage is a cash and carry: short the forward, and borrow to buy the spot. In the second case, the forward price "trades cheap" and the arbitrageur should conduct a reverse cash and carry trade: long forward and lend the cash received from shorting the spot.

Futures trades cheap: profit with REVERSE cash and carry		
Spot price of gold:	$900.00	
Interest rate:	10%	
Transaction:	0%	
Model (carry) price:	$990.00	No lower/upper bound since transaction cost = 0
Forward price:	$980.00	← "Trades cheap" as 980 < 990

Reverse cash & carry: short spot, lend cash, long forward

	T0	T1	Net
Spot commodity market	$900		
Transaction	$0		
Cash	-$900	$990	
Futures contract		-$980	
Net Cash Flow	$0	$10	+$10

If the Futures price is "*trading rich*," the arbitrage trade is cash and carry: borrow to buy the spot asset (buy the cheap thing) and short the forward (sell the expensive thing). If the Futures price is "trading cheap," the arbitrage trade is reverse cash and carry: sell short the spot asset & lend the cash (sell the expensive thing) and go long the forward (buy the cheap thing).

Describe some of the risks that can arise from the use of derivatives

There are three primary derivative uses:
1) Hedging
2) Speculation
3) Arbitrage

The key risk (danger) is that traders with mandates to hedge (or arbitrage) become speculators.

Lessons for Financial Institutions (Unassigned Hull, Chapter 34):

A bucket list of important points:

- Risk must be quantified and risk limits defined
- Exceeding risk limits not acceptable even when profits result, as this is unknown ex-ante.
- Do not assume that a trader with a good track record will always be right
- Be diversified
- Scenario analysis and stress testing is important
- Do not give too much independence to star traders
- Separate the front middle and back office

- Models can be wrong
- Be conservative in recognizing inception profits
- Do not sell clients inappropriate products
- Liquidity risk is very important
- There are dangers when many are following the same strategy
- Do not finance long-term assets with short-term liabilities
- Market transparency is important
- It is important to fully understand the products you trade
- Beware of hedgers becoming speculators
- It can be dangerous to make the Treasurer's department a profit center

Chapter Summary

An open outcry system is one where traders physically meet on exchange floor, shouting, using hand signals. Contrast this to an electronic trading system where trades are executes at lightening speed from banks, hedge funds and traders around the world by use of computers.

Two different markets exist: the OTC market and the Exchange-traded market. In terms of size, the OTC market is significantly larger than the Exchange-traded market measured in terms of volume of trading. The OTC market was around 8.4 times larger than the Exchange-traded market, with an estimated size of some $600 trillion in total principal amounts underlying the outstanding contracts.

The size of the two markets is difficult to measure, and it is likely that the Exchange-Traded market will grow at a much faster pace than the OTC market in years to come as the Dodd-Frank regulations forces certain financial institutions to have their trades on an exchange or to go through a clearinghouse. The regulations have yet to be finalized so the outcome over the next few years will be interesting, and will have a huge impact on those working in risk management and the financial sector.

Forward contracts and Futures contracts are similar in form, in that they both involve an agreement to buy or sell some asset in the future, but at a predetermined price. However, whereas forwards are transacted in the OTC market, Futures are highly standardized contract that trade on an exchange. Forwards can be customized in a way that Futures cannot, however, they also carry with them credit and counterparty risk - risk that a Futures contract is not nearly as exposed to. Futures contracts, on the other hand, while generally being less exposed to credit and counterparty risk, forces the owner of the Futures to post margin at the end of each trading day. We say that Futures are marked-to-market. It is this mechanism that reduces the counterparty risk. It does not however, reduce the market or liquidity risk, and forces the company to typically have a larger liquidity pool tied up for margining purposes – capital that could have been employed elsewhere.

Whereas forwards and Futures contracts involve an obligation to buy or to sell an asset at a predetermined price in the future, options on the other hand, gives you the right, but *not* the obligation to buy or sell the asset in the future at a predetermined price. Call options give you the right but not the obligation to buy an asset in the future, whereas put options give you the right but not the obligation to sell an asset in the future. A distinguishing feature of options as opposed to forwards and Futures is the fact that options require an initial outlay of money, whereas forwards and Futures do not.

The reading distinguishes between three broad categories of traders:

- Speculators: take a view on the market, and will often use leverage
- Hedgers: wish to use derivatives as insurance, or to minimize market risk
- Arbitrageurs: Seek out ephemeral pricing discrepancies in the market

1 Questions & Answers

Questions

1.1 Which of the following is LEAST likely to trade in an open outcry system?

a) Futures contract on investment commodity
b) Futures contract on consumption commodity
c) Exchange-traded call option on publicly traded stock
d) Over-the-counter (OTC) forward contract on consumption commodity

1.2 What is the chief advantage of a derivatives trade, that intends to hedge an exposure, on the OTC market over a similar trade on an exchange?

a) Greater liquidity
b) Lower counterparty risk
c) Lower basis risk
d) Ability to trade an option instead of a futures contract

1.3 A US-based importer knows it will need to pay EUR 10 million (Euros) to a European supplier in exactly three months. To hedge, the importer buys Euros in the three-month currency forward market. Which of the following is true?

a) The hedged outcome must be better than the un-hedged outcome
b) The hedged outcome could be worse than the un-hedged outcome
c) The payoff with currency forwards is identical to the payoff with currency options
d) If markets are efficient, there is no logical reason to use the forward contract

1.4 A long (plain vanilla) call option and a long futures position, both on the same underlying publicly traded stock, are similar in EACH of the following ways EXCEPT for:

a) Both can be priced analytically
b) Both have unlimited upside but a limited downside capped by the initial investment
c) Both forgo interim dividends
d) Both are leveraged relative to the corresponding cash (spot) position

1.5 Which BEST summarizes Hull's explanation for the ephemeral (short-lived) existence of arbitrage opportunities?

a) Efficient markets
b) Arbitrageurs conducting arbitrage
c) Transaction costs
d) Information technology

Answers

1.1 D. Open outcry occurs on the floor of an exchange; OTC markets are not standardize and do not utilize open outcry

1.2 C. Lower basis risk
Lower basis risk is the key hedging advantage afforded to the CUSTOMIZATION (or "tailoring") of the instrument to the underlying exposure; the exchange-traded instrument is standardized. In regard to (A) and (B), please note these are decidedly false: an exchange-traded market generally offers superior liquidity and lower counterparty risk.

1.3 B. The hedged outcome could be worse than the un-hedged outcome
If the Euro depreciates in the next three months, the un-hedged cost will be less: locking in forward cost cuts in both directions (Hull: "the purpose of hedging is to reduce risk. There is no guarantee that the outcome with hedging will be better than the outcome without hedging.")
In regard to (C), this is false: the option incurs a premium cost, and limits the downside, but unlike the forward allows the buyer to enjoy upside. Insurance has a totally different profile!
In regard to (D), this is false: the forward minimizes forex risk, even if the expected returns are identical.

1.4 B. The long futures contract has payoff = S(t) - K, such that while not uncapped on the downside, potential loss is (K). But as worst, the long option holder losses the premium.
In regard to (A), (C), and (D), please note these are TRUE of both options and futures.

1.5 B. Arbitrageurs conducting arbitrage
In regard to (A), this is tempting and an argument can be made. But EMH asserts the prices impound (incorporate) information; Hull makes a narrower argument that does not require EMH! He only requires the practice of arbitrage and supply/demand. Please notice this is related to the CAPM, which requires several restrictive assumptions, and the APT, which is less restrictive because it depends on no-arbitrage conditions.
Hull: "Arbitrage opportunities such as the one just described cannot last for long. As arbitrageurs buy the stock in New York, the forces of supply and demand will cause the dollar price to rise. Similarly, as they sell the stock in London, the sterling price will be driven down. Very quickly the two prices will become equivalent at the current exchange rate. Indeed, the existence of profit-hungry arbitrageurs makes it unlikely that a major disparity between the sterling price and the dollar price could ever exist in the first place. Generalizing from this example, we can say that **the very existence of arbitrageurs means that in practice only very small arbitrage opportunities are observed in the prices that are quoted in most financial markets.** In this book most of the arguments concerning futures prices, forward prices, and the values of option contracts will be based on the assumption that no arbitrage opportunities exist."

Hull, Chapter 2: Mechanics of Futures Markets

Learning Outcomes:

Define and describe the key features of a Futures contract including the asset, the contract price and size, delivery and limits.

Explain the convergence of Futures and spot prices.

Describe the rationale for margin requirements and explain how they work.

Describe the role of a clearinghouse in Futures transactions.

Describe the role of collateralization in the over-the-counter market and compare it to the margining system.

Identify and describe the differences between a normal and inverted Futures market.

Describe the mechanics of the delivery process and contrast it with cash settlement.

Define and demonstrate an understanding of the impact of different order types, including: market, limit, stop-loss, stop-limit, market-if-touched, discretionary, time-of-day, open, and fill-or-kill.

Compare and contrast forward and Futures contracts.

Define and describe the key features of a Futures contract including the asset, the contract price and size, delivery and limits.

A Futures contract is a standardized contract that trades on a Futures exchange to buy or to sell an underlying asset at a delivery date at a pre-set Futures price. The specifications of a Futures contract include, but are not limited to:

- Asset
- Contract Size
- Delivery Arrangement
- Delivery Months
- Price Quotes
- Price limits and position limits

For example, consider the underlying asset in the case of a Treasury bond/note:
A Treasury bond Futures contract is made on the underlying U.S. Treasury with maturity of at least 15 years and not callable within 15 years (15 years ≤ T bond).
A Treasury note Futures contract is made on the underlying U.S. Treasury with maturity of at least 6.5 years but not greater than 10 years (6.5 ≤ T note ≤ 10 years).
When the asset is a commodity (e.g., cotton, orange juice), the exchange specifies a grade (quality).
Contract Size

Contract size varies by the type of Futures contract:

Treasury bond Futures: contract size is a face value of \$100,000
S&P 500 Futures contract is index $\times$ \$250 (multiplier of 250X)
NASDAQ Futures contract is index $\times$ \$100 (multiplier of 100X)
Recently, “mini contracts” have been introduced: These have multipliers of 50X for the S&P and 20X for the NASDAQ. In other words, each contract is one-fifth the price in order to attract smaller investors.

A common test question involves S&P 500 Index Futures contract. Please note the multiple for the S&P 500 contract is \$250; e.g., if the index value is 1400, then one contract is worth \$350,00

Delivery Arrangement

The exchange specifies delivery location. The exchange must specify the delivery month; this can be the entire month or a sub-period of the month.
Delivery Months
The exchange must specify the precise period during the month when delivery can be made. For many Futures contracts, the delivery period is the whole month.
Price Quotes
The exchange defines how prices are quoted; e.g., crude oil is quoted in dollars and cents

Price limits and position limits

For most contracts, daily price move limits are specified by the exchange. Normally, if the limit is breached, trading stops for the day. Position limits are the maximum number of contracts that a speculator made hold (the purpose is to prevent speculators from an undue influence on the overall market for the commodity).

Example I: Futures Contract on Light Sweet Crude Oil

Asset	Light, Sweet, Crude Oil
Contract Size	1,000 barrels (42K gallons)
Delivery Arrangement	FOB Seller’s Facility
Delivery Months	Ratable over month
Price Quotes	U.S. dollars & cents
Price limits and position limits	Any one-month - 10,000 net Futures; all months - 20,000 net Futures; but not to exceed 3,000 contracts in the last three days of trading in the spot month.

Example II: Corn Futures

Asset	Corn (No. 2 Yellow..)
Contract Size	5000 bushels
Delivery Arrangement	Toledo, St. Louis
Delivery Months	Dec, Mar, May, Jul, Sep
Price Quotes	1/4 cent/bushel ($12.50/contract)
Price limits and position limits	Daily Price Limit: Thirty cent ($0.30) per bushel ($1,500/contract) above or below the previous day's settlement price. No limit in the spot month

Example III: S&P 500 Index Futures

Asset	S&P 500 Index
Contract Size	$250 x S&P 500 Futures Price
Delivery Arrangement	Cash settlement
Delivery Months	Mar, Jun, Sep, Dec
Price Quotes	0.05 index points = $12.50
Price limits and position limits	20,000 net long or short in all contract months combined

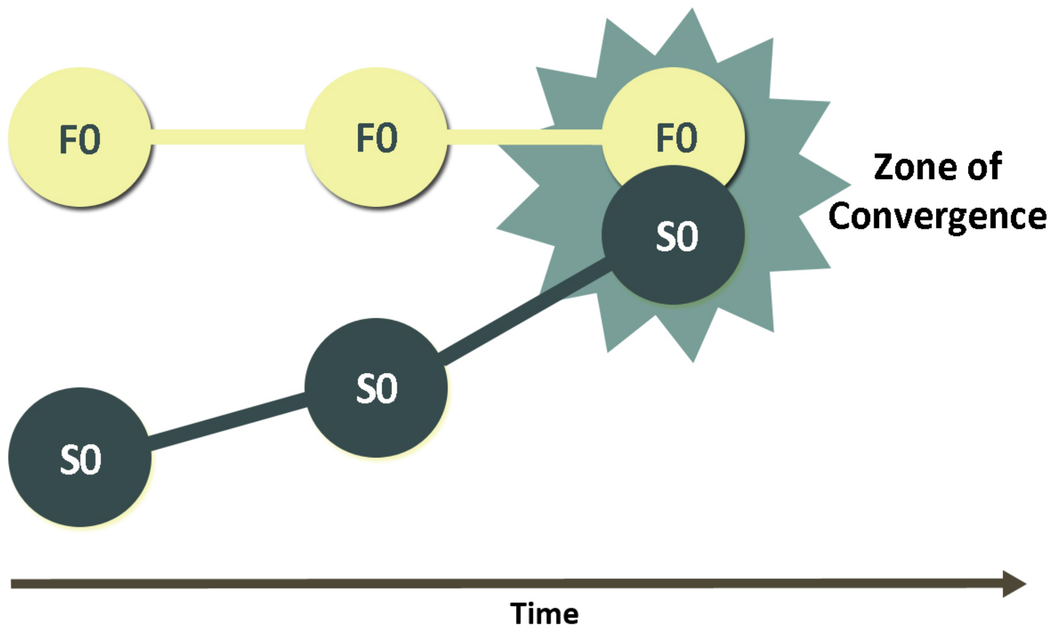
Mini contracts tend to be 1/5th the size
As the S&P 500 Futures contract is index $\times$ $250 (multiplier of 250X),
the S&P 500 “mini” = $50 x S&P Index
As the NASDAQ Futures contract is index $\times$ $100 (multiplier of 100X),
the NASDAQ “mini” = $20 x NASDQ (each contract is 1/5th price, to attract smaller investors).

Long versus Short Positions:
A long position agrees to buy in the future and a short position agrees to sell in the future. The price mechanism maintains a balance between buyers and sellers. For example, if there are more buyers than sellers, the price increases until new sellers enter the Futures market.

Most Futures contracts do not lead to delivery, because most trades “close out” their positions before delivery. Closing out a position means entering into the opposite type of trade from the original.

Explain the convergence of Futures and spot prices

At the Futures contract approaches maturity, the spot price should converge with the Futures price (at least to a so-called “zone of convergence”). Put another way, the basis (the difference between the spot and Futures price) should converge toward zero as the Futures contract approaches maturity.



In an (unrealistic) world where there is no risk premium, we can view this as the forward price representing an estimate of the expected future spot price, on the assumption that, as the maturity of its contract tends toward (shrinks) to zero, the forward price will converge on the spot price:

$$F_0 = E(S_T) \rightarrow F_0 = S_0 e^{rT}$$

Describe the rationale for margin requirements and explain how they work

Margin is one kind of credit risk mitigation (CRM)

Other CRM include:
- Netting
- Guarantees
- Credit Derivatives

A margin is cash or marketable securities deposited by an investor with his or her broker. The balance in the margin account is adjusted to reflect daily settlement. Margins thus minimize the possibility of a loss through a default on a contract because of the daily netting.

Operations of Margins:

1. Describe the marking to market procedure, the initial margin, and the maintenance margin.
2. Compute the variation margin.

When an investor enters into a Futures contract, the broker requires an initial margin deposit into the margin account. At the end of each trading day, the margin account is marked-to-market. If the account balance falls below the maintenance margin (i.e., typically lower than the initial margin), a margin call requires the investors to "top up" the account back to the initial margin amount.

Margin account: Broker requires deposit.

Initial margin: Must be deposited when contract is initiated.

Mark-to-market: At the end of each trading day, margin account is adjusted to reflect gains or losses.

Maintenance margin: Investor can withdraw funds in the margin account in excess of the initial margin. A maintenance margin guarantees that the balance in the margin account never gets negative (the maintenance margin is lower than the initial margin).

Margin call: When the balance in the margin account falls below the maintenance margin, broker executes a margin call. The next day, the investor needs to "top up" the margin account back to the initial margin level.

Variation margin: Extra funds deposited by the investor after receiving a margin call.
There is only a variation margin if and when there is a margin call.
Variation margin = initial margin – margin account balance

In the following example (Hull, 2012), the investor is long two contracts, the initial margin is \$4,000 (\$2,000 per contract) and the maintenance margin is \$3,000 (\$1,500 per contract). Note the margin call is triggered when the margin account balance breaches the maintenance margin; however, the investor must "top off" the account back to the initial margin, not just to the maintenance margin.

Contract Specifications:	
Contract Size (ounces)	100
# Contracts	2
Ounces:	200
Initial Futures	\$600

Margin	Per	Total
Initial margin	\$2,000	\$4,000
Maintenance margin	\$1,500	\$3,000

Date	Futures Price	Daily gain/loss	Cumulative Gain/loss	Margin Balance	Margin Call
	\$600			\$4,000	
5-Jun	\$597	-\$600	-\$600	\$3,400	\$0
6-Jun	\$596	-\$180	-\$780	\$3,220	\$0
7-Jun	\$598	\$420	-\$360	\$3,640	\$0
8-Jun	\$597	-\$220	-\$580	\$3,420	\$0
9-Jun	\$597	-\$80	-\$660	\$3,340	\$0
10-Jun	\$595	-\$260	-\$920	\$3,080	\$0
11-Jun	\$593	-\$420	-\$1,340	\$2,660	\$1,340
12-Jun	\$594	\$60	-\$1,280	\$4,060	\$0
13-Jun	\$592	-\$360	-\$1,640	\$3,700	\$0
14-Jun	\$593	\$180	-\$1,460	\$3,880	\$0
15-Jun	\$587	-\$1,140	-\$2,600	\$2,740	\$1,260
16-Jun	\$587	\$0	-\$2,600	\$4,000	\$0
17-Jun	\$588	\$220	-\$2,380	\$4,220	\$0
18-Jun	\$589	\$120	-\$2,260	\$4,340	\$0
19-Jun	\$591	\$460	-\$1,800	\$4,800	\$0
20-Jun	\$592	\$260	-\$1,540	\$5,060	\$0

June 11th: Because account falls below the maintenance, margin call (to "top up" to the initial margin) for \$1,340 = \$4,000 - \$2,660.
June 15th: Because account falls below the maintenance, margin call (to "top up" to the initial margin) for \$1,260 = \$4,000 - \$2,740.

IMPORTANT CONCEPT:

The maintenance margin is a trigger level—once triggered, the investor must “top up” to the *initial margin* rather than just to the *maintenance margin*.

Describe the role of a clearinghouse in Futures transactions

Clearinghouse acts as an intermediary in Futures transactions.
Guarantees performance of parties
Members must post funds with exchange
Main task to keep track of transactions, calculate net position of each member daily
The exchange clearinghouse is often a division of the exchange (e.g., the CME Clearing House is a division of the Chicago Mercantile Exchange) or an independent company. The clearinghouse serves as a guarantor, ensuring that the obligations of all trades are met.

Describe the role of collateralization in the over-the-counter market and compare it to the margining system

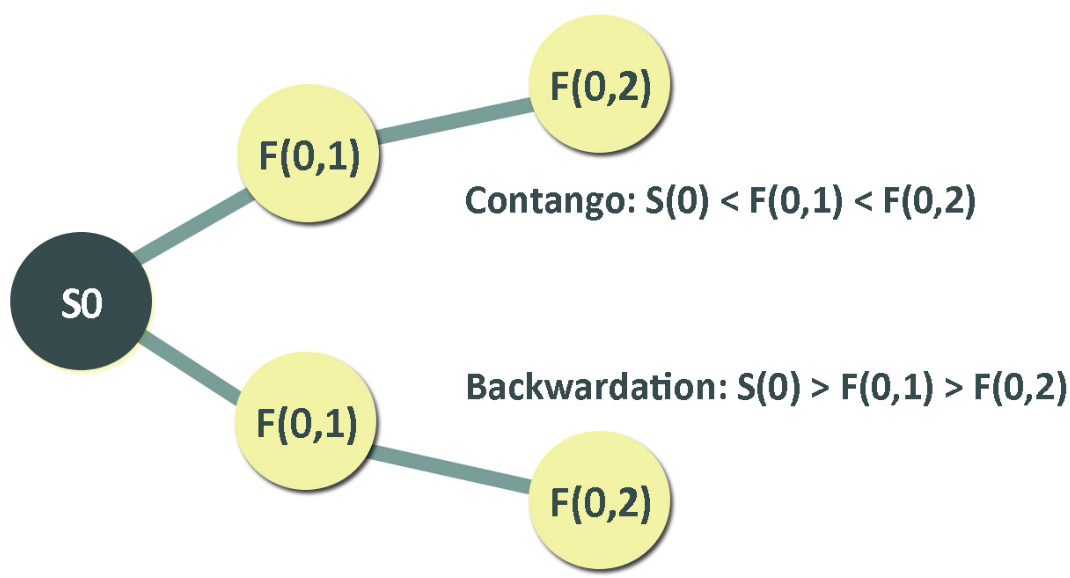
Over-the-counter (OTC) markets traditionally imply significant credit (counterparty) risk depending on the terms in the *credit support annex*.

Collateralization
Similar to margining system for exchanges
Value contract each day
OTC contract between Company A & Company B
If contract value to Company A increases, Company B pay cash equal to the increase
Interest paid on outstanding balances

"*Consider two participants in the over-the-counter market, company A and company B, with an outstanding over-the-counter contract. They could enter into a collateralization agreement where they value the contract each day. If from one day to the next the value of the contract to company A increases, company B is required to pay company A cash equal to this increase. Similarly, if the value of the contract to company A decreases, company A is required to pay company B cash equal to the decrease. Interest is paid on outstanding cash balances.*" –Hull

Identify and describe the differences between a normal and inverted Futures market

If the forward price is higher than the spot price (or the distant forward price is higher than the near forward price) the Futures curve is said to be normal, or in *Contango*.
If the forward price is less than the spot price (or the distant forward price is less than the near forward price), the Futures curve is said to be inverted, or in *Backwardation*



Describe the mechanics of the delivery process and contrast it with cash settlement

If a Futures contract is not closed out before maturity, it is usually settled by delivering the asset underlying the contract. When there are alternatives about what is delivered, where it is delivered, and when it is delivered, the party with the short position chooses.
Some contracts (e.g., those on stock indices and Eurodollar Futures) are settled in cash.

Define and demonstrate an understanding of the impact of different order types, including: market, limit, stop-loss, stop-limit, market-if-touched, discretionary, time-of-day, open, and fill-or-kill

Market order (guarantees the transaction, but not the price): The market order is a simple (the simplest) request to execute the trade immediately at the best available price.

Limit order (guarantees the price, but not the transaction): A limit order specifies a particular price. The order can be executed only at this price or at one more favorable to the investor. If the limit price is $30 for an investor wanting to buy, the order will be executed only at a price of $30: or less. There is no guarantee that the order will be executed at all, because the limit price may never be reached.

Stop-loss (if an asset reaches a specified price it, in effect, becomes a market order): The order is executed at the best available price once a bid or offer is made at that particular price or a less-favorable price. Suppose a stop-loss order to sell at $30 is issued when the market price is $35. It becomes an order to sell if and when the price falls to $30. The purpose of a stop order is usually to close out a position if unfavorable price movements take place. It limits the loss that can be incurred.

Stop-limit (combination of stop and limit: as soon as stop is breached, limit order applies): The order becomes a limit order as soon as a bid or offer is made at a price equal to or less favorable than the stop price. Two prices must be specified in a stop-limit order: the stop price and the limit price. If the stop price and the limit price are the same, the order is sometimes called a stop-and-limit order.
Suppose when the market price is $35, a stop-limit order to buy is issued with a stop price of $40 and a limit price of $41. As soon as there is a bid or offer at $40, the stop-limit becomes a limit order at $41.

Market-if-touched (a.k.a., board order): A market-if-touched (MIT)-order is executed at the best available price after a trade occurs at a specified price or at a price more favorable than the specified price. In effect, an MIT becomes a market order once the specified price has been hit.
Discretionary (a.k.a., market-not-held order): A market order except that execution may be delayed at the broker's discretion in an attempt to get a better price
Time-of-day: specifies a particular period of time during time
Open: in effect until executed or end of trading in contract
Fill-or-kill: immediately or not at all

Compare and contrast forward and Futures contracts

Key differences between a forward a Futures contract:

Forward vs. Futures Contracts	
Forward	Futures
Trade over-the-counter (OTC)	Trade on an exchange
Not standardized	Standardized contracts
One specified delivery date	Range of delivery dates
Settled at contract's end	Settled daily
Delivery or final cash settlement usually occurs	Contract usually closed out prior to maturity
Reduces basis risk due to tailored specifications but less liquid	High liquidity due to standardized specifications but more basis risk

Chapter Summary

The most important specifications of a Futures contract include: The asset, the contract size, the delivery arrangement, the delivery months, how prices are quoted as well as price and position limits.

As the Futures get closer and closer to the delivery period, the Futures price tends to converge to the Spot price. This must be the case, since Arbitrageurs would take advantage of this discrepancy by buying the Futures or Spot, depending on which is more expensive, and selling the opposite, depending on which is cheaper. As discussed in Chapter 1, Arbitrageurs make sure such opportunities are ephemeral.

The rationale for margin requirements is to avoid non-performance, or default on, e.g., a Futures contract. This is achieved by having the exchange acting as a financial intermediary. At inception there is an *initial* margin paid to the exchange as security. At the end of each trading day, the position is marked-to-market and the investor must provide additional funds if his position has lost money. In the event that the position falls below the *maintenance margin*, the investor must top-up the account to the *initial margin* (note: not just to the maintenance margin).

A clearinghouse functions as a financial intermediary through which members post margin, just as individual investors post margin with their brokers. Non-members must transact through members and post margin with them. The role of the clearinghouse is to reduce the credit risk.

In the OTC market there is no central clearinghouse function. Participants in the OTC market thus have traditionally relied on collateral. As mentioned in Chapter 1, regulation now requires certain OTC transactions to be cleared.

If the forward price is higher than the spot price the Futures curve is said to be *normal*, or in *Contango*. If the forward price is less than the spot price the Futures curve is said to be *inverted*, or in *Backwardation*.

Futures contracts are traded on exchanges and the counterparty to any deal may change at any time, whereas for a forward transaction one cannot close out the position by passing it on to a third party since it is between two defined counterparties and collateral is posted. For Futures, collateral is not needed due to the margining mechanism employed by the clearinghouses. However, Futures contract do not allow for the kind of customization that forward contracts do. Moreover whereas Futures contracts are often closed out prior to delivery, forward contracts usually do lead to delivery – and the delivery is typically specified to be a range of days, rather than a specific date, as for the Futures. Although some interest is paid on the margin account, this is typically much lower than the return a company could earn elsewhere, thus Futures also require a liquidity buffer of idle capital.

2 Questions & Answers

Questions

2.1 Which is a feature of a futures contract?

a) Range of delivery dates
b) Usually one specified delivery date
c) Significant counterparty risk
d) Settled at end of contract

2.2 The roll return is the return due to the change in the price of the futures contract. If the commodity forward curve is in contango, and the spot price will be constant over time, what is the roll return on a short position in a futures contact on the commodity?

a) Negative roll return (a.k.a., roll yield)
b) Approximately zero roll return
c) Positive roll return
d) Need more information

2.3. Assume a long position in a gold futures contract has the same terms as a long position in a gold forward contract; e.g., asset quality, quantity, size and delivery exactly the same. Should there be any theoretical difference in the price of the future and forward contract?

a) No, cost of carry is same
b) No, both lack a convenience yield
c) Yes, if gold has a storage cost
d) Yes, if interest rates vary unpredictably

2.4 Which of the following is TRUE about a normal/inverted futures market?

a) A futures market is either normal or inverted but cannot be a mixture
b) The roll return (roll yield) is profitable during an inverted futures market
c) A falling futures price necessarily implies backwardation
d) Gold must always be a normal market (assuming positive interest rates) because it has storage cost but does not pay a dividend

2.5 Yesterday, there were 1,000 open long positions in a futures contracts for a certain consumption commodity, and also 1,000 open short positions. Today, 100 contracts were physically delivered. What is today's open interest?

a) 800
b) 900
c) 1,800
d) 1,900

Answers

2.1 A. Range of delivery dates

2.2 C. Positive roll return
In contango, $F(T) > S(0)$, such that basis convergence implies $F(T)$ decreases as maturity decreases. A long futures position profits (loses) on the roll return in a backwardation (contango); a short futures position profits on the roll return in a contango.

2.3 D. Yes, if interest rates vary unpredictably
If interest rates are constant and flat, the prices should be the same. But if interest rates vary, the DAILY SETTLEMENT (mark to market) creates interim cash flow volatility with uncertain reinvestment risk: this impacts the futures position.

2.4 B. The roll return (roll yield) is profitable during an inverted (backwardation) futures market; i.e., the futures are rolled into higher prices as the futures price increases while maturity shortens.

In regard to (A), this is FALSE: commodities are often in alternating contango/backwardation; e.g., natural gas, corn due to seasonality in demand/production.

In regard to (C), this is tempting but FALSE: in the Metallgesellschaft, a backwardation experienced falling futures price yet went into contango because the SPOT price dropped more.

In regard to (D), this is FALSE for two reasons: 1. It omits the lease rate and convenience yield; and 2. It omits technical factors; supply/demand could conceivably create backwardation.

2.5. B. 900. An open contract has a long and a short, by definition. Yesterday, there was an open interest of 1,000 contracts. Today, it was reduced by the settlement of 100.

Hull, Chapter 3: Hedging Strategies Using Futures

Learning Outcomes:

Define and differentiate between short and long hedges and identify appropriate use.

Describe the arguments for and against hedging and the potential impact of hedging on firm profitability.

Define the basis and the various sources of basis risk, and explain how basis risks arise when hedging with Futures.

Define cross hedging, and compute and interpret the minimum variance hedge ratio and hedge effectiveness.

Define, compute and interpret the optimal number of Futures contracts needed to hedge an exposure, and explain and calculate the "tailing the hedge" adjustment.

Explain how to use stock index Futures contracts to change a stock portfolio's beta.

Describe what "rolling the hedge forward" means and describe some of the risks that arise from such a strategy.

Define and differentiate between short and long hedges and identify appropriate use.

Short hedge

A short forward or Futures hedge is an agreement to *sell in the future* and is appropriate when the hedger *already owns the asset*. The classic example is a farmer who wants to lock in a sales price for her crop of, e.g. corn, and therefore protect herself against a price decline. That is, by nature of owning the corn crop, she effectively has a long position in corn. We know that to offset that long position, she needs to enter into a corresponding short position, which is exactly what a short hedge accomplishes. Her long position in corn + a short position in the forward cancels out her exposure: her price has been locked in today.

Long Hedge

A long forward (or Futures) hedge is an agreement to *buy in the future* and is appropriate when the hedger does *not currently own the asset* but expects to purchase in the future. An example is an airline, which depends on jet fuel and enters into a forward or Futures contract (a long hedge) in order to protect itself from exposure to high oil prices.

A long forward (or Futures) hedge is an agreement to buy in the future	A short forward (or Futures) hedge is an agreement to sell in the future
Hedger does not currently own the asset. Expects to purchase in the future.	Hedger already owns the asset.
An airline depends on jet fuel. Enters into Futures contract (a long hedge) to protect from exposure to rising oil prices.	Farmer wants to lock in a sales price to protect against a price decline.

Describe the arguments for and against hedging and the potential impact of hedging on firm profitability

In favor of hedging:
Hedging is a way of reducing risk. Rather than focus on market forces, over which a firm has *no control*, and financial instruments, over which they have *little knowledge*, companies should focus on the main business they are in; where they do have specialized knowledge. Accordingly, the firm should take steps to minimize risks arising from market variables such as interest rates, exchange rates and other prices.

Against hedging:
In theory, there is no reason for the firm to try to minimize risk by hedging since shareholders can make their portfolios well diversified and can make their own hedging decisions. In practice though, this can be questioned due to frictions and transaction costs associated with owning *the market portfolio* as explored in Topic 1. It is nevertheless a common argument against hedging.

Another reason against hedging is the fact that it may increase risk to hedge when competitors do not. For example, if you are an airline and you hedge your exposure to jet fuel while your competitors do not and the price of jet fuel drops, you are stuck with "overpaying" for the jet fuel. This argument can however, be turned on its head. In other words, one might gain or lose on any hedge. Hedging, while reducing one type of risk, may add exposure to another risk.

In some industries margins stay roughly the same because the cost of inputs can be passed on directly to consumers in the form of higher prices. In that case one can easily end up in a situation where your margins get squeezed if you entered into a hedge and prices move against you. Conversely, you may gain if price move in your favor. However, in this case the hedging may be seen more as price speculation than actual hedging.

Finally, the person responsible to the owners may be reluctant to enter into a hedge in case prices do not move in their favor. It is likely that questions will be asked as to why the company entered into the hedge when there is a loss. However, if there is a gain, this might not be recognized. This informational asymmetry will tend to lead to principal-agent problems, whereby management may choose the safer option of not hedging, even though it may actually be optimal for the firm to hedge.

Define the basis and the various sources of basis risk, and explain how basis risks arise when hedging with Futures.

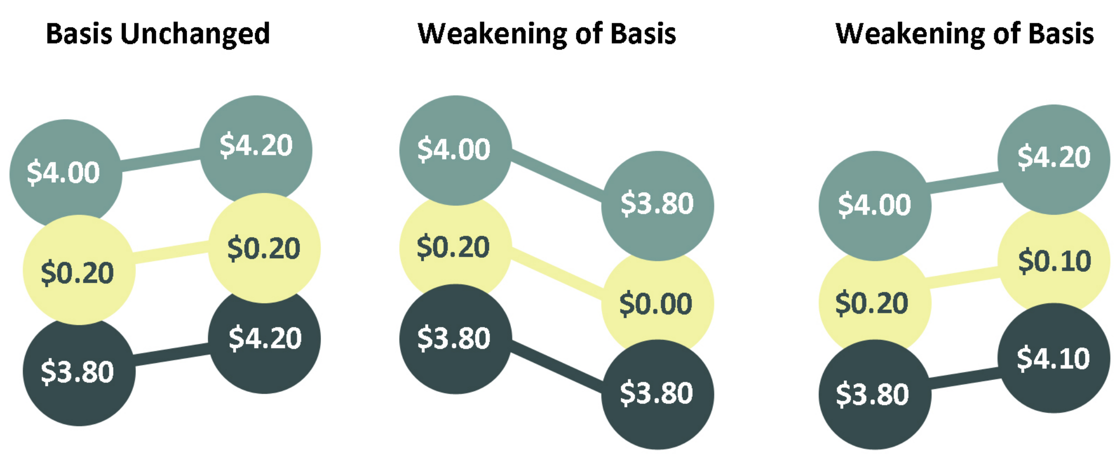
Define and compute the basis

Remember that the basis itself converges to zero over time, as the spot price converges toward the Futures price[2]. We can represent the basis as so,

$$Basis = Spot\ price_{Hedged\ Asset} - Futures\ price_{Futures\ contract} = S_0 - F_0.$$

Financial commodities often express basis risk in the reverse: Future price – Spot Price. The direction of your subtraction is not critical: *the basis is the difference in price*.

Green represents the spot price (today and subsequent). Blue is the Futures/forward price (today and subsequent). Basis (yellow) is the difference between spot and Futures price.



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[2] Refer back to page 18 and the *zone of convergence* for a refresher.

Consider a company that, in May 2013, will need to purchase 25,000 pounds of copper in four months' time (September 2013). On May 2013, the spot and September 2013 Futures prices are, $1.90 and $2.00, respectively. Three scenarios are illustrated, but in all scenarios the basis converges to zero.

Company will buy:		25,000	Lbs. of copper in Sep-13	
Contract (pounds)		25,000		
Number of contracts		1		

		Forward in Time: Basis converges		
	May-13	Sep-13	Sep-13	Sep-13
Spot (S)	$1.90	$1.95	$2.00	$2.05
Futures (F)	$2.00	$1.95	$2.00	$2.05
Basis (S-F)	($0.10)	$0.00	$0.00	$0.00
Un-hedged Cost				
Cost		($48,750)	($50,000)	($51,250)
Long Hedge				
Futures gain, per lbs.		($0.05)	$0.00	$0.05
Total Futures Gain		($1,250)	$0	$1,250
Net Cost		($50,000)	($50,000)	($50,000)

Now consider two scenarios in which the basis does not converge. These scenarios illustrate how the intended hedge, via *unexpected* basis weakening or strengthening, can contribute to a profit or loss.

Company will buy:		25,000	Lbs. of copper
Contract (pounds)		25,000	
Number of contracts		1	

		Basis Weakens	Basis Strengthens
	May-13	Sep-13	Sep-13
Spot	$1.90	$2.00	$2.00
Futures	$2.00	$2.05	$1.95
Basis	($0.10)	($0.05)	$0.05
Unhedged Cost			
Cost		($50,000)	($50,000)
Long Hedge			
Futures gain, per lb.		$0.05	($0.05)
Total Futures Gain		$1,250	($1,250)
Net Cost		($48,750)	($51,250)

Define the various sources of basis risk and explain how basis risks arise when hedging with Futures

When the spot price increases by more than the Futures price, the basis increases and this is said to be a, “strengthening of the basis.” When **unexpected**, this strengthening is favorable for a short hedge and unfavorable for a long hedge. In our example of the farmer, we can see how this is favorable: the farmer can close out the Futures contract and sell in the spot market. She will lose on the Futures contract since it has risen however, since the spot price has risen even more the difference, the basis, is her gain[3]. When the Futures price increases by more than the spot price, the basis declines and this is said to be a, “weakening of the basis.” When **unexpected**, this weakening is favorable for a long hedge and unfavorable for a short hedge. Going back to our example of the farmer who has a short hedge, we can see that an unexpected weakening of the basis is unfavorable: had the farmer for example waited until today to enter into the hedge, she could have locked in a higher price.

Basis risk arises when hedging with Futures

Basis risk arises because the characteristics of the Futures contract often differ from the underlying position. In particular, a scenario where Contract ≠ Commodity: the asset to be hedged is not exactly the same as the asset underlying the Futures contract.
Contract is standardized (e.g., WTI oil Futures)
Commodities are not exactly the same (they have different qualities or grades)
Timing (uncertainty vis-a-vis asset). The hedger may be uncertain as to the exact date when the asset will be bought or sold.
Timing (uncertainty vis-a-vis Futures contract). The hedger may require the Futures contract to be closed out before its delivery month.
Basis risk may be sub-classified in various ways. For example, changes in the cost of carry model, over time, may give rise to basis risk. But generally, any particular type of basis risk reduces to one key fact: the asset being hedged is typically not identical, in all respects, to the commodity underlying the Futures contract.
There is an inherent trade-off between liquidity and basis risk: to reduce basis risk is to require a tailored hedge.

[3] It is here assumed that she has the corn available at the time, e.g. stored in a silo.

Define cross hedging, and compute and interpret the minimum variance hedge ratio and hedge effectiveness

Define cross hedging

A cross hedge is when the asset underlying the hedge is different from the asset being hedged. For example, an airline may hedge the cost of jet fuel with Futures contracts. Cross-hedges are necessary because Futures are standardized contracts for commodities. The classic cross-hedge example alluded to above is that of an airline hedging its jet fuel costs. Jet fuel Futures are indeed traded on the CME, however, the open interest and volume is so limited, to the extent that airlines resort to cross-hedging by using highly correlated heating oil Futures instead.

Define, compute and interpret the minimum variance hedge ratio and hedge effectiveness

If the spot and future positions are perfectly correlated, then a 1:1 *hedge ratio* results in a *perfect* hedge. However, this is rarely the case. Indeed, with cross-hedging the *optimal* hedge ratio need not equal 1.0. Rather the *optimal* hedge ratio is the ratio that minimizes the variance of our hedge. This is the minimum variance hedge ratio (MVR), and is given by:

$$h^* = \rho_{S,F}\frac{\sigma_s}{\sigma_F}, (note\ the\ equivalence\ to\ \frac{\sigma_{S,F}}{\sigma_F^2} = \beta_{w.r.t.\ Futures})$$

where σ_s is the standard deviation of the changes in the price of the asset we are hedging, σ_F is the standard deviation of the changes in the Futures, and $\rho_{S,F}$ is the coefficient of correlation between the two price changes.

Note that the MVR is equal to 1.0 when $\rho_{S,F}$ = 0.5 and σ_s = 2σ_F, for example. As a corollary, while a *perfect hedge* implies a correlation equal to 1, in our example the correlation $\rho_{S,F} \neq 1.0$, yet the hedge is still *optimal* because it is the *minimum variance ratio*. There is a subtle difference between a *perfect hedge* that implies a correlation of 1, and an *optimal hedge,* which is the MVR and is the optimality criteria that the hedge typically wants to satisfy.

IMPORTANT CONCEPT

The minimum variance ratio is the slope of the regression line, and is the optimal hedge ratio. Note that the correlation can take on a range of different values and does not need to equal 1 in order for the hedge to be optimal.

The hedge effectiveness, that is, just how good our hedge is, can be measured by a regression of the change in spot prices against the change in Futures prices, typically using historical, non-overlapping data[4]. The resulting $R^2 = \sigma_{S,F}$ tells us how good our hedge is.

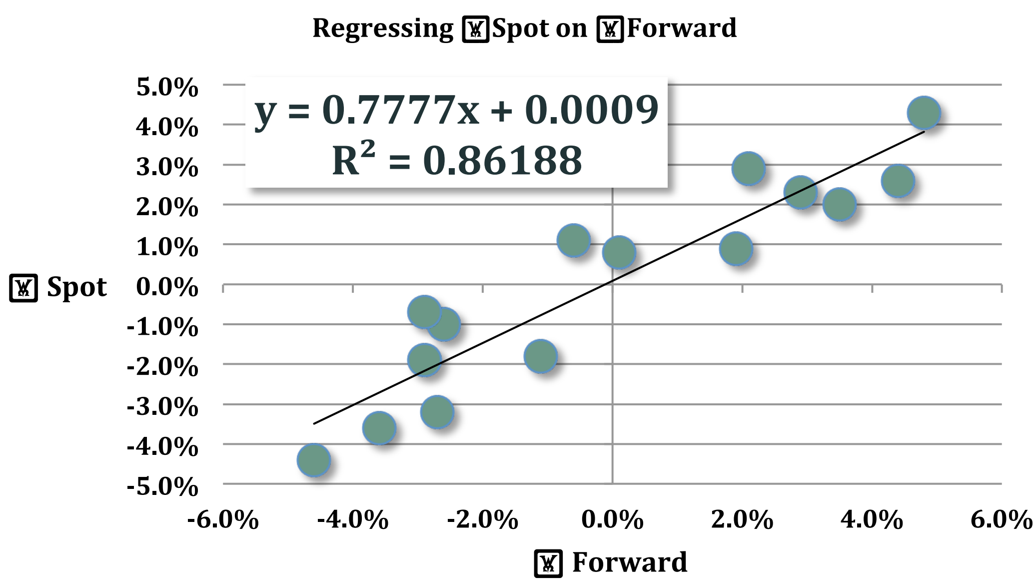
For example:
If the volatility of the spot price is 20%, the volatility of the Futures price is 10%, and their correlation is 0.4, then the optimal hedge ratio, h*, is given by:

$$h^* = (0.4)\frac{20\%}{10\%} = 0.8$$

And the number of Futures contracts is given by N* when N_A is the size of the position being hedged and QF is the size of one Futures contract:

$$N^* = \frac{h^* N_A}{Q_F}$$

Hull's Example: Airline cross-hedges the future purchase of jet fuel with heating oil Futures contracts
The historical change in spot price (jet fuel) is regressed against the change in Futures price (heating oil Futures). Note: the slope of the regression line equals the optimal hedge ratio.



[4] Current FASB 133 accounting rules regarding Cash-Flow hedging requires that you regress 36 months of price changes of your hedging instrument against the asset you wish to hedge, and that the beta-coefficient be between 0.85 and 1.2. You may choose the time interval (days, weeks, months) however, once chosen it cannot be changed. This hedge effectiveness testing must be done both on a retrospective (historical) and prospective (future expected) basis every month.

	(heating oil Futures)	(jet fuel spot)
Standard Dev	$0.0313	$0.0263
Correlation		0.928
(MV) Hedge ratio (h*)		0.7777
Airline will purchase		2,000,000
NYMEX oil Futures (gallons)		42,000
Number of contracts (N*)		37.01

$$h^* = \rho \frac{\sigma_S}{\sigma_F}$$

$$N^* = \frac{h^* Q_A}{Q_F}$$

Another Example: Spot volatility = 2.83, Futures volatility = 3.38, Correlation = 0.814. Airline plans to purchase 1 million gallons.

	Futures	Spot
Standard Deviation	$3.38	$2.83
Correlation		0.814
Minimum Variance Hedge ratio		**0.68**
Airline will purchase		1,000,000
NYMEX oil Futures (gallons)		42,000
Number of contracts		**16.25**
Scenario where jet fuel increases by $1/gallon (testing the hedge):		
Spot price of jet fuel		$1.00
Futures price gain		$1.47
Increase fuel cost		$1,000,000
Gain on Futures contracts		$1,000,000

Define, compute and interpret the optimal number of Futures contracts needed to hedge an exposure, and explain and calculate the "tailing the hedge" adjustment.

When Futures are used, a small adjustment, known as "*tailing the hedge*" can be made to allow for the impact of daily settlement. The only difference here is to replace the units with values. Instead of using quantities, as in:

$$N^* = \frac{h^* Q_A}{Q_F}$$

We use the dollar value of the position being hedged and the dollar value of one Futures contract, as in:

$$N^* = \frac{h^* V_A}{V_F}$$

The effect is to multiply the original ratio by the ratio of [spot price/Futures price].

Explain how to use stock index Futures contracts to change a stock portfolio's beta

Given a portfolio beta (β), the current value of the portfolio (P), and the value of stocks underlying one Futures contract (A), the number of stock index Futures contracts (i.e., which minimizes the portfolio variance) is given by:

$$N^* = \beta \frac{P}{A}$$

By extension, when the goal is to shift portfolio beta from (β) to a target beta (β*), the number of contracts required is given by:

$$N = (\beta^* - \beta)\frac{P}{A}$$

For example:
$10 million equity portfolio has a beta of 1.2. S&P 500 Index value is 1500 (one Futures contract is for delivery of $250 multiplied by the index). The optimal number of contracts is given by:

$$N^* = \beta \frac{P}{A} = (1.2)\frac{\$10 \text{ million}}{(1500)(250)} = 32$$

The hedge trade is short 32 Futures contracts. The above essentially changes the beta to zero. Now assume that, instead of hedging the net beta to zero, we want to change the beta of the portfolio to 2.0:

$$N = (\beta^* - \beta)\frac{P}{A} = (2.0 - 1.2)\frac{\$10 \text{ million}}{(1500)(250)} = 21.33$$

The hedge trade here is to enter into a long position on 21.33 Futures contracts. Note we could have used (beta minus target beta) in which case the result would be negative (-) 21.33. But in either case, we must buy (go long) Futures contracts because we are increasing the beta. If we are reducing the beta, then we short Futures.
Final example:
Assume our $10 million portfolio has a beta of 1.5, but we want to reduce the beta to 1.2:
Value of portfolio is $10 million
S&P 500 Futures Price = 1240
Portfolio beta (β) is 1.5
Contract = $250 × Index
Target beta = 1.2

$$N = (\beta^* - \beta)\frac{P}{A}$$

$$= (1.2 - 1.5)\frac{\$10{,}000{,}000}{(1240)(250)} \cong -97$$

We short ~ 97 contracts.
(-) = short
(+) = long

Describe what "rolling the hedge forward" means and describe some of the risks that arise from such a strategy

When the delivery date of the Futures contract occurs prior to the expiration date of the hedge, or there is little liquidity but in the spot and prompt month, the hedger can *roll forward* the hedge: close out a Futures contract and take the same position on a new Futures contract with a later delivery date. This is also known as a *stack and roll* strategy.

Risk arising from a stack and roll strategy

Rolling the hedge forward exposes the company, or hedger to:
Basis risk (original hedge)
Basis risk (each new hedge) = also called "rollover basis risk." That is, if the price of the asset we are long declines, such that there are margin calls, or at least cash-outflows in the near-term the firm might experience a liquidity squeeze. If the firm has ample liquidity, this is not a problem, however, with insufficient liquidity this can cause major problems. This is largely due to unfortunate timing: in the short-run we have cash outflows due to the loss on the Futures contract, however, since the firm employs the stack and roll strategy every month, it can readily expect to buy Futures, and thus hedge its cost, the following months at a lower price.

Chapter Summary

A short hedge is an agreement to *sell in the future* and is appropriate when the hedger *already owns the asset*. A long hedge is an agreement to *buy in the future* and is appropriate when the hedger does *not currently own the asset* but expects to purchase in the future.

Hedging is a way of reducing risk. Rather than focus on market forces, over which a firm has *no control*, and financial instruments, over which they have *little knowledge*, companies should focus on the main business they are in; where they do have specialized knowledge. In theory, there is no reason for the firm to try to minimize risk by hedging since shareholders can make their portfolios well diversified and can make their own hedging decisions. In practice though, this can be questioned due to frictions and transaction costs associated with owning *the market portfolio.*

$Basis = Spot\ price_{Hedged\ Asset} - Futures\ price_{Futures\ contract} = S_0 - F_0$. Hull states that, "basis arises from uncertainty as to what the basis will be at maturity of the hedge." This is not entirely satisfactory however, as it begs the question as to what drives this uncertainty in the first place. A more precise answer is that basis risk reduces to one key fact: the asset being hedged is typically not identical to the commodity underlying the Futures contract. There is an inherent trade-off between liquidity and basis risk: to reduce basis risk is to require a tailored hedge, which is why basis risk is not an issue when hedging using forwards.

A cross hedge is when the asset underlying the hedge is different from the asset being hedged. The *hedge ratio* is the Futures position taken over the total exposure. A *correlation* of 1 implies a *perfect hedge*. However, the criteria for optimality may be to minimize the variance of the hedge, in which case the *optimal hedge* is the beta of a regression of price changes of the spot against price changes in the Futures price. This is the *minimum variance hedge ratio*. The effectiveness of the hedge is measured by the aforementioned regression's R^2.

When Futures are used, a small adjustment, known as "*tailing the hedge*" can be made to allow for the impact of daily settlement. The only difference here is to replace the units with values. Instead of using quantities, we use the dollar value of the position being hedged and the dollar value of one Futures contract. The effect is to multiply the original ratio by the ratio of spot price/Futures price.

Stock index Futures may be used to change a stock portfolio's beta by going long or short stock index Futures depending on whether one wants to reduce or increase the portfolio's beta.

When the delivery date of the Futures contract occurs prior to the expiration date of the hedge, or there is little liquidity but in the spot and prompt month, the hedger can *roll forward* the hedge: close out a Futures contract and take the same position on a new Futures contract with a later delivery date.

3 Questions & Answers

Questions

3.1 In theory, futures contract can hedge commodity price risk. According to Hull, however, EACH of the following is a valid reason for a company to AVOID such a commodity price hedge EXCEPT for:

a) Hedge may be unnecessary if shareholders are well diversified
b) Hedge may be counterproductive if hedging is not the norm and commodity prices changes ripple from raw material to wholesale and retail
c) Treasurer may incur career risk if senior management (and Board of Directors) does not under fully understand the nature of the hedge
d) Hedge may be inappropriate if shareholders are focused on gross margins and the cost of hedging would reduce margins to an unacceptable level

3.2 EACH of the following is a source of basis risk EXCEPT:

a) The asset whose price is to be hedged may not be exactly the same as the asset underlying the futures contract.
b) The hedger cannot be 100% certain that the counterparty in the forward contract will meet their obligation (even the exchange-traded futures contract has a similar but smaller risk)
c) The hedger may be uncertain as to the exact date when the asset will be bought or sold.
d) The hedge may require the futures contract to be closed out before its delivery month.

3.3. Assume the volatility (standard deviation) of the change in prices for the spot price of oil and the futures price, respectively, are 20% and 32%. The (coefficient of) correlation between changes in the two prices is 0.80. What is the variance of the basis?

a) 0.04
b) 0.08
c) 0.12
d) 0.16

3.4 A wheat farmer hedged her future sale of 100,000 bushels of wheat by selling forward 10 contracts (each for 5,000 bushels). The standard deviation of monthly changes in the spot and futures price of wheat is, respectively, $0.60 and $0.90. What was her correlation assumption?

a) 0.67
b) 0.75
c) 0.80
d) 0.90

Answers

3.1 D.
Commodity futures are considered to have very low transaction costs; and unlike options, no up-front premium is incurred.
In regard to (A), (B) and (C), these are the three practical reasons Hull gives for the tendency of many companies to avoid hedges.
3.2. B. This is counterparty credit risk, not basis risk. Basis risk is a market (price differential) risk.

3.3. A. 0.04
Since basis = S-F, variance of basis = Variance(S-F) = variance(S) + variance(F) - 2*volatility(S)*volatility(F)*correlation(S,F);
Same as: Variance(S-F) = variance(S) + variance(F) - 2*covariance(S,F).
In this case, variance of basis = 20%^2 + 32%^2 - 2*20%*32%*0.8 = 0.04.

3.4 B. 0.75
The optimal hedge ratio = correlation * spot standard deviation / futures standard deviation.
The optimal # of contracts = optimal hedge ratio * quantity being hedged / quantity of contract.
Correlation = (optimal # of contracts * quantity of contract / quantity being hedged) * futures standard deviation / spot standard deviation.
In this case, correlation = (10 * 5,000 / 100,000) * 0.90 / 0.60 = 0.75.

Hull, Chapter 4: Interest Rates

Learning Outcomes:

Describe Treasury Rates, LIBOR, Repo Rates, and what is meant by the risk-free rate.

Calculate the value of an investment using daily, weekly, monthly, quarterly, semiannual, annual, and continuous compounding. Convert rates based on different compounding frequencies.

Calculate the theoretical price of a coupon-paying bond using spot rates.

Calculate forward interest rates from a set of spot rates.

Calculate the value of the cash flows from a forward rate agreement (FRA).

Describe the limitations of duration and how convexity addresses some of them.

Calculate the change in a bond's price given duration, convexity, and a change in interest rates.

Describe the major theories of the term structure of interest rates.

Describe Treasury Rates, LIBOR, Repo Rates, and what is meant by the risk-free rate.

Treasury rates

Treasury rates are the rates an investor earns on Treasury bills and Treasury bonds; i.e., the instruments used by a government to borrow in its own currency. For example, Japanese Treasury rates are the rates at which the Japanese government borrows in yen; US Treasury rates are the rates at which the US government borrows in US dollars. It is often, but not always, assumed that there is virtually no chance that a government will default on an obligation denominated in its own currency. Treasury rates are therefore totally risk-free rates in the sense that an investor who buys a Treasury bill or Treasury bond is certain that interest and principal payments will be made as promised.

LIBOR (London Interbank Offered Rate)

A LIBOR quote by a particular bank is the rate of interest at which the bank is prepared to make a large wholesale deposit with other banks. Large banks and other financial institutions quote LIBOR in all major currencies for maturities up to 12 months: l-month LIBOR is the rate at which 1-month deposits are offered, 3-month LIBOR is the rate at which 3-month deposits are offered, and so on.

Repo rates

Sometimes, trading activities are funded with a repo or repurchase agreement: a contract where a dealer (who owns securities) agrees to sell them to another company now and buy them back later at a slightly higher price. The other company is lending a collateralized loan. The difference between selling price (today) and the repurchased price (tomorrow or later) is called the repo rate. If structured carefully, the loan involves very little credit risk. If the borrower does not honor the agreement, the lending company simply keeps the securities. The most common type of repo is an overnight repo, in which the agreement is renegotiated each day. However, longer-term arrangements, known as term repos, are sometimes used.

Risk-Free Rate

Derivative traders have typically used LIBOR rates as short-term risk-free rates. For a AA-rated financial institution, LIBOR is the short-term opportunity cost of capital. Traders argue that Treasury rates are too low to be used as risk-free rates because:
Market demand: financial institutions to fulfill a variety of regulatory requirements must purchase Treasury bills/bonds. This increases demand for these Treasury instruments driving the price up and the yield down.

Regulatory relief: The amount of (regulatory) capital required to support an investment in Treasury bills/bonds is substantially smaller than the capital required to support a similar investment in other instruments

Tax treatment: In the United States, Treasury instruments are given a favorable tax treatment because they are not taxed at the state level.

Due to the global financial crisis (GFC), many dealers have switched to using the *overnight indexed swap*[5] (OIS) rate as a proxy for the risk-free rate.

[5] We discuss the OIS rate in more detail in Chapter 7 on Swaps.

Calculate the value of an investment using daily, weekly, monthly, quarterly, semiannual, annual, and continuous compounding. Convert rates based on different compounding frequencies.

Calculate the value of an investment using daily, weekly, monthly, quarterly, semi-annual, annual, and continuous compounding.

Assuming:
R c rate of interest with continuous compounding
R m rate of interest with discrete compounding (m per annum)
n is the number of years

$$Ae^{R_c n} = A\left(1+\frac{R_m}{m}\right)^{mn}$$

$$e^{R_c} = \left(1+\frac{R_m}{m}\right)^{m}$$

$$R_m = m\left(e^{R_c/m}-1\right)$$

$$R_c = m\cdot\ln\left(1+\frac{R_m}{m}\right)$$

Convert rates based on different compounding frequencies

The present value is discretely discounted at (m) periods per year (e.g., m=2 for semi-annual compounding) over n years by using the formula on the left. The continuous equivalent is the right. Note that if the future value is one dollar (FV = $1), then the PV is the discount factor (DF).

Discrete	Continuous
$PV = \frac{FV}{\left(1+\frac{r}{m}\right)^{m\cdot n}}$	$PV = FV\cdot e^{-r\cdot n}$
$PV = \frac{\$1}{\left(1+\frac{r}{m}\right)^{m\cdot n}}$	$PV = \$1\cdot e^{-r\cdot n}$

Discount Factor (DF), 10 years @ 8% semi-annual	Discount Factor, 10 years @ 8% continuous
$DF(PV) = \dfrac{\$1}{\left(1+\dfrac{8\%}{2}\right)^{2\cdot 10}}$ $DF = 0.4564$	$DF(PV) = \$1 \cdot e^{-8\% \cdot 10}$ $DF = 0.4493$

We also must be able to convert from a discrete rate into a continuous rate, and vice-versa:

Semi-annual equivalent of 8% continuous:	Continuous equivalent of 8.162% semi-annual:
$R_m = m(e^{R_c/m} - 1)$ $R_m = 2(e^{8\%/2} - 1)$ $= 8.162\%$	$R_c = m \ln\left(1 + \dfrac{R_m}{m}\right)$ $R_c = 2\ln\left(1+\dfrac{8.162\%}{2}\right)$ $= 8\%$

Some selected conversions from the learning spreadsheet:

Continuous rate	Years	PV of \$1 grows to FV of:	PV of \$1 received in the future:	Discrete Periods/Yr	Discrete Rate	Equivalent Continuous	Discount Factor (discrete)
1 9.758%	1	\$1.10	\$0.907	2	10.00%	9.758%	0.9070
2 8.000%	1	\$1.08	\$0.923	4	8.081%	8.000%	0.9231
3 8.000%	3	\$1.27	\$0.787	4	8.08%	8.000%	0.7866
4 9.000%	5	\$1.57	\$0.638	4	9.10%	9.000%	0.6376
5 9.998%	1	\$1.11	\$0.905	252	10.00%	9.998%	0.9049
6 11.000%	1	\$1.12	\$0.896	12	11.051%	11.000%	0.8958

We advise that you practice these conversions: fluency in this regard is a fundamental skill that you can use often. Referring to the table above, for example:

What is a semi-annual rate of 10.00% converted into its continuous equivalent?
Answer: $\ln(1.05) * 2$ = 9.758%

What is continuous rate of 11.00% converted into its monthly equivalent?
Answer: $[e^{0.11/12} - 1] * 12 = 11.0506\%$

What is a quarterly rate of 8.00% converted into its bond-equivalent (semi-annual) rate?
Answer: we can take the long way and find the continuous equivalent, which is equal to

$\ln(1.02) * 4$ = 7.92105%. Then convert that to the semi-annual rate, which is equal to $[e^{7.92105\%/2} - 1] * 2$ = 8.080%

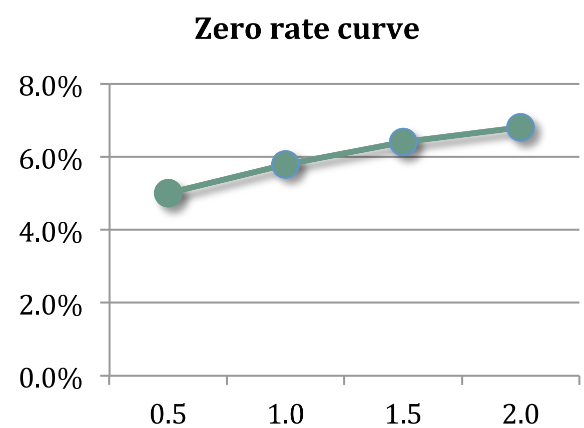
Calculate the theoretical price of a coupon-paying bond using spot rates

To calculate the price of a coupon-paying bond, each cash flow is discounted by the appropriate discount factor (or, equivalently, by using the corresponding spot rate). For example, given the zero rate curve below, the one-year zero rate is 5.8%. Under continuous compounding, the present value (PV) of the coupon cash flow of \$3.00 (i.e., a semi-annual installment on a 6% coupon, where the bond has face value of \$100) is \$3 * $e^{-5.8\%*1}$ = \$2.83.

Each cash flow is discounted by its respective zero rate. The theoretical model price is the sum of the present values (PVs) of the cash flows.

Hull Table 4.2

Face	\$100		
Coupon	6%		
Maturity	Zero Rate (CC)	FV CF	PV CF
0.5	5.0%	\$3.00	\$2.93
1.0	5.8%	\$3.00	\$2.83
1.5	6.4%	\$3.00	\$2.73
2.0	6.8%	\$103.00	\$89.90
			\$98.39



Calculate forward interest rates from a set of spot rates

Hull assumes a continuous compounding/discount frequency. Given the zero rate curve below, we solve for the implied forward rates. For example, the one-year implied forward rate = $\frac{[4\% * 2 - 3\% * 1]}{[2-1]}$ = 5%.

Year (n)	Zero rates (inputs)	Forward (solve for)
1	3.0%	
2	4.0%	5.0%
3	4.6%	5.8%
4	5.0%	6.2%
5	5.3%	6.5%

$$R_F = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

$$6.2\% = \frac{5.0\% \cdot 4 - 4.6\% \cdot 3}{(4-3)}$$

In the following exhibit (from a key learning spreadsheet), the forward rates are extracted from the spot rate curve:

Par	\$100.00					
Coupon	6.00%					
Yield to maturity (YTM)	2.72%					
Years to Maturity	0.5	1.0	1.5	2.0	2.5	
Cash flows	\$3.0	\$3.0	\$3.0	\$3.0	\$103.0	
Spot rates	1.50%	2.00%	2.25%	2.50%	2.75%	
Continuous Compounding (Hull)						
Discount function	0.993	0.980	0.967			
Six-month forward rate	1.500%	2.500%	2.750%	3.250%	3.750%	
Discounted (spot)	\$2.98	\$2.94	\$2.90	\$2.85	\$96.16	\$107.83
Discounted (function)	\$2.98	\$2.94	\$2.90	\$2.85	\$96.16	\$107.83
Discounted (forward)	\$2.98	\$2.94	\$2.90	\$2.85	\$96.16	\$107.83

It is good practice to extract these forward rates. For example, given the zero (spot) rate curve above, what is the six-month continuous forward rate starting in 1.5 years, $F_{1.5,2}$?

$$3.25\% = \frac{2.5\% \cdot 2 - 2.25\% \cdot 1.5}{(2 - 1.5)}$$

Calculate the value of the cash flows from a forward rate agreement (FRA).

A forward rate agreement (FRA) is an agreement that a certain rate will apply to a certain principal during a certain future time period. An FRA is equivalent to an agreement where interest at a predetermined rate, R_k is exchanged for interest at the market rate. An FRA can be valued by assuming that the forward interest rate is certain to be realized.

The value of a forward rate agreement (FRA) where a fixed rate, R_k, will be received on a principal (L) between times $T_1$and T_2 is given by:

$$V_{FRA,\text{fixed rate received}} = L(R_K - R_F)(T_2 - T_1)e^{-R_2 T_2}$$

The value of FRA where a fixed rate is paid is

$$V_{FRA,\text{fixed rate paid}} = L(R_F - R_K)(T_2 - T_1)e^{-R_2 T_2}$$

R_F is the forward rate for the period and R_2 is the zero rate for maturity T_2.
Hull departs here from continuous compounding and assumes (per practice) compound frequency equal to period; e.g., quarterly where $T_2 - T_1$= 3 months or 0.25 years.

For example, a company enters a 36 v 39 FRA to receive 4% ("sell FRA") on $100 MM principal for a three-month period, 3 years forward (Hull's example 4.3). The other interpretation is that the company will receive the fixed rate (4%) and pay LIBOR. If LIBOR is 4.5% in 3 years, company ends up paying. The counterparty is the buyer and receives the payment. That is, 100MM x (4% - 4.5%) x 3/12 = -$125,000, which is the future value at time t = 3.25 years. Discounting this to the present value at year 3, we get $\frac{-125{,}000}{(1+\frac{4.5\%}{4})}$ = -123,609.

FRA Principal	$100,000,000
Rate	4.00%

Year (n)	3 Mo. LIBOR		
1.00			
2.00			
3.00	4.5%	-$123,609	PV @ 3.0 years
3.25		-$125,000	FV @ 3.25 years

FRA Notation

There are two notations used for FRAs. Consider an FRA entered into on Jan 1st. The FRA will expire in six months, at which time ACME Corp. will pay the 6-month LIBOR interest rate and receive a fixed rate of 5% (the annualized rate, but only six months' worth of interest will be collected).

The FRA is entered on Jan 1st; the FRA expires on June 30th and the net payment is determined at that time based on the LIBOR rate at that time. If for example, LIBOR happens to be 5%, there is no net settlement. If the 6-month LIBOR, in six months' time, happens to be 4%, then ACME Corp. will receive a payment as follows:
Receive-fixed: 5% x ½ Year x Notional Principal ($)
Pay-floating:(-) 4% x ½ Year x Notional Principal ($)
= +1% x ½ Year x Notional Principal ($)

The first notation method to describe this swap is given by:
6 × 12 5%
[Term to Expire, Months] × [Term to End of Period Covered by FRA] [Fixed Rate]

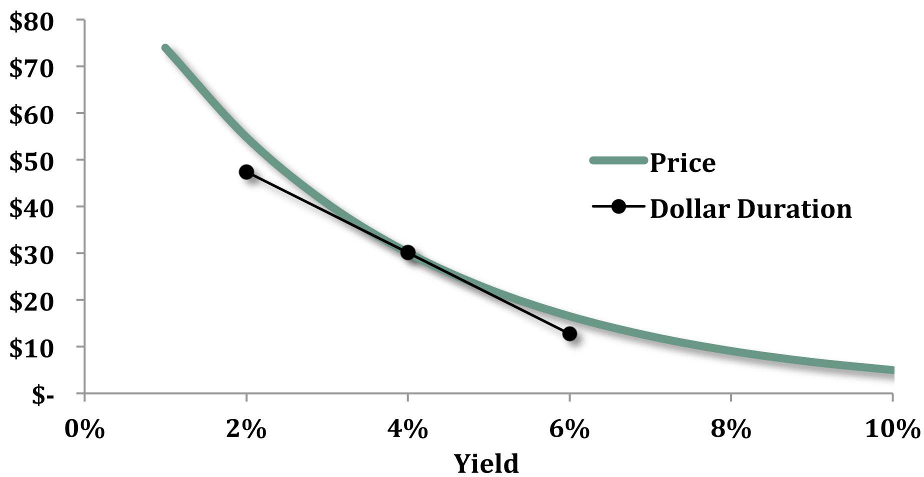
The second notation method to describe this (same) swap:

$FRA_{6,12}$ = 5%

Describe the limitations of duration and how convexity addresses some of them

Limitations of duration

By hedging a portfolio to achieve a net duration of zero, exposure is eliminated only with respect to *small parallel shifts* in the yield curve.



Thus although we have made an effort to hedge our position, we are still exposed to shifts that are either large in magnitude, non-parallel or both.

How convexity can help alleviate the problem

Convexity, as a function of the second derivative, adjusts for some but not all of the "gap" between duration and the actual price change. It does however; help address the issue of non-parallel shifts and larger changes by more effectively hedging our position against non-linearity.

It is important to note that even with the convexity adjustment, this remains a single-factor model, i.e., the yield to maturity is *the* single factor, with limitations:

- Duration is a first-order linear approximation
- Duration is only accurate for small, parallel shifts in the yield curve (i.e., unrealistic)
- Convexity adds a term to adjust for the curvature in the price/yield curve
- Convexity is still imprecise
- Both utilize the *Taylor Series approximation*: duration is a function of the first term and convexity is a function of the second term.

Calculate the change in a bond's price given duration, convexity, and a change in interest rates

Hull's duration is *Macaulay duration*: the weighted-average time to receipt of cash flows. The exhibit below calculates the bond's (Macaulay) duration. The first four columns simply itemize the future cash flows. The fifth column contains discount factors that are used to compute the present values (PVs) of the bond's cash flows. The sixth column weights each cash flow as a fraction of the total; e.g., the final return of principal earns a weight of 0.778 ($73.26 / $94.21). The total of the weights must equal 1.0. The final column multiplies the time period by the weight (e.g., the final row = 3.0 years * 0.778 weight = 2.333 weighted years). The sum of this column's components equals the bond's (Macaulay) duration.

Par value	$100.00
Coupon, %	10.0%
Yield	12.0%

Hull Table 4.6

S.A. Period	Coupon	Principal	Cash Flow	Discount Factor	PV of CF	Weight	Time * Weight
0.5	$5.00		$5.00	0.942	$4.71	0.050	0.025
1.0	$5.00		$5.00	0.887	$4.43	0.047	0.047
1.5	$5.00		$5.00	0.835	$4.18	0.044	0.066
2.0	$5.00		$5.00	0.787	$3.93	0.042	0.083
2.5	$5.00		$5.00	0.741	$3.70	0.039	0.098
3.0	$5.00	$100.00	$105.00	0.698	$73.26	0.778	2.333
			$130.00		$94.21	1.000	2.653
							↑ Duration

To use the duration, please note we have two examples below (the first column assume continuous rates, the second assumes semi-annual).

First, we convert the Macaulay duration into *modified duration*. This is done by using the relation Modified Duration = Macaulay Duration / (1 + yield/k) where k = compound periods per year. Or, equivalently: Modified Duration * (1+yield/k) = Macaulay Duration. Note that Modified Duration can be less than or equal to Macaulay Duration, but never greater than! In the case of continuous compounding, this reduces to Modified Duration = Macaulay Duration.

Given the modified duration, we select a yield "shock" (e.g., 10 bps). The estimated price change is then given by: estimated price change = (-)*Bond Price * Modified Duration * Yield Shock.
The "dollar duration" (at bottom) is equal to the modified duration multiplied by the bond price.

Example 4.5 & 4.6		
	Continuous	Semi-Annual
	12.000%	12.367%
(Macaulay) Duration	2.6530	2.6530
Modified Duration	2.6530	2.4985

Duration + Convexity			
Duration (above)	2.65		
Convexity (for example)	150		
Shock to yield	+ 0.25%		
Basis points	+ 25 bps		
Change in bond price (%)	-0.62%		
Bond Price	$94.21		
Change in bond price ($)	($0.58)		
Estimated Price	$93.63		
Yield change (bps)		10	10
Change in Bond Price		(0.2499)	(0.2354)
Estimated New Bond Price		$93.963	$93.978
Dollar duration		$249.948	

Using duration (same modified duration of 2.65) and convexity (assume 150) together:

$$\frac{\Delta B}{B} = -D\Delta y + \frac{1}{2}C(\Delta y)^2$$

For example, for a 25 bps increase in yield, the estimated bond price change is -0.62%:

$$\frac{\Delta B}{B} = -2.65(0.0025) + \frac{1}{2}150(0.0025)^2$$

Describe the major theories of the term structure of interest rates

A number of theories have been proposed to explain the shape of the zero (spot rate) curve:

Expectations Theory: Long-term interest rates should reflect expected future short-term interest rates. More exactly, a forward interest rate corresponding to a certain future period is equal to the expected future zero interest rate for that period
Market Segmentation Theory:
Short, medium & long rates are independent of each other:
"*There need be no relationship between short-, medium-, and long-term interest rates. Under the theory, a major investor such as a large pension fund invests in bonds of certain maturity and does not readily switch from one maturity to another. The short-term interest rate is determined by supply and demand in the short-term bond market; medium-term interest rates are determined by supply and demand in the medium-term bond market; and so on.*"

Liquidity Preference:
Forward rates higher than expected future zero rates:
The theory that is most appealing […] argues that forward rates should always be higher than expected future zero rates.
Investors prefer to preserve their liquidity and invest funds for short periods of time. Borrowers, on the other hand, usually prefer to borrow at fixed rates for long periods of time. Liquidity preference theory leads to a situation in which forward rates are greater than expected future zero rates. It is also consistent with the empirical result that yield curves tend to be upward sloping more often than they are downward sloping.

Chapter Summary

Important interest rates include the Treasury rates, the LIBOR rates, the Repo Rates and the OIS rate. The risk-free rate is an important part of both financial theory and practice. Traditionally the LIBOR rate has been seen as a proxy for the risk-free rate, however; the OIS rate is increasingly being used as a proxy for the risk-free rate.

Interest rates may be compounded at a variety of frequencies, of which quarterly, semi-annual, annual and continuous are the most important. It is imperative that you are able to quickly covert a rate from one compounding frequency to another. This is frequently a requirement in order to solve questions on the FRM exam.

The theoretical price of a coupon-paying bond can be calculated using spot rates, by discounting the nth cash flow with the corresponding nth year spot rate, n years back in time. This is also known as the nth year zero-rate.

Treasury rates and zero rates can be determined by using a technique known as *bootstrapping*. Forward interest rates can easily be inferred from a set of spot, or zero-rates. The forward interest rate is the rate implied for the period of time between the end of the nth and (nth-1) period of zero rates, e.g. the rate between the end of the 5^{th} and the 4^{th} zero rate.

A forward rate agreement (FRA) is an agreement, in which an investor will pay or receive a pre-determined rate in the future on a given principal, for a given period. An FRA is equivalent to an agreement where interest at a predetermined rate, R_k is exchanged for interest at the market rate. An FRA can be valued by assuming that the forward interest rate is certain to be realized.

Simple duration is the weighted-average time to receipt of cash flows. Duration hedging (slope) helps protect out portfolio against *small parallel shifts* in the yield curve. We are however still exposed to shifts that are either large in magnitude, non-parallel or both. Convexity, as a function of the second derivative (curvature), adjusts for some shortcomings of duration only hedging. However, hedging using both duration and convexity still implies that the yield to maturity remains *the* single factor in our model.

Duration typically comes in three forms: Macaulay Duration, Effective Duration and Dollar Duration. Changes in a bond's price can be calculated given duration, convexity and a change in interest rates using the following formula $\frac{\Delta B}{B} = -D\Delta y + \frac{1}{2}C\ (\Delta y)^2$.

The main theories of the term structure of interest rates are the *expectations theory,* the *market segmentation theory*, and the *liquidity preference theory* of which the latter is described as, "the most appealing."

4 Questions & Answers

Questions

4.1 Suppose a lender quotes the interest rate on a $1,000 loan as 9.0% per annum with continuous compounding but the interest is actually paid monthly. What are the monthly interest payments?

a) $7.47
b) $7.50
c) $7.53
d) $7.59

4.2 Assume the following theoretical continuously compounded spot rates: 2.0% at 0.5 years; 3.0% at 1.0 year; 4.0% at 1.5 years; and 5.0% at 2.0 years. What is the two-year PAR YIELD with continuous compounding?

a) 4.88%
b) 4.94%
c) 5.00%
d) 5.04%

4.3 Which of the following is TRUE about a forward rate agreement (FRA)?

a) It is an exchange-traded instrument
b) It can be cash or physically settled
c) A borrower who intends to borrow cash at LIBOR in the future will hedge by receiving the fixed interest rate, R(k), in an FRA
d) A bank that intends to lend cash at LIBOR in the future will hedge by receiving the fixed interest rate, R(k), in an FRA

4.4 Each of the following is TRUE with respect to duration and convexity EXCEPT:

a) Both modified and Macaulay duration are denoted in units of "years"
b) To estimate bond price change with both duration and convexity, per two-term Taylor series, is still to employ a single-factor measure of sensitivity that assumes a parallel shift in the yield curve
c) With respect to a plain vanilla bond (without embedded options), bond convexity increases with maturity, decreases with coupon rate and decreases with yield
d) At low yields, a callable bond exhibits negative convexity and therefore negative duration

Answers

4.1 C. $7.53

The equivalent monthly rate = 12 * $e^{0.09/12}$ - 1 = 9.0338%.

Each interest payment = $1,000 * 9.0338%/12 = $7.53

4.2 B. 4.94%

A 5.0% semiannual coupon rate is the solution that prices that bond exactly at par, given this theoretical spot rate curve.

The continuous par yield is therefore = 2*LN(1+5%/2) = 4.94%

4.3. D. A bank that intends to lend cash at LIBOR in the future will hedge by receiving the fixed interest rate, R(k), in an FRA

In the future, the bank's cash flows will be:

(+) receive LIBOR on the lent cash

(+) receive fixed rate on the FRA

(-) pay LIBOR on the FRA

= net (+) receive fixed rate on the FRA

In regard to (A), FRA is OTC. In regard to (B), FRA is cash settled. In regard to (C), to hedge the future LIBOR, the borrower wants to pay fixed and receive LIBOR (i.e., the gain/loss on LIBOR in the FRA offsets the future borrowing).

4.4 D. At low yields, a callable bond exhibits negative convexity but this does not imply negative duration; rather, it implies only that duration is increasing rather than decreasing (more specifically, as low yields increase, negative convexity implies the dollar duration [the slope of the tangent line] is decreasing from negative to more negative. As the negative convexity gives over to "regular" convexity, the dollar duration [the slope] continues to be negative but increasing to less negative. Regardless, even with the negative convexity, the duration is always positive (i.e., the slope of the tangent is always negative)

In regard to (A), (B) and (C), each of these are true.

In regard to (B), please note that although two terms are involved (duration + convexity), the model is still a single- or one-factor model, as the single factor remains the yield (yield to maturity). For example, a two-factor model might introduce a short- versus a long-term yield or interest rate.

Hull, Chapter 5: Determination of Forward and Futures Prices

Learning Outcomes:

Differentiate between investment and consumption assets.

Define short-selling and short squeeze.

Describe the differences between forward and Futures contracts and explain the relationship between forward and spot prices.

Calculate the forward price, given the underlying asset's price, with or without short sales and/or consideration to the income or yield of the underlying asset.

Describe an arbitrage argument in support of these prices.

Explain the relationship between forward and Futures prices.

Calculate the value of the cash flows from a forward rate agreement (FRA).

Define income, storage costs, and convenience yield.

Calculate the Futures price on commodities incorporating storage costs and/or convenience yields.

Define and calculate, using the cost-of-carry model, forward prices where the underlying asset either does or does not have interim cash flows.

Describe the various delivery options available in the Futures markets and how they can influence Futures prices.

Assess the relationship between current Futures prices and expected future spot prices, including the impact of systematic and nonsystematic risk.

Define contango and backwardation, interpret the effect contango or backwardation may have on the relationship between commodity Futures and spot prices, and relate the cost-of-carry model to contango and backwardation.

Differentiate between investment and consumption assets

An *investment asset* is an asset that is held for investment purposes by a significant number of investors; e.g., stocks, bonds, gold, silver, ETFs.
A *consumption asset,* on the other hand, is held primarily for consumption; e.g., copper, oil, pork bellies, gold, silver. Note that gold and silver, which are both used for industrial applications as well as a store of value are examples of commodities that are both investment AND consumption assets. On the exam, it is not unusual to see a question regarding what assets are consumption assets, or what assets are investment assets. It's important to not fall into the trap of classifying gold and silver as one or the other.

Investment	Consumption
[Theory] No-arbitrage implies that the forward price is a function of the spot price and any dividends paid.	Because of convenience yield, storage cost and the lease rate, the forward price is not a simple function of spot.

Define short-selling and short squeeze

In a short sale, the investor wants to profit from a decline in the price of the security. The short-seller borrows shares of stock from the broker in order to sell the shares. Subsequently, the short-seller purchases the shares in order to replace the borrowed shares. This is known as covering the short position as you effectively cancel out the short position with the stocks you buy. However, the short-seller can experience what is known as a *short squeeze*. In a short-squeeze, the contract is open, the broker runs out of shares to borrow, and the investor is forced to cover, i.e., close out the position.

Time		Cash Flow
0	Borrow shares, Sell shares	+ Price
1	Pay dividend	- Dividend
2	Buy shares to close short position	- Ending Price

Describe the differences between forward and Futures contracts and explain the relationship between forward and spot prices

Differences between forward and Futures contracts

While both forwards and Futures are agreements to buy or sell an asset in the future (at a pre-determined price), a forward contract is traded over-the-counter (OTC) and the forward is not standardized. The Futures contract, on the other hand, is traded on an exchange, and has standardized contract specifications. The Futures position is also often closed out before maturity, rather than taking delivery of the asset. As we will see later on in the reading, it is imperative to know the difference between a forward and a Futures contract, and the implications it has on the pricing of the assets.

Forward vs. Futures Contracts	
Forward	**Futures**
Trade over-the-counter	Trade on an exchange
Not standardized	Standardized contracts
One specified delivery date	Range of delivery dates
Settled at the end of a contract	Settled daily
Delivery or final cash settlement usually occurs	Contract usually closed out prior to maturity

Explain the relationship between forward and spot prices

Notation
The following notations apply to forward contracts:
T: Time until delivery date in a forward/Futures contract (in years)
S_0: Price of the underlying asset (spot price)
F_0: Today's forward or Futures price
K: Delivery price
r: Risk-free rate—annual rate but expressed with continuous compounding
r_f: Foreign risk-free interest rate
I: Present value of income received from asset (in dollar terms)
q: Dividend yield rate (in percentage terms; e.g., 2% dividend yield)
U, u: Storage cost. U = dollar cost and u = cost in % terms
y: convenience yield

Cost of Carry Model

The cost-of-carry model sets a Futures price as a function of the spot price: the Futures price (F) equals the spot price (S0) compounded at the interest rate (r, required to finance the asset) plus the storage cost of the asset less any income earned on the asset.
For a non-dividend-paying investment asset (i.e., an asset which has no storage cost) the cost of carry model says the Futures price is given by:

$$F_0 = S_0 e^{cT} \Longrightarrow F_0 = S_0 e^{rT}$$

The equations for forward prices are essentially similar to Futures prices. The generalized forward price (F0) is either case (Futures or forwards) is therefore given by:

$$F_0 = S_0 e^{rT}$$

If the asset provides interim cash flows (e.g., a stock that pays dividends), then let (I) equal the present value of the cash flows received and the cost-of-carry model is then given by:

$$F_0 = (S_0 - I)e^{rT}$$

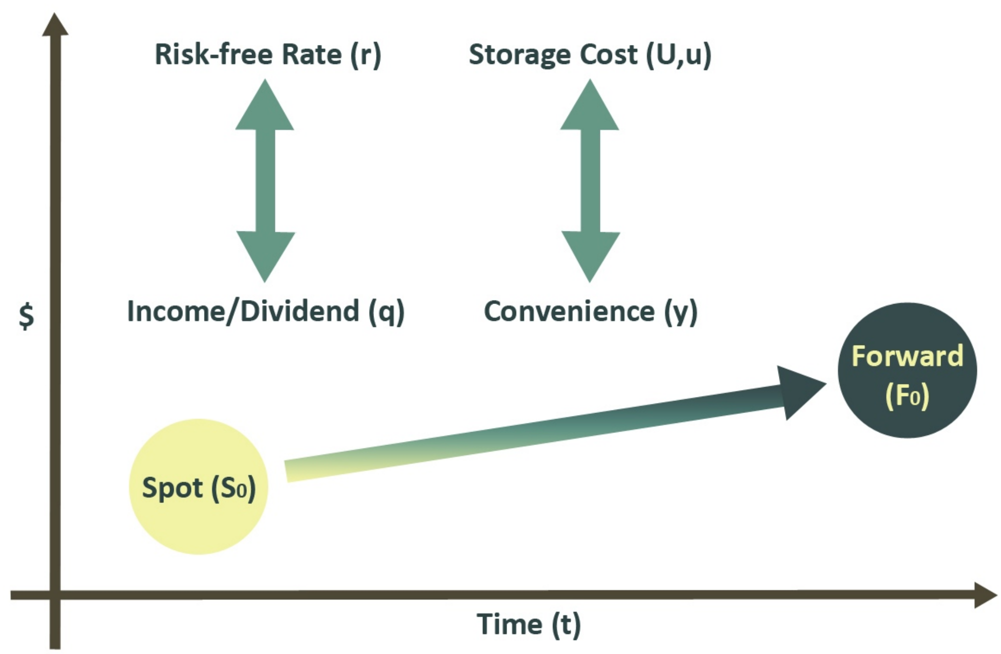
If the asset provides income (e.g., a stock that pays dividends), where the income can be expressed as a constant percentage of the spot price (given by q), then the model is given by:

$$F_0 = S_0 e^{(r-q)T}$$

If the asset has a storage cost and produces a convenience yield (where the convenience yield is a constant percentage of the spot price, denoted by 'y'), the cost-of-carry model expands to:

$$F_0 = S_0 e^{(r+u-y)T}$$

Where r is the risk-free rate, u is the storage cost as a constant percentage, and y is the convenience yield.



As the chart above shows, storage costs increase the value of the forward contract and storage costs work in the opposite direction of income (or in the case of a financial asset, dividends). The risk-free rate is the cost of financing and increases the value of the forward contract; it works in the opposite direction (i.e., is offset by) any convenience yield.

In summary, the cost of carry links the spot price to the forward price:

Commodity

$$F_0 = S_0 e^{(r+u-q-y)T} \quad \Longleftrightarrow \quad F_0 = (S_0 + U - I)e^{(r-y)T}$$

Constant Rates as %	Present Values
u = storage costs	U = present value, storage costs
q = income (dividend)	I = present value, income
y = convenience yield	

Financial Asset (eg: stock index)

$$F_0 = S_0 e^{(r-q)T} \quad \Longleftrightarrow \quad F_0 = (S_0 - I)e^{rT}$$

Constant Rates as %	Present Values
q = income (dividend)	I = present value, income

Calculate the forward price, given the underlying asset's price, with or without short sales and/or consideration to the income or yield of the underlying asset. Describe an arbitrage argument in support of these prices

The following exhibit (from a key learning spreadsheet) locates several cost-of-carry examples together into the same template (one textbook example per column).

> IMPORTANT CONCEPT:
>
> Note that the "four forces" are represented. Interest rate and storage costs *increase* the price of the forward. Dividend and convenience yield *decrease* the price of the forward.

Cost of Carry (XLSX)	McDonald Corn	Hull Stock Forward	Long Bond Forward	Hull 5.2
Spot (S0)	$2.50	$40.00	$900.00	$50.00
Time to maturity (months)	12	3	9	10
Interest rate (per annum)	6.00%	5.00%	4.00%	8.00%
Interest rate (per month)	0.50%	0.42%	0.33%	0.67%
Storage costs, as % (per month)	1.50%	0.00%	0.00%	0.00%
Yield/Dividend, as % (per month)				
Convenience Yield, as % (per month)	0%	0%	0%	0%
Implied Forward Price (F0)	$3.18	$40.50	$886.60	$51.14
Income/Cost as Lump Sum				
FV of income/cost (+ income, - cost)			$40.00	
Time to Lump Sum (months)			4	
Discount Rate			3%	
PV of Income (I)	$0.00	$0.00	$39.60	$2.16

Value of a forward contract

The value of a forward contract (f) is given by either equation below:

$$f = (F_0 - K)e^{-rT}$$

$$f = S_0 e^{-qT} - Ke^{-rT}$$

These are equivalent because the second equation replaces the forward price, F_0, with a spot price that is continuously compounded "forward in time." When a forward contract is first entered into, it has no value[6] because when first negotiated, the delivery price, K, equals the forward price F_0. Only as time passes and the forward price changes does the forward contract gain or lose value.

For example:
A long forward contract on a non-dividend-paying stock has three months left to maturity. The delivery price is \$8 and the stock price is \$10. Also, the risk-free rate is 5%.
The forward price (because t = 0.25 or one-fourth of a year) is given by:

$$F_0 = S_0 e^{rT} = 10e^{(5\%)(0.25)} \cong \$10.126$$

And the value of the forward contract is given by:

$$f = (F_0 - K)e^{-rT} = (10.126 - 8)e^{(-5\%)(0.25)} \cong \$2.10$$

For example, Question:
A stock's price today is \$50. The stock will pay a \$1 (2%) dividend in six months. The risk-free rate is 5% for all maturities. What is the price of a (long) forward contract, F(0), to purchase the stock in one year?
Answer:

$$\begin{aligned} F_0 &= (S_0 - I)e^{rT} \Rightarrow F_0 \\ &= (\$50 - [(\$1)e^{(-0.05)(6/12)}])e^{(.05)(1)} \\ &= \$51.538 \end{aligned}$$

Explain the relationship between forward and Futures prices

If the risk-free rate, r_f, is constant across all maturities, then the forward price should equal the Futures price (forward = Futures price).
But this will vary where there is a correlation between the underlying asset (S) and interest rates: If the correlation is strongly positive: Futures > forward
If the correlation is strongly negative: Futures < forward
The other factor relates to contract life:
For short contracts, price differences should be negligible
For long contracts (e.g., 10 year Eurodollar Futures), the price difference can be significant
Calculate the value of the cash flows from a forward rate agreement (FRA).
Define income, storage costs, and convenience yield
Income refers to a commodity that pays cash the owner (holder) of the asset; the party who is long the Futures or forward contract forgoes the income. Examples include:
Stocks paying known dividends
Coupon-bearing bonds

---

[6] Strictly speaking, we are here referring to the objective value using risk-neutral, or objective, probabilities. Using subjective probabilities, this contract does have value to both parties, or it would not have been entered into: both parties expect some benefit. This is a subtle but important general concept.

Storage costs are the cost to store or carry the asset; storage costs are typically associated with physical commodities.

Convenience Yield

The convenience yield reflects the "excess benefits" conferred by taking physical ownership of the asset (i.e., as opposed to holding a Futures contract). The convenience yield is generally not relevant for financial assets. But for commodities (physical assets), ownership may confer positive benefits or may decrease perceived risk.

The convenience yield is the "plug variable" that validates the cost of carry model. The convenience yield impounds benefits of holding/owning the physical asset. This includes any real optimality benefits of commodity ownership (i.e., owning the asset gives the owner some future real option).

For a consumption asset—where (y) is the convenience yield and (c) is the cost of carry—the Futures price is given by:

$$F_0 = S_0 e^{(c-y)T}$$

Note this is essentially similar to the forward price if we replace the cost of carry (c) with the risk-free rate (r).

If a non-dividend-paying stock offered a "convenience yield" then its forward price calculation would mirror the above formula:

$$F_0 = S_0 e^{(r-y)T}$$

Except that a non-dividend-paying stock does not offer a convenience yield, so we are left with the original formula:

$$F_0 = S_0 e^{(r)T}$$

Storage costs is economically like negative (-) income. Convenience yield is economically like income/dividend.

Calculate the Futures price on commodities incorporating storage costs and/or convenience yields

The Futures price for a commodity can be given by two formulae:

$$F_0 = (S_0 + U)e^{rT}$$

Where U is the present value of storage costs

$$F_0 = S_0 e^{(r+u-y)T}$$

Where
u is the storage costs as a proportion of the spot price
y is the convenience yield
Two additional examples (Hull 5.6 and Hull 5.8):

	Hull 5.6	Hull 5.8
Spot (S0)	$0.6200	$450.00
Time to maturity (months)	24	12
Interest rate (per annum)	7.00%	7.00%
Interest rate (per month)	0.58%	0.58%
Storage costs, as % (per month)	0.00%	0.00%
Yield/Dividend, as % (per month)	0.42%	0.00%
Convenience Yield, as % (per month)	0%	0%
Implied Forward Price (F0)	$0.6453	$484.63
Income/Cost as Lump Sum		
FV of income/cost (+ income, - cost)		-$2.00
Time to Lump Sum (months)		12
Discount Rate		7%
PV of Income (I)	$0.00	($1.86)

Define and calculate, using the cost-of-carry model, forward prices where the underlying asset either does or does not have interim cash flows

(This AIM is largely redundant with a prior AIM). Here are examples from the text where the cost-of-carry model is applied. The implied forward price is shown in the last column.

			Cost			Benefit	Forward
			Risk free Rate			Lease/Div	
	Spot	Time	Year	Per Month	Store Cost	% Per Month	Cost of Carry Implies
Culp's Corn	$2.50	12	6.00%	0.50%	1.50%		$3.18
Hull's Stock Forward	$40.00	3	5.00%	0.42%	0.00%		$40.50
Hull's Long Forward Bond (1)	$900.00	9	4.00%	0.33%	0.00%		$886.60
Hull's 5.3: Long Forward on asset paying income	$25.00	6	10.00 %	0.83%	0.00%	0.33%	$25.77
Hull's 5.5: S&P Futures	$800.00	3	6.00%	0.50%	0.00%	0.08%	$810.06
Hull's 5.6: Foreign Currency	$0.62	24	7.00%	0.58%	0.00%	0.42%	$0.6453
Hull's 5.8: Investment with Storage Cost (2)	$450.00	12	7.00%	0.58%	0.00%	0.00%	$484.63

Bond pays a $40 coupon in four months, discount rate = 3%
Investment requires lump sum storage outlay of $2 in 12 months (rate of 7%)
For example,
In regard to Culp's corn, $\$3.18 = \$2.50 * e^{(0.5\%+1.5\%)*12}$
In regard to Hull's 5.8 Investment with storage cost,
the PV of storage cost = ($2 storage) $* e^{-7\%*12/12}$ = $1.86.
Then, forward price of $484.63 = ($450 spot + $1.86 PV of storage cost) $* e^{(0.58\%)*12}$

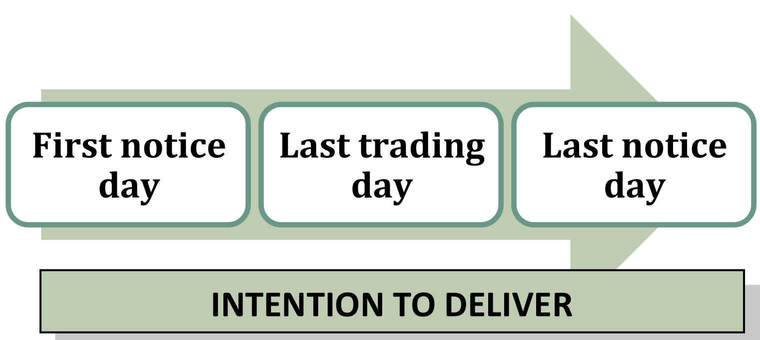
In the example below, a bond with price of $900 pays a $40 coupon in 4 months:

	Long Bond Forward
Spot (S0)	$900.00
Time to maturity (months)	9
Interest rate (per annum)	4.00%
Interest rate (per month)	0.33%
Storage costs, as % (per month)	0.00%
Yield/Dividend, as % (per month)	Lump sum, below
Convenience Yield, as % (per month)	0%
Implied Forward Price (F0)	$886.60

Income/Cost as Lump Sum	
FV of income/cost (+ income, - cost)	$40.00
Time to Lump Sum (months)	4
Discount Rate	3%
PV of Income (I)	$39.60

Describe the various delivery options available in the Futures markets and how they can influence Futures prices

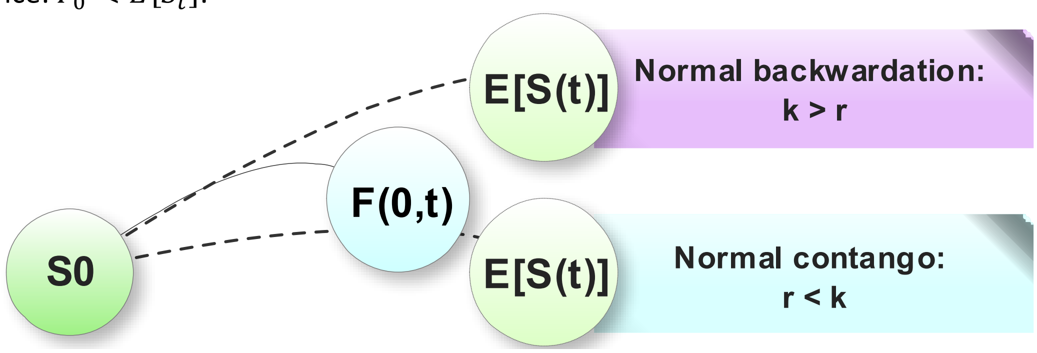
Although a forward contract typically specifies the day of delivery, a Futures contract often allows (short position) for delivery during a certain period. If the Futures price is an increasing function of time to maturity, the short should deliver as early as possible. (And for modeling purposes, here we assume delivery at beginning of period.)
If the Futures price is a decreasing function of time to maturity, the short should deliver as late as possible. (And for modeling purposes, here we assume delivery at end of period.)



Analyze the relationship between current Futures prices and expected future spot prices

Normal Backwardation and Normal Contango
"Normal backwardation" and "normal contango" refer to an unobserved relationship between the spot price and the expected future spot price. In normal contango, the forward price is greater than the expected future spot price (note the curve may or may not be inverted!). In normal backwardation, the forward price is less than the expected future spot price (again, the curve may or may not be inverted!).

Normal contango refers to a forward price that is greater than the expected future spot price:.$F_0 > E[S_t]$
Normal backwardation refers to a forward price that is less than the expected future spot price: $F_0 < E[S_t]$.



A classic model would predict a normal backwardation (i.e., compensation to the long forward position) during contango (i.e., positive cost of carry).
Theory of Normal Backwardation
The theory of normal backwardation assumes that hedgers want to be net short and speculators want to be net long. As such, the Futures price will fall below the expected future spot price and this is known as normal backwardation. If, instead, hedgers are net long and speculators are net short, Futures prices are greater than expected future spot prices and this is known as contango.
Normal backwardation is expected under the "theory of normal backwardation. Why? Under this theory, the long forward position (the speculator) expects more than a zero profit, so the long must expect $E[S_t] > F_{0,t}$. The expected positive non-zero profit, $E[S_t] - F_{0,t}$, is compensation for bearing systemic risk.

The impact of systemic and non-systemic risk

$$F_o = E(S_T)e^{(r-k)T}$$

$$k_{CAPM} = r_{riskfree} + \beta * ERP$$

If the investment has positive systematic risk, the future price should be less than the expected future spot price $E[S_t] > F_{0,t}$: the long position expects compensation for the assumption of systemic risk!

Define contango and backwardation, interpret the effect contango or backwardation may have on the relationship between commodity Futures and spot prices, and relate the cost-of-carry model to contango and backwardation

Contango

Contango refers to an upward-sloping ("normal") forward curve: long-term forward prices are greater than near-term forward prices (and the spot price).

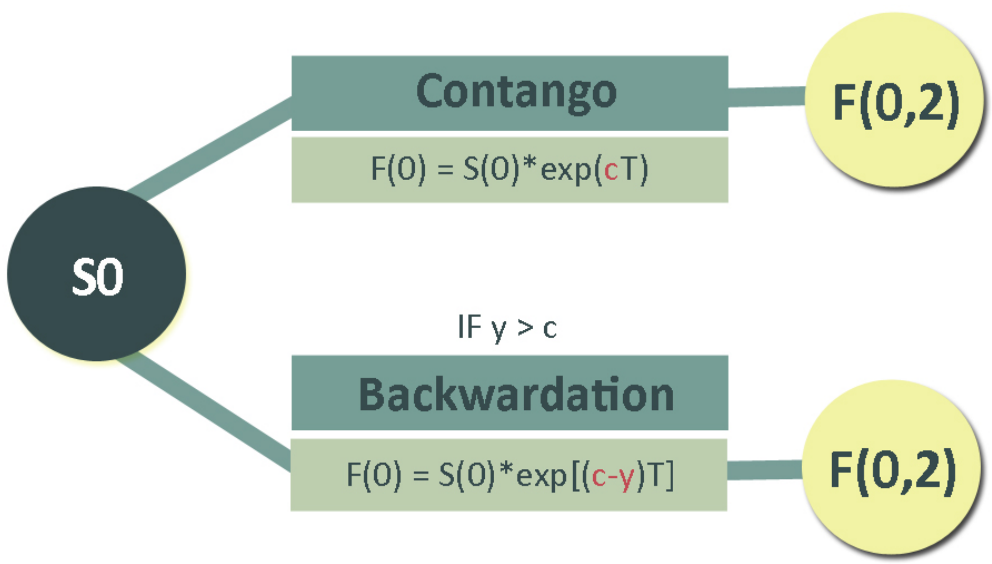
Backwardation

Backwardation refers to an inverted forward curve: long-term forward prices are less than near-term forward prices (and the spot price).

Relationship between commodity Future and spot prices, and how it relates to the cost of carry model

Contango and backwardation describe the shape of an observed forward curve. As the shape is a function of spot, S(0), and forward (0), prices, these are not model-driven but simply observed in the pricing term structure.

Backwardation (an inverted forward curve) may be due to high convenience yield is greater than the interest rate (or greater than the interest rate plus the storage costs):



Chapter Summary

We distinguish between investment assets and consumption assets. The former is held for purposes of investment whereas the latter is held primarily for consumption. Gold and silver are examples of assets that can be thought of as both investment and consumption assets.

An investor engaged in short selling is hoping to profit from a decline in the price of the security. The short-seller borrows shares of stock from the broker in order to sell the shares. The investor then covers his short position by buying the shares in the market to close out his position.

Forwards and Futures are agreements to buy or sell an asset in the future at a pre-determined price. However, a forward contract is traded over-the-counter (OTC) while a Futures contract is standardized and traded on an exchange. The forward position will typically take delivery of the underlying while Futures are often closed out before the delivery period.

The forward and Futures prices are the same only if the risk-free interest rate is constant and the rate curve is flat; and if the counterparty (credit) risk on the forward contract is virtually zero.

The cost-of-carry model sets a Futures price as a function of the spot price. Note that the “four forces” are represented in the general cost-of-carry model. Interest rate and storage costs *increase* the price of the forward. Dividends and convenience yield *decrease* the price of the forward. There is also a fifth factor that is sometimes lumped together with convenience yield: the lease rate. It is important to be clear about the reason a lease payment is required for a commodity and not for a financial asset[7].

The value of a forward contract with time to maturity T, risk-free rate r, and dividend yield q is given by either equation below:

$$f = (F_0 - K)e^{-rT}$$

$$f = S_0 e^{-qT} - Ke^{-rT}$$

These are equivalent because the second equation replaces the forward price, F_0, with a spot price that is continuously compounded “forward in time.”

Normal contango refers to a forward price that is greater than the expected future spot price:$F_0 > E[S_t]$
Normal backwardation refers to a forward price that is less than the expected future spot price: $F_0 < E[S_t]$.

[7] Refer to chapter 10 in this reading, based on Robert McDonald’s Chapter 6.

5 Questions & Answers

Questions

5.1 Under what condition should the price of a FORWARD contract equal the price of a FUTURE contract, if the commodity and specifics of the contracts (e.g., maturity) are otherwise identical? (Best answer)

a) If the financing cost (r), storage cost (u), income (q) and convenience yield (y) are identical
b) If the counterparty (credit) risk on the forward contract is virtually zero
c) If the risk-free interest rate is constant and the rate curve is flat
d) If the risk-free interest rate is constant and the rate curve is flat; and if the counterparty (credit) risk on the forward contract is virtually zero

5.2 Each of the following is TRUE about the cost of carry approach (model) to pricing commodity forwards EXCEPT:

a) If the storage cost of a consumption commodity exceeds the risk-free rate, the forward curve must exhibit contango
b) Forward curve backwardation implies a convenience yield that is greater than the cost of carry ($y > c$)
c) The convenience yield is economically like a dividend and therefore like a negative storage cost
d) A non-dividend-paying stock has a cost of carry equal to the risk-free rate

5.3 If currency futures are quoted in US dollars per unit of foreign currency, and if foreign exchange futures prices are increasing with maturity, what does interest rate parity (IRP) imply?

a) US risk interest rates are greater than foreign interest rates ($r > rf$)
b) Foreign interest rates and greater than US risk interest rates ($rf > r$)
c) Spot exchange rates are greater than foreign interest rates
d) Foreign interest rates are greater than spot exchange rates

5.4 Assume that corn has the following properties: positive storage cost, no convenience yield, and positive systemic risk (i.e., beta > 0). According to Hull, which is most likely with respect to, respectively, the observed forward curve (contango = normal; backwardation = inverted) and the relationship between the futures price, F(0,X), and the expected future spot price, E[S(X)]?

a) Contango and normal contango
b) Contango and normal backwardation
c) Backwardation and normal contango
d) Backwardation and normal backwardation

Answers

5.1 D.
If the risk-free interest rate is constant and the rate curve is flat; and if the counterparty (credit) risk on the forward contract is virtually zero
(C) is a fine answer and faithful to the text: the key theoretical pricing difference, immaterial at short maturities, is the marginal cash flow volatility implied by the daily (mark to market) settlement of the futures contract; this difference is a function of the interest rates at which the marginal cash flows are borrowed/reinvested.
But (D) is better because counterparty risk is another key difference between the forward and the futures contract.

5.2 A.
Finance cost plus storage $(r + u)$ alone implies contango but convenience yield might be large enough to induce backwardation; i.e., unclear with convenience yield.

In regard to (B), (C), and (D), each are TRUE.

5.3 A.
US risk interest rates are greater than foreign interest rates ($r > rf$). In regard to (C) and (D), these are meaningless comparison.

5.4. B. Contango and normal backwardation
With respect to the forward curve, per the cost of carry, we observe $F(X) = S(0)*EXP(r + u)$; i.e., contango.
With respect to the expected future spot price, positive systematic risk implies normal backwardation, $F(x) < E[S(x)]$; i.e., speculators expect a profit as compensation for assuming the systematic risk

Hull, Chapter 6: Interest Rate Futures

Learning Outcomes:

Identify the most commonly used day count conventions, describe the markets that each one is typically used in, and apply each to an interest calculation.

Calculate the conversion of a discount rate to a price for a U.S. Treasury bill.

Differentiate between the clean and dirty price for a US Treasury bond; calculate the accrued interest and dirty price on a US Treasury bond.

Explain and calculate a US Treasury bond Futures contract conversion factor.

Calculate the cost of delivering a bond into a Treasury bond Futures contract.

Describe the impact of the level and shape of the yield curve on the cheapest-to-deliver bond decision.

Calculate the theoretical Futures price for a Treasury bond Futures contract.

Calculate the final contract price on a Eurodollar Futures contract.

Describe and compute the Eurodollar Futures contract convexity adjustment.

Explain how Eurodollar Futures can be used to extend the LIBOR zero curve.

Calculate the duration-based hedge ratio and describe a duration-based hedging strategy using interest rate Futures.

Explain the limitations of using a duration-based hedging strategy.

Identify the most commonly used day count conventions, describe the markets that each one is typically used in, and apply each to an interest calculation.

Day count conventions are important for computing accrued interest:

- Actual/actual: U.S. Treasury bonds
- 30/360: U.S. corporate and municipal bonds
- Actual/360: U.S. Treasury bills and other money market instruments

For example, if coupons pay on March 1st and September 1st, then actual/actual computes based on actual/184 days while 30/360 would assume 180 days between coupons.

Money Market instruments include:

- Short-term financial instruments
- Treasury bill (government)
- Certificate of deposit (bank)
- Commercial paper (CP)
- Repurchase agreement (repo)

Capital Market Instruments include:

- Long-term securities
- US Treasury notes (1-10 yrs) and bonds (> 10 yrs)
- Domestic and Eurobonds (issued internationally)
- Euro bond (denominated in Euros)

The following illustrates three different day count conventions applied to the accrued interest (AI) on a bond that settles in-between coupons, which pay ever six months on March 1st and September 1st. In all cases, the coupon for the six-month is $4, but since the bond settles on July 3rd, the accrued interest varies depending on the day count convention:

Hull 6.1: Day Count Conventions

Principal		$100					
Coupon		8%					
Start Period	Settle	End Period	Day Count	Days Since	Total Days	Coupon	AI
Mar 1	Jul 3	Sep 1	Actual/Actual	124	184	$4.00	$2.696
Mar 1	Jul 3	Sep 1	30/360	122	180	$4.00	$2.711
Mar 1	Jul 3	Sep 1	Actual/360	124	180	$4.00	$2.756

Calculate the conversion of a discount rate to a price for a U.S. Treasury bill.

A US Treasury bill is a discount instrument: the discount rate is expressed as a percentage of the face value. Consequently, the discount rate is not a true yield.

Consider the following example. The face value of the Treasury bill is $100 and the cash price is 98.00. As the maturity is 0.25 years (90 days/360), the discount rate is 8. In other words, $8 = 360/90 * \ (100 - 98)$. The 8% is the annualized (2%*4) interest as a percentage of the face ([$2*4]/$100). Therefore, it is *not* the true yield. The true yield is 8.16%.

Discount Rate for Treasury Bill		
Face	$100.00	
n (days)	90	
P (discount)(%)	8.0	
Y (cash price)	97.98	
Solving for P	8.0	← $P_{Quoted} = \frac{360}{n} * (100 - Y_{Cash\ price})$

True Yield versus Discount Rate		
Discount rate	8.00%	
True Yield	8.1633	← $\frac{100.00*8.00\%}{98.00} = 8.1633$

Which is the quarterly compounded rate: $\$98 * (1 + \frac{8.1633\%}{4})^{n*4}$ = $100, for n = 0.25.

Differentiate between the clean and dirty price for a US Treasury bond; calculate the accrued interest and dirty price on a US Treasury bond.

Clean Price

The clean price (i.e., the quoted price) does not reflect the cash price if interest has accrued.

Dirty Price

The dirty price (i.e., the full price or the cash price) adds the accrued interest to the clean price. In short: Cash Price = Quoted Price + Accrued Interest since last coupon date.
The illustration below computes the dirty price by adding the accrued interest (which, recall, depends on the day count convention) to the quoted (clean) price.

Hull 6.1: Dirty Price of US Treasury	
Principal	$100
Coupon	11%
Quoted Price (Clean)	$95.50

Start Period	Settle	End Period	Day Count	Days Since	Total Days	Coupon	AI	Dirty (Full) Price
1/10/2010	3/5/2010	7/10/2010	ACT/ACT	54	181	$5.50	$1.641	$97.141

The flat price is also called the "clean price" or simply the "price". The full price is also called the "dirty price" or the "invoice price".
When the accrued interest of a bond is zero (i.e., when the settlement date is a coupon payment date) the flat and full prices of the bond are equal. When accrued interest is not zero, the amount paid/received for a bond (i.e., its full price) should equal the present value of its cash flows.

If P is a bond's flat price, and AI is the accrued interest, then the full price is the present value (PV) which is given by: P + AI = PV (future cash flows), where AI equals the accrued interest and is given by:

$$AI = \text{coupon}\frac{\text{\# of days from last coupon to settlement}}{\text{\# of days between coupons}}$$

To summarize, the full price of the bond equals the flat price plus accrued interest (if any). The invoice price is the amount paid by the buyer and received by the seller; therefore, it is the face amount multiplied by the full price. Here is another example:

Par	$100.00
Yield	4%
Coupon (%)	8%
Coupon ($)	$4.00

Last Coupon	1/01/2013
Settlement	5/16/2013
Next Coupon	7/01/2013
Matures	7/01/2016

Years (from last coupon)	2.50
Price (at last coupon)	$109.43
w (days to next coupon)	45
days in coupon period	180
w periods	0.25
Accrued interest (AI)	$3.00

Full price	$111.06
Accrued interest (AI)	$3.00
Clean price	$108.06

Present value "street method"

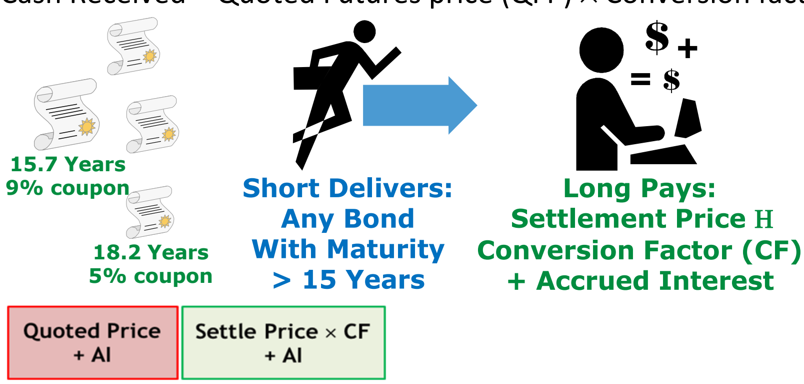
Period	CF	PV
1	$4.00	$3.98
2	$4.00	$3.90
3	$4.00	$3.83
4	$4.00	$3.75
5	$104.00	$95.61
Full Price (PV)		$111.06

Full price (PV) is also the recent price (at last coupon) compounded forward:
PV = ($109.43)[(1.02)^(1-w)]

Explain and calculate a US Treasury bond Futures contract conversion factor

The Treasury bond Futures contract allows the party with the short position to deliver any bond with a maturity of more than 15 years and that is not callable within 15 years. The short here has flexibility in delivery! When the chosen bond is delivered, the conversion factor defines the price received by the party with the short position:

Cash Received = Quoted Futures price (QFP) $\times$ Conversion factor (CF) + Accrued interest



Calculate the cost of delivering a bond into a Treasury bond Futures contract

The cost to deliver is the dirty price, which is the bond quoted price plus accrued interest (AI). The short position will receive the settlement multiplied by the conversion factor plus accrued interest (AI). The cheapest to deliver (CTD) is:

The bond that minimizes → MIN: Quoted Bond Price - (Settlement)(CF), or similarly
The bond that maximizes → MAX: (Settlement)(CF) - Quoted Bond Price

Short receives:	(Settlement)(CF) + AI
Cost ("dirty price"):	Quoted Bond Price + AI
CTD Minimizes	Quoted Bond Price - (Settlement)(CF)

Hull example 6.1:
Settlement **$93.25**

Bond		Price	CF	Cost
1		$99.50	1.0382	$2.69
2	**(CTD)**	**$143.50**	**1.5188**	**$1.87**
3		$119.75	1.2615	$2.12
				$1.87

Describe the impact of the level and shape of the yield curve on the cheapest-to-deliver bond decision

Bond yields < 6% favors delivery of high-coupon, short-maturity bonds.
Upward-sloping yield curve favors long time-to-maturity bonds.
Downward-sloping yield curve favors short time-to-maturity bond. Because the cheapest-to-deliver (CTD) is based on standardizing the yields at 6%, long
Because the cheapest-to-deliver (CTD) is based on standardizing the yield at 6%, long-maturity bonds will be favored if the yield is high and/or there is a long time-to-maturity:
Bond yields > 6% favors delivery of low-coupon, long-maturity bonds

Calculate the theoretical Futures price for a Treasury bond Futures contract

Assume the following (Hull example 6.2):
Cheapest to deliver bond is a 12% coupon bond with a conversion factor of 1.6 and delivery is in 270 days. Coupons pay semi-annually and the last coupon was paid 60 days ago, the

next coupon pays in 122 days, and the coupon thereafter is in 305 days. The term-structure is flat at 10%.

Calculations are show below:

Cash price = Accrued interest + Quoted bond price = $116.978

PV of $6 coupon to be received in 122 days = $5.803

Cash Futures price = (116.978 – 5.803) * $e^{10\%*270/365}$= $119.711

Quoted Futures price = $119.711 – (6 * 148/(148+35)) = $114.859

Quoted Futures price (CTD) = 114.859 / 1.6 = 71.79

Hull 6.2: Theoretical Price of Treasury Bond Futures Contract	
Cheapest to Deliver (CTD)	
Face	$100.00
Current Quoted Price	$115.00
Coupon	12%
Interest rate	10%
Conversion Factor	1.6
Delivery (days)	270
Last Coupon (-days)	60
Next Coupon (+ days)	122
Cheapest to Deliver Calculation	
Accrued Interest	$1.978
Cash (Dirty Price)	$116.978
PV of coupon	$5.803
Cash Futures Price	$119.711
Days Accrue, @ delivery	148
Days Remain, @ delivery	35
Quoted FP, 12% bond	$114.859
Quoted FP, CTD	$71.79

Calculate the final contract price on a Eurodollar Futures contract

If (R) is the LIBOR interest rate; the contract price is set to 100 – R.
In the following example, in June, the investor buys a Eurodollar contract at quote of 94.79 (implied LIBOR = 100 – 94.79 = 5.21%). Go forward to June, when contract settles: LIBOR is 4%, so quote is 96 (100 – 4 = 96).
Since contract price = 10,000 * [100 – 0.25 * (100 – Quote)]
January contract price = 10,000 * [100 – 0.25 * (100 – 94.790)] = $986,975
June (settlement) contract price = 10,000 * [100 – 0.25 * (100 – 96.000)] = $990,000
For gain on this long position of $3,025
By design, each contract pays $25 per basis point: 1.21 * $25 * 100 = $3,025 gain

Hull Table 6-1: Eurodollar Futures Contract			
	Quote	LIBOR	
January (June 2007 contract)	94.790	5.21	986,975
June (Settlement)	96.000	4.00	990,000
	1.210		3,025
Gain/Loss Per 1 bps	25.00		

IMPORTANT CONCEPT:

If the Eurodollar Futures quote increases by 1 basis point, long position gains $25 and short position loses $25.
If the Eurodollar Futures quote decreases by 1 basis point, long position loses $25 and short position gains $25.

Describe and compute the Eurodollar Futures contract convexity adjustment

The convexity adjustment assumes continuous compounding. Given that (σ) is the standard deviation of the change in the short-term interest rate in one year, t1 is the time to maturity of the Futures contract and t2 is the time to maturity of the rate underlying the Futures contract. Under the Hull-Lee model, the forward rate is less than the Futures rate as a function of variance:

$$\text{Forward} = \text{Futures} - \frac{1}{2}\sigma^2 t_1 t_2$$

The primary difference is due to daily settlement of Futures contracts: as they settle daily, this leads to interim cash flows (i.e., margin calls or excess margin).

Hull 6.3: Convexity Adjustment

Volatility of short rate	2.0%	
Eurodollar Futures price	95	
T1	4.00	
T2 (three month rate)	4.25	
Convexity adjustment	0.3400%	0.5 * 2%^2*(4)*(4.25)
Futures rate (ACT/360)	5.000%	= 100 – 95 price
	1.250%	Per 90 days
	5.038%	= LN (1.0125)*365/90; i.e., continuous & actual/365
Forward (continuous)	4.698%	= Future – convexity adjustment

Explain how Eurodollar Futures can be used to extend the LIBOR zero curve

The bootstrap procedure can be used to extend the LIBOR zero curve:

Start	Days	Forward (F)	Zero(R)
0	400	4.80%	
400	90	5.30%	4.800%
491	90	5.50%	4.893%
589	90	5.60%	4.994%

$$R_{i+1} = \frac{F_i(T_{i+1} - T_i) + R_i T_i}{T_{i+1}}$$

For example:
We use the second forward rate to obtain the 589-day rate: 4.994% = [(491 days * 4.893% zero rate) + (98 day difference * 5.5% forward rate)] / 589 days

Calculate the duration-based hedge ratio and describe a duration-based hedging strategy using interest rate Futures

The number of contracts required to hedge against an uncertain change in the yield, given by Δy, is given by:

$$N^* = \frac{PD_P}{F_C D_F}$$

Note: FC = contract price for the interest rate Futures contract. DF = duration of asset underlying Futures contract at maturity. P = forward value of the portfolio being hedged at the maturity of the hedge (typically assumed to be today's portfolio value). DP = duration of portfolio at maturity of the hedge.

For example, assume a portfolio value of $10 million. The manager hedges with T-bond Futures (each contract delivers $100,000) with a current price of $98. She thinks the duration of the portfolio at hedge maturity will be 6.0 and the duration of Futures contract will be 5.0. How many Futures contracts should be shorted?

$$N^* = \frac{PD_P}{F_C D_F} = \frac{(\$10 \text{ million})(6)}{(98{,}000)(5)} \cong 122$$

Explain the limitations of using a duration-based hedging strategy

Portfolio immunization or duration matching is when a bank or fund matches the average duration of assets with the average duration of liabilities.
Duration matching protects or “immunizes” against small, parallel shifts in the yield (interest rate) curve. The limitation is that it does not protect against nonparallel shifts. The two most common nonparallel shifts are:

1. A twist in the slope of the yield curve, or
2. A change in curvature

Chapter Summary

Different day-count conventions apply to U.S. Treasury bonds, U.S. corporate and municipal bonds, and U.S. Treasury bills and other money market instruments. Knowing which day count convention applies to each instrument is important in order to correctly calculate their respective prices.

As we have seen US Treasury bill is a discount instrument: the discount rate is expressed as a percentage of the face value, and consequently we need to employ some simple calculations in order to obtain the true yield.

Market conventions and participant tend to differentiate between the clean price and the dirty price of an instrument. The cash price = quoted price + accrued interest since last coupon date. The dirty price (the cash price) adds the accrued interest to the clean price.

A Treasury bond Futures contract allows the party with the short position to deliver any bond with a maturity of more than 15 years and that is not callable within 15 years. This flexibility leads to the Futures contract being less valuable.

The cheapest to deliver (CTD) Treasury bond Futures contract is:
The bond that minimizes → MIN: Quoted Bond Price - (Settlement)(CF), or similarly
The bond that maximizes → MAX: (Settlement)(CF) - Quoted Bond Price

It is important to understand how to calculate the theoretical Futures price for a Treasury bond Futures contract.

The Eurodollar Futures contract is important in building interest rate curves, and knowing how to price a Eurodollar Futures contract is imperative. Generally speaking, the price of a contract is 100 – LIBOR. By design, each contract pays $25 per basis point.

We have seen how to calculate a convexity adjustment to a Eurodollar Futures contract, using the Hull-Lee model, which gives us a formula that explains and enables us to compute the difference between a forward and a Futures contract. The primary difference between the two is due to the daily settlement process of Futures contracts. The bootstrap procedure that was mentioned in previous chapters was highlighted again in the context of extending the LIBOR zero curve using the Eurodollar Futures contracts.

Hedging small parallel shifts in the yield curve with duration hedging was once again explored, along with sensitivity analysis and the number of contracts necessary to hedge our exposure to such parallel shifts. As discussed in earlier chapters, duration based hedging is not a panacea as it fails to take into account such as non-linearity. Since short term rates are often more volatile than longer term rates, the assumption of parallel shifts is unlikely to hold up in actual markets.

6 Questions & Answers

Questions

6.1 If French money market instrument pays in Euros with an interest rate of 5.0% per annum with (discrete) annual compounding and under an actual/360 day count (ACT/360) convention, what is the equivalent rate under continuous compounding under an actual/365 day count?

a) 4.879%
b) 4.947%
c) 5.000%
d) 5.069%

6.2 Interest rates (bond yields) are currently below 6.0%. Which of the following bonds will the short position in U.S. Treasury bond futures contract be most likely to deliver; i.e., which will be CTD?

a) Short-maturity with low coupon
b) Short-maturity with high coupon
c) Long-maturity with low coupon
d) Long-maturity with high coupon

6.3 Each of the following is TRUE about the Eurodollar futures contract EXCEPT:

a) A Eurodollar is a dollar denominated deposit in a bank that is not located in the United States
b) The long position in a Eurodollar future contract promises to borrow $1,000,000 at maturity and repay this principal three months later
c) The notional value of a single Eurodollar futures contract is $1,000,000 with delivery months of March, June, September and December
d) The short position in a Eurodollar futures contract gains when the LIBOR interest rate increases

6.4 If a bond portfolio with a duration of 9.0 years is hedged with futures contracts in which the underlying asset has a duration of only 3.0 years, but the volatility of the 3-year interest rate is greater than the volatility of the 9-year interest rate, what is the likely impact on a duration-based hedge?

a) The portfolio is likely to be over-hedged
b) The portfolio is likely to be under-hedged
c) The portfolio will be correctly hedged because volatility does not enter into the duration hedge
d) The portfolio will be correctly hedged because more hedge contracts (~ 3X) will simply equate the value (dollar) durations of the portfolio and the hedging instruments

Answers

168.5. B. 4.947%
365/360*LN (1+5%) = 4.947%

171.4. B. Short-maturity with high coupon
If yields are low (< 6%), favors low duration bonds;
If yields are high (>6%), favors high duration bonds;

172.4. B. The $1 MM is a notional reference with the contract settling at maturity; the long does not actually borrow the $1 MM.
In regard to (A), (C) and (D), EACH is TRUE

173.4. A. The portfolio is likely to be over-hedged
In regard to (D), it is true that the hedge will equate the dollar durations and this would be correct under the naive assumption of SMALL, PARALLEL shifts in the yield curve. But the higher volatility of the short-rate is a realistic violation of the assumption and will imply over-hedging.

Hull, Chapter 7: Swaps

Learning Outcomes:

Explain the mechanics of a plain vanilla interest rate swap and compute its cash flows.

Explain how a plain vanilla interest rate swap can be used to transform an asset or a liability and calculate the resulting cash flows.

Explain the role of financial intermediaries in the swaps market.

Describe the role of the confirmation in a swap transaction.

Describe the comparative advantage argument for the existence of interest rate swaps and discuss some of the criticisms of this argument.

Explain how the discount rates in a plain vanilla interest rate swap are computed.

Calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions.

Calculate the value of a plain vanilla interest rate swap from a sequence of forward rate agreements (FRAs).

Explain the mechanics of a currency swap and **compute** its cash flows.

Describe the comparative advantage argument for the existence of currency swaps.

Explain how a currency swap can be used to transform an asset or liability and **calculate** the resulting cash flows.

Calculate the value of a currency swap based on two simultaneous bond positions.

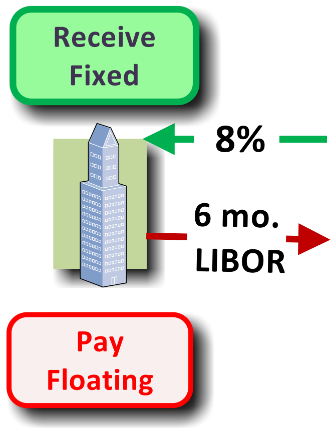
Calculate the value of a currency swap based on a sequence of FRAs.

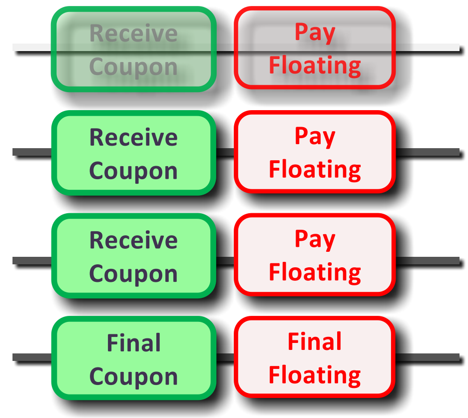
Describe the role of credit risk inherent in an existing swap position.

Identify and describe other types of swaps, including commodity, volatility and exotic swaps.

Explain the mechanics of a plain vanilla interest rate swap and compute its cash flows

In a plain-vanilla interest-rate swap, one company agrees to pay a fixed interest rate and receive a variable interest rate (i.e., “pay-fixed and receive-floating”) and the other company (a.k.a., the counterparty) agrees to pay a variable rate and receive a fixed interest rate (i.e., “pay-floating and receive-fixed”).





An illustration with the following assumption:
Notional principal: $100 million (it is called notional principal because, in this type of swap, the principal is not exchanged)
Swap agreement: Pay fixed rate of 5% and receive LIBOR
Term: 3 years with payments every six months

End of Period (6 months)	LIBOR at the Start of Period	Pay Fixed Cash Flow	Receive Floating Cash Flow	Net Cash Flow
1	5.0%	-2.5	+2.5	0.0
2 (Year 1)	5.2%	-2.5	+2.6	+0.1
3	5.4%	-2.5	+2.7	+0.2
4 (Year 2)	5.0%	-2.5	+2.5	0.0
5	4.8%	-2.5	+2.4	-0.1
6 (Year 3)	4.6%	-2.5	+2.3	-0.2

The notional is not exchanged in the plain vanilla interest rate swap. Also, the first floating rate payment is known at inception because the floating rate that applies is the rate that prevails at the start of the six month swap interval.

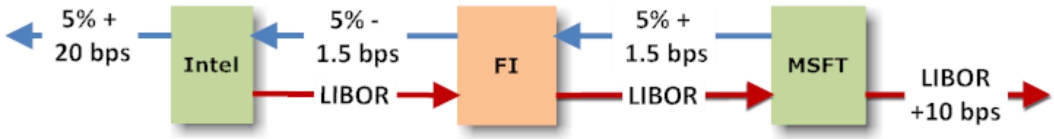
A note on discounting in wake of the LIBOR scandal and financial crisis

Throughout the assigned readings and in the examples presented in these notes, the LIBOR rate is used both to *infer* the future cash flows, as well as the risk-free rate to *discount* the future cash flows. In practice this is not the way the market operates. In wake of the LIBOR scandal, in which Governmental agencies tasked with overseeing banks and financial markets have investigated banks, and several banks have admitted to sending in artificially high or low LIBOR rates to the BBA during the financial crisis, in order to appear more creditworthy, the LIBOR rate has received a lot of negative attention. Moreover it shed light on shoddy practices by banks, as well as the vulnerability of the LIBOR rate to be manipulated. Banks and financial institutions today do not use LIBOR as the risk free rate at which they discount cash flows. Rather they use the Overnight Indexed Swap rate (OIS). The OIS rate is perceived to be nearly risk-free, and the spread between LIBOR – OIS is closely monitored in markets to gauge banks' willingness to lend to one another, as well as overall liquidity and financial conditions. As Hull states in the assigned reading, "In normal market conditions, it [the LIBOR – OIS spread] is about 10 basis points." However, during and in the wake of the financial crisis the spread experienced unprecedented volatility, as well as spikes as high as several hundred basis points. Clearly, this rate is not risk-free by any means, as the risk-free rate is supposed to be fairly constant. The OIS rate on the other hand did remain fairly constant compared to LIBOR. As a result banks today discount cash flows at the OIS rate. This has led to a phenomenon called *dual-curve stripping*. Dual curve stripping arises from the fact that, while the LIBOR $\pm$any spread is used to *infer* the future cash flows on the floating leg of, e.g. a swap, the rate which is used to discount the future cash flows is the OIS rate, hence the name *dual-curve* stripping. As a consequence, at inception, the value of a swap may not equal zero, although it is likely to be fairly close[8]. There is however a logical fallacy in Hull's statement. Although the historical average has been a LIBOR-OIS spread of roughly 10 basis points that does *not* imply that this will continue in the future. Moreover, the notion of, "normal market conditions" is at best highly stylized; it presupposes that there is such a thing as normal market conditions, AND that we as observers are able to infer what is and what is not *normal.* That is a bold assumption indeed.

[8] One can of course also force the value to be zero at inception by solving recursively for the spread.

Explain how a plain vanilla interest rate swap can be used to transform an asset or a liability and calculate the resulting cash flows

If a company owns a fixed-rate bond (e.g., say the company receives 5% fixed), then it can enter into an interest-rate swap in order to transform its asset. Under the swap, the company may pay 5% fixed and receive LIBOR +. After the swap, the company is earning a variable rate.



Explain the role of financial intermediaries in the swaps market

Usually two non-financial swap counterparties do not deal with each other directly
Financial intermediary may earn 3 or 4 basis points (0.03% or 0.04%) on a pair of offsetting transactions
In practice, intermediary is prepared to enter swap without having offsetting swap (warehousing)

Describe the role of the confirmation in a swap transaction

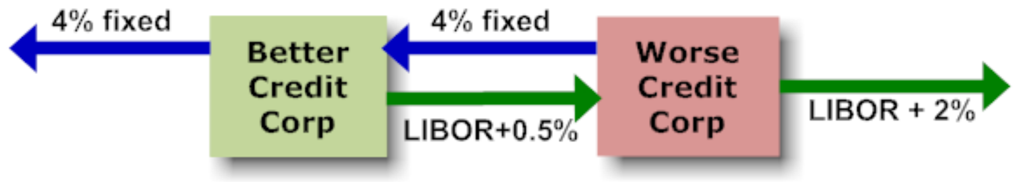
Confirmation is a legal agreement underlying a swap and is signed by representatives of two parties.
Drafting of confirmations is facilitated by the ISDA
ISDA has produced a number of master agreements and so called credit support annexes that include well-defined clauses. In the US, it is commonplace for companies to require an ISDA be entered into before entering into a swap transaction. This is then supplemented with a credit support annex, which stipulates further terms specific to the transaction. An ISDA can include description of the terms of the swap contracts including such as netting arrangements, collateral and non-performance clauses. It would be highly unusual for a firm to enter into a swap transaction in the OTC market without some form of agreement such as the ISDA.

Describe the comparative advantage argument for the existence of interest rate swaps and discuss some of the criticisms of this argument

The comparative-advantage argument is used to explain the popularity (or utility) of swaps. Consider two companies: BetterCreditCorp has a better credit rating than WorseCreditCorp. Their respective borrowing rates are given below:

	Fixed	Floating
BetterCreditCorp	4%	LIBOR + 1%
WorseCreditCorp	6%	LIBOR + 2%

Now assume that these two corporations enter into an interest rate swap. BetterCreditCorp will pay LIBOR + 0.5% to WorseCreditCorp and WorseCreditCorp will pay 4% fixed to BetterCreditCorp (we are ignoring transaction costs):



Under this swap, BetterCreditCorp is paying LIBOR + 0.5% (since the fixed payments from WorseCreditCorp essentially pass-through) and WorseCreditCorp is paying 5.5% fixed (i.e., 4% fixed to BetterCreditCorp plus 1.5 on the additional LIBOR). Notice that both have improved their cost of capital:

- BetterCreditCorp pays LIBOR + 0.5%: 0.5% less than its "competitive" floating rate, whilst
- WorseCreditCorp pays 5.5% fixed: 0.5% less than its "competitive" fixed rate

Notice that BetterCreditCorp has an advantage in *both* the fixed and the floating market. When one company has an advantage in a market, it is called an *absolute advantage*. It is important to note that it is a *comparative advantage* and not an *absolute advantage* that make the above savings possible: by engaging in a mutually beneficial transaction, both parties are better off. Why is it that even BetterCreditCorp, which has an absolute advantage in both markets can benefit from transacting with WorseCreditCorp? Because the 2% spread in their fixed rates (6% - 4% = 2%) is greater than the 1% spread in their variable rates (LIBOR + spread). WorseCreditCorp is said to have a *comparative advantage* in the floating-rate market; and BetterCreditCorp is said to have a *comparative advantage* in the fixed-rate market. To generalize, we can say that a *comparative advantage* exists when two companies face different interest rate markets: the difference in fixed rate markets (i.e., between the companies; call this "a") is greater than the difference in floating rate markets (call this "b"). Under these circumstances, a swap arrangement can produce a total gain, that is, to both parties, before any transaction costs, equal to: a–b. Consequently, if both firms could borrow in the fixed and the floating market at exactly the same terms, neither firm would have a comparative advantage, and no transaction would take place.

> IMPORTANT CONCEPT:
>
> A *comparative advantage* exists when [two] companies face different interest rates in the market. Both companies benefit from entering into a swap, even if one company has an *absolute advantage* in both the fixed and the floating market. This is a common test question so be sure you know the difference, and how to calculate the gain to each party.

Criticism of the comparative advantage argument

The contrary view concerns arbitrage: if markets were efficient, we would expect the differentials, that allow for the comparative advantage in the first place, to erode. A further

criticism is the duration difference between the typical market rates: the floating rate is typically LIBOR + a spread, and is adjusted, or *reset*, every six months; while the fixed rate loan may be longer. The comparative advantage argument assumes that the floating rates will not adjust and converge, an assumption, which in practice does not hold up.
The 4.0% and 5.2% rates available to AAA Corp and BBB Corp in fixed rate markets are 5-year rates, however; the LIBOR−0.1% and LIBOR+0.6% rates available in the floating rate market are six-month rates. BBB Corp's fixed rate depends on the spread above LIBOR it borrows at in the future.

Explain how the discount rates in a plain vanilla interest rate swap are computed

LIBOR rates are observable only for short time periods, typically from one day to one year. For longer durations, typically 1 year to 3 years, Eurodollar Futures are used, and then from year 3 to year 30, the "swap curve" is used. The reason multiple curves are used has to do with the liquidity of the instrument at the different time-horizons. To compute the discount rate for the LIBOR/swap zero curve, we can use the bootstrap method.

For example:
Assume that LIBOR/swap zero rates are given: six-month = 3%, one-year = 3.5%, and eighteen months = 4%.
The 2-year swap rate is 5%, which implies that a $100 face value bond with a 5% coupon will sell exactly at par (why? Because the 5% coupons are discounted at 5%[9])
We can solve for the two year zero rate (R) because it is the unknown

Period	Cash flow	LIBOR/swap zero rates	Present Value of Cash Flow
0.5	$2.5	3.0%	$2.46
1.0	$2.5	3.5%	$2.41
1.5	$2.5	4.0%	$2.35
2.0	$102.50	X?	102.5e-2R
		Total PV	**$100.00**

We solve for R as follows:

$$2.5e^{(-.5)(3\%)} + 2.5e^{(-1)(3.5\%)} + 2.5e^{(-1.5)(4\%)} + 102.5e^{-2R} = 100$$
$$2.46 + 2.41 + 2.35 + 102.5e^{-2R} = 100$$
$$e^{(-2R)} = 0.90506 \quad \rightarrow \quad R = 4.99\%$$

9 We will shortly see that this is not strictly true in practice.

Calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions

Interpretation of Swap

If two companies enter into an interest rate swap arrangement, then one of the companies has a swap position that is equivalent to a long position in floating-rate bond and a short position in a fixed-rate bond.

$$V_{Swap} = B_{Float} - B_{Fixed}$$

The counterparty to the same swap has the equivalent of a long position in a fixed-rate bond and a short position in a floating-rate bond:

$$V_{Swap\ Counterparty} = B_{Fixed} - B_{Float}$$

For a fixed-rate payer (who therefore receives floating), the value of an interest rate swap is:

$$V_{Swap} = B_{Float} - B_{Fixed}$$

Here is the notation:	Symbol
Time until i^{th} payments are exchanged:	t_i
Notional principal in swap agreement:	L
Fixed payment made on each payment date:	k
The next floating-rate payment to be made on the next payment date:	k*

The swap is the present value of receive-floating cash flow stream minus the present value of the pay-fixed cash flow stream:

$$V_{Swap} = B_{Float} - B_{Fixed}$$

The value of the fixed rate cash flows requires the discounting of each coupon and the final payment:

$$B_{Fixed} = \sum_{i=1}^{n} ke^{-r_i t_i} + Le^{-r_n t_n}$$

The floating-rate stream is easier! We only need to discount the sum of the notional principal (L) and the next floating-rate payment:

$$B_{Float} = (L + k^*)e^{-r_1 t_1}$$

Let’s look at an example:
We assume a notional principal of $100 million (L = $100 million). In this case, we will receive fixed-rate payments at 4% per annum. The LIBOR rates at 3-months, 9-months, and 15-months are, respectively, 5%, 6% and 7%. The 6-month LIBOR is 5.5% and our company is going to “pay floating” such that the first floating payment is based on this six-month LIBOR. The swap expires in 15 months. We’ll assume the LIBOR curve does not shift over time.

Assumptions	
Notional	100
Receive Fixed	4.0%

LIBOR Rates	T	
3 Months	0.25	5.0%
6 Months	0.50	5.5%
9 Months	0.75	6.0%
12 Months	1.00	6.5%
15 Months	1.25	7.0%

	LIBOR	Disc.	Fixed Cash Flows		Floating Cash Flows	
Time	Rates	Factor	FV	PV	FV	PV
0.25	5.0%	0.988	$2.0	$1.98	$102.75	$101.47
0.75	6.0%	0.956	$2.0	$1.91		
1.25	7.0%	0.916	$102.0	$93.45		
			Total	$97.34		$101.47
	Value (swap) =		$97.34 - $101.47 =		-$4.13	

First, we compute the present value of the (received or incoming) fixed cash flow stream. That requires us to discount the $2 (1/2 of the 4% per annum) to be received in three months (4% annual = $2 semi-annual) and again in nine months; finally, we discount the lump sum to be received in fifteen months ($102). The present value of the fixed cash flow stream is $97.34.

For the present value of the floating-rate cash flow stream, we only need to value one cash flow at three months: 2.75% of the notional ($2.75 because it’s a semi-annual payment on 5.5%) plus the notional ($100) equals $102.75. That’s the future value in three months, so we discount it to get $101.47.

The value of the swap, to our floating-rate payer, is the difference between the present value of the fixed cash flow stream they are receiving (97.34) and the present value of the floating rate stream they are paying (101.47).

Another example (using our learning spreadsheet):
In this case, the fixed-payments are 8% per annum, paid semi-annually on $100 million notional. The swap is being valued at the midpoint between swap settlements; i.e., the last swap was three months prior and the next swap occurs three months' forward.

Assumptions			
Notional		100	
Receive Fixed		8.0%	
LIBOR @ last coupon		10.2%	<< 1st floating rate in semi-annual

Time	0.25	0.75	1.25
LIBOR	10.0%	10.5%	11.0%
Discount Factor	0.975	0.924	0.872

We only need to value one cash flow on the floating-rate side because, when the next coupon pays, the floating-rate bond must be priced at par!

Time	0.25	0.75	1.25	
LIBOR	10.0%	10.5%	11.0%	
Discount Factor	0.975	0.924	0.872	
Value Interest Rate Swap as Two Bonds				
Floating Cash Flows				
Future value (FV)	$105.10			
Present value (PV)	$102.51			$102.51
Fixed Cash Flows				
Future value (FV)	$4.00	$4.00	$104.00	
Present value (PV)	$3.90	$3.70	$90.64	$98.24
Net Value				$4.27

Calculate the value of a plain vanilla interest rate swap from a sequence of forward rate agreements (FRAs)

To value the swap as a sequence of forward rate agreements (FRAs), the procedure is essentially similar. In this case, assume (this example replicates the John Hull example in the text) a notional of $100 million, a receive-fixed rate of 8%, and the following LIBOR rates: 3-month @ 10%, 6-month @ 10.2%, 9-month @ 10.5% and 15-months @ 11%.
The key difference here is that we calculate the forward rate in three months and in nine months; specifically, we want to calculate the 3 x 9 (6 month rate when contract expires in three months) and the 9 x 15 (6 month rate when contract expires in 9 months). So the first forward rate (3 x 9) is given by [(10.5%)(0.75) – (10%)(0.25)]/(0.5) = 10.75% in continuous terms; this is converted to a semi-annual basis: 11.04% = (2)[EXP(10.75%/2)-1].

Value Interest Rate Swap as Forward Rate Agreements (FRA)				
Time	0.25	0.75	1.25	
LIBOR (copied)	10.00%	10.50%	11.00%	
Forward rates (continuous)		10.75%	11.75%	
Forward (semi-annual)	10.20%	11.04%	12.10%	
Floating CF (FV)	$5.10	$5.52	$6.05	
Fixed CF (FV)	$4.00	$4.00	$4.00	
Net Cash flows (FV)	$1.10	$1.52	$2.05	
Net Cash flows (PV)	$1.07	$1.41	$1.79	$4.27

Explain the mechanics of a currency swap and compute its cash flows

A currency swap exchanges principal and interest in one currency for principal and interest in another currency[10]. The valuation of currency swap is given by:

$$V_{SWAP} = B_D - S_o B_F$$
$$V_{SWAP} = S_o B_F - B_D$$

So, in the first case, the valuation of a swap that pays in US dollars and receives a foreign currency bond involves subtracting the foreign bond after translating its value based on the exchange rate. In effect, the exchange rate allows you to “standardize” on US dollars and take the difference in values.

[10] In practice, the principal is often not actually exchanged in a plain vanilla swap; rather it merely serves as the basis from which interest rate payments are calculated. In a currency swap however, the principal is generally exchanged.

For example:
In this case, our company enters in a currency swap where it receives yen (¥) at 8% and pays US dollars at 5% (once per year, to keep it simple). In regard to principal amounts, we have $10 million US dollars and 1,000 million yen. The LIBOR/swap interest rates (we need these to discount the cash flows – these are the relevant market interest rates) are flat at 6% in the US and 4% in Japan. The swap matures in three years. The assumptions are:

Assumptions	
Principal, Dollars ($MM)	10
Principal, Yen (MM)	¥ 1,000
FX rate	110
US rate	6.0%
Japanese rate	4.0%

SWAP:	
PAY dollars @	5%
RECEIVE yen @	8%

And the calculations are given by:

	Dollars (MM)		Yen (MM)	
Time	FV	PV	FV	PV
1	0.5	$0.47	80	¥ 77
2	0.5	$0.44	80	¥74
3	0.5	$0.42	80	¥71
3	10	$8.35	1000	¥887
		$9.68		¥1,109

Yen bond	¥ 1,109
Yen bond in US dollars	$10.08
Dollar bond	$9.68
Swap, yen bond - dollar bond	$0.39

Our company is paying dollars, specifically 5% of $10 million or $500,000 (0.5 million) for each year until the third year, when the $10 million principal is also paid. These cash flows are discounted at the U.S. rate of 6%. (We assume a flat interest rate curve, otherwise we would discount at the relevant spot rate). The sum of the discounted dollars is about $9.68. A similar calculation is performed on the yen that are received. Our company receives 8% of 1,000 yen or 80 million yen per year, and 1,000 million yen in principal on the third year. These cash flows are discounted at the Japanese interest rate of 4% (also a flat yield curve for simplicity's sake). The present value of the yen bond is about ¥1,109. But that is denominated in yen, so we convert that amount—based on our exchange rate of 110 yen to the dollar—to get a dollar value of $10.08 for the cash flows that are received.
The final step is to deduct the present value of the dollar bond (i.e., that will be paid) from the present value of the yen-based bond (i.e., that will be received).

Describe the comparative advantage argument for the existence of currency swaps

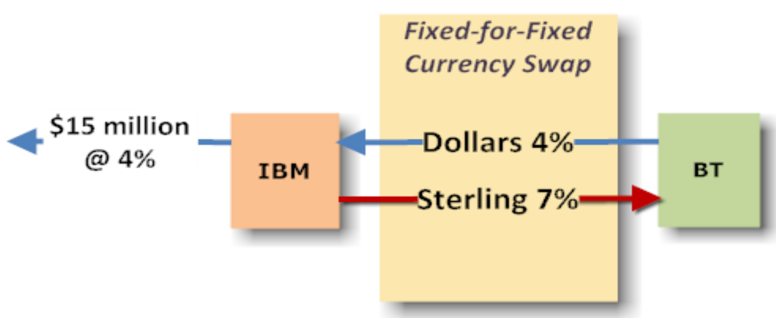
General Electric wants to borrow AUD
Qantas wants to borrow USD

	USD	AUD
General Electric	5.0%	7.6%
Qantas	7.0%	8.0%

Hull argues that comparative advantages for plain vanilla interest rate swaps are, "largely illusory." But in a currency swap, advantages can indeed be genuine. The example from the assigned reading presents the case of General Electric and Qantas Airways, and their respective borrowing rates in USD and AUD as represented in the table above. A situation like this can arise since General Motors is a US based company and Qantas is an Australian based company. Thus, in light of their consolidated income being in USD and AUD, respectively, the companies may benefit from a currency swap due to the potentially lower taxes it affords the companies. Although not mentioned specifically in the reading, perhaps the greatest source of this tax savings in practice arise in the case when there is an intercompany loan from, e.g. the US entity to the Australian entity. The currency swap can then be part of a structured transaction in order to avoid double taxation, that is, being taxed in both countries.

Explain how a currency swap can be used to transform an asset or liability and calculate the resulting cash flows

A currency swap entails the exchange of principal and interest in one currency for the principal and interest in another currency. Unlike the plain vanilla interest rate swap, principal typically is exchanged at the start and end of a currency swap.



Calculate the value of a currency swap based on two simultaneous bond positions

A currency swap exchanges principal and interest in one currency for principal and interest in another currency. The valuation of currency swap is given by:

$$V_{SWAP} = B_D - S_o B_F$$
$$V_{SWAP} = S_o B_F - B_D$$

Assumptions	
Principal, Dollars ($MM)	10
US rate	9.0%
Principal, Yen (MM)	¥1,200
Japanese rate	4.0%
FX rate (yen/dollar)	110
FX rate (dollar/yen)	0.009091
Swap:	
PAY dollars @	8.0%
RECEIVE yen @	5.0%

Value Interest Rate Swap as Two Bonds					
Pay Dollars					
Future value (FV)	$0.80	$0.80	$0.80	$10.00	
Present value (PV)	$0.73	$0.67	$0.61	$7.63	$9.64
Pay Yen					
Future value (FV)	¥60.00	¥60.00	¥60.00	¥1,200.00	
Present value (PV)	¥57.65	¥55.39	¥53.22	¥1,064.30	¥1,230.55
Present value (PV), US Dollars					$11.19
Net Value					$1.54

Calculate the value of a currency swap based on a sequence of FRAs

In this case, forward rates are calculated.

Value Interest Rate Swap as Forward Rate Agreements (FRA)				
Pay Dollars				
Future value (FV)	$0.80	$0.80	$0.80	10
Pay Yen				
Future value (FV), Yen	¥60.00	¥60.00	¥60.00	¥1,200.00
Forward Rate (IRP)	0.009557	0.010047	0.010562	0.010562

Dollar value of Yen CF	$0.57	$0.60	$0.63	$12.67	
Net Cash Flow (FV)	-$0.23	-$0.20	-$0.17	$2.67	
Net Cash Flow (PV)	-$0.21	-$0.16	-$0.13	$2.04	$1.54

Describe the role of credit risk inherent in an existing swap position

Because a swap involves offsetting position, there is no credit risk when the swap has negative value. Credit risk only exists when the swap has positive value. Further, because principal is not exchanged at the end of the life of an interest rate swap, the potential default losses are much less than those on an equivalent loan. On the other hand, in a currency swap, the risk is greater because currencies are exchanged at the end of the swap.
Note the distinction between credit risk and market risk. In the case of a swap, credit risk includes the risk the counterparty will defaults; market risk includes the risk that interest rates and/or currency exchanges rates vary unfavorably.

Identify and describe other types of swaps, including commodity, volatility and exotic swaps

The examples in the text refer to a typical "plain vanilla" interest rate swap:

LIBOR is the floating reference rate
Tenor (payment frequency) is six months

Other types of swaps include:

- **Amortizing Swap:** principal reduces in predetermined way
- Step-up swap: principal increases
- Deferred (forward) swap: parties begin exchange in future
- Variations on vanilla fixed-for-floating interest rate swap: tenor is 1 month, 3 months, or 12 months; floating tenor does not match fixed-rate tenor; rates other than LIBOR (commercial paper)
- Principal can vary throughout the swap term.
- Amortizing swap: principal reduces in a predetermined way.
- Step-up swap: principle increases in a predetermined way.
- Forward (deferred) swap: parties do not begin exchange until some future period.
- **Constant maturity swap (CMS swap):** an agreement to exchange a LIBOR rate for a swap rate. In a constant maturity Treasury swap (CMT swap), the counterparties agree to swap a LIBOR rate for a Treasury rate.
- Constant maturity Treasury swap: LIBOR for T-rate
- Compounding swap: interest on one or both sides is compounded forward to the end of the swap's life.
- LIBOR-in arrears: LIBOR rate used for current (not next) payment.
- Accrual: interest on one side accrues if floating rate within a range.

- Variations on fixed-for-fixed currency swap. A cross-currency swap is essentially a fixed-for-floating interest rate swap plus a fixed-for-fixed currency swap. Also, both sides can float in a floating-for-floating currency swap.
- In a quanto, a rate observed in one currency is applied to a principal amount in another currency.
- Equity swaps: agreement to exchange total return (gains plus dividends) realized on an equity index in exchange for LIBOR (both on the same principal).
- Options embedded in swaps:
    - Extendable swap (one counterparty has the option to extend the swap
    - Puttable (one party has the option to terminate early)
    - **Swaptions** (options on swap).

Chapter Summary

Swaps are instruments entered into by two or more parties in order to swap one cash flow for another. In theory we discount the cash flows from the swaps by the respective LIBOR rates, but in practice, the OIS (Overnight Indexed Swap) rate is used as the risk-free rate, while LIBOR is used as the index for one of the swaps' legs to infer the future cash flows, which leads to what is called *dual-curve* stripping.

The most common forms of swaps by far are interest rate swaps and currency swaps. In an interest rate swap a company swaps a cash flow based on either a fixed or floating rate, in exchange for a cash flow based on another rate or index, which can be either fixed or floating. The most common form of interest rate swap is the fixed-float swap, however, fixed-fixed and float-float swaps are also common. In a currency swap it is more common [than for an interest rate swap] that both legs are floating, based on their respective currencies LIBOR rate.

We explored why companies might enter into a swap in the first place, and discussed the theory of *comparative advantage* in which both parties can benefit from exchange, even though on party may have an *absolute advantage* in both fixed and floating rates. This is a powerful economic theory, which applies universally to all forms of exchange, and is, in particular a key argument for the benefits of trade [between nations]. We saw that for swaps in particular, the theory of comparative advantage was questionable as applied to plain vanilla swaps, however, firms may derive real tax advantages from using currency swaps.

The *bootstrap* method for calculating interest rates was used to explain how we might compute the discount rates in a plain vanilla interest rate swap.

Two techniques may be employed in order to value vanilla interest swaps, as well as a currency swaps. These include:

1. Calculating the value of the swap based on two simultaneous bond positions (long one bond, short another), and
2. Calculating the value of the swap from a sequence of Forward Rate Agreements (FRAs).

Swaps can be trade both on an exchange or OTC. The OTC volume by far exceeds the exchange-traded volume, albeit recently introduced legislation might shift the balance by forcing such as sell-side firms to use an exchange/clearinghouse for their transactions. When traded OTC, financial intermediation is typically performed by brokers or banks such as to reduce the Credit risk and match the parties involved in the transaction. This means that the financial intermediary takes on credit risk as it guarantees payment, even in the event of non-performance by one of the parties.

A currency swap is valued just like a plain vanilla swap, only with an exchange rate component attached to it.

7 Questions & Answers

Questions

174.4. Company A can borrow at 7.0% in fixed rate markets and LIBOR plus 100 basis points in floating rate markets. If Company B, which is riskier, can borrow at 7.8% in fixed markets, at which borrowing rate in the floating rate market would any comparative borrowing advantage be neutralized such that NEITHER company has a comparative advantage in the fixed nor floating rate market?

a) LIBOR plus 20 basis points
b) LIBOR plus 80 basis points
c) LIBOR plus 120 basis points
d) LIBOR plus 180 basis points

175.4. Consider four statements about the 5-year swap rate:

a) The 5-year swap rate is the average of the bid and offer fixed rates that a market maker is prepared to exchange for LIBOR in a standard plain vanilla 5-year swap
b) The 5-year (par) swap rate is the 5-year LIBOR/swap par yield; i.e., the fixed rate that makes the value of the swap equal to zero
c) The 5-year swap rate should be greater than the 5-year AA-rated borrowing (lending) rate
d) The 5-year swap rate is a pure risk-free (riskless) rate

Which are true?
1. a only; 2. a & b only; 3. b, c & d; 4. all of the above

176.6. Consider two companies. Company A can borrow euros (EUR) at 5.0% or dollars (USD) at 4.0%. Riskier Company B can borrow EUR at 6.5% or USD at 6.0%. EACH of the following is TRUE except:

a) Company A has an absolute advantage in both markets
b) Company A has a comparative advantage borrowing dollars (USD)
c) Company B has a comparative advantage borrowing euros (EUR)
d) Company B has a comparative advantage in NEITHER market\

178.5. With respect to a generic commodity swap, EACH of the following is true EXCEPT:

a) A commodity swap is effectively a series of forward contracts on a commodity with different maturity dates and the same delivery prices
b) A Commodity swap is financially settled and does not involve any physical delivery
c) Like a vanilla interest rate swap, notional is not exchanged
d) Unlike a vanilla interest rate swap, exchanged payments are NOT netted

Answers

175.4. b. I. and II. only
Both (I.) and (II.) are true but:
in regard to (III.) the X-year swap rate should be LESS THAN the X-year lending/borrowing rate;
in regard to (IV.), like LIBOR, the swap rate is NOT riskless as there is some counterparty (default) risk. They are "near riskless."

175.4. 2. a & b only
Both (a.) and (b.) are true but:
in regard to (c.) the X-year swap rate should be LESS THAN the X-year lending/borrowing rate;
in regard to (d.), like LIBOR, the swap rate is NOT riskless as there is some counterparty (default) risk. They are "near riskless."

176.6. D. As (C) is true, (D) must be false.
If the differential in differentials is non-zero, both must have a comparative advantage. The differential in differentials, in this case, is:
(USD 6.0% - USD 4.0%) - (EUR 6.5% - EUR 5.0%) = 2.0% - 1.5% = 0.5%.

178.5. D. Payments are netted: the generic commodity swap is just like an interest rates swap, the only difference is that the underlying good is a commodity rather than cash such that the reference is a spot price rather than an interest rate.

Hull, Chapter 10: Properties of Stock Options

Learning Outcomes:

Identify the six factors that affect an option's price and discuss how these six factors affect the price for both European and American options.

Identify, interpret and compute upper and lower bounds for option prices.

Explain put-call parity and calculate, using the put-call parity on a non-dividend-paying stock, the value of a European and American option, respectively.

Explain the early exercise features of American call and put options on a non-dividend-paying stock and the price effect early exercise may have.

Explain the effects of dividends on the put-call parity, the bounds of put and call option prices, and the early exercise feature of American options. Conversion of a discount rate to a price for a U.S. Treasury bill.

Identify the six factors that affect an option's price and discuss how these six factors affect the price for both European and American options

In the chart below, we show the directional impact of each input on the value of a call or put:

Directional Impact of Each Input on Call or Put

Factor	Symbol	Call	Put
Stock price (↑)	S	↑+	↓ -
Strike price (↑)	X	↓ -	↑+
Time to expire (↑)	T	↑+ (American) ⇔ (European)	↑+ (American) ⇔ (European)
Volatility (↑)	σ	↑+	↑+
Risk-free rate (↑)	r_f	↑+	↓ -
Div. yield (↑)	D	↓ -	↑+

Stock price: For a call option, a higher stock price implies greater intrinsic value
Strike price: For a call option, a higher strike price implies less intrinsic value
Time to expiration: For an American option (call or put), option value is an increasing function with greater time to expiration. For a European option, while typically value with

increase with greater time to expiration, the timing of dividends makes the relationship ambiguous (on dividend payout, the stock tends to drop).
Volatility: Greater volatility increases the value of both a call and a put option
Risk-free rate: For a European call option, consider that the minimum value of an option is the stock minus the discounted strike price. A higher risk-free rate implies a lower discounted strike price; therefore, a higher risk-free rate increases the value of the call option.
Dividend yield: As the option holder forgoes the dividend, a higher dividend reduces the call option's value.
Please note that the stock price, the strike price, the riskless rate and the time to expiration (T) are easily seen in the Black-Scholes-Merton; volatility is contained in the d1 and d2.

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

Identify, interpret and compute upper and lower bounds for option prices

Upper Bounds $\quad c \le S_0 \quad C \le S_0$

$\quad p \le X \quad P \le X$

Lower bound for European CALL non-dividend paying stock:

$$c \ge \max(S_0 - Ke^{-rT}, 0)$$

Lower bound for European PUT on non-dividend paying stock:

$$p \ge \max(Ke^{-rT} - S_0, 0)$$

To illustrate, assume a call option (c) with a strike price of $10 (K = $10) where the current stock price is also $10 (S = K = $10. We say here the option is granted at FMV). The upper bound on the call option is $10: why would you pay more for an option than you could pay for the stock? The lower bound is the maximum of zero (0) and (10 - 10e-rT). So if the option has a one-year term and the risk-free rate is 5%, then the lower bound is [10-10e(-5%)(1)] ≅ $0.488. That is the so-called minimum value: you would be willing to pay at least $0.49.

The lower bound on a European call is the stock price minus discounted strike price. This is called the call option's minimum value and is the price the Black-Scholes gives if the volatility input is equal to zero. The lower bound on a European put is discounted strike price minus the stock price.

Explain put-call parity and calculate, using the put-call parity on a non-dividend-paying stock, the value of a European and American option

Put–call parity is based on a no-arbitrage argument; it can be shown that arbitrage opportunities exist if put–call parity does not hold. Put–call parity is given by:

$$c + Ke^{-rT} = p + S_o$$

$$c = p - Ke^{-rT} + S_o$$

To illustrate, assume two portfolios:

The first portfolio is a call option with a strike of $10 combined with a $10 par bond. The second portfolio is a put with a strike of $10 and a single share of stock priced at $10 (i.e., a protective put). Now consider the payoff of each portfolio if the stock increases to $13. The payoff on the first portfolio = $3; the option gain plus $10 bond = $13. The payoff on the second portfolio = $13 stock price. Now consider the payoff of each portfolio if the stock drops to $7. The payoff on the first portfolio = $10 bond. The payoff on the second portfolio = $3 gain on put option + $7 stock = $10. The portfolios have the same payoff regardless of the stock price!

Please be ready to re-arrange put-call parity. For example:

$$c + Ke^{-rT} = p + S_o$$

$$\rightarrow c - p = S_o - Ke^{-rT}$$

Typical question:

The typical application is to solve for the price of a call or put given the other variables. For example, assume we know that a one-year European put is valued at $2. If the risk-free rate is 4%, what the value of the corresponding European call (i.e., one-year term) if the strike price is $10 (K = $10) and the stock price is $11 (S = $11)?

$$c = S_o + p - Ke^{-rT} \Rightarrow 11 + 2 - (10)e^{-(.04)(1)} = \$3.39$$

The following shows two examples (Hull Ex 8.1 and Hull Ex 8.2) that apply put-call parity. Note in the first case (Ex 8.1) the lower bound on the call option is given by the stock price ($51) minus the discounted strike price: lower bound = $\$51_{Stock} - \$50 * e^{-12\%*0.5} = \$3.91$. The second example (second column, Ex 8.2) computes the lower bound of a European put. The lower bound here is given by $\$40 * e^{-10\%*0.25} - 38 = \$1.01$

	Ex. 8.1 (Call)	Ex 8.2 (Put)
Stock	$51.00	$38.00
Strike	$50.00	$40.00
Risk-free rate	12%	10%
T	0.5	0.25
Volatility	20%	20%

Scenarios: Don’t worry about the Black-Scholes (BS) price for now, just follow the examples and the

Stock goes up to	$60	$45	
Long call + lend strike	$60.00	$45.00	
Protective put	$60.00	$45.00	
Stock goes down to	$40	$35	
Long call + lend strike	$50.00	$40.00	
Protective put	$50.00	$40.00	
Lower Bound, Call	**$3.91**	-$1.01	
Lower Bound, Put	$3.91	**$1.01**	
d1	0.64	(0.21)	
d2	0.49	(0.31)	
Call (c) Black-Scholes (BS)	$5.15	$1.08	don’t worry about the BS calculations
Put (p) Black-Scholes (BS)	$1.24	$2.09	for now

Put-Call Parity

Discounted Strike	$47.09	$39.01
Long call	$5.15	$1.08
$c + Ke^{-rT}$	**$52.24**	**$40.09**
Stock	$51.00	$38.00
Long put	$1.24	$2.09
$S_0 + p$	**$52.24**	**$40.09**

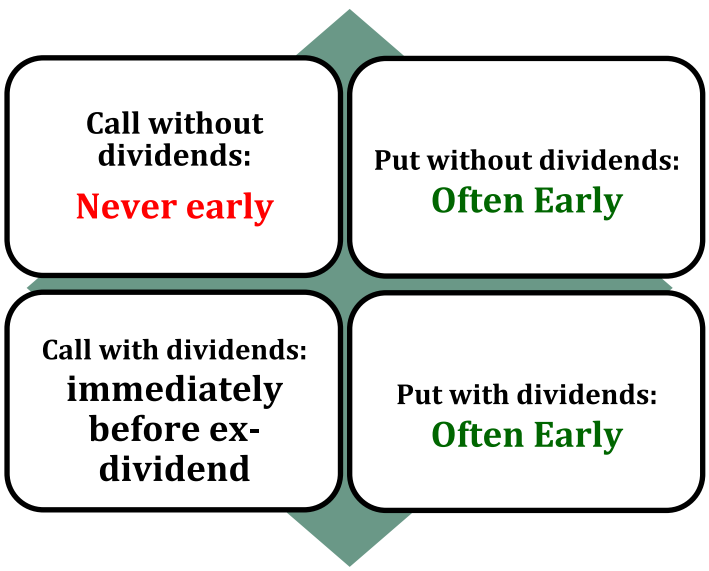
Explain the early exercise features of American call and put options on a non-dividend-paying stock and the price effect early exercise may have

An American-style option can be exercised prior to expiration:
American option: can be exercised before expiration (early exercise)
European option: can only be exercised on the expiration date itself
All other things being equal, the value of an Amerian style option must be at least as great as a European option with the same features; Value [American option] ≥ Value [European option]
Strictly speaking, the put–call parity relationship applies to European options (note the use of small ‘c’ and small ‘p’ in the equation). An American call on a non-dividend paying stock must be worth at least its European analogue.
The difference between an American call and an American put (C–P) is bounded by the following:

$$S_0 - K \leq C - P \leq S_0 - Ke^{-rT}$$

From a mathematical standpoint, it is never optimal to execute an early exercise on an American call option on a non-dividend paying stock. However, it can be optimal to execute an early exercise on an American put. In general, we can say that for an American put, the early exercise becomes more attractive as:
Stock price (S0) increases,
Risk-free (r) rate increases, and/or
Volatility (σ) decreases.

Quoted Price + AI
Settle Price × CF + AI



Explain the effects dividends have on the put-call parity, the bounds of put and call option prices, and on the early exercise feature of American options

The ex-dividend date is specified when a dividend is declared. Investors who own shares of the stock as of the ex-dividend date receive the dividend.

An American option should never be exercised early in the absence of dividends. In the case of a dividend-paying stock, it would only be optimal to exercise immediately before the stock goes ex-dividend. Specifically, early exercise would remain sub-optimal if the following inequality applied:

$$D_i \leq K\left(1 - e^{-r(t_{i+1} - t_i)}\right)$$

Further, this inequality applies unless the dividend yield is “either close to or above the risk-free rate of interest”, which typically is not the case. Therefore, early exercise remains sub-optimal in most cases.

Put–call parity applies to European options (note the use of small ‘c’ and small ‘p’ in the equation).

An American call on a non-dividend paying stock must be worth at least its European analogue

The difference between an American call option and an American put option, (C–P), is bounded by the following:

$$S_o - K \leq C - P \leq S_o - Ke^{-rT}$$

Chapter Summary

In this chapter we explored the factors that affect the price of European and American put and call options. These factors, along with the boundary conditions that were established for the prices of the respective options should become second nature to you, as it has high-testability. Furthermore, understanding how these factors affect options prices is fundamental to understanding this and the following chapters.

The factors that affect the prices of an options were presented earlier in this chapter in this table, however, they are so important that it is repeated here. You should *know* and *understand* these by heart (not memorize).

Factor	Symbol	Call	Put
Stock price (↑)	S	↑+	↓ -
Strike price (↑)	X	↓ -	↑+
Time to expire (↑)	T	↑+ (American) ⇔? (European)	↑+ (American) ⇔? (European)
Volatility (↑)	σ	↑+	↑+
Risk-free rate (↑)	r_f	↑+	↓ -
Div. yield (↑)	D	↓ -	↑+

A [call] option is an option but not an obligation to purchase the underlying instrument at a predetermined price. It should therefore not be surprising that the price of the option can never exceed that of the underlying. This is the upper bound for both American and European call options. Analogously, the most one may gain from a put option is the strike price, K. This is the upper bound for the American put option; for the European equivalent, it is K discounted to the present value.

Put-call parity allows us to combine the stock price, strike price, and risk-free rate, along with, e.g., the put option price in order to uniquely solve for the call price. The same applies for solving for any of the other variables. This is a strict equality, which hold whether or not there are dividends involved or not.

For American options, the early exercise feature complicates the matter slightly. That is, we cannot create a strict equality but rather a set of inequalities that bounds the price of our put and call options. When there is no dividend being paid on an American call option, we can show by arbitrage arguments that it is never optimal to exercise early. For an American call option with dividends, it can be optimal to exercise immediately before the ex-dividend date if the dividend is sufficiently large. For dividend and non-dividend paying put options it can be optimal to exercise. The maximum we can ever hope to gain is max(K – S, 0), thus when the stock price is zero it is clearly optimal to exercise.

You should be comfortable with solving and re-arranging the following equation:
$S_o - K \leq C - P \leq S_o - Ke^{-rT}$ which bounds the American Call and Put option.

8 Questions & Answers

Questions

8.1 EACH of the following is true about option volatility EXCEPT:

a) The lower bound for a European call option is equal to the Black-Scholes-Merton option value where the volatility input is zero
b) The historical volatility, as in input into Black-Scholes-Merton, tends to converge on implied volatility as the option is nearer to being “at the money” (ATM)
c) Option value is an increasing function with volatility for an American or European CALL on a both a dividend- or non-dividend-paying stock
d) Option value is an increasing function with volatility for an American or European PUT on a both a dividend- or non-dividend-paying stock

8.2 What is the lower bound for the price of a nine (9)-month American PUT option on a non-dividend-paying stock when the stock price is $14.00, the strike price is $20.00, and the risk-free interest rate is 4.0% per annum?

a) zero (0)
b) $5.22
c) $5.41
d) $6.00

8.3 The price of a non-dividend-paying stock is $14.00 and the price of a three (3)-month European put option on the stock with a strike price of $15.00 is $1.85. The risk free rate is 4.0% per annum. What is the price of a three (3)-month European call option with a strike price of $15.00?

a) $0.65
b) $1.00
c) $1.15
d) $1.88

8.4 Each of the following statements is true EXCEPT for (which is the false statement?):

a) In the case of American options on a dividend-paying stock, put-call parity holds (can incorporate dividends)
b) In the case of European options on a dividend-paying stock, put-call parity holds (can incorporate dividends)
c) While is is never optimal to exercise early an American option on a NON-dividend-paying stock, it can be optimal to exercise early an American option on a dividend-paying stock
d) In the case of a dividend-paying stock, we can obtain upper and lower bounds for the difference between the price of an American call and an American put (i.e., C - P)

Answers

8.1 B. Historical volatility is backward-looking and does not necessarily correspond to forward-looking implied volatility for OTM/ATM/ITM options (the skew/smile of implied volatility is a different issue).
In regard to (A), this is true: BSM option value (if volatility = 0) gives same value as minimum value (lower bound).
In regard to (C) and (D), option value is always an increasing function of volatility.

8.2 D. $6.00
For a European put, the lower bound is p >= K*exp(-rT) - S(0), but
For an American put, the lower bound is P >= K - S(0). In this case, P >= 20 - 14 = $6.00

8.3 B. $1.00
c = p + S(0) - K*exp(-rT) = 1.85 + 14 - 15*exp(-4%*0.25) = $1.00

8.4 A. In the case of American options on a dividend-paying stock, put-call parity does not hold.
In regard to (B), (C) and (D), EACH are TRUE.

9 Hull, Chapter 11: Trading Strategies Involving Options

Learning Outcomes:

Explain the motivation to initiate a covered call or a protective put strategy.

Describe and explain the use and payoff functions of spread strategies, including bull spread, bear spread, calendar spread, butterfly spread, and diagonal spread.

Calculate the pay-offs of various spread strategies.

Describe and explain the use and payoff functions of combination strategies, including straddles, strangles, strips, or straps.

Compute the pay-offs of combination strategies.

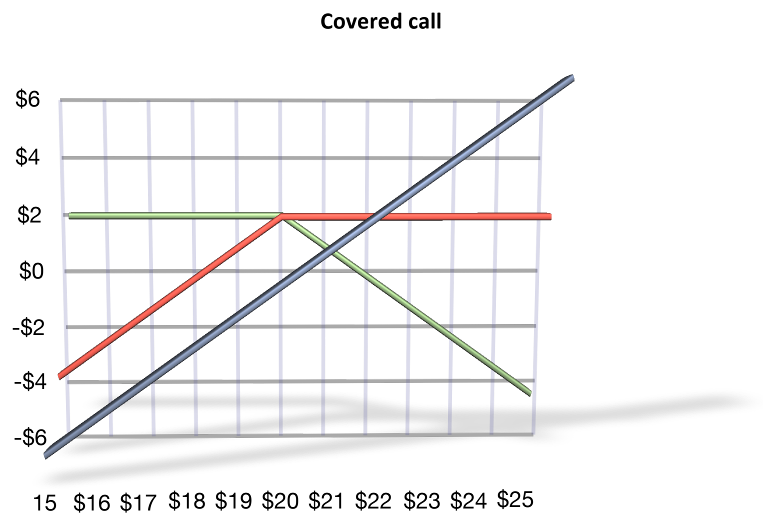
9.1 Explain the motivation to initiate a covered call or a protective put strategy and calculate the payoff functions of the respective strategies.

9.1.1 Covered Call

To "write a covered call" is to combine a long stock position with a short position in a call option. Writing a covered call = long stock + short call option. In many cases, the call option is out-of-the-money. The rationale of the covered call is either:

1. To generate income via the sale of the short call, or
2. To cover the cost of the potential short call payoff with the stock.

Covered call; Long stock @ $20 + Short call Strike @ $20 (premium = $1.99)
Writing a covered call is an income strategy.
Outlook is neutral to bullish.



Payoff: Red, **Option:** Green, **Stock:** Blue
Covered call: Long stock @ \$20 + Short call ($\boldsymbol{K_{Short\ call}} = \$\mathbf{20}$, premium = \$1.99)

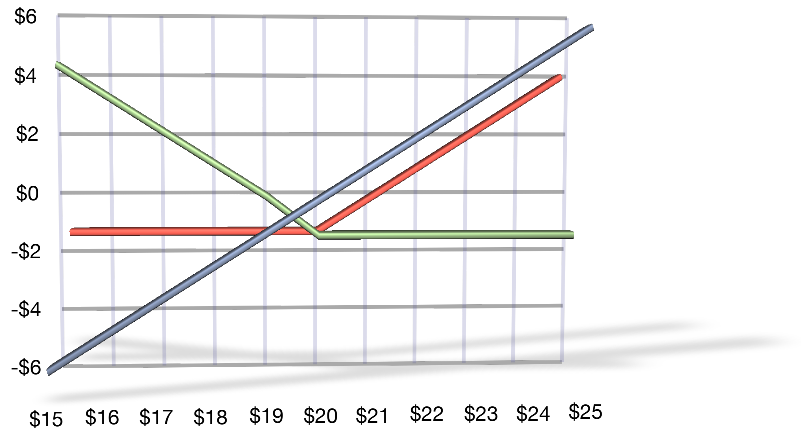
Writing a covered call is an income strategy, that is, we will enter into this trade if our outlook is neutral to bullish. That is, if the stock rises significantly, the call option will be exercised; however, we have the underlying to protect ourselves from that scenario. On the other hand, by writing a call option we collect the premium. Thus if the stock price stays about the same or rises modestly, we collect the premium. We can then use this strategy by rolling over the options each month, collecting our premium. At first look we might view this as an easy way to generate a healthy income each month. However, chances are that we are not the only ones who have thought of this; hence, it will be priced into the option. Accordingly, over time, when we take into account the transaction costs we incur, as well as the fact that from time to time the option will get exercised, this is a poor income generating strategy.

IMPORTANT CONCEPT:

If the payoff strategy looks like a short put option, that's because it is! Remember the put-call parity $S_0 - c = Ke^{-rt} - p$. We can infer from this that going long the stock and short a call is the same as going short a put with the PV of the strike price in the bank. By knowing the intuition behind the put-call parity you can often reason your way to the answer!

9.1.2 Protective Put

A protective put can be thought of as a form of insurance. Indeed, looking at the strategy and the payoff, it looks like we have created a synthetic call option! Again, the put-call parity comes in handy. It is tempting to think that having protective puts on your portfolio and rolling them over at maturity is a great way to benefit from the potential increase in the stock price while having our losses capped. However, the premium paid and transaction costs incurred dilute the profits form such a strategy, just like in the case of the Covered call. We can generalize this concept further by noting that, after adjusting for risk, there is no one strategy that offers an easy way to make money in a [weak form] efficient market. There is no such thing as a free lunch.



Payoff: Red; Option: Green; Stock: Blue
Protective put: Long stock @ $20 + Long put ($K_{long\ put}$ = $20, premium = $1.20)

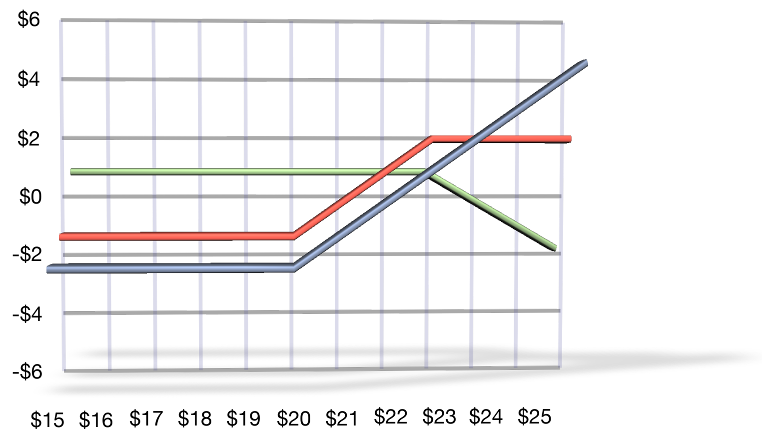
> Important Concept:
>
> The covered call generates income (the short call option premium) when the (long) stockholder does not expect further price appreciation on the long position. The protective put forfeits some income (the long put option premium) in exchange for downside protection.

9.2 Describe and explain the use and payoff functions of spread strategies, including bull spread, bear spread, calendar spread, butterfly spread, and diagonal spread.

A spread strategy is a position with two or more options of the same type, i.e., two or more calls; or, two or more puts.

9.2.1 Bull spread (type of vertical spread)

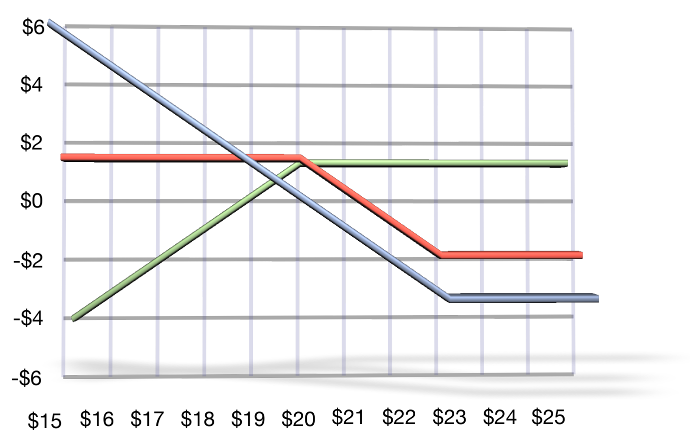
A bull spread can be created by writing a call with, e.g. strike $K_{short} = \$23$, while going long a call with a lower strike price, $K_{long} = \$20$, on the same stock (with the same expiration). In this example, we go long a call @ strike = $20, with a premium = $1.99 + short call @ strike = $23, with a premium = $0.83. When creating a bull spread using call options there is always a cash outflow; conversely, when creating a bull spread using put options there is always a cash inflow. In both cases we are bullish, as the name implies, and thus expect the price of the underlying to increase.



Payoff: Red; **put option:** Green; **call option:** Blue
Protective put: Long call ($K_{long\ call}$ = \$20, premium = \$1.99) + short call ($K_{short\ call}$ = \$23, premium = \$0.83)

9.2.2 Bear spread (type of vertical spread)

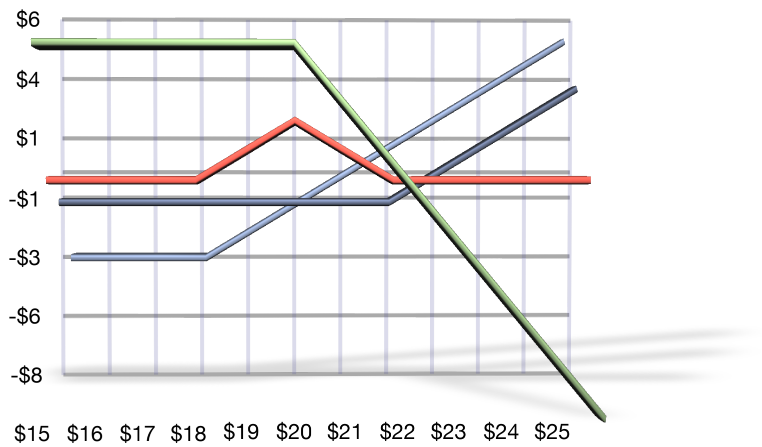
A bear spread can be created by writing a put option with, e.g., strike $\boldsymbol{K_{short}} = \$\mathbf{20}$, while going long a put with a higher strike price $\boldsymbol{K_{long}} = \$\mathbf{23}$, on the same stock (with the same expiration). It is important to note that we can create a bear spread either using put options or call options (think about the payoff profile in terms of the put-call parity). When creating a bear spread using put options there is always a cash outflow, whereas when creating a bear spread with calls we have a cash inflow. In this latter case we would simply buy a call and sell a call but our strike price on the long call is higher than on the short call: $\boldsymbol{K_{long\ call}} > \boldsymbol{K_{short\ call}}$. In both cases we are bearish as the name implies and thus expect the underlying stock price to decline. As the graph below show, we have a range of possible payoff scenarios, however; our loss is capped at \$0 (excluding the premium), while our gain is capped at $\boldsymbol{K_2} - \boldsymbol{K_1}$ (excluding the premium).



Payoff: Red; **Option:** Green; **Stock:** Blue
Long put $\boldsymbol{K_{long\ put}} = \$\mathbf{23}$, premium = -\$2.93) + short put $\boldsymbol{K_{short\ put}} = \$\mathbf{20}$, premium = \$1.20)

9.2.3 Butterfly spread (sideway strategy)

Buy a call option at low strike price K1, buy a call option with high strike price K3, and sell two call options at strike price K2 halfway between K1 and K2. In this example, the butterfly spread: Long call ($K_{long\ call} = \$18$, premium = \$3.21), long call ($K_{long\ call} = \$22$, premium = \$1.13) short two calls (strike @ \$20, premium = \$1.99). **Why the butterfly?** The investor expects low volatility (range-bound), and wants to cap her risk.



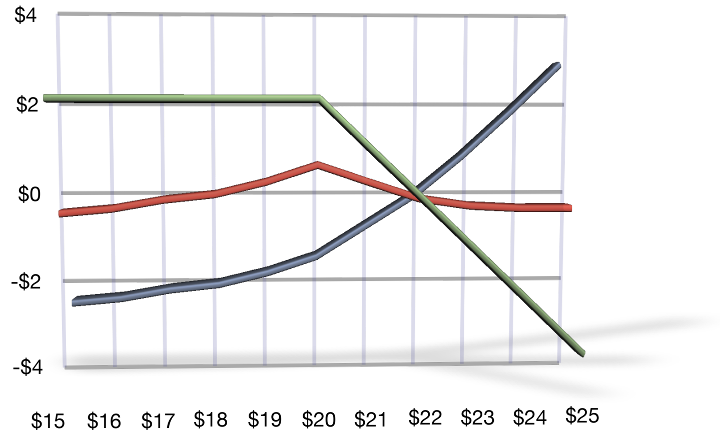
Payoff: Red; **put option:** Green; **call options 2 & 3:** Blue

9.2.4 Calendar spread

In a calendar spread, the options have the same strike price but different expiration dates. The calendar spread can be created with calls or puts.

- Two calls: sell a call option with strike price K1 and buy a call option with same strike price K1 but with a longer maturity term.
- Two puts: sell a put option with strike price K1 and buy a put option with same strike price K1 but with a longer maturity term.

Calendar Spread



Payoff: Red; Option: Green; Stock: Blue
Short call with 1y maturity ($K_{short\ call} = \$20$, premium = \$1.99) + Long call with 1.25y maturity ($K_{long\ call} = \$20$, premium = \$2.27)

9.2.5 Diagonal spread

In a diagonal spread, both the expiration date and the strike price of the calls are different.

9.2.6 Box spread

A box spread is a combination of a bull call spread with strike prices K1 and K2 and a bear put spread with the same two strike prices. The payoff from a box spread is always K2 – K1. The value of the box spread is always the present value of its payoff or $(K_2 - K_1)e^{-rT}$.

IMPORTANT CONCEPT:

"A box spread arbitrage only works with European options" (Hull)

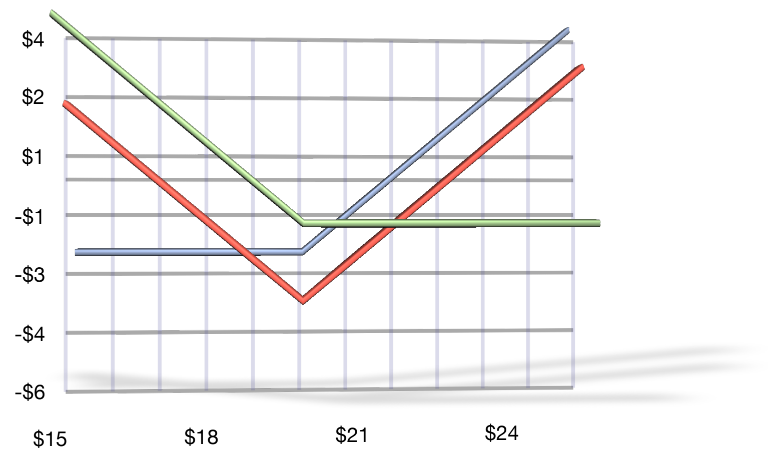
9.3 Describe and explain the use and payoff functions of combination strategies, including straddles, strangles, strips, or straps

A combination strategy involves taking a position in both call(s) and put(s) on the same stock

9.3.1 Straddle

To straddle is to buy a call and buy a put on the same stock with same strike price and expiration date. **Why the (bottom) straddle?** The investor expects a large move in either direction. The worst-case scenario is that the stock settles at the strike price: the investor has paid two premiums but does not receive any payoffs. This illustrated straddle consists of a long call (strike @ $20, premium = $1.99) plus a long put (strike $20, premium = $1.20). This straddle is a "bottom straddle."

Straddle



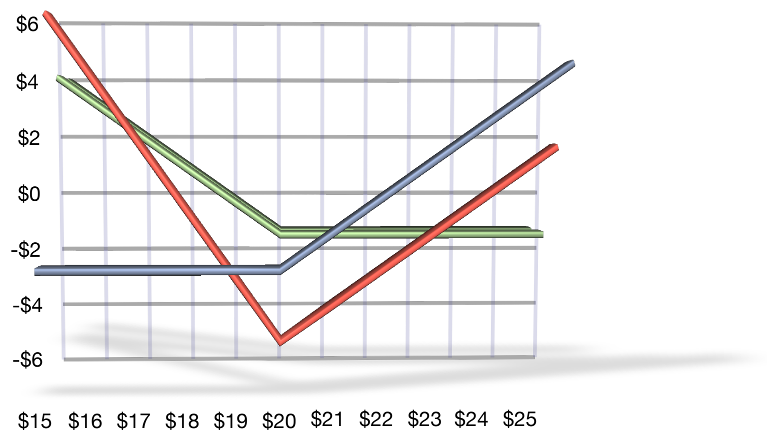
Payoff: Red; **put option:** Green; **call option:** Blue
Long call ($K_{long\ call}$ = $20, premium = $1.99) plus a long put ($K_{long\ put}$ = $20, premium = $1.20)

> IMPORTANT CONCEPT:
>
> A straddle is a **direction neutral volatility strategy**: we don't mind which way the underlying moves. As an example, a **bottom straddle** involves buying both a call and a put with the same strike price and expiration date. In this case, as long as the price moves sufficiently, we are invariant to which way it moves. Conversely, for **a top straddle**, we want the price to deviate from the strike as little as possible.

A top straddle (or straddle write) is to sell a call and sell a put on the same stock with same strike price and expiration date. **Why the top straddle?** The investor is highly confident that the stock will not stray from the strike price in either direction. If the stock price equals the strike price, the investor has collected two premiums for profit. This is a very risky strategy however, because the potential loss is unlimited. As you can see, a top straddle is also a direction neutral volatility strategy; however, unlike with the bottom straddle, we want little to no movement in the underlying.

9.3.1 Strip

Strip: To take a long position in one call and two puts with same strike price and expiration date. **Why the strip?** The investor bets on a large stock price move but considers a decrease more likely than an increase. This illustrated strip consists of a long call (strike @ $20, premium = $1.99) plus two long puts (strike @ $20, premium = $1.20).

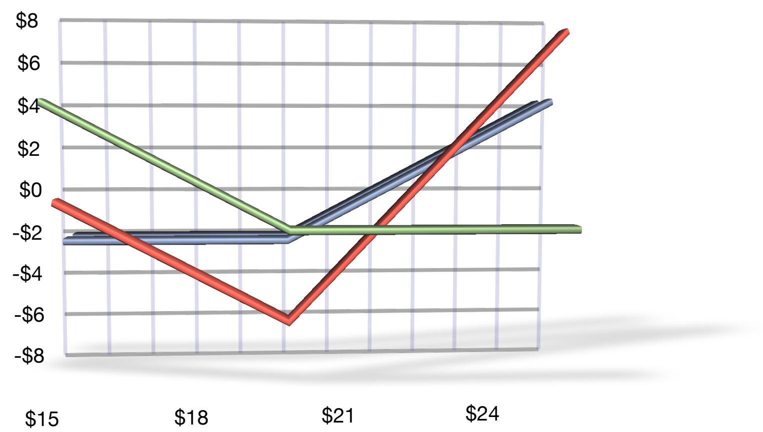


Payoff: Red; **put option:** Green**; call option:** Blue
Long call ($K_{long\ call} = \$20$, premium = $1.99) plus two long puts $K_{long\ put} = \$20$, premium = $1.20)

9.3.2 Strap

Strap: To take a long position in two calls and one put with same strike price and expiration date. Why the strap? Like the strip, the investor bets on a large stock price movement, but instead considers an increase more likely. In this regard a strap is also similar to a straddle, but in this case we are biased upwards. This illustrated strap consists of two long calls (strike @ $20, premium = $1.99) plus a long put (strike @ $20, premium = $1.20).

Strap



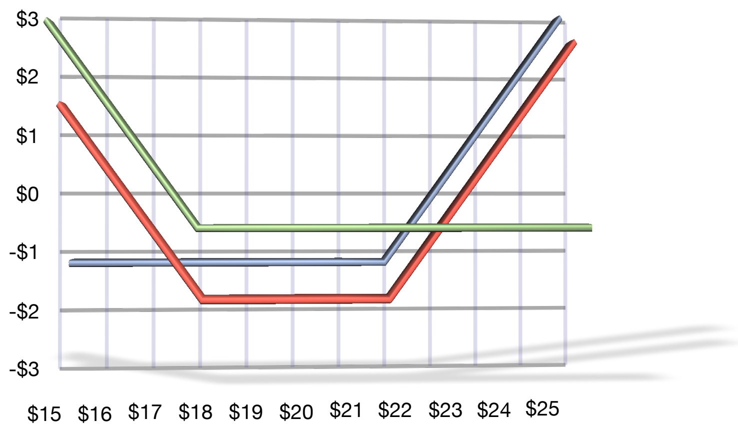
Payoff: Red; **put option:** Green**; call option:** Blue
Two long calls ($K_{long\ call} = \$20$, premium = $1.99) plus a long put ($K_{long\ put} = \$20$, premium = $1.20)

9.3.3 Strangle

Strangle: To buy a put and a call with the same expiration and different strike prices. **Why**

the strangle? The investor is betting on a large price movement (similar to the straddle). This illustrated strangle is a long call (strike @ \$22, premium = \$1.13) plus a long put (strike \$18 premium = \$0.51). A strangle is similar to a straddle but cheaper to install, however; this comes at the cost of requiring more extreme price movements than with the straddle. Consequently, this is a strategy that is bullish on volatility.

Strangle



Payoff: Red; **put option:** Green**; call option:** Blue

9.3.4 Collar and costless collar

A collar (sometimes also referred to as a fence) is a combination strategy where we own the underlying, sell a call option with a strike price greater than the current price of the underlying ($K_{call} > S$), and buy a put option with a strike price less than the current price of the underlying ($K_{put} < S$). This strategy gives us a range of profit or losses. When the premium collected from writing the call exactly matches the premium paid for the put, we have what is called a zero cost, or *costless collar*.

9.4 Compute the pay-offs of combination strategies.

Please see the practice question (PDF) set.

Chapter Summary

The options strategies we have seen are but a few of the many that are used on a daily basis. However, they form the basis of many such strategies. That is, a combination of the strategies we have reviewed can be used to construct any options strategy - your imagination (and wallet) is the limit. It is important to note that, while several of the strategies seem to lock in a guaranteed profit, or have a seemingly high probability of making money, this is largely an illusion due to the transaction costs involved - and the spread between the bid and the ask price actually observed in the market. We can generalize this further: typically, only when your expectations differ from that of the market will there be a genuine moneymaking opportunity. However, going against the market can pose a significant risk as options enable you to leverage your positions in a way that regular stocks and bonds do not. That being said, there are many scenarios in which putting on any of the aforementioned options strategies would makes sense. The primary reason why we would want to do this is to hedge our risk, and the secondary reason is that we simply hold a different view than the market and are making an informed bet. Investment banks are typically more than willing to act as market makers as, on average, they end up with a net profit due to this sort of market making.

After reading this chapter (and completing the accompanying exercises in the separate PDF) you should be able to define and calculate the payoff of a:

1) Principal-protected note
2) Covered call
3) Protective put
4) Bull spread
5) Bear spread
6) Butterfly spread
7) Calendar spread
8) Diagonal spread
9) Box spread
10) Straddle
11) Strap
12) Strip
13) Strangle
14) Collar

You should also be able to specify under what circumstances each of these strategies would be appropriate to put on, and you should know which strategies are direction neutral with respect to a risk factor, and which ones are not.

9.6 Questions & Answers

9.6.1 Questions

1. What are some of the benefits of “options strategies” over a single call or put option investment?

2. You believe volatility is about to drop. What is the appropriate strategy?

3. Of the following options strategies, which one is the most risky?
    a. Buying a bull spread
    b. Buying a bear spread
    c. Writing (selling) naked put options
    d. Writing (selling) naked call options

4. An asset manager tells you he wants to go long a straddle because he has a bullish view on volatility (and the movement of the stock price). The asset manager tells you that this strategy is always superior to a strangle, since you need less of a price movement in order to be net In-the-Money. Is the asset manager right? Why or why not?

5. An investor wants to hedge her bond portfolio. Being sensitive to interest rate the investor decides to sell a call option with $K_{call} \geq S$ to finance entirely the cost of going long a put option where $K_{put} \leq S$ which hedges against an increase in interest rate (remember the value of a portfolio of bonds decreases when the payments are discounted more heavily). Which option strategy is the investor employing?
    a. Butterfly
    b. Costless collar
    c. Strangle
    d. Box spread

6. You are bearish on the market (you think the market will go down) so you decide to use the following options, all with the same maturity: buy a put with $K_{put} = \$49$ for \$8, sell two puts with $K_{put} = \$40$ for \$5 each, and buying one put where $K_{put} = \$49$ for \$3. If at maturity the underlying trades at S = \$34, what is the profit of your trade?
    a. \$3
    b. \$4
    c. \$5
    d. \$6

7. Why does a box-spread arbitrage only work with European options?

9.6.2 Answers

1. One of the primary goals of the various options strategies is to hedge against exposure from different risk factors. The different options strategies also enable investors to take a view on risk factors other than those associated with just the underlying stock. Indeed, we have seen that there are a number of ways to take advantage of a view on e.g., volatility.

2. When volatility is about to drop, you want to enter into a direction neutral volatility strategy, hoping that the underlying will barely move. In this case the appropriate strategy is a top straddle.

3. Both a, b and c have bounded losses, whereas the potential loss from d is unbounded (limitless). Thus, writing naked call options is the most risky strategy.

4. The asset manager is wrong. While it is true that the straddle requires less of an uptick in volatility (and movement in the stock price in either direction) to make money than with a strangle, this comes at the cost of higher risk downside risk if the stock is OTM at expiration.

5. The investor is employing a costless-collar. Notice that in a costless collar, the premium from selling the call option exactly matches the cost of the put option. The investor accordingly locks in a range of profit and losses since the investor is already long the underlying, buys a [protective] put and sells a call.

6. The profit from the trade is $4, thus (c) is correct. We can calculate this as follows:$S_{maturity} < K$, meaning that at maturity the underlying is worthless than the strike of all the options. Accordingly, all the put options will get exercised. [($49 - $34) - 2 x ($40 - $34) + ($36 - $34)] = $5. The initial option premiums sum up to: (-$8 +2x5 -3) = -$1. Thus $5 - $1 = $4.

7. A box spread arbitrage only works with European options as the payoff is always K2 - K1, while the present value of a European option is thus $(K_2 - K_1)e^{-rT}$.

10 McDonald, Chapter 6: Commodity Forwards and Futures

Learning Outcomes:

Define commodity terminology such as storage costs; carry markets, lease rate, and convenience yield.

Explain the basic equilibrium formula for pricing commodity forwards and Futures.

Describe an arbitrage transaction in commodity forwards and Futures, and **compute** the potential arbitrage profit.

Define the lease rate and explain how it determines the no-arbitrage values for commodity forwards and Futures.

Define carry markets, and explain the impact storage costs and convenience yields have on commodity forward prices and no-arbitrage bounds.

Compute the forward price of a commodity with storage costs.

Compare the lease rate with the convenience yield.

Identify factors that impact gold, corn, natural gas, and crude oil Futures prices.
Define and compute a commodity spread.

Explain how basis risk can occur when hedging commodity price exposure.

Evaluate the differences between a strip hedge and a stack hedge and analyze how these differences impact risk management.

Describe examples of cross-hedging, specifically hedging jet fuel with crude oil and using weather derivatives.

Explain how to create a synthetic commodity position and use it to explain the relationship between the forward price and the expected future spot price.

Define commodity terminology such as storage costs; carry markets, lease rate, and convenience yield.

Storage Costs

Investment assets such as securities do not have any physical storage costs associated with them, however assets that are both investment assets and consumption assets, such as gold and silver do have a storage cost associated with them, such as the cost of storing the commodity in a vault. Consumption assets in general do have storage costs associated with them, e.g. one can think of the cost of storing corn in a silo. Moreover, certain consumption assets such as, e.g. coal deteriorate over time, so there is a real cost to not utilizing it immediately.

Carry Markets

Above, we mentioned that there is no physical storage cost for pure investment assets. One does however; “store” financial assets although it might only be stored electronically in an exchange’s database. The old adage, “time is money,” is nevertheless true: the owner of a forward or a Futures contract expects to be compensated for this time-value of money. Thus investment assets are always compensated for their “storage cost,” which is reflected in a higher price for the asset. This phenomenon – that the forward or Futures price reflect the costs of storage is called a *carry market.* Analogously, markets for consumption commodities, such as e.g. corn, where the owner incurs storage costs are also *carry markets*. Electricity, on the other hand, which cannot be easily stored, is not a carry market. Later in this chapter we will explore the financial implications of storage costs and carry markets on the forward and Futures price.

Lease rate

The lease rate is to commodities what the dividend is to financial assets: it is the rate received by the owner of a consumption asset from the investor for borrowing the asset. Lease rate payment is clearly a benefit to the owner of the asset. Accordingly, it has the effect of lowering forward price. To see this, just imagine that the forward price was not impacted by the lease rate. The owner of the commodity could exploit this by leasing his commodity to a short seller, and turn around in the market and sell the forward at the higher price, thus earning a risk-free payment equal to the present value of the lease rate.

> “It is important to be clear about the reason a lease payment is required for a commodity and not for a financial asset” (McDonald)

Convenience yield

Convenience yield also affect the pricing relation for a forward or a Futures contract. To see this, think of a commodity, such as oil, where the owner of the oil can derive immediate benefits by using it in, e.g. production. This certainly has some value, and in fact, it serves to reduce the cost of storing the oil. The forward price will thus decrease in the case when there is a convenience yield.

10.2 Explain the basic equilibrium formula for pricing commodity forwards and Futures

The forward price is equal to the expected spot price in the future, but discounted to the present.

$$F_{0,T} = E_0(S_T)e^{(r-\alpha)T}$$

Where:

$E_0(S_T)$	Spot price of S at time T, as expected at time 0
$F_{0,T}$	Forward price
r	Risk-free rate
α	Discount rate for commodity S

The discount rate is a function of the risk premium on the commodity: the risk premium is the difference between the discount rate on the commodity and the riskless rate.

10.3 Describe an arbitrage transaction in commodity forwards and Futures, and compute the potential arbitrage profit

If the forward price is too high, say $0.21, we can buy a pencil and sell it forward:

Cash and Carry				
Spot	$0.20			
Forward	$0.21			
			Cash Flows	
	Transaction		Time 0	Time 1
	Short forward @	$0.20	0	$0.010
	Buy pencil	$0.20	($0.20)	$0.200
	Lend pencil @	10%	0	$0.021
	Borrow @	10%	$0.200	($0.221)
				$0.010

Consider the following example. The spot price is $10 and the implied forward price is about $10.30 (i.e., the forward price implied by the cost of carry model). If the observed forward price is $10.30, then both arbitrage attempts (at right) produce no profit.

McDonald Commodity Forwards

Spot (S0)	$10.00
Riskless (r)	4%
Time	1.0
Commodity discount rate	6%
expected growth rate (g)	5%
Lease rate	1%
Exp. future spot price	$10.5127
Implied forward price	$10.3045
Observed forward price	$10.3045
Implied lease rate	1.00%

Cash-and-carry arbitrage

	Cash Flows	
Transaction	Time 0	Time T
Short forward @	0	($0.20
Buy commodity	($9.90)	$10.51
Borrow @ riskless rate	$9.900	($10.3
		$0.000

Reverse cash-and-carry arbitrage

	Cash Flows	
Transaction	Time 0	Time T
Long forward @ F0	0	$0.208
Short commodity	$9.90	($10.5
Lend @ riskless rate	$9.900	$10.30 5
		$0.000

Now instead assume the forward price is $10, such that the forward is “cheap” relative to its (cost-of-carry) model price of $10.30. Now, the reverse cash-and-carry arbitrage is profitable. If the forward is “cheap” then the trade is:
Buy the cheap thing: go long the forward
Sell the expensive (in a relative sense thing): short the commodity and lend the short proceeds

McDonald Commodity Forwards

Spot (S0)	$10.00
Riskless (r)	4%
Time	1.0
Commodity discount rate	6%
expected growth rate (g)	5%
Lease rate	1%
Exp. future spot price	$10.5127
Implied forward price	$10.3045
Observed forward price	$10.00
Implied lease rate	4.00%

Cash-and-carry arbitrage

	Cash Flows	
Transaction	Time 0	Time T
Short forward @	0	($0.513)
Buy commodity	($9.90)	$10.513
Borrow @ riskless rate	$9.900	($10.305)
		($0.3045)

Reverse cash-and-carry arbitrage

	Cash Flows	
Transaction	Time 0	Time T
Long forward @ F0	0	$0.513
Short commodity	$9.90	($10.513)
Lend @ riskless rate	$9.900	$10.305
		$0.3045

Now assume the observed forward price is "trading rich" at $11.00; i.e., higher than the model implied price of $10.30. Now the cash-and carry arbitrage is profitable:
Buy the cheap thing: borrow to buy the commodity on the (cash) spot market
Sell the expensive: short the forward

McDonald Commodity Forwards	
Spot (S0)	$10.00
Riskless (r)	4%
Time	1.0
Commodity discount rate	6%
expected growth rate (g)	5%
Lease rate	1%
Exp. future spot price	$10.5127
Implied forward price	$10.3045
Observed forward price	$11.00
Implied lease rate	-5.53%

Cash-and-carry arbitrage

	Cash Flows	
Transaction	Time 0	Time T
Short forward @	0	$0.487
Buy commodity	($9.90)	$10.513
Borrow @ riskless rate	$9.900	($10.305
		$0.6955

Reverse cash-and-carry arbitrage

	Cash	Flows
Transaction	Time 0	Time T
Long forward @ F0	0	($0.487)
Short commodity	$9.90	($10.513
Lend @ riskless rate	$9.900	$10.305
		($0.6955

10.4 Define the lease rate and how it determines the no-arbitrage values for commodity forwards and Futures.

If the lease rate is given by δ, then the forward price is given by:

$$F_{o,T} = S_o e^{(r-\delta)T}$$

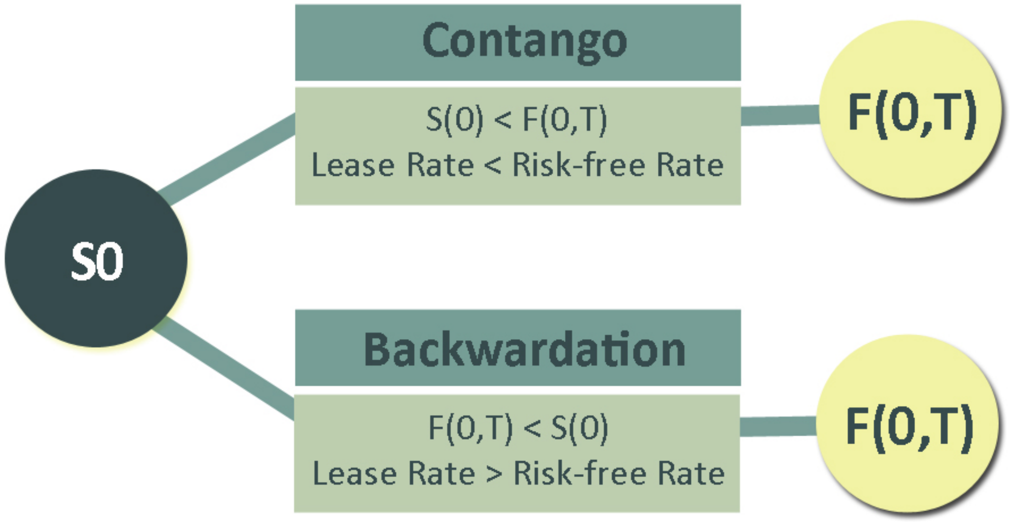
The lease rate formula may look familiar. In an earlier section, we saw that the value of a stock index Futures contract was given by $F_0 = S_0 e^{(r-q)T}$ where (q) equals the dividend yield rate. That's because the lease payment is essentially a dividend.
The lease rate = commodity discount rate – growth rate:

$$\delta = \alpha - g$$

The lease rate is economically like a dividend yield.

10.4.1 Contango and Backwardation

Contango refers to an upward-sloping forward curve, which must be the case if the lease rate is less than the risk-free rate.
Backwardation refers to a downward-sloping forward curve, which must be the case if the lease rate is greater than the risk-free rate.



10.5 Define carry markets, and explain the impact storage costs and convenience yields have on commodity forward prices and no-arbitrage bounds.

10.5.1 Define carry markets

A commodity that is stored is in a carry market. Storage is carry. Storage permits consumption throughout the year

$$F_{0,T} = S_0 e^{(r+\lambda-c)T}$$

10.5.2 Explain the impact storage costs and convenience yields have on commodity forward prices and no-arbitrage bounds

Carry is the cost of storage (a.k.a., holding cost). In an earlier section, we saw the "cost-of-carry" model, as given by:

$$F_0 = S_0 e^{(r+u-y)T}$$

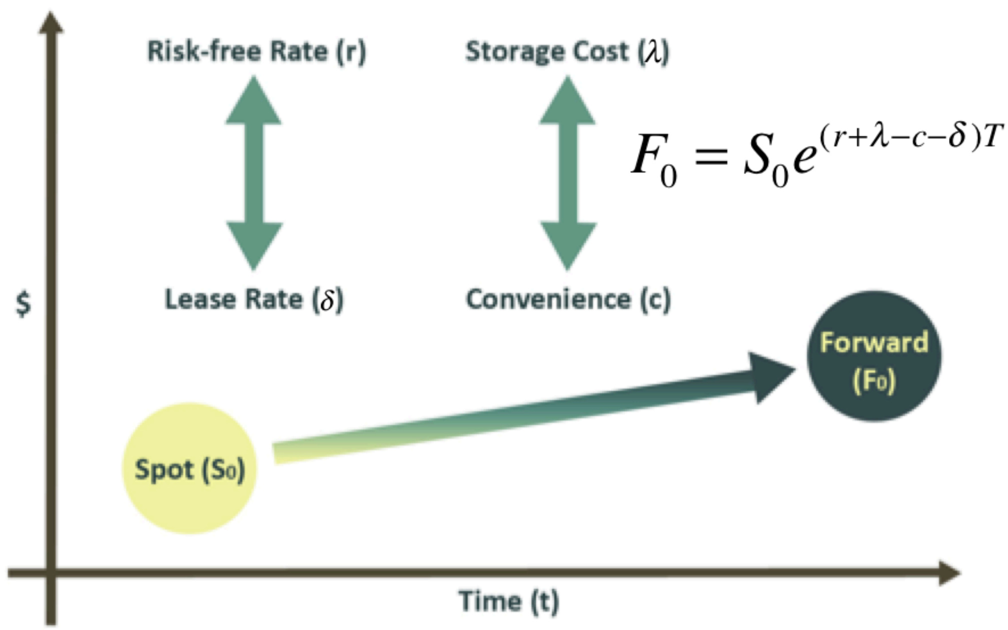
This model says that the forward price is a function of the spot price, compounded forward as a function of three variables: the riskless rate (r), the carry cost (u) and the convenience yield (y).

We can use the same equation if we insert lambda (λ) for the carry cost and (c) is used for the convenience yield. Under that notation, the cost-of-carry model is still:

$$F_o = S_o e^{(r+\lambda-c)T}$$

These two cost-of-carry formulas (one, really) are the master formulas because the others are subsets of these. Memorize this dynamic! Start with exponential function. It compounds the spot rate to the forward rate. The "base case" is to compound the spot rate by the riskless rate (spot $\times$ er).

To expand on the exponential function, ask whether there are benefits or costs to holding the asset. Costs get added to the risk-free rate (because you'd pay less today for that!) and benefits reduce the risk-free rate. So, if it's a dividend paid on the stock, that's a benefit and you've got (r-q) instead of (r). If it's a storage cost, that's a cost, so you've got (r+u) instead of (r) and so on. If it's a storage costs (+u) but also convenience (-y), then we have (r+u-y).



10.5.3 Explain the impact storage costs and convenience yields have on no-arbitrage price bounds

Given convenience yield (c) and storage costs (λ), the no-arbitrage price range is given by:

$$S_o e^{(r+\lambda-c)T} \leq F_{o,T} \leq S_o e^{(r+\lambda)T}$$

10.6 Compute the forward price of a commodity with storage costs.

[Needs Content]

10.7 Compare the lease rate with the convenience yield

If we are given the forward price, we only need to re-arrange the above formula to solve for the implicit lease rate. We re-arrange as follows:

$$F_{0,T} = S_0 e^{(r-\delta)T} \Rightarrow \frac{F_{0,T}}{S_0} = e^{(r-\delta)T} \Rightarrow$$

$$\ln\left(\frac{F_{0,T}}{S_0}\right) = (r-\delta)T \Rightarrow \frac{1}{T} l\text{n}\left(\frac{F_{0,T}}{S_0}\right) = (r-\delta) \Rightarrow$$

$$\delta = r - \frac{1}{T} l\text{n}\left(\frac{F_{0,T}}{S_0}\right)$$

For example, assume the spot price, S_0, is \$9.8 and the forward price in six months (T=0.5) is \$10 $F_{0,0.5}$. Given further a risk-free rate of 6%, the implicit lease rate is about 2%:

$$\delta = r - \frac{1}{T} l\text{n}\left(\frac{F_{0,T}}{S_0}\right) = 6\% - \frac{1}{0.5} l\text{n}\left(\frac{10}{9.8}\right) \cong 2\%$$

Both are benefits of ownership, however; convenience yield is hard to quantify in practice. The observed lease rate δ, depends on both storage costs λ, and the convenience yield c. This implies a no-arbitrage region (zone) rather than a specific price point estimate.

$$S_0 e^{(r+\lambda-c)T} < F_0 < S_0 e^{(r+\lambda)T}$$

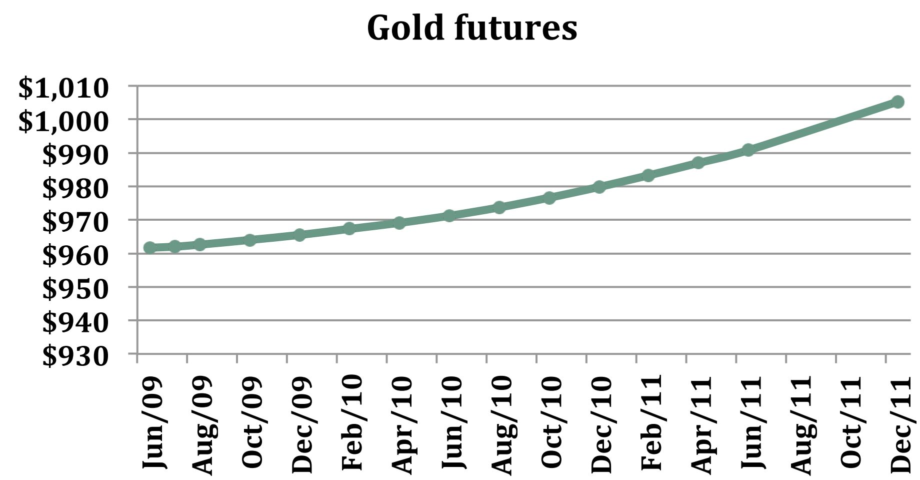
You may not need to memorize this formula if the derivation is natural

$$F_{0,T} = S_0 e^{(r-\delta)T}$$

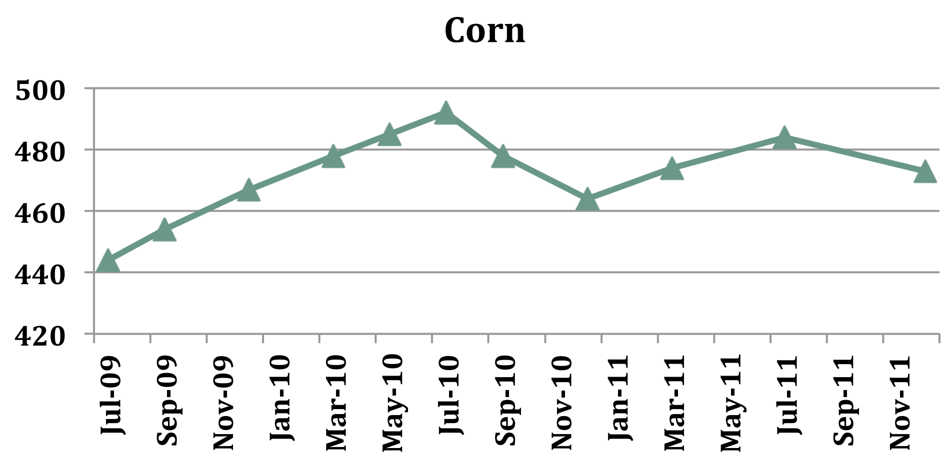
$$\delta = r - \frac{1}{T} \ln\left({F_{0,T}}/{S} \right)$$

10.8 Identify factors that impact gold, corn, natural gas, and crude oil Futures prices

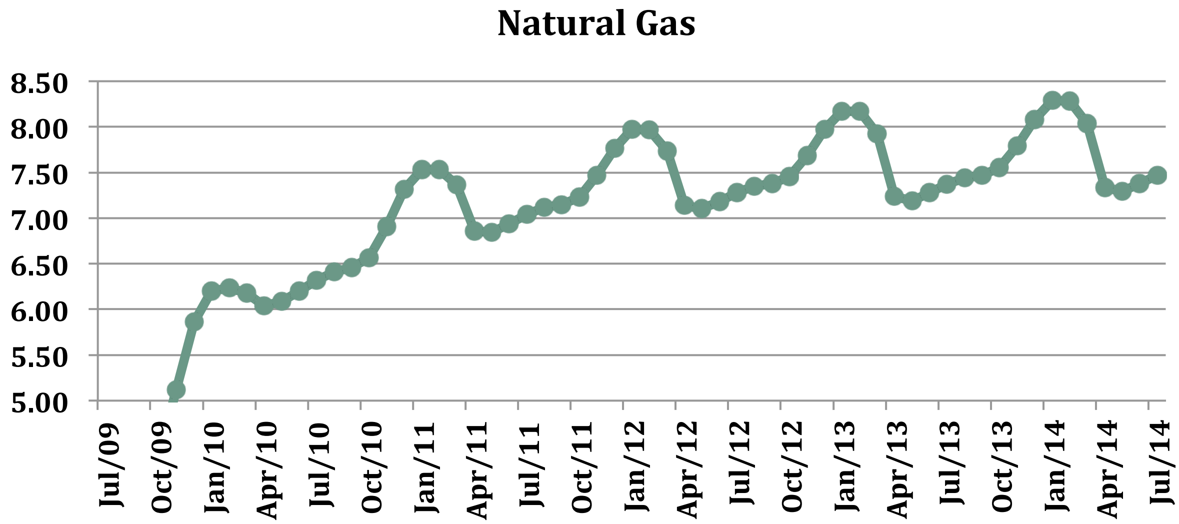
Gold is durable with low storage costs. The forward price tends to be a gradually increasing function of maturity; this implies a lease rate. Exposure to gold can be achieved by ownership or (indirectly) by a long position in gold Futures.
If you own physical gold directly: you forgo a “lease rate” but you also bear storage costs.
If instead you have a synthetically long position in gold: you have no storage costs, but you are exposed to credit risk. The text says that synthetic exposure is preferable, assuming you ignore credit (counterparty) risks.



Corn is seasonal. In theory, the price should rise between harvests (rises to reward storage) due to storage costs. In reality, the price varies year to year.

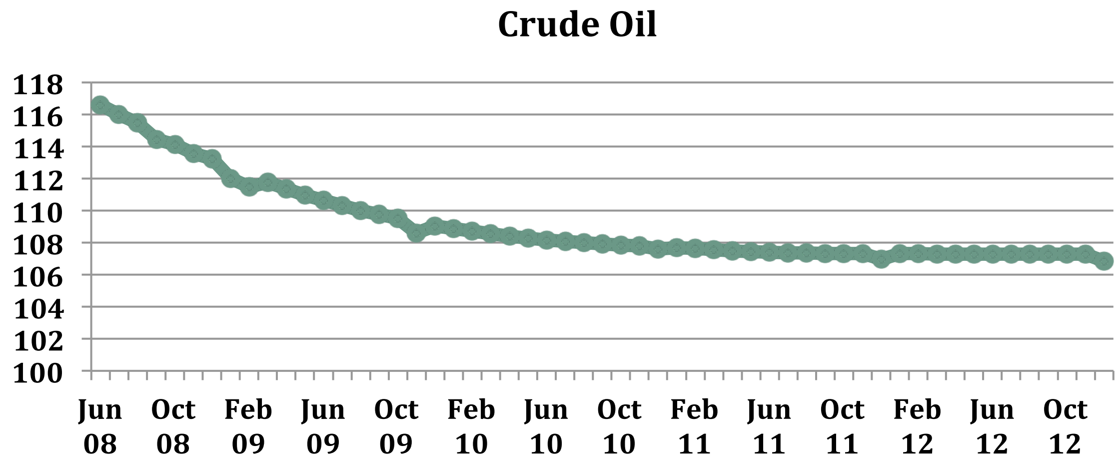


Natural gas is largely impacted by seasonality and storage costs. Gas is (i) expensive to transport overseas, (ii) costly to store, (iii) exposed to seasonal demand with a characteristic peak in the winter. While corn is seasonally produced and constantly demanded; gas is constantly produced and seasonally demanded.

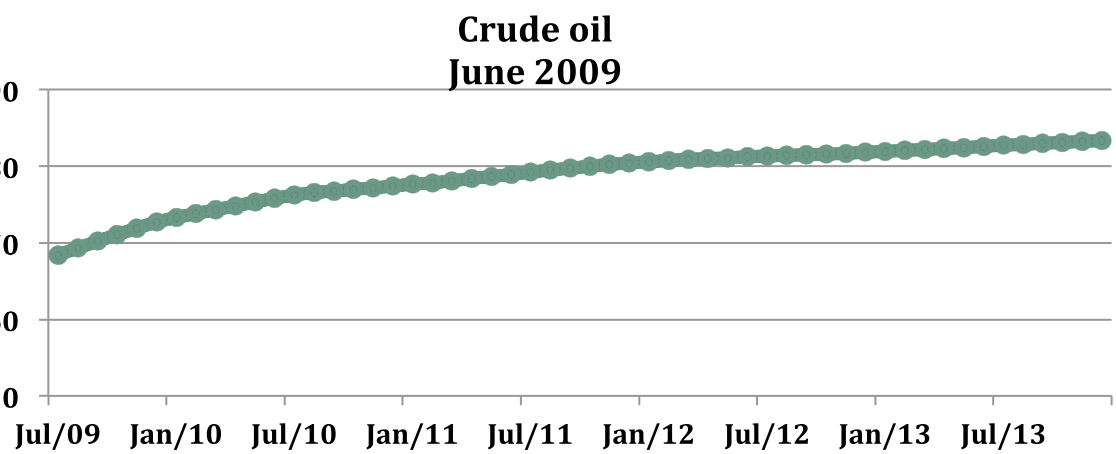


Natural Gas
8.50
8.00
7.50
7.00
6.50
6.00
5.50
5.00
Jul/09
Oct/09
Jan/10
Apr/10
Jul/10
Oct/10
Jan/11
Apr/11
Jul/11
Oct/11
Jan/12
Apr/12
Jul/12
Oct/12
Jan/13
Apr/13
Jul/13
Oct/13
Jan/14
Apr/14
Jul/14

Oil is less expensive than gas to transport and easier to store. Historically crude oil forward (Futures) curve was in backwardation…



But, for example in June 2009, oil Futures switched to contango:



10.9 Define and compute a commodity spread

If we can take a long position in one commodity that is an input (e.g., oil) into another commodity that is an output (e.g., gas or heating oil), then we can take a short position in the output commodity and the difference is the *commodity spread*.
Assume oil is \$2 per gallon, gasoline is \$2.10 per gallon and heating oil is \$2.50 per gallon. If we take a long position in 2 gallons of gasoline and one gallon of heating oil, plus a short position in three gallons of oil, the commodity spread =
(2 long gasoline $\times$ \$2.10) + (1 long heating oil $\times$ \$2.50) – (3 oil $\times$ \$2) = +\$0.70

10.10 Explain how basis risk can occur when hedging commodity price exposure

The basis is the difference between the price of the Futures contract and the spot price of the underlying asset. Basis risk is the risk (to the hedger) created by the uncertainty in the basis.
The Futures contract often does not track exactly with the underlying commodity; i.e., the correlation is imperfect. Factors that can give rise to basis risk include:
Mismatch between grade of underlying and contract
Storage costs
Transportation costs
Basis = Spot Price Hedged Asset – Futures Price Futures Contract = S0 – F0
The basis converges to zero over time, as the spot price converges toward the future price.
When the spot price increases by more than the Futures price, the basis increases and this is said to be a “strengthening of the basis” (and when unexpected, this strengthening is favorable for a short hedge and unfavorable for a long hedge).
When the Futures price increases by more than the spot price, the basis declines and this is said to be a “weakening of the basis” (and when unexpected, this weakening is favorable for a long hedge and unfavorable for a short hedge).

10.11 Evaluate the differences between a strip hedge and a stack hedge and analyze how these differences impact risk management

A strip hedge is when we hedge a stream of obligations by offsetting each individual obligation with a Futures contract that matches the maturity and quantity of the obligation. For example, if a producer must deliver X number of commodities per month, then the strip hedge entails entering into a Futures contract for X commodities, to be delivered in one month; plus a Futures contract for X commodities to be delivered in two months. The strip hedger matches a series of Futures to the obligations.

A stack hedge is front-loaded: the hedger enters into a large future with a single maturity. In this case, our hedger would take a long position in a near-term Futures contract for 12X commodities (i.e., a year’s worth). The stack hedge may have lower transaction costs but it entails speculation (implicit or deliberate) on the forward curve: if the forward curve gets steeper, the stack hedger may lose. On the other hand, if the forward curve flattens, then the stack hedger gains because he/she has locked in the commodity at a relatively lower price.
Oil producer to deliver 10K barrels per month
Strip hedge: contract for each obligation
Stack hedge: Single maturity, “stack and roll.”

10.12 Describe examples of cross-hedging, specifically hedging jet fuel with crude oil and using weather derivatives.

Jet fuel Futures do not exist in the United States, but firms sometimes hedge jet fuel with crude oil Futures and/or Futures for related petroleum products. In order to cross-hedge, we need to understand the relationship between crude oil and jet fuel prices. If we own a quantity of jet fuel and hedge by holding (H) crude oil Futures contracts, our mark-to-market profit depends on the change in the jet fuel price and the change in the Futures price: $[P(t) - P(t-1)] + H[F(t) – F(t-1)$, where $P(t)$ is the price of jet fuel and $F(t)$ the crude oil Futures price. We can estimate (H) by regressing the change in the jet fuel price (denominated in cents per gallon) on the change in the crude Futures price (denominated in dollar per barrel).

Weather derivatives give another example of cross hedging. Weather as a business risk can be difficult to hedge. For example, weather can affect both the prices of energy products and the amount of energy consumed. If a winter is colder than average, homeowners and businesses will consume extra electricity, heating oil, and natural gas, and the prices of these products will tend to be high as well. Conversely, during a warm winter, energy prices and quantities will be low. While it is possible to use Futures markets to hedge prices of commodities such as natural gas, hedging the quantity is more difficult. There are many other examples of weather risk: ski resorts are harmed by warm winters, soft drink manufacturers are harmed by a cold spring, summer, or fall, and makers of lawn sprinklers are harmed by wet summers. In all of these cases, firms could hedge their risk using weather derivatives—contracts that make payments based upon realized characteristics of weather—to cross-hedge their specific risk.
The payoffs for weather derivatives are based on weather-related measurements. For example:

The degree-day index Futures contract trades on the Chicago Mercantile Exchange. A heating degree-day is the maximum of zero and the difference between the average daily temperature and 65 degrees Fahrenheit. A cooling degree-day is the maximum of the difference between the average daily temperature and 65 degrees Fahrenheit, and zero. Sixty-five degrees is a moderate temperature. At higher temperatures, air conditioners may be used, while at lower temperatures, heating may be used. A monthly degree-day index is constructed by adding the daily degree-days over the month. The Futures contract then settles based on the cumulative heating or cooling degree-days (the two are separate contracts) over the course of a month. The size of the contract is $100 times the degree-day index. As of September 2004, degree-day index contracts were available for over 20 cities in the United States, Europe, and Japan. There are also puts and calls on these Futures.

10.13 Explain how to create a synthetic commodity position and use it to explain the relationship between the forward price and the expected future spot price

$E_0(S_T)$ Spot price of S at time T, as expected at time 0
$F_{0,T}$ Forward price
r Risk-free rate
α Discount rate for commodity S

Consider the following investment strategy: enter into a long forward contract plus a zero coupon bond that pays F(0,T) at time T. Since the forward contract is costless, the cost of this investment strategy at time 0 is just the cost of the bond: the discounted price of the face value of the bond = EXP[(-r)(T)]*F(0,T). Again, the idea is to lend at the risk-free rate in order to receive back, at future time T, the exact amount need to meet the long forward obligation. At time T, this strategy (long forward on the commodity plus invest in zero coupon bond) has the same payoff as the future spot price. By using the forward, the unfunded position is synthetic but otherwise equivalent to buying the commodity on the cash market:

$$S_{T-F_{0,T}} - F_{0,T} + F_{0,T} = S_T$$

Here is the key step: we equate the price paid for the synthetic strategy (i.e., the amount we need to invest at the riskless rate in order to receive future proceeds to meet the long forward obligation) with the price we should be willing to pay for the commodity today. That price is the expected future spot price, discounted to today using the discount rate (α):

$$e^{-rT}F_{0,T} = E_0(S_T)e^{-\alpha T}$$

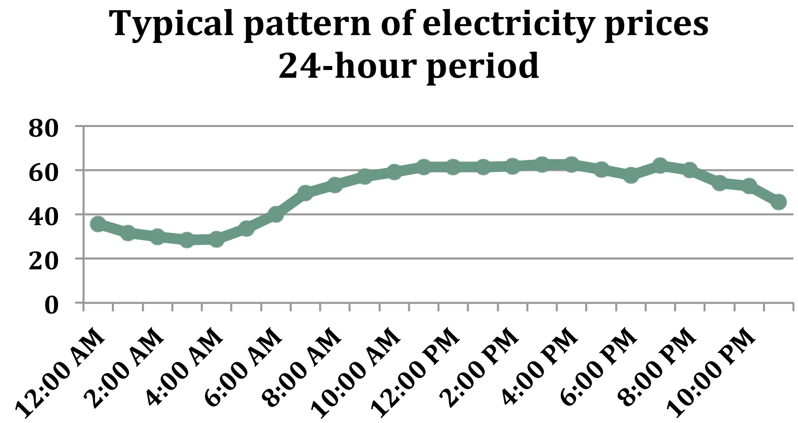
Then we solve for the forward price:

$$F_{0,T} = e^{rT}E_0(S_T)e^{-\alpha T}$$

And end up with the essential formula that links the forward price to the expected future spot price:

$$F_{0,T} = E_0(S_T)e^{(r-\alpha)T}$$

And, as McDonald says, the forward price [F0] is a biased estimate of expected spot price [E(St)], where the bias is due to the risk premium on the commodity (risk premium = α – r).
Explain the effect non-storability has on electricity prices
Because electricity cannot (mostly) be stored, the forward market provides “invaluable price discovery.” Price changes largely reflect, “[consensus] changes in the expected future spot price.


Typical pattern of electricity prices
24-hour period
80
60
40
20
0
12:00 AM
2:00 AM
4:00 AM
6:00 AM
8:00 AM
10:00 AM
12:00 PM
2:00 PM
4:00 PM
6:00 PM
8:00 PM
10:00 PM

Chapter Summary

This chapter re-visits several themes previously seen in Hull's chapter 5 *Determination of Forward and Futures prices,* thus a lot of the concepts should already be familiar. One important thing to note is the difference in notation between Hull and McDonald, e.g. Hull uses u for storage costs and y for convenience yield, whereas McDonald uses lambda and c, respectively for the same. The underlying theory remains the same though.

What is new and important in this reading is how the theory is applied specifically to commodities. One of the key concepts encountered is that of a *carry market,* that is, when commodities are, e.g., stored, the forward or Futures price must reflect the fact that the investor must bear both financing costs as well as storage cost (a so called *cash-and-carry*). More succinctly, a commodity that is stored is in a carry market. Storage is carry.
In addition to our usual terms of the risk-free rate, storage costs the notion of a *lease-rate* and *convenience yield* are introduced. The lease-rate can be thought of as the commodity market equivalent of a financial dividend yield. In particular, the owner of a commodity expects to be compensated in order to lend, e.g., a short seller the commodity. On the other hand, the convenience yield reflects the fact that holders of a given commodity may derive some benefit from having physical ownership over the commodity. Both the convenience yield and lease rate are benefits to the owner, and thus will reduce the price of the forward.

One thing to note is that it can often be difficult to ascertain what the convenience yield for a commodity is. This gives us arbitrage bounds for the forward price, where we effectively solve for the lease rate, but the lease rate depends on both the storage cost and the convenience yield. You should be comfortable with the derivation of the following inequality:

$$S_o e^{(r+\lambda-c)T} < F_o < S_o e^{(r+\lambda)T}$$

When taking, e.g., a long position in one commodity that is an input factor for another commodity that is an [final] output, then we can take a short position in the output commodity and the difference is the *commodity spread*.

As we saw in Hull's chapter 3 on Hedging Strategies using Futures, the basis is the difference between the price of the Futures contract and the spot price of the underlying asset. Basis risk is the risk (to the hedger) created by the uncertainty in the basis. We elaborated slightly on the concept (page 44).

The forward curves of various commodities exhibit patterns that are often idiosyncratic to that commodity. We can often infer these factors from the forward curve and what we know about the commodity. McDonald says, the forward price [F0] is a biased estimate of expected spot price [E(St)], where the bias is due to the risk premium on the commodity (risk premium = $\alpha - r$).

Synthetic commodities can be constructed using default-free bonds and commodity Futures, and will always be preferred over their physical equivalent, except for in a carry-market where the investor will be indifferent between the two.

10.14 Questions & Answers

10.14.1 Questions

10.1 Consider the following statements (McDonald) about the commodity lease rate:

I. A positive lease rate, convenience yield and dividend yield are similar in that they all, ceteris paribus, tend to push the commodity forward curve down (toward backwardation)

II. A lease rate (on a consumption commodity) is like a dividend yield (on a financial commodity like a stock index) in that both are observable, require a storable commodity, and are earned by the owner regardless of whether the commodity is loaned

a) I. only
b) II. Only
c) Both I. and II.
d) Neither I. nor II.

10.2 Each of the following is TRUE about the natural gas forward (futures) curve EXCEPT:

a) Natural gas forward curves ought to vary by region; i.e., ought to defy a single global forward curve
b) Like corn, natural gas has relatively constant year-round demand but seasonal production
c) Any natural gas forward curve ought to contain swings, even "large and predictable swings," of both contango and backwardation segments
d) Even with swings, the underlying natural gas forward curve can be upward-sloping or inverted to incorporate the market's expectation of current spot prices vis-a-vis current spot price

10.3 An oil producer has an obligation to deliver 100,000 barrels per month at a fixed price for the next three years. Which of the following circumstances, ceteris paribus, favors a stack hedge over a strip hedge?

a) Bid-ask spreads (on the oil futures contracts) are atypically constant over time; i.e., spreads do not widen with maturity
b) The oil futures curve exhibits contango
c) There is uncertainty with respect to the future shape of the oil futures curve
d) There is justifiable conviction that current oil futures curve is too steep and will flatten

10.14.2 Answers

10.1 a) I. only
II. is false, while both are directionally similar in the carry model, the lease rate is not directly observable, does not require a storable commodity, and is earned (unlike a stock) only if the commodity is loaned.

10.2 B. False
Corn is constant in demand, seasonal in production; natural gas is seasonal in demand, with relatively constant supply.
In regard to (A), (C), and (D), EACH is TRUE.

10.3 D. There is justifiable conviction that current oil futures curve is too steep and will flatten.
According to McDonald, the strip hedge is superior because it matches the timing of obligations with maturity of contracts EXCEPT for two reasons. One, higher transaction costs (lower liquidity) in distant contracts may favor the stack. Two, the hedger may wish to speculate on the shape of the forward curve.

Geman, Chapter 1: Fundamentals of Commodity Spot and Futures Markets

Learning Outcomes:

Define "bill of lading".

Define the major risks involved with commodity spot transactions.

Differentiate between ordinary and extraordinary transportation risks.

Explain the major differences between spot, forward, and Futures transactions, markets, and contracts.

Describe the basic characteristics and differences between hedgers, speculators, and arbitrageurs.

Describe an "arbitrage portfolio" and explain the conditions for a market to be arbitrage-free.

Describe the structure of the Futures market.

Define basis risk and the variance of the basis.

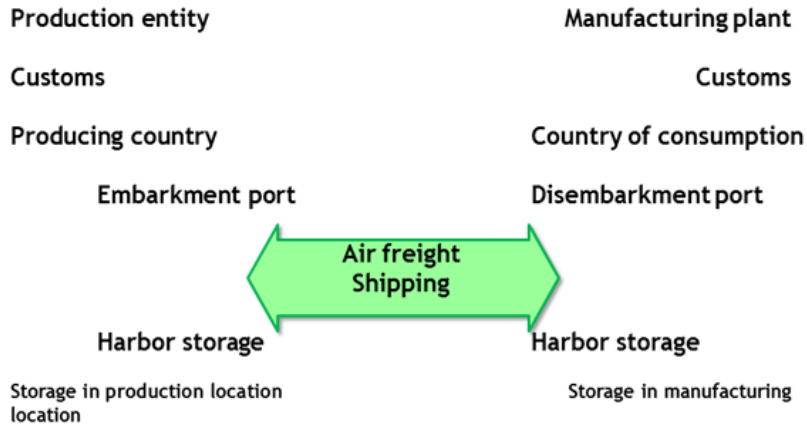
Identify a commonly used measure for the effectiveness of hedging a spot position with a Futures contract; use this measure to **compute** and compare the effectiveness of alternative hedges.

Define and differentiate between an Exchange for Physical and agreement and an Alternative Delivery Procedure.

Describe volume and open interest and their relationship to liquidity and market depth.

Define "bill of lading"

The document that represents the ownership of the good is called a bill of lading. It is issued either by the captain of the transportation ship or by the transporter in charge.
That transportation contract may eventually be traded. It can bear the label ''shipped'' or ''to be shipped''; the latter terminology indicates that the merchandise has been embarked, leading to the qualification clean on board.



Define the major risks involved with commodity spot transactions

Four major types of risk are identified in commodity spot markets:

1. Price risk
2. Transportation risk
3. Delivery risk.
   a. Quality of the delivered commodity.
   b. There is "no financial hedge" for delivery risk. Only "coverage" is (i) very customized contract or (ii) solid long-term relationship with the originator.
4. Credit risk: Always present until trade completion

Differentiate between ordinary and extraordinary transportation risks

According to Geman, "The first category of risks concerns the deterioration, partial or total, of goods during transportation. Two types of risks are usually recognized in this category:

1. Ordinary risks
2. Extraordinary risks such as wars, riots and strikes.

The expeditor of the goods or the FOB (Free On Board) buyer directly holds the transportation risk, unless they purchase an insurance contract to be covered.
Different companies specializing in freight insurance propose various types of contracts."

Explain the major differences between spot, forward, and Futures transactions, markets, and contracts

Spot trading

Any transaction where delivery either takes place immediately, which is rarely the case in practice or if there is a minimum lag, due to technical constraints, between the trade and delivery. Beyond a minimal lag, the trade becomes a forward agreement between the two parties and is properly documented by a written contract.

Forward Contract

A forward contract is an agreement signed between two parties A and B at time 0, according to which party A has the *obligation* of delivering, at a fixed future date T, an underlying asset; and party B has the obligation of paying at that date an amount fixed at date 0, denoted FT(0) and called the forward price for date T for the asset.

$$S(T) = F^T(T)$$

Futures Contract

Futures contracts are analogous to a forward contract but there are some key differences:

- Standardized in terms of their characteristics (maturity, quantity of the underlying commodity, quality or variety).
- They are traded on an exchange, such as NYMEX or the IPE; hence, they carry minimal counterparty risk since both the buyer and the seller of the Futures deal with the clearinghouse of the exchange that is *in principle* fully trustworthy.
- They require the payment of margin deposits in order to be able to start placing orders on the exchange.
- They are marked-to-market daily and the participants have to adjust their positions

Spot	Forward	Futures
Commercial contract	Bilateral agreements	Standardized instrument
Flexible covenants	Flexible covenants	Buyer & Seller only refer to clearinghouse
Illiquid and discontinuous market	Often replaces spot transactions Credit risk fully present	Central clearing generates market prices Liquidity

Describe the basic characteristics and differences between hedgers, speculators, and arbitrageurs

Hedgers

Futures markets were originally designed to meet the needs of hedgers typically farmers who wanted to lock a fixed price for their harvests in advance. The classic example of a hedger is: an airline knows it will *buy jet fuel in the future* so the airline enters a long position in Futures contracts (to hedge). As an alternative, the airline could also buy call options. Indeed, this is “strictly superior” for the hedger *at maturity*, but requires a premium at inception. Another classic example is a commodity producer who knows she will *sell crop in the future*: the producer enters into a short position in Futures contracts in order to hedge, effectively canceling out her price exposure.

Speculators

Speculators wish to get exposure to commodity price moves. For example, Bank ABC (which has no ‘’natural’’ exposure to the price of fuel) decides to take a position (e.g., Futures contract on fuel, option on fuel). The position “expresses a view” on subsequent moves in the fuel price. Commodities are increasingly attractive to investors who view them as an alternative asset class allowing them to improve return/risk profile, and further diversify their portfolios. Futures are instruments that serve their [speculators'] purpose well: There is often ample liquidity in many of the actively traded Futures contracts, the transaction costs on the exchange are low, and due to the central clearing mechanism, there is little credit risk to speak of.

Arbitrageurs

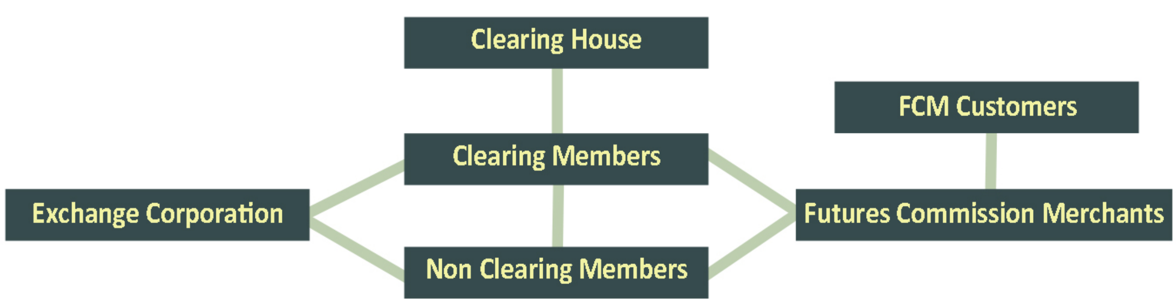
This is an important (but smaller in size) group of participants. An arbitrage is a riskless profit realized by simultaneously entering into several transactions in two or more markets. Arbitrage opportunities are very desirable but not easy to uncover and they do not persist. Arbitrageurs play an important role: they make sure that capital is allocated efficiently in the market, while providing liquidity.

Describe an "arbitrage portfolio" and explain the conditions for a market to be arbitrage-free

If a portfolio requires a null investment and is riskless (there is no possible loss at the horizon H), then its terminal value at date H has to be zero.
"No free lunch property:" if you start with no money and take no risk, your final wealth will be zero.
The assumption of ''riskless'' is crucial
Note: In practice, traders searching for arbitrage opportunities are looking for ''quasi-riskless'' strategies generating profits. There will always be some risk involved.

Describe the structure of the Futures market.

Most Futures exchanges are incorporated as membership associations and operate for the benefit of their members. The purpose of an exchange is to provide an organized marketplace, with uniform rules and standardized contracts. Like any corporation, a Futures exchange has shareholders, a board of directors and executive officers. An exchange may operate markets for spot commodities, options and other financial securities in addition to Futures contracts and provide other services to the public (in particular, price discovery). An exchange funds its activities by membership dues and by transaction fees paid on the contracts traded on the exchange.



Define basis risk and the variance of the basis

$$\text{Basis}_{t,T} = \text{Spot Price}_t - F^T(t)$$

There are several types of basis risk:
In the case of a trading desk which needs to cut at date t (to avoid negative margin calls) – a position in Futures which was meant to hedge a position in the spot commodity – the basis risk is represented by the quantity define above.
More generally, basis risk exists when Futures and spot prices do not change by the same amount over time and, possibly, will not converge at maturity T
Because the Futures contracts were written on an underlying similar but not identical to the source of risk, such as an airline company hedging exposure to a rise in jet fuel prices with NYMEX heating oil Futures contracts;
Because of the optionality left to the seller at maturity in the physical settlement of the Futures contract: grade of the commodity, location, chemical attributes.
The variance of the basis (which is Geman's definition of basis risk) is given by:

$$\sigma^2(S_t - F^T(t)) = \sigma^2(S_t) + \sigma^2(F^T(t)) - 2\rho\sigma(S_t)\sigma(F^T(t))$$

This equation shows that basis risk is zero when the variance between the Futures and spot prices are identical, and the correlation coefficient between spot and Futures prices is equal to one.
In practice, the second condition is the most stringent one and the magnitude of basis risk depends mainly on the degree of correlation between cash and Futures prices.

Identify a commonly used measure for the effectiveness of hedging a spot position with a Futures contract; use this measure to compute and compare the effectiveness of alternative hedges

The classical measure of the effectiveness of hedging a spot position with Futures contracts is given by:

$$h = 1 - \frac{\sigma^2(basis)}{\sigma^2(S_t)}$$

The nearer h is to one, the better (more perfect) the hedge.

Define and differentiate between an Exchange for Physical and agreement and an Alternative Delivery Procedure

Exchange For Physical

An EFP is an agreement between a party holding a long Futures position and a party with an equal size short position to enter a bilateral contract specifying the terms of physical delivery (location and price).
The two parties notify the clearing house of the quantity and price negotiated between them and both Futures positions are then terminated under the terms of the EFP.
In grain markets, this type of transaction is called ''ex pit''.

Alternative Delivery Procedure

An ADP is available to buyers and sellers who have been matched by the exchange subsequent to the termination of trading in the spot month contract. If buyer and seller agree to achieve delivery under terms different from those prescribed in the contract specifications, they may proceed on that basis after submitting a notice of their intention to the exchange.

Describe volume and open interest and their relationship to liquidity and market depth

An important feature shared by all commodity Futures is that the highest liquidity is observed for short maturities, of the order of a few months. Even though liquidity modeling is one of the topics on which financial theory needs more findings … some qualitative concepts have emerged that can be extended to commodity markets:

- Liquidity may be measured by the size of the trade it takes to move the market.
- Market depth may be measured by the time it takes for an order of a standard size to be executed.

Open interest in Futures market

Open interest refers to the number of Futures contracts outstanding at a particular moment, i.e., the number of contracts that have not been canceled by an offsetting trade. Official figures on open interest are released by the exchange in a daily manner for the day before. One needs to keep in mind that it represents the total number of contracts held by buyers or sold short by sellers since these two numbers are always equal. The size of the open interest reflects the determination of the two groups, longs and shorts, to hold to their positions. It is a major indicator for technical analysts who derive buy-and-sell rules from the combined effect of an upward or downward trend with a rising or falling open interest.

Chapter Summary

This chapter covers topics that have largely already been covered in the Hull and McDonald readings, thus few concepts here are truly new. The few things that are new are primarily the commodity specific terms and terminology used.

Bill of lading is the document that represents the ownership of the good. We also encountered terms such as Free On Board (FOB), which means the seller is responsible for delivering goods on board a vessel designated by the buyer. Once the goods have been loaded (or technically, have passed over the ship's rail) the seller has fulfilled his part of the contract. Contrast this to Cost, Insurance and Freight (CIF) where the seller is required not only to load a vessel (not necessarily designated by the buyer), but rather is required to arrange for the delivery of the goods at a port of destination, at which time the buyer takes ownership. These terms are used extensively in nearly all commodity trading and are thus important.

This reading recapitulates the difference between futures and Forward contracts, and the role of the clearinghouse and clearing members, as well as hedge effectiveness – all terms that you should be familiar with by now.

One important note is how Geman defines basis risk. The other authors (Hull and McDonald) both describe basis risk and how basis risk arises – as does Geman. However Geman is slightly more specific in his definition of basis risk. In particular, basis risk is defined as *the variance of the basis* and provides the following formula as so,

$$\sigma^2(S_t - F^T(t)) = \sigma^2(S_t) + \sigma^2(F^T(t)) - 2\rho\sigma(S_t)\sigma(F^T(t))$$

Finally, there is also the distinction between ordinary risk and extraordinary risks (such as war).

11 Questions & Answers

Questions

11.1 According to Geman, each of the following is a major risk in commodity SPOT markets EXCEPT:

a) Price risk
b) Basis risk
c) Delivery risk
d) Credit risk

11.2 Which risk is most difficult to hedge or insure, according to Geman?

a) Ordinary transportation risk
b) Extraordinary transportation risk
c) Cost of transportation risk
d) Delivery risk

11.3 Each of the following contributes to basis risk EXCEPT:

a) Optionality left to the seller at maturity in the physical settlement of the futures contract; e.g., commodity grade, location
b) Futures contracts reference underlying commodity that is not identical to the hedged exposure
c) The counterparty who is long the forward position may fail to make the promised purchase on the delivery date
d) Trading desk is forced to trim or "cut" a position earlier than planned; e.g., to avoid negative margin calls

Answers

11.1 B. Basis Risk (basis risk is a price risk that can only exist in the context of a hedge)
Geman’s four commodity spot market risks are:

1) Price risk
2) Transportation risk
3) Delivery risk, which concerns the quality of the commodity that is delivered
4) Credit risk, which is present all along until the final completion of the trade.

11.2 D. Delivery risk: “for which there is no financial hedge that may be put in place. The only coverage is provided by a very customized contract or by a solid long-term relationship with the originator.”

11.3 C. This is counterparty risk (an acute risk in forwards but even a futures contract exhibits some counterparty exposure to the CCP).
Basis risk is a price (market) risk that invariably is created when an underlying exposure is hedged.

Saunders, Chapter 14: Foreign Exchange Risk

Learning Outcomes:

Calculate a financial institution's overall foreign exchange exposure.

Explain how a financial institution could alter its net position exposure to reduce foreign exchange risk.

Calculate a financial institution's potential dollar gain or loss exposure to a particular currency.

Identify and describe the different types of foreign exchange trading activities.

Identify the sources of foreign exchange trading gains and losses.

Calculate the potential gain or loss from a foreign currency denominated investment.
Explain balance-sheet hedging with forwards.

Describe how a non-arbitrage assumption in the foreign exchange markets leads to the interest rate parity theorem; use this theorem to calculate forward foreign exchange rates.

Explain why diversification in multicurrency asset-liability positions could reduce portfolio risk.

Describe the relationship between nominal and real interest rates.

Foreign Exchange Rates
Direct quote (US$ Equivalent)
U.S. dollars per one unit of foreign currency
0.9079 USD / CAD
Indirect quote (Currency per US$)
Foreign currency per one US dollar
1.1015 CAD / $USD

Calculate a financial institution's overall foreign exchange exposure.

FX position exposure

Net exposure_i = (FX assets_i - FX liabilities_i) + (FX bought_i - FX sold_i)
= Net foreign assets_i + Net FX bought_i, where i = i^{th} currency.

- Positive net exposure ⇒ net long a currency.
- Negative net exposure ⇒ net short a currency.

Explain how a financial institution could alter its net position exposure to reduce foreign exchange risk

To reduce its foreign currency exposure

Bank can match its foreign currency assets to its liabilities.
Bank can match buys and sells in trading book.
Financial holding companies can aggregate their foreign exchange exposure under one umbrella, commercial bank, insurance company, and pension fund.

Calculate a financial institution's potential dollar gain or loss exposure to a particular currency

The potential size of a bank's FX exposure given by:
Dollar loss/gain in currency i = Net exposure in foreign currency i measured in US dollars $\times$ Shock (volatility) to the $/foreign currency i exchange rate.

Identify and describe the different types of foreign exchange trading activities

A bank's position in the FX markets generally reflects four trading activities. The purchase and sale of foreign currencies in order to facilitate customers to:

1. Participate in international commercial trade transactions
2. Take positions in foreign investments (real or financial assets)
3. Hedge FX exposure —i.e., to offset currency exposure
4. Take a view on the market in the form of speculating

Identify the sources of foreign exchange trading gains and losses

In the first two activities (To allow customers to participate in international commercial trade transactions; and to allow customers to take positions in foreign investments, real or financial assets), the bank normally acts as an agent of its customers for a fee but does not assume the FX risk itself. In the third activity (For hedging purposes—i.e., to offset currency exposure), the bank acts defensively to reduce FX exposure. Consequently, the primary FX exposure "essentially relates to *open positions* taken as a principal by the bank for speculative purposes (the 4th activity)."

Calculate the potential gain or loss from a foreign currency denominated investment

Baseline Scenario: Un-hedged Balance Sheet is exposed to FX Risk.

In this scenario (Saunders Example 14-1), a US institution raises $200 million in liabilities that fund $200 million in assets, but $100 million (50%) are loaned (invested) in the foreign currency (British pound Sterling). Suppose the British pound depreciates from $1.60 to 1.45. Then the ROA is 6.16% and the ROI is negative because the cost of funds (COF) is 8%:

Assets (loans)		Liabilities (CDs)	
	Invest:		Lend:
$100.00	US $ @ 9%	$200.00	US $ @ 8%
$100.00	UK £ @ 15%	$0.00	UK £ @ 11%
	$/£		
Start	$1.60		
End	$1.45		
$100.00	£62.50	$0.00	£0.00
$104.22	£71.88	$0.00	£0.00
4.22%		0.00%	
ROA	6.61%	COF	8.00%
ROI	-1.39%		

On Balance Sheet Hedge: Liabilities match FX Exposure of Assets

The UK Pound Depreciates: both ROA and Cost of Funds lower.

The difference here is that, instead of funding $200 million with US deposits, $100 million is funded by deposits via U.K. CDs. Now the $100 million asset exposure is “matched” (not duration matching!) with $100 million in liabilities. Now, if the British pound depreciates, the on-balance sheet hedge works because the cost of funds (COF) is lower, too:

Assets (loans)		Liabilities (CDs)	
	Invest:		Lend:
$100.00	US $ @ 9%	$100.00	US $ @ 8%
$100.00	UK £ @ 15%	$100.00	UK £ @ 11%
	$/£		
Start	$1.60		
End	$1.45		
$100.00	£62.50	$100.00	£62.50
$104.22	£71.88	$100.59	£69.38
4.22%		0.59%	
ROA	6.61%	COF	4.30%
ROI	2.31%		

On Balance Sheet Hedge: Liabilities match FX Exposure of Assets

UK Pound Appreciates: Both ROA and Cost of Funds (COF) higher!

This scenario has British pound appreciating from $1.60 to $1.70:

Assets (loans)		Liabilities (CDs)	
	Invest:		Lend:
$100.00	US $ @ 9%	$100.00	US $ @ 8%
$100.00	UK £ @ 15%	$100.00	UK £ @ 11%
	$/£		
Start	$1.60		
End	$1.70		
$100.00	£62.50	$100.00	£62.50
$122.19	£71.88	$117.94	£69.38
22.19%		17.94%	
ROA	15.59%	COF	12.97%
ROI	2.63%		

Explain balance-sheet hedging with forwards

Off balance sheet hedge with forwards

In the case, the bank “locks in” the future exchange rate with a forward currency contract. In this example, although the foreign currency depreciates (e.g., $1.45), the bank converts at $1.55 per the forward contract.

Assets (loans)		Liabilities (CDs)	
	Invest:		Lend:
$100.00	$ @ 9%	$200.00	$ @ 8%
$100.00	£ @ 15%	$0.00	£ @ 11%

	$/£		
Spot	$1.60		
Discount	$0.05		
Forward	$1.55		
$100.00	£62.50		
Loan @	15%		
Returned (£)	£71.88		
Returned ($)	$111.41		
Loan Return	11.41%		
ROA	10.20%	COF	8.00%
ROI:	2.20%		

Describe how a non-arbitrage assumption in the foreign exchange markets leads to the interest rate parity theorem; use this theorem to calculate forward foreign exchange rates

The interest rate parity theorem (IRPT) s given by the following equation:

$$1 + r_{ust}^{D} = \frac{1}{S_t} \times \left[1 + r_{ukt}^{L}\right] \times F_t$$

where,

$$1 + r_{ust}^{D} \quad = \quad 1 + \text{interest rate on US CDs}$$

$$S_t \quad = \quad {}^{\$}\!/_{\pounds} \text{ spot exchange rate at time t}$$

$$1 + r_{ukt}^{L} \quad = \quad 1 + \text{interest rate on U.K. loans}$$

$$F_t \quad = \quad {}^{\$}\!/_{\pounds} \text{ forward exchange rate at time t}$$

The IRPT is motivated by arbitrage arguments: if interest rates are higher in, say the UK, than in the US, it is attractive for investors to invest in UK assets, earning a higher rate of return. Indeed, investors would borrow cheaply in the US and invest in the UK and make a profit on the interest rate differential. However, as you have probably already figured, such arbitrage opportunities cannot persist. If US investors borrow USD to buy Pound Sterling denominated CDs, the cost of Pound Sterling in terms of USD appreciates (becomes more expensive). As the spot price increases, the forward exchange rate simultaneously decreases. Thus, it becomes less attractive to but Pound Sterling denominated CDs as you get fewer Pounds per USD, and you know that the exchange rate one year from now will be lower (you will receive less for your Pound Sterling). These forces will ensure that Covered Interest Rate Parity holds, with any deviation quickly being seized upon by Arbitrageurs.

Explain why diversification in multicurrency asset-liability positions could reduce portfolio risk

To the degree that domestic and foreign interest rates (or stock returns) are not perfectly correlated, potential gains from asset-liability portfolio diversification can offset risk of asset-liability currency mismatch. This is just basic portfolio theory from Volume 1 regarding diversification effects, applied to a setting where we essentially have assets that can help us reduce the overall systemic risk to our portfolio without reducing the returns, or vice-versa.

Describe the relationship between nominal and real interest rates

Nominal interest rate ≅ real interest rate + [expected] inflation rate. This can also be defined slightly more mathematically precise as so,
Nominal interest rate = [(1 + real interest rate) x (1 + expected inflation)] -1.

$r_i = rr_i + i_i^e$
$r_i =$ The nominal interest rate in country i
$rr_i =$ The real interest rate in country i
$i_i^e =$ The expected one-period inflation rate in country i

We see from the equation above that the interest rate has two components, inflation (or expected inflation) and the real interest rate. The nominal interest rate is observed in the market. In the US for example, the Bureau of Labor Statistics tries to collect data on inflation in order to help determine monetary policy. If one applies their Consumer Price Index (CPI) to the equation above, we could also find what would be a proxy for real interest rates. It is important to understand that CPI does not equal inflation. It is just one measure of it. Indeed, it is a chain-weighted average of a basket of goods, making it a cost-of-living index, since when relative prices fluctuate, consumers will substitute their “typical” basket of goods with another. CPI does not take this into account, nor does it take into account improvements in the quality of products.

Chapter Summary

Companies and financial institutions are often faced with exposure to foreign currency, due to trading with other countries in the form of imports or exports, or in the form of investments. A company may wish to reduce, or hedge its exposure to the foreign currency and can do so using, e.g., FX forwards. The net FX exposure can be decomposed as follows:

$Net\ exposure_i$ = (FX assets_i - FX $\text{liabilities}_\text{i}$) + (FX bought_i- FX sold_i)

= Net foreign assets_i+ Net FX bought_i, where i = i^{th} currency.

A financial institution's potential gain or loss to an FX position can be calculated as so,

$ gain/loss in currency i =

Net exposure in currency i in $ $\times$ Shock (volatility) to the $/currency i exchange rate.

For a bank or financial institution there are four main areas from which foreign currency exposure arises. As seen, the **primary** FX exposure relates to ***open positions*** taken as a principal by the bank for speculative purposes.

Purchasing Power Parity is defined in Saunders as the relative PPP; however, in financial equilibrium models this is rarely useful and is rather substituted for the more powerful *law of one price* which states that each commodity should have the same price after converting to ones respective currency. When this holds for all commodities, PPP also holds. The Purchasing Power Parity, simply stated, compares the purchasing power of the currency in one country with that of the currency in another country. Since one can engage in physical arbitrage, buying in the cheap currency and selling in the expensive currency, one would expect this to lead to equilibrium in the exchange rates. In practice, PPP is highly correlated with exchange rates, however, exchange rates can deviate significantly from that suggested by PPP. This is possible as there are frictions in real markets that need to be taken into account. However, in the long run (10+ years), PPP is a good proxy for how exchange rates move.

Interest Rate Parity (IRP) or IRPT describes an equilibrium relationship between interest rates in two countries, and the Spot and forward exchange rate between their currencies. This is what is known as Covered Interest Rate Parity, and it tends to hold in general, with deviations usually quickly being corrected by arbitrageurs (although significant shocks to financial markets can introduce other factors which causes CIRP not to hold). It is also worth to mention that there is also a theory of Uncovered Interest Rate Parity (UIRP), which is similar to CIRP but the no-arbitrage condition is satisfied without forward or Futures contracts. When both the CIRP and the UIRP hold the forward exchange rate is an unbiased predictor of the future spot exchange rate.

To the degree that domestic and foreign interest rates (or stock returns) are not perfectly correlated, potential gains from *asset-liability portfolio diversification* can offset risk of asset-liability currency mismatch.

Candidates often find exchange rates confusing. It is highly recommended that you do the exercises in the separate practice question set.

Questions & Answers

Questions

12.1 A US bank has the following pound sterling exposures: GBP 10.0 billion in assets, GBP 7.0 billion in liabilities, GBP 5.0 billion bought, GBP 6.0 billion sold. The bank is concerned that the pound sterling will fall in value relative to the US dollar. Which of the following will reduce the bank's exposure to pound sterling depreciation?

a) Nothing, its net exposure implies a benefit if GBP depreciates
b) Add +2 billion in assets to the balance sheet that are denominated in pound sterlings
c) Add +2 billion in liabilities to the balance sheet that are denominated in pound sterlings
d) Add + 2 billion in long forward exposure to the pound sterling; i.e., promises to buy GBP in the future

12.2 According to Saunders, which of the four trading activities most contributes to foreign exchange (FX) risk exposure?

a) Open positions in a currency
b) Purchase and sale of currencies for hedging purposes
c) Purchase and sale of currencies to complete international transactions.
d) Facilitating positions in foreign real and financial investments

12.3 Which of the following is true about the use of an ON-BALANCE-SHEET HEDGE to control a bank's foreign exchange (FX) exposure?

a) The hedge will lock-in (guarantee) a specific, predetermined net return
b) The hedge can ensure a positive, but nevertheless volatile, net return
c) The hedge cannot ensure a positive net return
d) By employing a forward foreign currency contract, the on-balance-sheet hedge can ensure a positive return that is also not volatile

12.4 A US bank raises USD $10 million (liabilities) and invests this amount into a Russian project denominated in Russian rubles (asset) with an expected foreign rate of return of 12%. The bank remains unhedged with respect to this currency risk. If there is an sudden increase in the Russian inflation rate, without any corresponding impact on the project's nominal, foreign 12% return on the project, according to purchasing power parity (PPP), what is the impact on the bank?

a) No impact
b) Ruble should appreciate, translating into a gain for the bank
c) Ruble should depreciate, translating into a gain for the bank
d) Ruble should depreciate, translating into a loss for the bank

Answers

12.1 C. Add +2 billion in liabilities to the balance sheet that are denominated in pound sterling
The net exposure = (10 - 7) + (5 - 6) = +2 GBP; i.e., the bank is net long pound sterling and faces the risk of GBP depreciation.
Each of answers (A), (B) and (D), increase the net long exposure to a greater net long exposure.

12.2 A. Open positions in a currency
In regard to (C) and (D), please note Saunders says here, “the bank [FI] normally acts as an agent of its customers for a fee but does not assume the FX risk itself.”

12.3 B. The hedge can ensure a positive, but nevertheless volatile, net return
As illustrated by the examples, the hedge ensure directional protection and the net return will tend to cluster near the net return earned under a scenario of: un-hedged with no currency changes. However, due to the spread differentials, volatility will remain.
Please note (D) is nonsensical.

12.4. D. Ruble should depreciate, translating into a loss for the bank
As the bank is net invested in ruble-denominated assets, the bank is long the Russian ruble. Per PPP, inflation in Russia should lead to depreciation of the Russian ruble, which will create a loss on the long currency position.

Fabozzi, Chapter 12: Corporate Bonds

Learning Outcomes:

Describe a bond indenture and explain the role of the corporate trustee.

Explain a bond's maturity date and how it impacts bond retirements.

Describe the main types of interest payment classifications.

Describe zero-coupon bonds, the relationship between original-issue-discount and reinvestment risk, and the treatment of zeroes in bankruptcy.

Describe the various security types relevant for corporate bonds, including:
Mortgage bonds
Collateral trust bonds
Equipment trust certificates
Debenture bonds (including subordinated and convertible debentures)
Guaranteed bonds

Describe the mechanisms by which corporate bonds can be retired before maturity, including:
Call provisions
Sinking-fund provisions
Maintenance and replacement funds
Tender offers

Describe, and differentiate between credit default risk and credit-spread risk.

Describe event risk and what may cause it in corporate bonds.

Define high-yield bonds; describe types of high-yield bond issuers, and some of the payment features peculiar to high yield bonds.

Define and differentiate between an issuer default rate and a dollar default rate.

Define recovery rates and describe the relationship between recovery rates and seniority.

Describe a bond indenture and explain the role of the corporate trustee

Bond indenture
The contract that contains corporate bond issuer promises and investors' rights. The indenture is made out to corporate trustee, who represents bondholders' interests.

Corporate trustee

The corporate trustee is a third party to the contract. The trustee acts in a fiduciary (legal) capacity on behalf of the investors. Typically, the trustee is a bank or trust company with a corporate trust department and officers who are experts in performing trustee functions.

The corporate trustee's responsibilities include authenticating the bonds issued. Moreover, acting on behalf of the bondholders, the trustee must ensure that the bond issuer is in compliance with the covenants of the indenture at all times. These covenants are often many and technical, and they must be watched during the entire period that a bond issue is outstanding.

Corporate Bonds

The five broad categories of corporate bonds sold in the United States based on the type of issuer are:

1. Public utilities,
2. Transportations,
3. Industrials,
4. Banks and finance companies; and
5. International or Yankee issues.

Explain a bond's maturity date and how it impacts bond retirements

Bond's maturity: date on which the issuer's obligation to satisfy the terms of the indenture is fulfilled. Principal is repaid with any premium and accrued interest that may be due. However, many issues can be retired prior to maturity.

Describe the main types of interest payment classifications

The 3 main interest payment classifications of domestically (US) issued corporate bonds are:

1. Straight-coupon bonds (sometimes also referred to as plain-vanilla bonds),
2. Zero-coupon bonds, and
3. Floating-rate, or variable-rate, bonds.

Describe zero-coupon bonds, the relationship between original-issue-discount and reinvestment risk, and the treatment of zeroes in bankruptcy

Zero-coupon bonds

Zero-coupon bonds are bonds without coupons or interest rate payments. Zero-coupon bonds pay only the principal portion at some future date.
Zero-coupon bonds are issued at discounts to par; the difference is the return to the bondholder. Furthermore, zero-coupon rates play an important role in the construction of a discount curve: it informs the rate of return for the date at which it matures since there is no re-investment risk.

Original-issue-discount

The difference between the face amount and the offering price when first issued is called the original-issue discount (OID). The rate of return depends on the amount of the discount and the period over which it accrues.

Zero reinvestment risk in a zero

A zero-coupon bond eliminates reinvestment risk (shifting all of the risk to interest rate risk; e.g., duration) because there is no coupon to reinvest. A zero is considered beneficial in declining-interest-rate markets: the zero is discounted less heavily AND we do not worry about having to reinvest any coupon payments at a low rate of interest. When interest rates are rising, the opposite is true, we discount the zero more heavily, but we do not receive any coupons that we can reinvest at the higher rate, thus we are stuck with a low-rate bond. Recall Tuckman: a bond investor faces a tradeoff between reinvestment risk (the reinvestment of coupons) and interest rate risk. Consequently, investors tend to find zeros less attractive in lower-interest-rate markets. Moreover, the lower the rates are, the more likely it is that they will rise again, making a zero-coupon investment worth less in the eyes of potential holders.

Treatment of zeroes in bankruptcy

In bankruptcy, zero-coupon bond creditor claim original offering price plus accrued and unpaid interest, but not the principal amount of $1,000.
Zero-coupon bonds are sold at (deep) discounts: liability of the issuer at maturity may be substantial and there are no sinking funds on most of these issues. The potentially large

balloon repayment creates a cause for concern among investors. Thus it is most important to invest in higher-quality issues so as to reduce the risk of a potential problem[11].

Describe the various security types relevant for corporate bonds:

Mortgage bonds

A mortgage bond grants bondholders a first-mortgage lien on substantially all its properties. The issuer is thus able to borrow at a lower interest rate than would be the case with unsecured debt. A lien is a legal right to sell mortgaged property to satisfy unpaid obligations to bondholders. Foreclosure and sale of mortgaged property are not typical. In default, there is typically a financial reorganization and provisions are made for settlement of the debt to bondholders. However, mortgage lien gives bondholders a very strong bargaining position relative to other creditors in determining the terms of reorganization.

Collateral trust bonds

When companies cannot pledge fixed assets or other real property, they pledge securities of other companies or Treasury's instead. To satisfy the desire of bondholders for security, they pledge stocks, notes, bonds, or whatever other kinds of obligations they own. If they are holding companies, the other companies may be their subsidiaries. These assets are termed collateral (or personal property), and bonds secured by such assets are collateral trust bonds.

Equipment trust certificates

Although railroads have issued the largest amount of equipment trust certificates, airlines have also used this form of financing. The legal arrangement is one that vests legal title to railway equipment in a trustee, which is better from the standpoint of investors than a first-mortgage lien on property. A railway company orders some cars and locomotives from a manufacturer. When the job is finished, the manufacturer transfers the legal title of the equipment to a trustee. The trustee leases it to the railroad that ordered it and at the same time sells equipment trust certificates (ETCs) in an amount equal to a large percentage of the purchase price, normally 80%. Money from sale of certificates is paid to the manufacturer.

Debenture bonds (including subordinated and convertible debentures)

Unsecured bonds are called debentures. With the exception of the utilities and structured products, nearly all other corporate bonds issued are unsecured. Debenture bondholders do have the claim of general creditors on all assets of the issuer not pledged specifically to secure other debt. Subordinated debenture bonds: issue ranks after secured debt, after debenture bonds, and often after some general creditors in its claim on assets and earnings. Owners of this bond "stand last in line". Because subordinated debentures are weaker in their claim on assets, issuers must offer a higher rate of interest, unless they also offer some special inducement to buy the bonds. The inducement may be in the form of an option to convert bonds into stock of the issuer at the discretion of bondholders, subject to certain restrictions.

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[11] This is not entirely true, as one would readily expect to be compensated for the higher risk in terms of higher returns on the bond issue (a deeper discount at the time of issuance).

This conversion privilege also may be included in the provisions of debentures that are not subordinated.
The bonds may be convertible into the common stock of a corporation other than that of the issuer. Such issues are called exchangeable bonds. There are also issues indexed to a commodity's price or its cash equivalent at the time of maturity or redemption.

Guaranteed bonds

Guaranteed bonds: a corporation may guarantee the bonds of another corporation.
The guarantee, however, does not mean that these obligations are free of default risk. The safety of a guaranteed bond depends on the financial capability of the guarantor to satisfy the terms of the guarantee, as well as the financial capability of the issuer. The terms of the guarantee may call for the guarantor to guarantee the payment of interest and/or principal repayment

Describe the mechanisms by which corporate bonds can be retired before maturity, including:

Retiring bonds before maturity

Frequently, bonds are retired early, before maturity. There are several mechanisms by which a corporation may go about retiring their debt. Included below are some of the most important and commonly used retirement mechanisms. The mechanisms with a star (*) are particularly important.

- Call and refunding provisions*
- Fixed-price call provision
- Make-whole call provision
- Sinking-fund provision*
- Maintenance and replacement funds*
- Redemption through sale of assets
- Tender offers*

Fixed price

Bond issuer has the option to buy back some or all of the bond issue prior to maturity at a fixed price ("call price").
Call prices generally start at a substantial premium over par and decline toward par over time; in the final years of a bond's life, the call price is usually par.

Make-whole

Call price is calculated as the present value of the bond's remaining cash flows subject to a floor price equal to par value. The discount rate used to determine the present value is the yield on a comparable maturity.

Call provision

Corporate bonds that contains an embedded option that gives the issuer the right to buy the bonds back at a fixed price prior to maturity, either in whole or in part.

The ability to retire debt before its scheduled maturity date is a valuable option for which bondholders will demand compensation ex-ante.

Ceteris paribus, bondholders will pay a lower price for a callable bond than an otherwise identical option-free (i.e., straight) bond.

The difference between the price of an option-free bond and the callable bond is the value of the embedded call option

Sinking-fund provisions

Money applied periodically to redemption of bonds before maturity. Two advantages from the bondholder's perspective:

Default risk is reduced

If bond prices decline as a result of an increase in interest rates, the issuer or its fiscal agent may provide price support because it must enter the market on the buy side in order to satisfy the sinking-fund requirement.

Disadvantage is the bonds may be called at the special sinking-fund call price at a time when interest rates are lower than rates prevailing at time of issuance.

Maintenance and replacement funds

Maintenance and replacement fund (M&R) provisions first appeared in bond indentures of electric utilities subject to regulation by the Securities and Exchange Commission (SEC) under the Public Holding Company Act of 1940. It remained in the indentures even when most of the utilities were no longer subject to regulation under the act. The original motivation for their inclusion is straightforward. Property is subject to economic depreciation, and the replacement fund ostensibly helps to maintain the integrity of the property securing the bonds. An M&R differs from a sinking fund in that the M&R only helps to maintain the value of the security backing the debt, whereas a sinking fund is designed to improve the security backing the debt. Although it is more complex, it is similar in spirit to a provision in a home mortgage requiring the homeowner to maintain the home in good repair.

Tender offers

At any time a firm may execute a tender offer and announce its desire to buy back specified debt issues. Firms employ tender offers to eliminate restrictive covenants or to refund debt. Usually the tender offer is for "any and all" of the targeted issue, but it also can be for a fixed dollar amount that is less than the outstanding face value. An offering circular is sent to the bondholders of record stating the price the firm is willing to pay and the window of time during which bondholders can sell their bonds back to the firm.

Describe, and differentiate between credit default risk and credit-spread risk

Credit default risk

Any bond investment carries with it the uncertainty as to whether the issuer will make timely payments of interest and principal as prescribed by the bond's indenture.
Credit default risk is the risk that a bond issuer will be unable to meet its financial obligations.

Credit-spread risk

The credit spread is the difference between a corporate bond's yield and the yield on a comparable-maturity benchmark Treasury security.

What explains the difference?

The difference in yields is due primarily to the corporate bond's exposure to credit risk, but not exclusively. The risk profile of corporate bonds differs from Treasuries on other dimensions; corporate bonds are less liquid and may have embedded options. Credit-spread risk is the risk of financial loss resulting from changes in the level of credit spreads used in the marking-to-market of a fixed income product. Credit spreads driven by: Macroeconomic forces include such things as the level and slope of the Treasury yield curve, the business cycle, and consumer confidence. Issue-specific factors include the corporation's financial position and the future prospects of the firm and its industry.

Describe event risk and what may cause it in corporate bonds

Event risk is the risk that a transaction (or corporate event) will devalue bondholder's position. Restructurings, recapitalizations, mergers, acquisition, leveraged buyouts, and share repurchase often cause substantial changes in a corporation's capital structure, greatly increased leverage and decreased equity. Event risk has caused some companies to include other special debt- retirement features in their indentures. An example is the maintenance of net worth clause included in the indentures of some lower-rated bond issues.

Define high-yield bonds; describe types of high-yield bond issuers, and some of the payment features peculiar to high yield bonds

High-yield bonds are those rated below investment grade by the ratings agencies, these issues are also known as junk bonds.

Types of high-yield bond issuers

Original Issuers
Fallen Angels
Restructurings and Leverage Buyouts

Payment features peculiar to high-yield bonds (deferred-coupon structures)

Deferred-interest bonds,
Step-up bonds, and
Payment-in-kind bonds.

Define and differentiate between an issuer default rate and a dollar default rate

Issuer default rate

Number of issuers that default divided by total number of issuers. Gives no recognition to amount defaulted nor amount of issuance.

Dollar default rate

Par value of all defaulted bonds divided by total par value of bonds outstanding during the year (Altman uses this method).

Average annual default rate
Cumulative $ value of all defaulted bonds ÷ Cumulative $ value of all issuance ´ by weighted average number of years outstanding

Cumulate default rate
Cumulative $ value of all defaulted bonds ÷ Cumulative $ value of all issuance

Define recovery rates and describe the relationship between recovery rates and seniority

Measuring the amount recovered is non-trivial. The final distribution to claimants may consist of cash and securities. However, it can often be difficult to track what was received and then additionally, determine the present value of any noncash payments received. Moody's use the trading price at the time of default as a proxy for the amount recovered. The recovery rate is thus the trading price at that time divided by the par value. Moody's found that the recovery rate was 38% for all bonds. While default rates are the same regardless of the level of seniority, recovery rates differ. The study found that the higher the level of seniority, the greater is the recovery rate.

Chapter Summary

The bond's *indenture* is the contract that contains corporate bond issuer promises and investors' rights.

The corporate trustee is a third party to the contract and acts in a fiduciary (legal) capacity for, or on behalf of, investors. It is the role of the trustee to certify that the bond issuer is in compliance with the covenants set forth in the bond's indenture.

The maturity date of the bond is the date on which the issuer's obligation to satisfy the terms of the indenture are fulfilled. Moreover, the principal must be repaid along with any premium and accrued interest.

The 3 main interest payment classifications of domestically (US) issued corporate bonds are:

1. Straight-coupon bonds (sometimes also referred to as plain-vanilla bonds),
2. Zero-coupon bonds, and
3. Floating-rate, or variable-rate, bonds.

Zero-coupon bonds are bonds without coupons or interest rate payments. They are issued at discounts to par; the difference is the return to the bondholder. Furthermore, zero-coupon rates play an important role in the construction of a discount curve: it informs the rate of return for the date at which it matures since there is no re-investment risk. The difference between the face amount and the offering price when first issued is called the original-issue discount (OID).

The security types relevant for corporate bonds include mortgage bonds, collateral trust bonds, equipment trust certificates, debenture bonds, including subordinated and convertible bonds, and guaranteed bonds.

Firms often retire their bonds before maturity. The mechanisms by which they do this includes: **call and refunding provisions**, fixed-price call provision, make-whole call provision, **sinking-fund provision**, **maintenance and replacement funds**, redemption through sale of assets and **tender offers**.

Credit default risk is the risk that a bond issuer will be unable to meet its financial obligations. The *credit spread* is the difference between a corporate bond's yield and the yield on a comparable-maturity benchmark Treasury security. *Credit-spread risk* is the risk of financial loss resulting from changes in the level of credit spreads used in the MTM of a fixed income product.

The *issuer default* rate is the number of issuers that default divided by total number of issuers. The *dollar default* rate is the par value of all defaulted bonds divided by total par value of bonds outstanding during the year.

Questions & Answers

Questions

13.1 Each of the following is true about the corporate trustee in a corporate bond issuance EXCEPT:

a) The trustee is paid by bondholders
b) The trustee acts in a fiduciary capacity for investors who own the bond issue
c) The trustee must, at the time of issue, authenticate the bonds issued (i.e., keep track of all the bonds sold) and make sure that they do not exceed the principal amount authorized by the indenture
d) If a corporate issuer fails to pay interest or principal, the trustee may declare a default and take such action as may be necessary to protect the rights of bondholders

13.2 What is an advantage of a fixed-spread tender offer over a fixed-price tender offer?

a) No significant advantage
b) It is easier for bondholders to assess the value of a fixed-spread tender offer
c) Significantly less counterparty credit risk to bondholders in a fixed-spread tender offer
d) Fixed-spread tender offers eliminate the exposure to interest-rate risk for both bondholders and the issuer during the tender offer window

13.3 Consider which of the following statements are true about bond reinvestment risk and bond duration (interest rate risk):

I. Lower bond reinvestment implies higher interest rate risk (duration), ceteris paribus
II. Due to reinvestment risk, the yield-to-maturity on a bond is unlikely to equal the bond's realized return
III. Reinvestment risk is eliminated in a zero-coupon bond

a) I. Only
b) II and I.
c) II and III.
d) All three

13.4 Each of the following is an example of a high-yield bond issuer EXCEPT:

a) Issuer with a credit rating of "BBB-"
b) Original issuer
c) Fallen angel
d) Leveraged buyout

Answers

13.1 A. “It must be emphasized that the trustee is paid by the debt issuer and can only do what the indenture provides.”
In regard to (B), (C), and (D), each is true.

13.2 D. Fixed-spread tender offers eliminate the exposure to interest-rate risk for both bondholders and the issuer during the tender offer window
Fabozzi: “Recently, tender offers have been executed using a fixed spread as opposed to a fixed price. In a fixed-spread tender offer, the tender offer price is equal to the present value of the bond’s remaining cash flows either to maturity or the next call date if the bond is callable. The present-value calculation occurs immediately after the tender offer expires. The discount rate used in the calculation is equal to the yield-to-maturity on a comparable-maturity Treasury or the associated CMT yield plus the specified fixed spread. Fixed-spread tender offers eliminate the exposure to interest-rate risk for both bondholders and the firm during the tender offer window.”

13.3 D. All three
Fabozzi: “One important risk is eliminated in a zero-coupon investment—the reinvestment risk. Because there is no coupon to rein-vest, there isn’t any reinvestment risk. Of course, although this is beneficial in declining-interest-rate markets, the reverse is true when interest rates are rising. The investor will not be able to reinvest an income stream at rising reinvestment rates. Investors tend to find zeros less attractive in lower-interest-rate markets because compounding is not as meaningful as when rates are higher. Also, the lower the rates are, the more likely it is that they will rise again, making a zero-coupon investment worth less in the eyes of potential holders.”

13.4 A. “BBB-” is the lowest S&P Investment Grade rating; BB+ is the highest Speculative Grade (aka, high-yield, junk)
In regard to (A), (B), and (C), each are an example, given by Fabozzi, of high-yield bond issuers.

Caouette, Chapter 6: The Rating Agencies

Learning Outcomes:

Describe the role of rating agencies in the financial markets.

Explain market and regulatory forces that have played a role in the growth of the rating agencies.

Describe a rating scale, define credit outlooks, and explain the difference between solicited and unsolicited ratings.

Describe Standard and Poor's and Moody's rating scales and distinguish between investment and noninvestment grade ratings.

Describe the difference between an issuer-pay and a subscriber-pay model and describe concerns regarding the issuer-pay model.

Describe and contrast the process for rating industrial and sovereign debt and describe how the distributions of these ratings may differ.

Describe the ratings performance for corporate bonds.

Describe the relationship between the rating agencies and regulators and identify key regulations that impact the rating agencies and the use of ratings in the market.

Describe some of the trends and issues emerging from the current credit crisis relevant to the rating agencies and the use of ratings in the market.

Describe the role of rating agencies in the financial markets.

Responsibility to: inform investors of likelihood they will receive principal and interest payments as scheduled

In some markets (e.g., United States) the capital markets have replaced banks as the primary source of debt capital; and ratings agencies have assumed enormous importance in the management of credit risk.

Qualifier: agencies make no recommendations about buying, selling, or holding a particular security or about suitability for a particular investor. Their ratings **express nothing more than informed opinions** about creditworthiness

IMPORTANT CONCEPT

> Agency opinions are supposed to be “independent, objective, and produced through a transparent and high-quality analytic process”

Describe market and regulatory forces that have played a role in the growth of the rating agencies.

US Regulators

Encouraged use of ratings from designated agencies: *Nationally recognized statistical rating organizations* (NRSROs) – a “regulatory oligopoly”

SEC: broker-dealer net capital requirements; certain reporting exemptions

Fed Reserve/FDIC

DOL/States: Eligibility for pensions

Basel

Basel does not decide which agencies are eligible but rather sets forth the criteria to be used by national supervisors for the “recognition” of External credit assessment institutions (ECAIs)

Pillar I (Minimum capital requirements)

Credit risk: Credit risk mitigation (CRM),

Credit risk: Securitization

Pillar III (Market discipline)

Disclosures

BIS’ **Stocktaking on the use of credit ratings** (June 2009) found five key purposes of ratings:

1. Determine capital requirements;
2. Identify or classify assets, usually in the context of eligible investments or permissible asset concentrations;
3. Providing a credible evaluation of the credit risk associated with assets purchased as part of a securitization offering or a covered bond offering;
4. Determine disclosure requirements;

5. Determine prospectus eligibility

Describe what a rating scale is, what credit outlooks are, and the difference between solicited and unsolicited ratings.

Describe what a rating scale is

In rating long-term debt, each agency uses **alphanumeric letter grades**: locates issuer/issue on credit quality spectrum

Very highest (AAA/Aaa; extremely strong capacity to meet financial commitments)

Very lowest (C/D; has been a payment default).

Each letter grade has three notches

Fitch and S&P use + and − modifiers; e.g., BBB+

Moody’s uses numerical modifiers; e.g., Ba1

Investment grade: rated BBB/Baa or above

Speculative: rated BB/Ba or below

Define credit outlooks

Credit quality of an obligor can change dramatically over time; ratings are subject to revision. Agencies update their credit outlook for most issuers on a continuing basis.

Indicate whether …

Outlook is positive: rating may be raised,

Outlook is negative: rating may be lowered,

Stable: neutral outlook,

Developing/evolving: rating may change up or down.

Solicited and unsolicited ratings

Unsolicited or agency-initiated ratings: When agencies assign ratings at their own initiative

Such ratings represent “only a small minority” of the larger agencies’ coverage. Agencies maintain such ratings are assigned on same basis as compensated ratings.

Disclosure of unsolicited ratings varies.

Both Moody's & Fitch disclose agency-initiated ratings in press releases along with initial rating assignment

S&P discloses uncompensated ratings in each press release, but does not otherwise indicate the level of issuer participation

Identify Standard and Poor's and Moody's rating scales and distinguish between investment and noninvestment grade ratings.

TABLE 6.3: Long-Term Senior Debt Rating Symbols

	Investment Grade Ratings
Rating	**Interpretation**
AAA/Aaa	Highest quality; extremely strong, highly unlikely to be affected by foreseeable events.
AA/Aa	Very high quality; capacity for repayment is not significantly vulnerable to foreseeable events.
A/A	Strong payment capacity; more likely to be affected by changes in economic circumstances.
BBB/Baa	Adequate payment capacity; a negative change in environment may affect capacity for repayment.
	Below Investment Grade Ratings
Rating	**Interpretation**
BB/Ba	Considered speculative with possibility of developing credit risks.
B/B	Considered very speculative with significant credit risk.
CCC/Caa	Considered highly speculative with substantial credit risk.
CC/Ca	Maybe in default or wildly speculative.
C/C/D	In bankruptcy or default.

Describe the difference between an issuer-pay and a subscriber-pay model and what concerns the issuer-pay model engenders.

Even before the crisis, "the switch in payment arrangements from subscribers to issuers remains a point of some controversy, raising concerns about the independence of the agencies."

In truth, although they are paid for their services, the agencies generally behave more like academic research centers than businesses.

Describe and contrast the process for rating industrial and sovereign debt and describe how the distributions of these ratings may differ.

Industrial ratings

S & P focuses on:

- Business risk

- Industry characteristics
- Competitive positioning
- Management
- Financial risk
- Financial characteristics
- Financial policies
- Profitability
- Capitalization
- Cash flow protection
- Financial flexibility

Sovereign Debt:

A host of other qualitative factors: stability of political institutions, social/economic coherence, and integration into the world's economic system. These factors lead to the **greater dispersion** around quantitative estimates in sovereign ratings

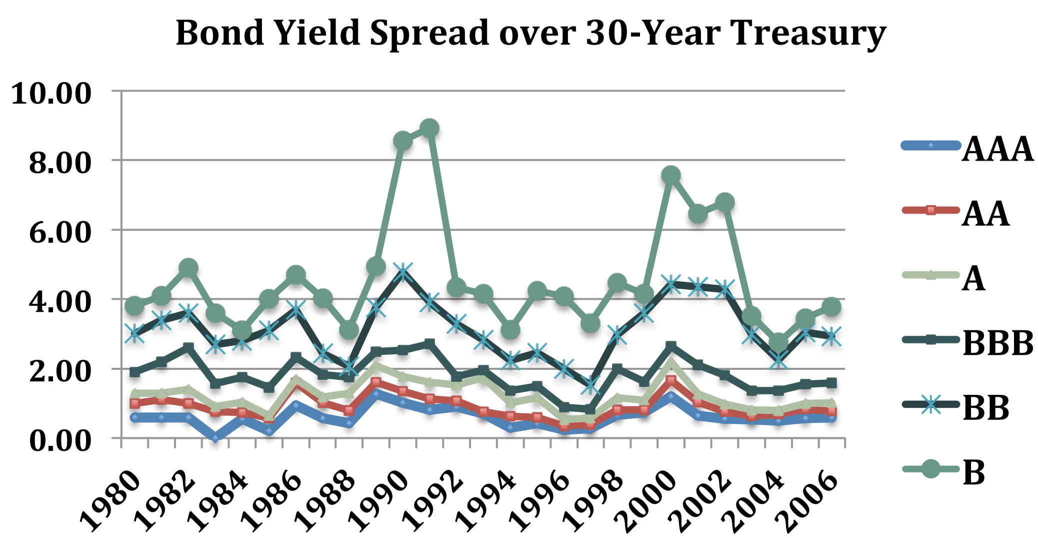
Discuss the ratings performance for corporate bonds

Default rates are "consistently inversely related to credit ratings over both short & long time periods."

From 1970–2005, the one-year default rate of Aaa bonds was zero

But it was more than 5 percent for B-rated bonds.

Ordinal ranking tends to hold up.



Describe the relationship between the rating agencies and regulators and identify key regulations that impact the rating agencies and the use of ratings in the market

Relationship is “deep and often ambiguous”

Regulators attracted to standard (widespread acceptance)

For agencies, regulator reliance is validation

Key regulations

NRSROs in US

Credit Rating Agency Reform Act of 2006

ECAI in Basel II

Discuss some of the trends and issues emerging from the current credit crisis relevant to the rating agencies and the use of ratings in the market.

Increasing regulatory involvement, “we expect [the credit crunch] will subject them to more intensive review and criticism than either the Asian crisis or Enron and WorldCom.”

“Credit rating agencies are going to remain a major influence in the capital markets and, if anything, their domination will grow stronger. Even though their performance has not been without blemish it is clear that global financial markets need their essential information services.”

Chapter Summary

The rating agencies play an important role in capital markets around the world. We can generalize and say that two credit ratings agency structures exist: the issuer-pay model, in which the issuer of a security pays the credit ratings agencies to rate their securities, or the subscriber model, in which the ratings are produced independently of the company which issues them, and paid for by subscriber to the ratings services.

The former of the two models have come under some criticism in part because of the high ratings that were given to firms prior to the financial crisis that started in 2007. This raised questions about the transparency and independency of the agencies. The agencies have categorically stated that their assessments are independent of the issuer, however, they have acknowledged that other factors such as liquidity could have played a bigger role in their ratings. Following the financial crisis, the major agencies have subsequently made strides to improve upon their modeling techniques and ratings.

It is important to recognize that the agencies' ratings do not constitute a recommendation regarding whether or not to buy, sell or hold a security. Their ratings express nothing more than informed opinions about creditworthiness.

The three major rating agencies in the US are Standard & Poor's, Moody's Investor Service and Fitch Ratings, and are all issuer-pay rating agencies. Standard & Poor's and Moody's use letter categories to rate firms. AAA/Aaa is the highest, while D and C are the lowest for S&P and Moody's, respectively. Investment grade issues range from AAA/Aaa to BBB/Baa. Issues below investment grade range between BB/Ba and D/C. The latter category represents companies in bankruptcy or default.

In the US only 4 companies have a AAA rating (Automatic Data Processing, Exxon Mobile Corp. and Johnson & Johnson, Microsoft), some companies such as Apple Inc. do not have a rating due to not having any debt or bond issues.

Agencies continuously update their credit outlook for most issuers and will indicate whether the outlook is positive, negative, stable or evolving.

Ratings are consistently inversely related to default over both short and long time periods.

Sovereign debt introduces a lot of factors such as socioeconomic and demographic factors, which make the ratings distribution for countries more dispersed. Sovereign debt cannot readily be compared to corporate ratings.

Questions & Answers

Questions

14.1 Which of the following is most nearly TRUE about agency credit ratings?

a) Agency ratings are investment advice; e.g., a credit downgrade is analogous to a equity-class sell recommendation
b) In theory, agency ratings have a longer term orientation (approximating or exceeding the maturity of the instrument) and through-the-cycle, not point-in-time; they do not undulate with business cycles
c) An agency credit rating is sufficient to price (or value) a debt instrument
d) Agency ratings, because they are measured on a cardinal scale, correspond directly to probabilities of default (PD; or, expected default frequencies)

14.2 Assume the following corporate bond credit rating migration matrix for a one-year period (same as prior: Fitch's actual updated matrix):

	AAA	AA	A	BBB	BB	B	CCC to C	D
AAA	94.50	5.50	-	-	-	-	-	-
AA	0.09	90.24	9.28	0.31	0.02	0.02	-	0.04
A	0.02	1.99	91.79	5.45	0.52	0.08	0.07	0.08
BBB	-	0.20	3.62	91.49	3.62	0.60	0.24	0.23
BB	0.02	0.05	0.10	8.49	81.35	7.04	1.66	1.29
B	-	-	0.24	0.45	9.50	83.18	4.28	2.35
CCC - C	-	-	-	0.20	2.40	18.76	50.90	27.74
	-	-	-	-	-	-	-	-

Source: Fitch, Global Corporate Avg Annual Transition Rates: 1990-2011

What is the cumulative probability that a BB obligor will default within the next two years, assuming Markovian independence?

a) 0.017%
b) 1.290%
c) 2.563%
d) 2.985%

14.3 The United Kingdom (UK) has an 'Aaa' rating from Moody's. However, Moody's recently announced they were attaching a negative outlook to the rating. Which of the following best summarizes the implication of the negative outlook?

a) UK retains the triple-A rating, but there is a possibility of a downgrade in the future
b) UK retains the triple-A rating, but a downgrade is inevitable while only the timing is uncertain
c) UK retains the triple-A rating, but is no longer considered "investment grade"
d) The UK rating technically becomes AAA- or Aaa3

14.4 In regard to the relationship between rating agencies and regulators, each of the following is true EXCEPT:

a) The Credit Rating Agency Reform Act of 2006 abolishes the registration, and designation, of Nationally Recognized Statistical Rating Organizations ("NRSROs")
b) The Credit Rating Agency Reform Act of 2006 vested the SEC with the authority to establish a registration and oversight program for credit rating agencies
c) Dodd-Frank removes references to ratings from the Securities Exchange Act and a number of other statutes.
d) Dodd-Frank requires federal agencies like the SEC and Federal Reserve to remove references to ratings from their own regulations when those ratings are used to assess the creditworthiness of a security or money market instrument

Answers

14.1 B. In theory, agency ratings have a longer-term orientation (approximating or exceeding the maturity of the instrument) and through-the-cycle, not point-in-time; they do not undulate with business cycles
In regard to (A), false: Agency ratings are opinions.
In regard to (C), false: Agency ratings measure credit risk, not market risk (interest rates).
In regard to (D), false: Agency ratings employ an ordinal scale and do not map directly (or precisely) to a PDF (or EDF) which is a continuous variable.

14.2 D. 2.985%
The 2-year cumulative PD is the probability that the obligor defaults in the first year plus the probability the bond defaults in the second year.
Pr[BB -> D in first year] = 1.29%;
Pr[default in second year] = Pr[BB->AAA->D] + Pr[BB->AA->D] + Pr[BB->A->D] + Pr[BB->BBB->D] + Pr[BB->BB->D] + Pr[BB->B->D] + Pr[BB->C->D] =
~0 + ~0 + ~0 + 0.0195% + 1.0494% + 0.1654% + 0.4605% = 2.9849%

In regard to (C), tempting is: 1 - (1-1.29%)^2 = 2.563%; and this is the cumulative PD if we are only given the conditional PD of 1.29%. However, this assumes a sequence of two Bernoulli variables and excludes the migration dynamic. Put another way, this approach does excludes a path such as: in the first year, the BB migrates to B, then defaults in the second year. In this way, by excluding migration, this will understate the cumulative PD.

14.3 A. UK retains the triple-A rating, but there is a possibility of a downgrade in the future
In regard to (C), this is obviously false; even if downgraded to Aa, the UK would remain investment grade
In regard to (D), this refers to notches; and, nevertheless, triple-A ratings don't attach with notches

14.4 A. The Credit Rating Agency Reform Act of 2006 did not abolish NRSROs; rather, it attempted to replace an "opaque" system with a more transparent registration, and limited the SEC's oversight to registered agencies.

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