

# Mutation-based Lifted Repair of Software Product Lines

Aleksandar S. Dimovski 

Mother Teresa University, st. Mirche Acev nr. 4, 1000 Skopje, North Macedonia

## Abstract

This paper presents a novel lifted repair algorithm for program families (Software Product Lines - SPLs) based on code mutations. The inputs of our algorithm are an erroneous SPL and a specification given in the form of assertions. We use variability encoding to transform the given SPL into a single program, called family simulator, which is translated into a set of SMT formulas whose conjunction is satisfiable iff the simulator (i.e., the input SPL) violates an assertion. We use a predefined set of mutations applied to feature and program expressions of the given SPL. The algorithm repeatedly mutates the erroneous family simulator and checks if it becomes (bounded) correct. Since mutating an expression corresponds to mutating a formula in the set of SMT formulas encoding the family simulator, the search for a correct mutant is reduced to searching an unsatisfiable set of SMT formulas. To efficiently explore the huge state space of mutants, we call SAT and SMT solvers in an incremental way. The outputs of our algorithm are all minimal repairs in the form of minimal number of (feature and program) expression replacements such that the repaired SPL is (bounded) correct with respect to a given set of assertions.

We have implemented our algorithm in a prototype tool and evaluated it on a set of `#ifdef`-based C programs (i.e., annotative SPLs). The experimental results show that our approach is able to successfully repair various interesting SPLs.

**2012 ACM Subject Classification** Software and its engineering → Software functional properties; Software and its engineering → Software creation and management; Theory of computation → Logic

**Keywords and phrases** Program repair, Software Product Lines, Code mutations, Variability encoding

**Digital Object Identifier** 10.4230/LIPIcs.ECOOP.2024.36

## 1 Introduction

A *program family* (Software Product Line - SPL) represents a set of similar programs, known as *variants*, generated from a common code base [2]. SPL engineering has been successfully applied in industry to meet the need for custom-tailored software. For instance, different variants from an SPL can target different platforms or may serve customization requirements for different customers. The variants are specified in terms of *features* selected for that particular variant. The popular `#ifdef` directives from the C preprocessor CPP [43] represent the most common way to implement such (annotative) program families. An `#ifdef` directive specifies under which presence conditions (i.e., feature selections or feature expressions), parts of code should be included or excluded from a variant at compile-time. SPLs are often used in the development of the embedded and safety-critical systems (e.g., mobile devices, cars, medicine, avionics), where their behavioral correctness is of primary interest. In particular, the focus is on applying various verification and analysis techniques from the field of formal methods, which can give stronger guarantees on the correctness of software systems. In the last decade, much effort has been invested in designing specialized so-called *lifted* (family-based) formal verification and analysis algorithms [4, 6, 9, 43, 30, 14, 23, 15, 20, 22, 25, 55], which allow simultaneous verification of all variants of an SPL in a single run by exploiting the commonalities between the variants. They usually return an error trace, which shows how the given specification is violated. However, the users still need to process the obtained result, in order to isolate the cause of the error to a small part of the code and subsequently

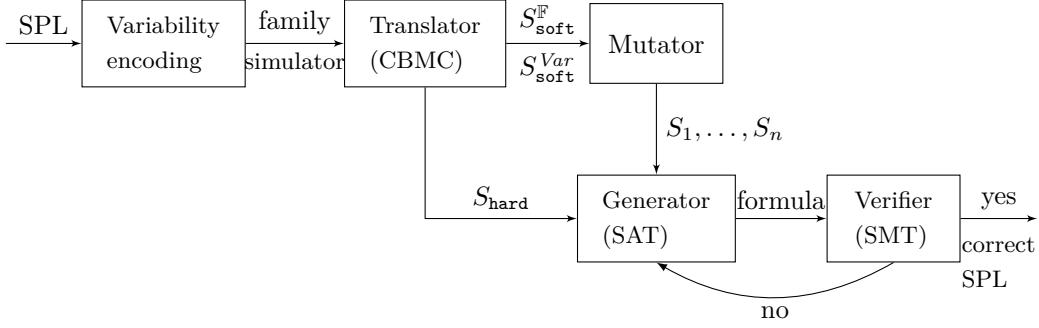


Figure 1 Diagram illustrating our lifted repair system.

to repair the given SPL. Here, we consider the problem of *SPL repair*, which is defined to be a code transformation such that the repaired SPL satisfies a given specification (e.g. assertion). Automatic SPL repair is an important problem since even if an error is identified in the verification phase, the manual error-repair is a nontrivial time-consuming task that requires close knowledge of the SPL. For instance, the error-repair of one variant may cause new errors to appear in other variants due to the feature interaction in the given SPL [3]. Recently, researchers have developed several successful single-program repair tools [28, 37, 40, 42, 45, 46, 48, 50, 51]. However, these tools cannot be directly applied to SPLs as they are only able to handle pre-processed single programs.

In this paper, we lift the mutation-based approach ALLREPAIR [50, 51] for repairing single programs to program families (SPLs). Figure 1 illustrates our lifted repair system. More specifically, we use *variability encoding* [30, 56] to transform program families to single programs, called family simulators, by replacing compile-time variability with run-time variability (non-determinism). The (family) simulator, which contains the computations of all variants of a program family, is then translated into a set of SMT formulas using the CBMC bounded model checker [8]. The conjunction of the obtained SMT formulas is *satisfiable* iff there is an assertion violation in the given simulator iff there is an assertion violation in at least one variant of the original program family. On the other hand, the conjunction of the obtained SMT formulas is *unsatisfiable* iff all assertions are valid in the given simulator iff all assertions are valid in all variants of the original program family. We use a bounded notion of correctness, since we consider only bounded computations in which each loop and recursive call are inlined at most  $b$  times. Each statement in the simulator corresponds to a formula in the obtained set of SMT formulas, which can be partitioned into subset  $S_{\text{hard}}$  encoding parts of the program that cannot be changed and subset  $S_{\text{soft}} = S_{\text{soft}}^{\mathbb{F}} \cup S_{\text{soft}}^{Var}$  encoding parts of the program that can be changed. Therefore, mutating a feature or program expression found in a statement that can be changed corresponds to changing the respective SMT formula from  $S_{\text{soft}}^{\mathbb{F}}$  or  $S_{\text{soft}}^{Var}$ , respectively. The *mutator* unit generates mutated family simulators (mutants) by using a predefined set of syntactic mutations/edits applied to feature and program expressions. Hence, in our repair model, we permit feature and program expressions to be changed but not statements. For example, we allow replacement of `#ifdef` guards (e.g., by applying  $\neg$  to features, replacing  $\wedge/\vee$  with  $\vee/\wedge$ ) and right-hand sides of assignments (e.g., by increasing or decreasing a constant, replacing  $+/-$  with  $-/+$ ). Thus, the size of the space of mutants depends on the choice of permissible mutations/edits used for repair. The mutants are explored in increasing number of mutations applied to the original family simulator, so that only minimal sets of mutations are considered. Hence, the search in the space of mutants reduces to searching for an unsatisfiable set of SMT formulas. This

82 search is performed using an iterative generate-and-verify process. The *generator* produces a  
 83 minimally changed mutant using a SAT solver and the *verifier* checks the bounded-correctness  
 84 of mutant using an SMT solver. This way, we find a solution with a minimal number of  
 85 syntactic changes/edits to the original (incorrect) program family. Therefore, the type of  
 86 errors that can be corrected is determined by the fixed set of syntactic mutations/edits, which  
 87 can be applied to feature and program expressions. Hence, our approach can make repairs  
 88 by replacing expressions in `#ifdef`-guards and right-hand sides of assignments with another  
 89 expressions of the same form, but it cannot make repairs by replacing (adding/deleting)  
 90 statements (e.g., replace assignment with if statement) or by replacing expressions with  
 91 another expressions of different form (e.g., replace expression 5 with  $x+y$ ). Both SAT and  
 92 SMT solvers are used in an incremental way, which means that learned information is passed  
 93 between successive calls. Since variants in a program family as well as mutated simulators  
 94 are very similar, their encodings as sets of SMT formulas will have a lot in common. Hence,  
 95 we can reuse the information that was gathered in checking previous mutated simulators to  
 96 expedite the solution of the current one. The incremental solving was implemented via the  
 97 mechanisms called assumptions and guard variables [26].

98 We have implemented our algorithm for repairing `#ifdef`-based C program families in a  
 99 prototype tool, called SPLALLREPAIR, which is built on top of the ALLREPAIR tool [50, 51].  
 100 The tool uses the CBMC model checker [8] for translating single programs to SMT formulas,  
 101 as well as the MINICARD [39] and Z3 [11] tools for SAT and SMT solving. We illustrate  
 102 this approach for automatic repair on a number of C program families from the literature  
 103 [10, 37, 46, 50, 51], and we report very encouraging results. We compare performances  
 104 of two versions of our tool, with smaller and bigger sets of possible mutations, as well as  
 105 with the **Brute-force** approach that repairs all variants from a program family one by one  
 106 independently.

107 We summarize the contributions of this paper as follows:

108 **Lifted Algorithm for SPL Repair:** We propose a novel lifted algorithm based on variability  
 109 encoding and syntactic code mutations for repairing program families;

110 **Synthesizing Minimally Repaired SPLs:** We automatically compute all minimal repaired  
 111 program families (minimal in the number of code replacements) that are bounded correct  
 112 by mutating feature and program expressions;

113 **Implementation and Evaluation:** We build a prototype tool for automatically repairing  
 114 `#ifdef`-based C program families, and present experimental results by evaluating it on a  
 115 dozen of C benchmarks.

## 116 2 Motivating Example

117 We now present an overview of our approach using a motivating example. Consider the  
 118 `#ifdef`-based C program family `intro1`, shown in Fig. 2, which uses two Boolean features  
 119 A and B. They induce a family of four variants defined by the set of configurations  $\mathbb{K} =$   
 120  $\{A \wedge B, \neg A \wedge B, A \wedge \neg B, \neg A \wedge \neg B\}$ . For each configuration, a different variant (single program)  
 121 can be generated by appropriately resolving `#if` directives. For example, the variant for  
 122 configuration  $(A \wedge B)$  will have both features A and B enabled (set to true or 1), thus yielding  
 123 the body of `main()`: `int x=0; x=x+2; assert(x≥0); return x`. The variant for  $(\neg A \wedge \neg B)$   
 124 will have both features A and B disabled (set to false or 0), so it has the following body  
 125 of `main()`: `int x=0; x=x-2; assert(x≥0); return x`. In such program families, it may  
 126 happen that errors (e.g., assertion violations) occur in some variants but not in others. In the  
 127 `intro1` family, the assertion is valid for variants  $(A \wedge B)$ ,  $(A \wedge \neg B)$ ,  $(\neg A \wedge B)$  since the returned

---

```

int main(){
    ① int x := 0;
    ② #if (A) x := x+2; #endif
    ④ #if ( $\neg A \wedge \neg B$ ) x := x-2; #endif
    ⑥ assert(x ≥ 0);
    ⑦ return x;
}

```

---

**Figure 2** intro1.

---

```

int A := [0, 1];
int B := [0, 1];
int main(){
    int x := 0;
    if (A) x := x+2;
    if ( $\neg A \wedge \neg B$ ) x := x-2;
    assert(x ≥ 0);
    return x;
}

```

---

**Figure 3** intro2.

---

```

A0 := [0, 1];
B0 := [0, 1];
int main(){
    x0 := 0;
    g0 := A0;
    x1 := x0+2;
    x2 := g0?x1 : x0;
    g1 :=  $\neg A0 \wedge \neg B0$ ;
    x3 := x2-2;
    x4 := g1?x3 : x2;
    assert(x4 ≥ 0);
    return x4;
}

```

---

**Figure 4** intro3.

---

```

Sintro = {
    A0=[0, 1],
    B0=[0, 1],
    x0=0,
    g0=A0,
    x1=x0+2,
    x2=ite(g0,x1,x0),
    g1= $\neg A0 \wedge \neg B0$ ,
    x3=x2-2,
    x4=ite(g1,x3,x2),
     $\neg(x4 \geq 0)$ 
}

```

---

**Figure 5**  $S_{\text{intro}}$ 

128 value  $x$  will be 2, 2, 0, respectively. However, the assertion fails for variant  $(\neg A \wedge \neg B)$  since  
 129 the returned value  $x$  will be -2 in this case. The goal is to automatically repair this program  
 130 family, so that the assertion is valid for all its variants.

131 If we make mutations only to feature expressions, there are two possible repairs of `intro1`  
 132 that remedy the feature interaction  $(\neg A \wedge \neg B)$  responsible for the fault. First, the feature  
 133 expression  $(A)$  at loc. ② can be replaced with  $(\neg A)$ , thus making the assertion correct for  
 134 all variants: the returned value  $x$  will be 0 for variants  $(A \wedge B)$ ,  $(A \wedge \neg B)$ ,  $(\neg A \wedge \neg B)$ ; and  
 135 2 for  $(\neg A \wedge B)$ . Second, the feature expression  $(\neg A \wedge \neg B)$  at loc. ④ can be replaced with  
 136  $(A \wedge \neg B)$ , thus making the assertion correct for all variants: the returned value  $x$  will be 0  
 137 for variants  $(\neg A \wedge B)$ ,  $(A \wedge \neg B)$ ,  $(\neg A \wedge \neg B)$ ; and 2 for  $(A \wedge B)$ . If we make mutations only to  
 138 program expressions, then one possible repair is the program expression  $(x-2)$  at loc. ⑤ to  
 139 be replaced with  $(x+2)$ . The above three repairs are all minimal patched mutations obtained  
 140 by applying only one code mutation to the original program family. Note that the found  
 141 repairs depend on the sets of mutations applied to feature and program expressions. For  
 142 example, if we allow mutations of the arithmetic operator  $-$  to  $*$  and of the integer constant  
 143  $n$  to 0, we will also find additional minimal repairs that replace the expression  $(x-2)$  at loc.  
 144 ⑤ with  $(x*2)$  or  $(x-0)$ .

145 Our algorithm for repairing program families goes through four steps. We refer to the  
 146 running example `intro1` in Fig. 2 to demonstrate the steps.

147 (1) We transform the program family to a single program, called family simulator, using  
 148 variability encoding [30, 56], such that all features are first declared as global variables and  
 149 non-deterministically initialized to 0 or 1, and then all `#if` directives are transformed into  
 150 ordinary `if` statements with the same branch condition. For example, the single program

151     intro2 in Fig. 3 is a simulator for the program family intro1 in Fig. 2. Features A and  
 152     B are defined as non-deterministically initialized global variables and two #if directives  
 153     are replaced with if-s.

- 154     (2) The simulator is simplified (e.g., branch conditions are replaced with fresh Boolean  
 155     variables), unwinded by unrolling loops and recursive functions  $b$  times, and converted to  
 156     static single assignment (SSA) form. In the SSA form, time-stamped versions of program  
 157     variables are created: every time a variable is assigned, the time-stamp is incremented  
 158     by one and then the variable is renamed; every time a variable is read, it is renamed  
 159     using the current time-stamp. Thus, the single program intro3 in Fig. 4 is obtained by  
 160     simplifying and converting to SSA form the simulator intro2 in Fig. 3. For example, the  
 161     if condition ( $\neg A_0 \wedge \neg B_0$ ) is assigned to a fresh Boolean variable  $g_1$ , the first assignment  
 162     to  $x$  is replaced by an assignment to  $x_0$ , the second by an assignment to  $x_1$ , etc. We use  
 163      $\Phi$ -assignments to determine which copy of  $x$  will be used after if-s. For example, the  
 164      $\Phi$ -assignment  $x_2 := g_0?x_1 : x_0$  means that  $x_1$  is used if  $g_0$  is true, and  $x_0$  is used if  $g_0$  is  
 165     false.  
 166     (3) The simplified program in SSA form is converted to a program formula. Hence, the  
 167     program intro3 in Fig. 4 is converted to a set of SMT formulas  $S_{\text{intro}}$  shown in Fig. 5,  
 168     such that the corresponding program formula  $\varphi_{\text{intro}}$  is a conjunction of all SMT formulas  
 169     in  $S_{\text{intro}}$ . Note that the  $\Phi$ -assignment  $x_2 := g_0?x_1 : x_0$  is converted to the formula  
 170      $x_2 = \text{ite}(g_0, x_1, x_0)$ , which means  $(g_0 \wedge x_2 = x_1) \vee (\neg g_0 \wedge x_2 = x_0)$ , while assert(be) is  
 171     converted to  $(\neg \text{be})$ . Therefore, a program is correct (i.e., all assertions in it are valid) iff  
 172     the corresponding program formula is unsatisfiable.  
 173     (4) By making mutations in the set of SMT formulas, we aim to construct an unsatisfiable  
 174     program formula and report the corresponding program as repaired. In the running  
 175     example, if one of the following mutations:  $(g_0 = \neg A)$ ,  $(g_1 = A \wedge \neg B)$ , or  $(x_3 = x_2 + 2)$ , is applied  
 176     to the set of SMT formulas  $S_{\text{intro}}$  in Fig. 5, we obtain an unsatisfiable program formula.  
 177     This way, we generate a minimally mutated program family, which contains only one  
 178     code mutation, that is correct.

### 179     3 Background

180     In this section, we introduce the background concepts used in later developments. We  
 181     begin with the definition of syntax and semantics of program families. Then, we proceed to  
 182     introducing the bounded program analysis for translating single programs to SMT formulas.

#### 183     3.1 Program Families

184     Let  $\mathbb{F} = \{A_1, \dots, A_n\}$  be a finite set of *Boolean features* available in a program family. A  
 185     *configuration*  $k : \mathbb{F} \rightarrow \{\text{true}, \text{false}\}$  is a truth assignment or a *valuation*, which gives a truth  
 186     value to each feature. If  $k(A) = \text{true}$ , then feature  $A$  is enabled in configuration  $k$ , otherwise  
 187      $A$  is disabled. We assume that only a subset  $\mathbb{K}$  of all possible configurations are *valid*. Each  
 188     configuration  $k \in \mathbb{K}$  can also be represented by a formula:  $(k(A_1) \cdot A_1 \wedge \dots \wedge k(A_n) \cdot A_n)$ ,  
 189     where  $\text{true} \cdot A = A$  and  $\text{false} \cdot A = \neg A$ . We write  $\mathbb{K}$  for the set of all valid configurations.  
 190     We define *feature expressions*, denoted  $\text{FeatExp}(\mathbb{F})$ , as the set of propositional logic formulas  
 191     over  $\mathbb{F}$ :

$$192 \quad \theta (\theta \in \text{FeatExp}(\mathbb{F})) ::= \text{true} \mid A \in \mathbb{F} \mid \neg \theta \mid \theta \wedge \theta \mid \theta \vee \theta$$

193     We consider a simple sequential non-deterministic programming language, in which the  
 194     program variables  $\text{Var} = \{x_1, \dots, x_n\}$  are statically allocated and the only data type is the

## 36:6 Mutation-based Lifted Repair of Software Product Lines

195 set  $\mathbb{Z}$  of mathematical integers. To define program families, a new compile-time conditional  
 196 statement is introduced: “`#if ( $\theta$ )  $s$  #endif`”, such that the statement  $s$  will be included in  
 197 the variant corresponding to configuration  $k \in \mathbb{K}$  only if  $\theta$  is satisfied by  $k$ , i.e.  $k \models \theta$ . The  
 198 syntax is:

```

 $s (s \in Stm) ::= \text{skip} \mid x := ae \mid s; s \mid \text{if } (be) \text{ then } s \text{ else } s \mid \text{while } (be) \text{ do } s \mid$ 
 $\quad \#if (\theta) s \#endif \mid \text{assert}(be) \mid \text{assume}(be)$ 
199  $ae (ae \in AExp) ::= n \mid [n, n'] \mid x \mid ae \oplus ae,$ 
 $be (be \in BExp) ::= ae \bowtie ae \mid \neg be \mid be \wedge be \mid be \vee be$ 

```

200 where  $n \in \mathbb{Z}$ ,  $x \in Var$ ,  $\oplus \in \{+, -, *, \%, /\}$ ,  $\bowtie \in \{<, \leq, ==, !=\}$ , and integer interval  $[n, n']$   
 201 denotes a random integer in the interval. Without loss of generality, we assume that a  
 202 program family  $P$  is a sequence of statements followed by a single assertion, whereas a single  
 203 program  $p$  is a sequence of statements without `#if`-s followed by an assertion.

204 ▶ **Remark 1.** The C preprocessor CPP [32] also uses other compile-time conditional statements  
 205 that can be desugared and represented only by the `#if` construct we use in this work,  
 206 e.g. `#if ( $\theta$ )  $s_0$  #else  $s_1$  #endif` is translated into `#if ( $\theta$ )  $s_0$  #endif; #if ( $\neg\theta$ )  $s_1$  #endif`.  
 207 Compile-time conditional constructs can also be defined at the level of expressions, e.g.  
 208 `#if ( $\theta$ )  $ae_0$  #else  $ae_1$  #endif`, and they can be translated into compile-time conditional  
 209 statements by code duplication [32]. We use variability at the level of statements for  
 210 pedagogical reasons in order to keep the presentation focussed.

211 A program family is evaluated in two phases. First, the C preprocessor CPP [32] takes  
 212 a program family  $s$  and a configuration  $k \in \mathbb{K}$  as inputs, and produces a variant (single  
 213 program without `#if`-s) corresponding to  $k$  as output. Second, the obtained variant is  
 214 evaluated using the standard single-program semantics [20]. The first phase is specified  
 215 by the projection function  $\pi_k$ , which is an identity for all basic statements and recursively  
 216 pre-processes all sub-statements of compound statements. Hence,  $\pi_k(\text{skip}) = \text{skip}$  and  
 217  $\pi_k(s; s') = \pi_k(s); \pi_k(s')$ . The most interesting case is “`#if ( $\theta$ )  $s$  #endif`”, where the statement  
 218  $s$  is included in the variant  $k$  if  $k \models \theta$ ;<sup>1</sup> otherwise  $s$  is excluded from the variant  $k$ . That is:

$$219 \quad \pi_k(\#if (\theta) s \#endif) = \begin{cases} \pi_k(s) & \text{if } k \models \theta \\ \text{skip} & \text{if } k \not\models \theta \end{cases}$$

220 Given a program family  $P$ , the set of all variants derived from  $P$  is  $\{\pi_k(P) \mid k \in \mathbb{K}\}$ .

### 221 3.2 Bounded Program Analysis

222 Unbounded loops with memory allocation are the reason for the undecidability of the  
 223 assertion verification problem [24]. To avoid undecidability, we impose a bound on the loops  
 224 by discarding all executions paths in which a loop is iterated more than a pre-determined  
 225 number of times. That is, we analyze a new bounded program that under-approximates  
 226 the original program. Using such bounded program, we can build a SMT formula that  
 227 represents its semantics. We now briefly explain how a pre-processed program without `#if`-s  
 228 is translated into a set of SMT formulas using the CBMC bounded model checker [8]. We  
 229 present only the details that are important to understand our algorithm.

230 The given pre-processed (single) program undergoes three transformations: simplification,  
 231 unwinding, and conversion to SSA form. Recall from Section 2 that the simplification ensures

---

<sup>1</sup> Since  $k \in \mathbb{K}$  is a valuation function, either  $k \models \theta$  holds or  $k \not\models \theta$  holds for any  $\theta$ .

that all branch conditions are replaced with fresh Boolean variables, whereas the SSA-form guarantees that each local variable has a single static point of definition. More specifically, in SSA-form each assignment to a variable  $x$  is changed into an unique assignment to a new variable  $x_i$ . Hence, if variable  $x$  has  $n$  assignments to it throughout the program, then  $n$  new variables  $x_0$  to  $x_{n-1}$  are created to replace  $x$ . All uses of  $x$  are replaced by a use of some  $x_i$ . To decide which definition of a variable reaches a particular use after an **if**-statement with the guard  $g$ , we add the  $\Phi$ -assignment  $x_k := g?x_i : x_j$  after the **if**. This means that if control reaches the  $\Phi$ -assignment via the path on which  $g$  is true,  $\Phi$  selects  $x_i$ ; otherwise  $\Phi$  selects  $x_j$ . This way, all uses of  $x$  after an  $\Phi$ -assignment  $x_k := g?x_i : x_j$  become uses of  $\Phi$ -assignment  $x_k$  until the next assignment of  $x$ . The unwinding with bound  $b$  means that all **while** loops and recursive functions are unwound  $b$  times, so that we consider only so-called *b-bounded paths* that are going through them at most  $b$  times. For example, the statement “**while** ( $be$ ) **do**  $s$ ” after unwinding with  $b = 2$  will be transformed to:

$g := be; \text{if } (g) \text{ then } \{s; g := be; \text{if } (g) \text{ then } \{s; g := be; \text{assume}(\neg g); \} \}$

where we use **assume**( $\neg g$ ) to block all paths longer than the bound  $b$ . After the above three transformations, in the obtained simplified program all original expressions are right-hand sides (RHSs) of assignments, loops are replaced with **if**-s, and each variable is assigned once. For example, the simplified program **intro3** is obtained from **intro2** by the above three transformations.

The generated simplified program is converted to a set of SMT formulas  $S$  as follows. An assignment  $x := ae$  is converted to equation formula  $x = ae$ ; a  $\Phi$ -assignment  $x := be?x_1 : x_2$  is converted to formula  $x = \text{ite}(be, x_1, x_2)$ ; an **assume**( $be$ ) is converted to formula  $be$ ; and an **assert**( $be$ ) is converted to formula  $\neg be$ . A statement that is part of a **while** body may be encoded by several formulas  $\phi_1, \dots, \phi_k$  in  $S$  due to the unwinding. In this case, we remove  $\phi_1, \dots, \phi_k$  from  $S$ , and add instead one conjunctive formula  $(\phi_1 \wedge \dots \wedge \phi_k)$  in  $S$ . In effect, we obtain that one formula in  $S$  encodes a single statement in the original program. For example, the set  $S_{\text{intro}}$  is obtained from **intro3** by the above conversion.

The obtained set of formulas  $S$  is partitioned into three subsets:  $S_{\text{soft}}^{\text{Var}}$  that contains all formulas corresponding to statements containing original program expressions,  $S_{\text{soft}}^{\mathbb{F}}$  that contains all formulas corresponding to statements containing original feature expressions, and  $S_{\text{hard}}$  that contains the other formulas corresponding to assertions, assumptions,  $\Phi$ -assignments, and feature variable-assignments. Since all original program and feature expressions are RHSs of assignments after the simplification phase, all formulas in  $S_{\text{soft}}^{\text{Var}}$  and  $S_{\text{soft}}^{\mathbb{F}}$  are either single assignment formulas ( $x = ae$ ) or multiple assignment formulas  $((x_1 = ae_1) \wedge \dots \wedge (x_k = ae_k))$ . For example, the set  $S_{\text{intro}}$  in Fig. 5 is partitioned as follows:

$$\begin{aligned} S_{\text{soft}}^{\text{Var}} &= \{x_0=0, x_1=x_0+2, x_3=x_2-2\}, \\ S_{\text{soft}}^{\mathbb{F}} &= \{g_0=A_0, g_1=\neg A_0 \wedge \neg B_0\}, \\ S_{\text{hard}} &= \{A_0=[0, 1], B_0=[0, 1], x_2=\text{ite}(g_0, x_1, x_0), x_4=\text{ite}(g_1, x_3, x_2), \neg(x_4 \geq 0)\} \end{aligned}$$

Given a pre-processed (single) program  $p$ , the program formula  $\varphi_p^b$  is the conjunction of all formulas in  $S$ , where  $b$  denotes the unwinding bound used in the transformation phase of  $p$ . The formula  $\varphi_p^b$  encodes all possible  $b$ -bounded paths in the program  $p$  that go through each loop at most  $b$  times. We say that a program  $p$  is *b-correct* if all assertions in it are valid in all  $b$ -bounded paths of  $p$ .

► Proposition 2 ([8]). *A pre-processed (single) program  $p$  is b-correct iff  $\varphi_p^b$  is unsatisfiable.*

274 A satisfying assignment (model) of  $\varphi_p^b$  represents a  $b$ -bounded path of  $p$  that satisfies all  
 275 assumptions but violates at least one assertion. In the following, we omit to write  $p$  and  $b$  in  
 276 the program formula  $\varphi_p^b$  when they are clear from the context.

277 Our approach reasons about loops by unrolling them, so it is sensitive to the unrolling  
 278 bound. We now present an example, where the unrolling bound has impact on the assertion  
 279 validity.

280 ► Example 3. Consider the program:

```
281 int i:=0, x:=0; while (i<3) do {i:=i+1; x:=x+1; }
```

282 Suppose that the assertion to be checked is `assert(x≥3)` at the final location. If we use  
 283 the unrolling bound  $b = 2$ , we will find that the program is incorrect due to the spurious  
 284 execution path that runs the `while`-body 2 times. Hence, we will needlessly try to repair  
 285 this correct program. However, if we use the bound  $b \geq 3$ , then we will establish that the  
 286 program is correct and so no repair is needed.

287 Suppose that the assertion to be checked is `assert(x<3)` at the final location. If we use  
 288 the unrolling bound  $b = 2$ , we will find that the program is correct since the assertion is valid  
 289 for all 2-bounded paths, so no repair will be performed. However, if we use the bound  $b \geq 3$ ,  
 290 then we will truly establish that the program is incorrect and so a repair is needed.

291 To enable incremental SMT solving, the program formula  $\varphi$  is instrumented with Boolean  
 292 variables called *guard variables*. More specifically, a formula  $\varphi = \phi_1 \wedge \dots \wedge \phi_n$  is replaced with  
 293  $\varphi' = (x_1 \implies \phi_1) \wedge \dots \wedge (x_n \implies \phi_n)$ , where  $x_1, \dots, x_n$  are fresh guard variables. In effect,  
 294 the formula  $(x_i \implies \phi_i)$  can be satisfied by setting  $x_i$  to false. Some guard variables called  
 295 *assumptions* are conjuncted with  $\varphi'$  and passed to an incremental SMT solver. For example,  
 296  $\varphi' \wedge x_1 \wedge x_2$  is satisfiable iff  $\phi_1$  and  $\phi_2$  are satisfiable, since the satisfying assignment will set  
 297  $x_3, \dots, x_n$  to false thus making  $(x_3 \implies \phi_3), \dots, (x_n \implies \phi_n)$  true. Thus, an incremental  
 298 SMT-solver checking the satisfiability of  $\varphi' \wedge x_1 \wedge x_2$  will only check satisfiability of  $\phi_1$  and  
 299  $\phi_2$ , thus essentially disabling formulas  $\phi_3, \dots, \phi_n$ .

300 We will use formulas of the form `AtMost({l1, ..., ln}, k)` (resp., `AtLeast({l1, ..., ln}, k)`)  
 301 to require that at most (resp., at least)  $k$  of the literals  $l_1, \dots, l_n$  are true. They are called  
 302 *Boolean cardinality formulas* encoding that  $\sum_{i=1}^n l_i \leq k$  (resp.,  $\sum_{i=1}^n l_i \geq k$ ), where  $l_i$  is a  
 303 literal assigned the value 1 if true and the value 0 if false, and  $k \in \mathbb{N}$ . We will use the  
 304 MINICARD SAT-solver [39] to check their satisfiability.

## 305 4 Lifted Repair Algorithm

306 In this section, we present our lifted repair algorithm, called SPLALLREPAIR, for repairing  
 307 program families. We first give a high-level overview of the algorithm, and then describe its  
 308 components more formally.

### 309 High-level Description.

310 The SPLALLREPAIR is given in Algorithm 1. It takes as input a program family  $P$ , an  
 311 unwinding bound  $b$ , and a repair size  $r$  that limits the search space to only mutated programs  
 312 with at most  $r$  mutations (changes to the original code) applied at once. The algorithm goes  
 313 through an iterative generate-and-verify procedure, implemented using an interplay between  
 314 an SAT solver and an SMT solver. In particular, we use an SAT solver in the generate phase  
 315 to find a mutated program from the search space, whereas we use an SMT solver in the  
 316 verify phase to check if the mutated program is correct.

---

**Algorithm 1** SPLAllRepair( $P, b, r$ )

---

**Input:** Program family  $P$ , unwinding bound  $b$ , repair size  $r$

**Output:** Set of solutions  $Sol$

```

1  $p_{sim} := \text{VarEncode}(P)$  ;
2  $(S_{\text{hard}}, S_{\text{soft}}^{\text{Var}}, S_{\text{soft}}^{\mathbb{F}}) := \text{CBMC}(p_{sim}, b)$  ;
3  $(S_1, \dots, S_n) := \text{Mutate}(S_{\text{soft}}^{\text{Var}}, S_{\text{soft}}^{\mathbb{F}})$  ;
4  $(S'_1, \dots, S'_n, V_1, \dots, V_n, V_{\text{orig}}) := \text{InstGuardVars}(S_1, \dots, S_n)$  ;
5  $\varphi_{sim}^b := (\wedge_{s \in S_{\text{hard}}} s) \wedge (\wedge_{s \in S'_1 \cup \dots \cup S'_n} s)$  ;
6  $\varphi := (\wedge_{i=1}^n \text{AtMost}(V_i, 1)) \wedge (\wedge_{i=1}^n \text{AtLeast}(V_i, 1))$  ;
7  $k := 1$ ;  $Sol := \emptyset$  ;
8 while ( $k \leq n$ )  $\wedge$  ( $k \leq r$ ) do
9    $\varphi_k := \varphi \wedge \text{AtLeast}(V_{\text{orig}}, n - k)$  ;
10   $satres, V := \text{SAT}(\varphi_k)$  ;
11  if ( $satres$ ) then
12     $smtres := \text{IncrementalSMT}(\varphi_{sim}^b \wedge \wedge_{v \in V} v)$  ;
13    if ( $\neg smtres$ ) then
14       $Sol := Sol \cup V$  ;
15       $\varphi_k := \varphi_k \wedge (\vee_{v \in V \setminus (V_{\text{orig}})} \neg v)$  ;
16    else
17       $\varphi_k := \varphi_k \wedge (\vee_{v \in V} \neg v)$  ;
18  else
19     $k := k + 1$  ;
20 if ( $\text{Timeout}$ ) then return  $Sol$  ;
21 return  $Sol$ ;
```

---

317     The SPLALLREPAIR starts by generating the family simulator  $p_{sim}$  using the pre-  
318     processor **VarEncode** procedure (line 1). Then, the CBMC translation procedure calls the  
319     CBMC model checker to generate the triple  $(S_{\text{hard}}, S_{\text{soft}}^{\text{Var}}, S_{\text{soft}}^{\mathbb{F}})$  of sets of formulas cor-  
320     responding to  $p_{sim}$  as explained in Section 3.2 (line 2). By calling the **Mutate** procedure, we  
321     generate all possible mutations  $S_1, \dots, S_n$  of formulas in  $S_{\text{soft}}^{\text{Var}}$  and  $S_{\text{soft}}^{\mathbb{F}}$  (line 3). Here  $S_i$  is a  
322     set of formulas obtained by mutating some  $\phi_i \in S_{\text{soft}}^{\text{Var}} \cup S_{\text{soft}}^{\mathbb{F}}$ . Thus,  $S_1, \dots, S_n$  correspond to  
323      $n$  program locations where an error may occur. Next, we use the **InstGuardVars** procedure  
324     to instrument all formulas in  $S_1, \dots, S_n$  by fresh guard variables, so that the results are sets  
325     of instrumented formulas  $S'_1, \dots, S'_n$  and sets of fresh guard variables  $V_1, \dots, V_n$  used to guard  
326     formulas in  $S'_1, \dots, S'_n$  (line 4). Here  $S'_i = \{(x \implies \phi) \mid \phi \in S_i, x \text{ is a fresh guard variable}\}$ .  
327     The set  $V_{\text{orig}}$  contains guard variables corresponding to original formulas in  $S_{\text{soft}}^{\text{Var}}$  and  $S_{\text{soft}}^{\mathbb{F}}$ .  
328     The program formula  $\varphi_{sim}^b$  is then initialized to be the conjunction of all formulas from  $S_{\text{hard}}$   
329     and all instrumented formulas from  $S'_1 \cup \dots \cup S'_n$  (line 5). Subsequently, we search the space  
330     of all mutated formulas in increasing size order using the variable  $k$ , which is initialized to  
331     1 and increased after each iteration (lines 8–20). In particular, we generate the boolean  
332     formula  $\varphi_k$  [13] (line 9) expressing that  $k$  guard variables are not original, that is  $n - k$  are  
333     original (by using  $\text{AtLeast}(V_{\text{orig}}, n - k)$ ), and there is exactly one guard variable selected for  
334     each statement in the program (by using  $\varphi \equiv \wedge_{i=1}^n \text{AtMost}(V_i, 1) \wedge \wedge_{i=1}^n \text{AtLeast}(V_i, 1)$ , line 6).  
335     This means that every satisfying assignment of  $\varphi_k$  represents one mutated program formula  
336     of size at most  $k$  (i.e. with  $k$  changes to the original code). The boolean formula  $\varphi_k$  is fed to  
337     an SAT solver, which can handle Boolean cardinality formulas, to check its satisfiability. If

## 36:10 Mutation-based Lifted Repair of Software Product Lines

338  $\varphi_k$  is unsatisfiable, this means that there are no unexplored mutated program formulas of  
 339 size  $k$  so we increase  $k$  by one (line 19) and generate a new formula  $\varphi_k$ . Otherwise, if  $\varphi_k$   
 340 is satisfiable, we store in a set  $V$  all guard variables assigned true in the given satisfying  
 341 assignment of  $\varphi_k$  (line 10). To check the correctness of the mutated program corresponding  
 342 to the satisfying assignment  $V$  of  $\varphi_k$ , we call an incremental SMT solver to check  $\varphi_{sim}^b$  with  
 343 all guards in  $V$  passed as assumptions (i.e.,  $\varphi_{sim}^b \wedge \wedge_{v \in V} v$ ) (line 12). This is the same to  
 344 checking the conjunction of all formulas in  $S_{hard}$  and all soft formulas guarded by variables  
 345 in  $V$ , since all other soft formulas will get satisfied by setting their guard variables to false.  
 346 Notice that SMT formulas solved consecutively in the iteration are very similar, thus sharing  
 347 majority of their assumptions and all hard formulas. This means that most of what was  
 348 learnt in solving the previous formula can be reused to solve the current one. If the result of  
 349 incremental SMT solving is true, the mutated program is not correct so we block  $V$  from  
 350 further exploration (line 17). Otherwise, we report  $V$  as a possible solution (i.e., a repaired  
 351 program family) and block all supersets of  $V$  for further exploration (lines 14,15). The  
 352 algorithm terminates when either the whole search space of mutated programs is inspected,  
 353 i.e. all possible combinations of guard variables in  $n$  locations are explored as assumptions  
 354 ( $k > n$ , line 8), or the subspace of mutated programs with at most  $r$  mutations is inspected  
 355 ( $k > r$ , line 8), or a time limit is reached (line 20).

356 ▶ **Example 4.** Let  $p$  be a simulator with 4 statements that can be mutated. Let  $p_1$  be a  
 357 repaired mutant of  $p$  consisting of mutating statement 1 with mutation  $M_1^1$  (guard variable  
 358  $v_1^1$ ) and statement 3 with mutation  $M_3^2$  (guard variable  $v_3^2$ ). Then blocking any superset  
 359 of this mutation is done by adding the blocking clause  $(\neg v_1^1 \vee \neg v_3^2)$  to the Boolean formula  
 360  $\varphi_k$  representing the search space of all mutants. This means do not apply either  $M_1^1$  to  
 361 statement 1 or do not apply  $M_3^2$  to statement 3.

362 On the other hand, let  $p_2$  be a buggy mutant of  $p$  consisting of mutating statement 1  
 363 with mutation  $M_1^2$  (guard variable  $v_1^2$ ) and statement 4 with mutation  $M_4^2$  (guard variable  
 364  $v_4^2$ ). The guards for original statements 2 and 3 are  $v_2^{orig}$  and  $v_3^{orig}$ . Then the blocking clause  
 365  $(\neg v_1^2 \vee \neg v_2^{orig} \vee \neg v_3^{orig} \vee \neg v_4^2)$  will be added to prune from the search space exactly the mutant  
 366  $p_2$ . Note that smaller blocking clause (with smaller number of literals) will result in a larger  
 367 set of pruned mutants.

### 368 Pre-Processor: VarEncode.

369 The aim of the pre-processor **VarEncode** procedure is to transform an input program family  
 370  $P$  with sets of features  $\mathbb{F}$  and configurations  $\mathbb{K}$  into an output pre-processed (single) program  
 371 without **#if**-s, called family simulator. The set of configurations  $\mathbb{K}$  includes all possible  
 372 combinations of feature values. In the pre-transformation phase, we convert each feature  $A \in \mathbb{F}$   
 373 into the global variable  $A$  non-deterministically initialized to 0 or 1. Let  $\mathbb{F} = \{A_1, \dots, A_n\}$   
 374 be the set of available *features* in the program family  $P$ . We generate the following pre-  
 375 transformed program:

376  $\text{pre-t}(P) \equiv \text{int } A_1 := [0, 1], \dots, A_n := [0, 1]; P$

377 We now define a rewrite rule for eliminating **#if**-s from  $\text{pre-t}(P)$ . Let  $\mathbb{K}$  be the set of  
 378 configurations in the family  $P$  that can be equated to a propositional formula  $\kappa = \vee_{k \in \mathbb{K}} k$ .  
 379 Note that if  $\mathbb{K}$  contains all possible combinations of feature values, then  $\kappa \equiv \text{true}$ . The  
 380 rewrite rule replaces **#if**-s with ordinary **if**-s whose guards are strengthened with the feature  
 381 model  $\kappa$ .

382  $\text{\#if } (\theta) s \text{ \#endif } \rightsquigarrow \text{if } (\theta \wedge \kappa) \text{ then } s \text{ else skip} \quad (R-1)$

If the current program family being transformed matches the abstract syntax tree node of the shape `#if ( $\theta$ ) s #endif`, then replace it with the RHS of rule (R-1). We write  $\text{VarEncode}(P)$  to be the final transformed single program obtained by repeatedly applying rule (R-1) on  $\text{pre-t}(P)$  and on its transformed versions until we reach a point at which this rule can no longer be applied.

A memory state  $\sigma : \Sigma = \text{Var} \rightarrow \mathbb{Z}$  is a function mapping each program variable to a value. Given a single program  $p$  and a memory state  $\sigma$ , we write  $\llbracket p \rrbracket \sigma$  for the set of final states that can be derived by executing all terminating paths (computations) of  $p$  starting in the input state  $\sigma$ . Note that the result is a set of states since our language is non-deterministic. We define  $\llbracket p \rrbracket = \bigcup_{\sigma \in \mathcal{P}(\Sigma)} \llbracket p \rrbracket \sigma$  to be the set of final states that can be reached by  $p$  from any possible input state  $\sigma \in \mathcal{P}(\Sigma)$  (where  $\mathcal{P}(\Sigma)$  is the powerset of  $\Sigma$ ). The following result shows that the set of final states from terminating computations of  $\text{VarEncode}(P)$  coincides with the union of final states from terminating computations of all variants derived from the program family  $P$ .

► Proposition 5 ([30]). *For a program family  $P$ ,  $\llbracket \text{VarEncode}(P) \rrbracket = \bigcup_{k \in \mathbb{K}} \llbracket \pi_k(P) \rrbracket$ .*

► Example 6. Consider the program family `intro1` in Fig. 2 and its family simulator `intro2≡VarEncode(intro1)` in Fig. 3. The states  $\sigma$  contain only one program variable `x`. Hence, the semantics of all variants of `intro1` is:

$$\begin{aligned} \llbracket \pi_{A \wedge B}(\text{intro1}) \rrbracket &= [x \mapsto 2], \quad \llbracket \pi_{A \wedge \neg B}(\text{intro1}) \rrbracket = [x \mapsto 2] \\ \llbracket \pi_{\neg A \wedge B}(\text{intro1}) \rrbracket &= [x \mapsto 0], \quad \llbracket \pi_{\neg A \wedge \neg B}(\text{intro1}) \rrbracket = [x \mapsto -2] \end{aligned}$$

On the other hand, the semantics of `intro2≡VarEncode(intro1)` is:

$$\llbracket \text{VarEncode}(\text{intro1}) \rrbracket = \{[x \mapsto -2], [x \mapsto 0], [x \mapsto 2]\}$$

#### 404 **Mutate.**

As explained in Section 3.2, the SMT formulas in  $S_{\text{soft}}^{\text{Var}}$  and  $S_{\text{soft}}^{\mathbb{F}}$  correspond to statements containing program and feature expressions, so our goal is to repair the given erroneous program family by applying mutations to those formulas. A *mutation* is a replacement of a program/feature expression with another expression of the same type. For example, feature expressions  $A$  and  $A \wedge B$  can be replaced with  $\neg A$  and  $(A \vee \neg B)$ , while program expressions  $x$  and  $x + 2$  can be replaced with  $0$  and  $x - 2$ . We maintain a fixed list of syntactic mutations for each type of program and feature expressions. Let us assume that mutations  $M_1, \dots, M_j$  can be applied to a formula  $\phi \in S_{\text{soft}}^{\text{Var}} \cup S_{\text{soft}}^{\mathbb{F}}$ . Then,  $\text{Mutate}(\phi) = \{\phi, M_1(\phi), \dots, M_j(\phi)\}$ . Finally, we have  $\text{Mutate}(S_{\text{soft}}^{\text{Var}}, S_{\text{soft}}^{\mathbb{F}}) = \prod_{\phi \in S_{\text{soft}}^{\text{Var}} \cup S_{\text{soft}}^{\mathbb{F}}} \text{Mutate}(\phi)$ .

We now present the variability-specific mutations applied to feature expressions:  $A \rightarrow \neg A$  (read: feature  $A$  is replaced by  $\neg A$ ) and  $\neg A \rightarrow A$  for features  $A \in \mathbb{F}$ , as well as  $\{\wedge, \vee\}$  (read: logical operator  $\wedge$  can be replaced with  $\vee$ , and vice versa).

► Example 7. Recall that  $S_{\text{soft}}^{\mathbb{F}} = \{g0=A0, g1=\neg A0 \wedge \neg B0\}$  for our running example `intro1`. If we use the variability-specific mutations  $A \rightarrow \neg A$ ,  $\neg A \rightarrow A$  for  $A \in \mathbb{F}$  and  $\{\wedge, \vee\}$ , we obtain:

$$\begin{aligned} \text{Mutate}(S_{\text{soft}}^{\mathbb{F}}) &= \{g0=A0, g0=\neg A0, g1=\neg A0 \wedge \neg B0, g1=A0 \wedge \neg B0, g1=\neg A0 \wedge B0, g1=A0 \wedge B0, \\ &\quad g1=\neg A0 \vee \neg B0, g1=A0 \vee \neg B0, g1=\neg A0 \vee B0, g1=A0 \vee B0\} \end{aligned}$$

#### 420 **Post-Processor: Interpreting results.**

The solutions obtained by calling the ALLREPAIR tool to repair  $\text{VarEncode}(P)$  are interpreted back on the original program family  $P$ . Any possible repair for  $\text{VarEncode}(P)$ , which consists

## 36:12 Mutation-based Lifted Repair of Software Product Lines

423 of replacing some feature and program expressions, represents a valid repair for  $P$  as well.  
424 This is due to the fact that our transformed program  $\text{VarEncode}(P)$  contains all possible  
425 paths that may occur in any variant  $\pi_k(P)$  for  $k \in \mathbb{K}$ . A single program (variant) is  $b$ -correct  
426 if it has no  $b$ -bounded path that leads to an assertion failure, while a program family is  
427  $b$ -correct if all its variants are  $b$ -correct. Therefore, the  $b$ -correctness and possible repair of  
428  $\text{VarEncode}(P)$  and  $P$  are isomorphic (identical).

429 More formally, by using Propositions 2 and 5, we can prove the following result.

- 430 ▶ **Corollary 8.** *Let  $P$  and  $b$  be a program family and an unwinding bound.*  
431 (i)  $\varphi_{\text{VarEncode}(P)}^b$  is unsatisfiable iff  $\forall k \in \mathbb{K}. \pi_k(P)$  is  $b$ -correct iff  $P$  is  $b$ -correct.  
432 (ii)  $\varphi_{\text{VarEncode}(P)}^b$  is satisfiable iff  $\exists k \in \mathbb{K}. \pi_k(P)$  is not  $b$ -correct iff  $P$  is not  $b$ -correct.

433 **Correctness.**

434 We first use Corollary 8 to show the  $b$ -correctness of the SPLALLREPAIR algorithm (where  $b$   
435 is the unwinding bound). That is, every solution returned by SPLALLREPAIR is minimal  
436 repaired program family ( $b$ -soundness), and every minimal repaired program family with  
437 respect to mutations we apply is eventually returned by SPLALLREPAIR ( $b$ -relative com-  
438 pleteness). Our algorithm explores all mutated programs in increasing size order starting  
439 with size 1. Every returned solution is minimally repaired due to the fact that it would have  
440 been blocked by another smaller solution in a previous iteration. Therefore, the  $b$ -correctness  
441 ( $b$ -soundness and  $b$ -relative completeness) of SPLALLREPAIR follows from the  $b$ -correctness  
442 of ALLREPAIR shown in [50] and Corollary 8 (i.e., the fact that  $\text{VarEncode}(P)$  and  $P$  are  
443 isomorphic with respect to  $b$ -correctness).

444 The SPLALLREPAIR always terminates, as there are only finitely many mutations that  
445 can be applied to any type of (feature and program) expressions so the algorithm enumerates  
446 all possible mutated programs (simulators) until it finds the minimal repaired ones if any.  
447 This way, we have proved the following result.

- 448 ▶ **Theorem 9.** *The algorithm  $\text{SPLAllRepair}(P, b, r)$  is  $b$ -bounded correct and terminates.*

## 449 5 Evaluation

450 We now evaluate our approach for mutation-based lifted repair of SPLs. We show that our  
451 approach can efficiently repair various interesting `#ifdef`-based C program families, and  
452 we compare the runtime performances and precision of two versions of our algorithm, with  
453 smaller and bigger sets of mutations, as well as with the `Brute-force` approach that repairs  
454 all variants of a program family one by one independently.

455 **Implementation.**

456 We have implemented our lifted repair algorithm SPLALLREPAIR in a prototype tool, which  
457 is built on top of the ALLREPAIR tool [50, 51] for repairing single programs. The pre-processor  
458 `VarEncode` procedure is implemented in Java, while the translation and mutation procedures  
459 (`CBMC` and `Mutate` in Algorihtm 1) are implemented by modifying the CBMC model checker  
460 [8] written in C++, where variability-specific mutations are defined. Moreover, we have  
461 experimented by defining various mutations to other types of program expressions (see below).  
462 The repair phase is implemented by calling the ALLREPAIR tool [50] written in Python. We  
463 also call the MINICARD SAT solver [39] and the Z3 SMT solver [11]. The altered CBMC  
464 (plus  $\sim 1$ K LOC) takes as input a family simulator, and generates a `gsmt2` file containing SMT

465 formulas for all possible mutations of the corresponding statements in the input program.  
 466 The ALLREPAIR ( $\sim 2K$  LOC) takes as input a `gsmt2` file, generates formulas for SAT and  
 467 SMT solving, and handles all calls to them.

468 The tool accepts programs written in C with `#ifdef/#if` directives. It uses three main  
 469 parameters: *mutation level* that defines the kind of mutations that will be applied to feature  
 470 and program expressions; *unwinding bound b* that shows how many times loops and recursive  
 471 functions will be inlined; and *repair size r* that specifies how many mutations will be applied  
 472 at most to buggy programs. We use two mutation levels: *level 1* contains simpler mutations  
 473 that are often sufficient for repairment, while *level 2* contains all possible mutations we apply.  
 474 For each type of feature and program expression, the list of syntactic mutations/edits in  
 475 level 1 and level 2 is given below:

type of exp.	level 1	level 2
arithmetic op.	$\{+, -\}, \{*, \%, \div\}$	$\{+, -, *, \%, \div\}$
relational op.	$\{<, \leq\}, \{>, \geq\}, \{==, !=\}$	$\{<, \leq, >, \geq, ==, !=\}$
logical op.	$\{\&\&,   \}$	$\{\&\&,   \}$
bit-wise op.	$\{>>, <<\}, \{\&,  , ^\}$	$\{>>, <<, \&,  , ^\}$
program vars		$x \rightarrow 0, x \rightarrow -x$
integer constants		$n \rightarrow n+1, n \rightarrow n-1, n \rightarrow -n, n \rightarrow 0$
feature vars	$A \rightarrow \neg A, \neg A \rightarrow A$	$A \rightarrow \neg A, \neg A \rightarrow A$

477 For example, for arithmetic operators in mutation level 1 we have two sets  $\{+, -\}$  and  
 478  $\{*, \%, \div\}$ , which means that  $+$  is replaced with  $-$  and vice versa, and  $*, \%, \div$  can be replaced  
 479 with each other. On the other hand, in mutation level 2 we have one set  $\{+, -, *, \%, \div\}$ ,  
 480 which means that any arithmetic operator from the set can be replaced with any other.  
 481 Mutations on feature variables  $A \in \mathbb{F}$  in both levels include negations of feature variables  
 482 ( $A \rightarrow \neg A, \neg A \rightarrow A$ ), whereas for program variables  $x \in Var$  in level 2 we have mutations for  
 483 replacing them with 0 ( $x \rightarrow 0$ ) and changing the sign ( $x \rightarrow -x$ ). Integer constants  $n \in \mathbb{Z}$  in  
 484 mutation level 2 can be increased by one, decreased by one, minused, or replaced with 0.

#### 485 Experimental setup and Benchmarks.

486 Experiments are run on 64-bit Intel®Core<sup>TM</sup> i7-1165G7 CPU@2.80GHz, VM Ubuntu 22.04.3  
 487 LTS, with 8 GB memory. We use a timeout value of 400 sec. The implementation, benchmarks,  
 488 and all obtained results are available from: <https://zenodo.org/records/11179373>. For  
 489 the aim of evaluation, we ran: (1) our tool with mutation level 1, denoted SPLALLREPAIR<sub>1</sub>;  
 490 (2) our tool with mutation level 2, denoted SPLALLREPAIR<sub>2</sub>; and (3) the Brute-force  
 491 approach that uses a preprocessor to generate all variants of a program family and then  
 492 applies the single-program repair tool ALLREPAIR to each individual variant independently.

493 The evaluation is performed on a dozen of C programs: two warming-up examples  
 494 (`intro1` in Fig. 2 and `feat-inter` in Fig. 6); four commonly known algorithms (`feat_power`  
 495 in Fig. 7, `factorial` in Fig. 8, `sum` in Fig. 9 and `sum_mton` in Fig. 10); `Codeflaws` [53], `TCAS`  
 496 [29], and `Qclose` [10] benchmarks that are widely used for evaluating program repair tools  
 497 [10, 37, 46, 50, 51]; as well as `MinePump` system [38] from the `product-lines` category of  
 498 `SV-COMP 2024` (<https://sv-comp.sosy-lab.org/2024>) that is often used to assess product-  
 499 line verification in the SPL community [4, 9, 56, 55]. `Codeflaws` consists of programs  
 500 taken from buggy user submissions to the programming contest site Codeforces (<http://codeforces.com>). For each program, there is a correct reference version and several buggy  
 501 versions. Traffic Alert and Collision Avoidance System (`TCAS`) represents an aircraft collision  
 502

---

```

void main(){
    int x := 0;
    #if (A) x := x+2; #endif
    #if (B ∧ C) x := x-2; #endif
    assert (x ≥ 0 && x < 4);
}

```

---

**Figure 6** feat-inter.

---

```

int feat_power(int n){
    assume(n ≥ 1);
    int res := 0;
    #if (¬A) int i := 1;
    #else int i := 0; #endif
    while(i < 3){
        res=res*n;
        i++; }
    #if (A) assert (sum==n*n*n);
    #else assert (sum==n*n*n*n); #endif
    return res; }

```

---

**Figure 7** feat\_power.

---

```

void main(int n){
    assume(n ≥ 0);
    int res1 := fact(n);
    int res2 := fact_correct(n);
    assert(res1 == res2);
}
int fact_correct(int x){
    int res=1;
    for (int i=2; i ≤ x; i++)
        res *= i;
    return res;
}

```

---

**Figure 8** factorial

---

```

int fact(int x){
    int res=1, i=2;
    while (#if (A) (i < x) #else (i ≤ x) #endif){
        res = mult(res,i);
        i++; }
    return res; }
int mult(int x, int y){
    int res=0;
    for (int i=1; i ≤ y; i++)
        #if (B) res-=x; #else res+=x; #endif
    return res;
}

```

---

503 detection system used by all US commercial aircrafts. The TCAS benchmark suite consists  
 504 of a reference (correct) implementation and 41 faulty versions. In our experiments, we use  
 505 10 faulty versions that can be repaired using the mutations we apply in our approach. The  
 506 Qclose benchmarks are used for evaluating the Qclose program repair tool [10], which consist  
 507 of a reference (correct) implementation and several faulty versions for each programming  
 508 task. In the case of *Codelaws*, TCAS, and Qclose, we have selected several faulty versions of  
 509 each benchmark and we have created a buggy program family out of them. For example,  
 510 we use *tcas\_v3* and *tcas\_v12* (resp., *tcas\_v16* and *tcas\_v17*) to create the *tcas\_spl1*  
 511 (resp., *tcas\_spl2*) program family. Then, we use assertions to check the equivalence of the  
 512 results returned by the program family and the reference (correct) version (for example, see  
 513 *main()* of *factorial* in Fig. 8). Note that the correct version is marked so that it will not  
 514 be mutated. The MinePump SPL system contains 730 LOC and six independent optional  
 515 features: *start*, *stop*, *methaneAlarm*, *methaneQuery*, *lowWaterSensor*, *highWaterSensor*.  
 516 When activated, the controller should switch on the pump when the water level is high, but  
 517 only if there is no methane in the mine. We consider two specifications of the MinePump  
 518 system encoded as assertions in SV-COMP 2024: *MinePump\_spec1* checks whether the pump is  
 519 not running if the level of methane is critical; and *MinePump\_spec3* checks whether the pump  
 520 is running if the level of water is high. Table 1 presents characteristics of the benchmarks,  
 521 such as: the file name (Benchmark), the number of features  $|F|$  (note that  $|K| = 2^{|F|}$ ), and  
 522 the lines of code (LOC).

---

```

int sum(int n){
    assume(n ≥ 1);
    int sum := 0, i := 0;
    #if (A) i := 1; #endif
    while(i < n){
        #if (B) sum+=i;
        #else sum-=i; #endif
        i++; }
    assert(sum==n*(n+1)/2);
    return sum;
}

```

---

**Figure 9** sum.

---

```

int sum_mton(int n, int m){
    assume(n ≥ 1&&m ≥ 1);
    #if (A) assume(n ≥ m);
    #else assume(m ≥ n); #endif
    int sum := 0;
    #if (A) int i := n;
    #else int i := m; #endif
    while(#if (A) (i ≤ n)#else (i ≤ m) #endif)
    { sum:=sum-i;
      i++; }
    #if (A) assert(sum==(n*(n+1)-m*(m-1))/2);
    #else assert(sum==(m*(m+1)-n*(n-1))/2);
    #endif
    return sum; }

```

---

**Figure 10** sum\_mton.

523    **Examples.**

524    We now present several of our examples in detail. Consider the program family **feat-inter**  
 525    in Fig. 6. The error occurs due to the feature interaction ( $\neg A \wedge B \wedge C$ ). In particular, the  
 526    variant ( $\neg A \wedge B \wedge C$ ) is: `int x=0; x=x-2; assert(x≥0 && x<4)`. So the assertion fails since  
 527    `x` has value -2 at the assertion location. The simplest fix from mutation level 1, which replaces  
 528    `x:=x-2` with `x:=x+2`, does not work as it introduces a new error in other variants. In this  
 529    case, the feature interaction ( $A \wedge B \wedge C$ ) causes the assertion failure since the value of `x` will be  
 530    4 at the assertion location for variant ( $A \wedge B \wedge C$ ). Therefore, SPLALLREPAIR<sub>1</sub> reports that  
 531    no repair is found by searching the space of 7 mutants in 0.254 sec. However, if we consider  
 532    mutations of level 2 then SPLALLREPAIR<sub>2</sub> successfully finds a repair, which replaces `x:=x-2`  
 533    with `x:=x-0`, by searching the space of 25 mutants in 0.315 sec. On the other hand, the  
 534    **Brute-force** approach applies mutations to all faulty variants independently. As the only  
 535    faulty variant is ( $\neg A \wedge B \wedge C$ ), it will report the repair that replaces `x:=x-2` with `x:=x+2`. This  
 536    is a correct repair for the variant ( $\neg A \wedge B \wedge C$ ), but not for the entire family. This example  
 537    shows that sometimes the **Brute-force** approach may not report correct results due to the  
 538    feature interaction.

539    The program family **feat\_power** in Fig. 7 should find the third power of `n` when feature  
 540    `A` is enabled and the fourth power of `n` when `A` is disabled. SPLALLREPAIR<sub>1</sub> suggests fixes  
 541    in 0.722 sec that replace the feature expression ( $\neg A$ ) with (`A`) when initializing variable `i`  
 542    and replace **while**-guard (`i < 3`) with (`i ≤ 3`). The **Brute-force** finds that variant (`A`) is  
 543    correct, but variant ( $\neg A$ ) is not correct and no fix is suggested as integer constants cannot  
 544    be mutated in level 1. Some possible repairs of variant ( $\neg A$ ) in level 2 will make variant (`A`)  
 545    incorrect. For example, changing the **while**-guard to (`i ≤ 3`) will make variant (`A`) incorrect  
 546    since it is initialized to 0 so it will return the fourth power of `n` instead of the third.

547    The program **factorial** in Fig. 8 contains two implementations of the factorial function:  
 548    a correct one, called **fact\_correct**, and a buggy one, called **fact**, that represents a program  
 549    family with four variants. The assertion requires that the results returned from each variant of  
 550    **fact** are equivalent with the result returned from **fact\_correct**. We do not apply mutations  
 551    to **fact\_correct**, but only to the program family **fact**. All three approaches suggest fixes  
 552    that replace the **while**-guard (`i < x`) with (`i ≤ x`) and the assignment `res-=x` with `res+=x`.

553    Consider the program family **sum** in Fig. 9, which computes the sum of all integers from 0

**Table 1** Performance results of SPLALLREPAIR<sub>1</sub> vs. SPLALLREPAIR<sub>2</sub> vs. Brute-force. All times in sec.

Benchmarks	F	LOC	SPLALLREPAIR <sub>1</sub>			SPLALLREPAIR <sub>2</sub>			Brute-force		
			Fix	Space	Time	Fix	Space	Time	Fix	Space	Time
intro1	2	20	✓	7	0.252	✓	25	0.304	✓	5	0.981
feat-inter	3	20	✗	7	0.254	✓	25	0.315	✗	9	2.110
feat_power	1	20	✓	16	0.722	✓	403	7.79	✗	8	0.882
factorial	2	50	✓	86	2.540	✓	1603	107.3	✓	81	4.196
sum	2	30	✓	17	0.376	✓	266	2.656	✓	18	1.147
sum_mton	1	20	✓	32	0.770	✓	681	15.22	✗	10	0.556
4-A-Codeflaws	2	95	✗	52	0.426	✓	1390	2.578	✗	36	1.180
651-A-Codeflaws	2	85	✓	180	3.394	✓	2829	38.53	✓	237	5.78
tcas_spl1	1	305	✗	37	0.99	✓	158	6.10	✗	37	1.41
tcas_spl2	1	305	✗	38	1.19	✓	164	8.94	✗	38	1.47
Qlose_multiA	3	32	✗	122	0.711	✓	5415	69.21	✗	65	5.781
Qlose_iterPower	2	30	✗	9	0.973	✓	38	2.921	✗	16	1.391
MinePump_spec1	6	730	✓	38	300.0	✗	-	timeout	✓	-	timeout
MinePump_spec3	6	730	✓	39	291.0	✗	-	timeout	✓	-	timeout

to a given input integer  $n$ . The specification indicates that given a positive input  $n$  ( $n \geq 1$ ), the output represented by the variable `sum` is  $n*(n+1)/2$ . The body of `sum` is implemented in an iterative fashion. There are two features A and B that enable different initializations of `i` and different updates of `sum`. Let us consider mutations of level 1. If the repair size is 1 (i.e., only one original expression can be mutated), our tool cannot find a repair by searching the space of 7 mutants in 0.321 sec. However, if the repair size is 2, then SPLALLREPAIR<sub>1</sub> suggests a fix that replaces the `while`-guard ( $i < n$ ) with ( $i \leq n$ ) and the assignment `sum:=i` with `sum+=i`. The search space contains 17 mutants and the tool explores it in 0.376 sec. The Brute-force approach reports a correct repair in 1.147 sec.

The program family `sum_mton` in Fig. 10 computes the sum  $m + (m + 1) + \dots + n$  when feature A is enabled and ( $n \geq m$ ), and the sum  $n + (n + 1) + \dots + m$  when feature A is disabled and ( $m \geq n$ ). The corresponding specifications assert that the returned value `sum` is equal to  $(n * (n + 1) - m * (m - 1))/2$  when A is on and  $(m * (m + 1) - n * (n - 1))/2$  when A is off. The programmer has made two mistakes: when initializing variable `i` and when updating variable `sum` in the `while`-body. SPLALLREPAIR<sub>1</sub> suggests fixes in 0.770 sec that replace the feature expression (A) with ( $\neg A$ ) when initializing variable `i` and replace `sum:=sum-i` with `sum:=sum+i` when updating `sum`. However, the Brute-force cannot fix any of the two variants since mutating variable `n` (resp., `m`) to other variable `m` (resp., `n`) is not allowed.

## Performance.

Table 1 shows performance results of running SPLALLREPAIR<sub>1</sub>, SPLALLREPAIR<sub>2</sub>, and the Brute-force approach on the given benchmarks. We use mutation level 1 for Brute-force. Note that the Brute-force approach calls translation, mutation, and repair procedures for each variant separately, whereas SPLALLREPAIR<sub>1</sub> and SPLALLREPAIR<sub>2</sub> call these procedures only once per program family. Moreover, the Brute-force approach can only find repairs by mutating program expressions. The default values for unwinding bound is

**Table 2** Performance results of SPLALLREPAIR<sub>1</sub> for different values of the unwinding bound  $b = 2, 5, 8$ . All times in sec.

Benchmarks	$b = 2$		$b = 5$		$b = 8$	
	Fix	Time	Fix	Time	Fix	Time
feat_power	✗	0.254	✓	0.722	✓	0.978
factorial	✗	1.231	✓	3.540	✓	6.524
sum	✗	0.304	✓	0.376	✓	0.456
sum_mton	✗	0.589	✓	0.770	✓	0.922
651-A-Codelfaws	✓	1.814	✓	3.394	✓	6.828

579  $b = 5$  and for repair size is  $r = 1$ . However, for some benchmarks whose repaired versions  
 580 contain more than one code mutation, we use the minimal value of repair size  $r$  that allows  
 581 one approach to find a correct solution. For example, we use repair size  $r = 2$  for `sum`. For  
 582 each approach, there are three columns: “Fix” that specifies with ✓ (resp., ✗) whether the  
 583 given approach finds (resp., does not find) a correct repair for a given benchmark; “Space”  
 584 that specifies how many mutants have been inspected; and “Time” that specifies the total  
 585 time (in seconds) needed for the given tasks to be performed.

586 From Table 1, we can see that SPLALLREPAIR<sub>1</sub> and SPLALLREPAIR<sub>2</sub> combined out-  
 587 perform the **Brute-force** approach with respect to both repairability and runtime. In  
 588 particular, SPLALLREPAIR<sub>2</sub> fully repairs 12 benchmarks, which is better than 8 full correct  
 589 repairs reported by SPLALLREPAIR<sub>1</sub> and 4 full correct repairs reported by the **Brute-force**  
 590 approach that use the same mutations of level 1 (see also Discussion below). Note that  
 591 SPLALLREPAIR<sub>2</sub> and the **Brute-force** timeout after 400 sec for the MinePump system.  
 592 Hence, they report only a partial list of possible repairs, denoted by ✓. On the other  
 593 hand, SPLALLREPAIR<sub>1</sub> achieves time speed-ups compared to **Brute-force** when report  
 594 the same results, that range from 1.2 to 4 times. If we compare SPLALLREPAIR<sub>1</sub> and  
 595 SPLALLREPAIR<sub>2</sub>, we can see that there is a trade-off between repairability and runtime.  
 596 That is, SPLALLREPAIR<sub>2</sub> is more precise (12 vs. 8 fixes) but slower (from 1.2 to 42 times  
 597 slow-down when report the same results) compared to SPLALLREPAIR<sub>1</sub>.

598 Table 2 shows performance results of running SPLALLREPAIR<sub>1</sub> on a selected set of  
 599 benchmarks for different unwinding bounds  $b$ . Recall that our approach reasons about loops  
 600 by unrolling (unwinding) them, so it is sensitive to the chosen unwinding bound. By choosing  
 601 larger bounds  $b$ , we will obtain more precise results (more genuine repairs), but we will  
 602 also obtain longer SMT formulas and slower speeds of the repairing tasks. We can see that  
 603 the running times of all repairing tasks grow with the number of bound  $b$ . This is due to  
 604 the fact that longer SMT formulas are generated, which need more time to be verified. Of  
 605 course, we will also obtain more precise results for bigger values of  $b$ , and less precise results  
 606 (i.e., some genuine repairs will not be reported) for smaller values of  $b$ . Hence, there is a  
 607 preision/speed tradeoff when choosing the unwinding bound  $b$ . We obtain similar results for  
 608 SPLALLREPAIR<sub>2</sub> and the **Brute-force**.

## 609 Discussion

610 In summary, our experiments demonstrate that our tool outperforms the **Brute-force**  
 611 approach, and moreover it can be used for repairing various SPLs with different sizes of LOC,  
 612 configuration space, and mutation space. Although SPLALLREPAIR<sub>1</sub> and **Brute-force**

613 have similar precision (8 vs. 4 fixes) due to the use of same sets of mutations, there is still  
 614 a difference in the quality of the reported results. As we argued before, SPLALLREPAIR<sub>1</sub>  
 615 and SPLALLREPAIR<sub>2</sub> report repaired program families obtained by fixing both feature and  
 616 program expressions, whereas **Brute-force** only reports the repaired variants obtained by  
 617 fixing program expressions. Hence, the results from **Brute-force** have to be analyzed by  
 618 the user to produce information comparable to that returned by SPLALLREPAIR<sub>1</sub> and  
 619 SPLALLREPAIR<sub>2</sub> in the form of repaired program families. Moreover, the fixes of individual  
 620 variants may cause errors in other variants as evidenced by **feat-inter** and **feat\_power**.

621 The main bottleneck for real-world SPLs, such as **MinePump** with 730 LOC and 6 features,  
 622 is the huge space of mutants. The problem is that the search space of mutants grows very  
 623 rapidly as the number of changeable expressions (statements) included in  $S_{\text{soft}}$  grows. For  
 624 example, the space of mutants for **MinePump** is  $\sim 10^{12}$  for mutation level 1 and  $\sim 10^{34}$  for  
 625 mutation level 2. Hence, to explore even the sub-space of mutants with only 1 edit ( $r = 1$ )  
 626 we need around 300 sec for SPLALLREPAIR<sub>1</sub> and >400 sec (timeout) for SPLALLREPAIR<sub>2</sub>.  
 627 One way to address this problem is to use variability fault localization [5, 47], which will first  
 628 identify feature and program expressions relevant for a variability bug, so that the SPL repair  
 629 algorithm will apply mutations only to those expressions. This way, we will significantly  
 630 reduce the space of all mutants without dropping any potentially correct mutant, and so we  
 631 will improve the performance of the SPLALLREPAIR algorithm.

632 The runtime performance results confirm that our lifted (family-based) repair algorithm  
 633 is indeed effective and especially so for large values of  $|F|$  and  $|K| = 2^{|F|}$ . That is, the focus of  
 634 lifted repair algorithm is to combat the configuration space explosion of SPLs, not their LOC  
 635 or mutation space sizes. As an experiment, we took **feat-inter**, and we have gradually  
 636 added optional features into it by conjoining them to the presence conditions of **#if**-s. For  
 637  $|F| = 3$ , SPLALLREPAIR<sub>1</sub> achieves speed-up of 8.3 times compared to **Brute-force**, whereas  
 638 for  $|F| = 4$  and  $|F| = 5$  we observe speed-ups of 14.7 and 26.7 times, respectively. The  
 639 key for those speed-ups is the linear growth of the running times of SPLALLREPAIR<sub>1</sub> with  
 640 the number of features  $|F|$  compared to the exponential growth of the running times of  
 641 **Brute-force** with  $|F|$ .

642 Finally, the evaluation shows that for bigger values of the unwinding bound  $b$ , we obtain  
 643 repairing tasks with slower runtime speed, but reporting more precise results.

## 6 Related Work

645 We divide our discussion of related work into two categories: lifted SPL analysis and program  
 646 repair.

### 647 Lifted SPL analysis

648 Formal analysis and verification of program families have been a topic of considerable research  
 649 in recent times. The challenge is to develop efficient techniques that work directly on program  
 650 families, rather than on single programs. Various lifted techniques have been introduced  
 651 that lift existing single-program analysis techniques to work on the level of program families.  
 652 Some examples are lifted syntax checking [27, 34], lifted type checking [7, 33], lifted static  
 653 analysis [6, 30, 15, 20, 55], lifted model checking [9, 16, 25], etc. There are two main lifted  
 654 techniques: to develop dedicated lifted (family-based) algorithms and tools (e.g. [9, 7, 6, 20]);  
 655 or to use specific simulators and variability encodings which transform program families into  
 656 single programs that can be analyzed by the standard single-program analysis tools. The  
 657 two approaches have different strengths and weaknesses. The advantage of the dedicated

lifted algorithms is that precise (conclusive) results are reported for every variant, but the disadvantage is that their implementation and maintenance can be labor intensive and expensive. For example, CBMC [8] is prominent (single-system) software model checker that contains many optimization algorithms, which are result of more than two decades research in advanced formal verification. Adapting and implementing all these algorithms in the context of lifted software model checking would require an enormous amount of work. Moreover, the performance of dedicated lifted algorithms still heavily depends on the size and complexity of the configuration space of the analyzed SPL.

On the other hand, the approaches based on variability encoding [30, 56] generate a family simulator that simulates the behaviour of all variants in an SPL. They re-use existing tools from single-program world, but some precision may be lost when interpreting the obtained results. The work [56] defines variability encoding on the top of TYPECHEF parser [34] for C and Java SPLs, while the work [30] defines variability encoding on the top of SUPERC parser [27] for C SPLs. The results of variability encoding have been applied to testing [35], software model checking [4], formal verification [30], and theorem proving [54] of SPLs. In this work, we pursue this line of research by presenting a lifted repair algorithm that is based on variability encoding of program families and an existing single-program mutation-based repair algorithm ALLREPAIR [50, 51].

## Program repair

Automated program repair has been extensively examined in software engineering as a way to efficiently maintain software systems [28, 37, 40, 42, 45, 46, 48, 50, 51]. These works aim to repair the buggy program, so that the transformed program does not exhibit any faults. Most of them use test suits as the only specification, so the correctness of a candidate is checked by running all tests in the test suite against it. They iteratively generate a candidate from the repair search space and check its validity by testing. Some examples are GENPROG [28], RsREPAIR [48], SPR [40]. The main problem of all testing-based approaches is the generation of overfitting repairs that pass all the test cases, but they break some untested required functionality of correct programs. This happens when the test suites do not cover all the functionality of a program.

In contrast to testing-based approaches, our work belongs to the category of repair tools that use formal techniques to guide the repair process. Several techniques, such as SEMFIX [45] and ANGELIX [42], use symbolic execution to find a repair constraint and then generate a correct fix based on it. Similarly to our work, Könighofer et al. [37] also use assertions as formal specifications, but instead of mutations they use on-the-fly concolic execution (a variant of symbolic execution that uses both symbolic and concrete input values) and templates (linear expressions of program variables with unknown coefficients) as repairs. The solutions for unknown coefficients are found by SMT solving, thus discovering the repaired program. The MAPLE tool [46] utilizes a formal verification system to locate buggy expressions, which are again replaced with templates in which the unknown coefficients are determined using constraint solving. The work [36] uses a deductive synthesis framework for repairing recursive functional programs with respect to specifications expressed in the form of pre- and post-conditions.

Finally, our approach is inspired by Rothenberg and Grumberg [50, 51] that have developed the ALLREPAIR tool for automatic program repair based on code mutations. In this paper, we pursue this line of work by applying it in a new context of SPL repair, which is done by taking into account all specific characteristics of SPLs. This way, we broaden the space of programs that can be repaired.

705        The QLOSE tool [10] introduces a quantitative program repair algorithm that finds the  
 706        “optimal” solutions by taking into account multiple quantitative objectives, such as the  
 707        number of syntactic edits and semantic changes in program behaviours/executions. The work  
 708        [41] proposes a semantic program repair technique that performs counterexample-guided  
 709        inductive repair loop via symbolic execution. In this work, we currently find a solution with  
 710        minimal number of syntactic changes to the original program family. The semantics of the  
 711        program family is encoded as an SMT formula that is mutated and checked for correctness  
 712        by an SMT solver. In the future, we plan to investigate some semantics-based learning  
 713        techniques that will use the counterexamples returned by the SMT solver to guide the  
 714        algorithm towards finding faster solutions.

715        Automated program repair has often been combined with fault localization. Fault  
 716        localization [31, 17] is a technique for automatically generating concise error explanations  
 717        in the form of locations/statements relevant for a given error that describe why the error  
 718        has occurred. The works [12, 49, 51] use fault localization to narrow down the repair search  
 719        space, followed by applying program repair. Firstly, fault localization suggests locations in  
 720        the erroneous program that might be the cause of the error. Subsequently, the program  
 721        repair attempts to change only those locations detected by the fault localization in order to  
 722        eliminate the error. This way, the original program repair procedure is speeded up without  
 723        incurring any precision loss. Recently, variability fault localization in buggy SPL systems  
 724        has also been a subject of research [5, 44, 47]. They use spectrum-based fault localization  
 725        (SBFL) metric [1] to calculate the suspiciousness scores for localizing variability bugs at the  
 726        level of features [5] and statements [44, 47] based on the test information (program spectra).  
 727        We can combine the variability fault localization and our variability-aware repair method  
 728        to additionally prune the search space of mutants, thus improving the performances of our  
 729        approach.

730        Program repair is also related to program sketching [52], where a program with missing  
 731        parts (holes) has to be completed in such a way that a given specification is satisfied. One  
 732        of the earliest and widely-known approach to solve the sketching problem is the SKETCH  
 733        tool [52], which uses SAT-based counterexample-guided inductive synthesis. It iteratively  
 734        performs SAT queries to find integer constants for the holes so that the resulting program is  
 735        correct on all possible inputs. The works [19, 21] introduce the FAMILYSKETCHER tool that  
 736        solves the sketching problem by using a lifted static analysis based on abstract interpretation.  
 737        The key idea is that all possible sketch realizations represent a program family, and so the  
 738        sketch search space is explored via an efficient lifted analysis of program families, which uses  
 739        a specifically designed decision tree abstract domain. The FAMILYSKETCHER also generates  
 740        an optimal solution to the sketching problem with respect to the number of execution steps  
 741        to termination. Furthermore, the approach [18] uses abstract static analysis and logical  
 742        abduction to solve the generalized program sketching problem where the missing holes can  
 743        be replaced with arbitrary expressions, not only with integer constants as in the case of  
 744        SKETCH and FAMILYSKETCHER tools.

## 7 Conclusion

745        In this paper, we have introduced an automated SPL repair framework using variability  
 746        encoding, bounded model checking and cooperation between SAT and SMT solvers. More  
 747        specifically, we utilize the CBMC bounded model checker to translate the family simulator of  
 748        a program family to a program formula. By checking the satisfiability of the program formula  
 749        using an SMT solver, we verify the correctness of the given program family. Then, each

751 formula corresponding to a buggy (feature or program) expression is replaced by a mutated  
 752 patch, to create a new SMT formula that is again checked for satisfiability. To ensure that  
 753 only minimally mutated programs are considered, we call a SAT solver. By experiments we  
 754 have shown that our prototype tool can discover interesting patches for various buggy SPLs.  
 755

756 The huge space of mutants can be a bottleneck when dealing with real-world SPLs that  
 757 have high sizes of LOCs and features. To overcome this problem, we can consider different  
 758 techniques for pruning the search space of all possible mutations in the future. One possibility  
 759 is to use variability fault localization [5, 47], which will find statements relevant for the  
 760 variability bug. The formulas corresponding to all other statements will be included in  $S_{\text{hard}}$   
 761 and so no mutations will be applied to them. By mutating only statements relevant for the  
 762 bug, we will significantly reduce the space of all mutants, thus speeding up the SPL repair  
 763 method without any precision loss.

---

## 763 References

- 764 1 Rui Abreu, Peter Zoeteweij, and Arjan J. C. van Gemund. Spectrum-based multiple fault  
     localization. In *ASE 2009, 24th IEEE/ACM International Conference on Automated Software  
     Engineering, Auckland, New Zealand, November 16-20, 2009*, pages 88–99. IEEE Computer  
     Society, 2009. doi:10.1109/ASE.2009.25.
- 765 2 Sven Apel, Don S. Batory, Christian Kästner, and Gunter Saake. *Feature-Oriented Software  
     Product Lines - Concepts and Implementation*. Springer, 2013.
- 766 3 Sven Apel, Hendrik Speidel, Philipp Wendler, Alexander von Rhein, and Dirk Beyer. Detection  
     of feature interactions using feature-aware verification. In *26th IEEE/ACM Int. Conf. on  
     Automated Software Engineering (ASE 2011)*, pages 372–375, 2011. URL: <http://dx.doi.org/10.1109/ASE.2011.6100075>, doi:10.1109/ASE.2011.6100075.
- 767 4 Sven Apel, Alexander von Rhein, Philipp Wendler, Armin Größlinger, and Dirk Beyer. Strategies for product-line verification: case studies and experiments. In *35th Inter. Conference  
     on Software Engineering, ICSE '13*, pages 482–491, 2013.
- 768 5 Aitor Arrieta, Sergio Segura, Urtzi Markiegi, Goiuria Sagardui, and Leire Etxeberria. Spectrum-  
     based fault localization in software product lines. *Inf. Softw. Technol.*, 100:18–31, 2018. URL:  
     <https://doi.org/10.1016/j.infsof.2018.03.008>, doi:10.1016/J.INFSOF.2018.03.008.
- 769 6 Eric Bodden, Társis Tolédo, Márcio Ribeiro, Claus Brabrand, Paulo Borba, and Mira Mezini. Splift: statically analyzing software product lines in minutes instead of years. In *ACM SIGPLAN Conference on PLDI '13*, pages 355–364, 2013.
- 770 7 Sheng Chen, Martin Erwig, and Eric Walkingshaw. An error-tolerant type system for variational  
     lambda calculus. In *ACM SIGPLAN International Conference on Functional Programming,  
     ICFP'12*, pages 29–40, 2012. URL: <http://doi.acm.org/10.1145/2364527.2364535>, doi:  
     10.1145/2364527.2364535.
- 771 8 Edmund M. Clarke, Daniel Kroening, and Flavio Lerda. A tool for checking ANSI-C programs.  
     In *Tools and Algorithms for the Construction and Analysis of Systems, 10th International  
     Conference, TACAS 2004, Proceedings*, volume 2988 of *LNCS*, pages 168–176. Springer, 2004.  
     doi:10.1007/978-3-540-24730-2\15.
- 772 9 Andreas Classen, Maxime Cordy, Pierre-Yves Schobbens, Patrick Heymans, Axel Legay, and  
     Jean-François Raskin. Featured transition systems: Foundations for verifying variability-  
     intensive systems and their application to LTL model checking. *IEEE Trans. Software Eng.*,  
     39(8):1069–1089, 2013. URL: <http://doi.ieee.org/10.1109/TSE.2012.86>,  
     doi:10.1109/TSE.2012.86.
- 773 10 Loris D’Antoni, Roopsha Samanta, and Rishabh Singh. Qlose: Program repair with quantitative  
     objectives. In *Computer Aided Verification - 28th International Conference, CAV 2016,  
     Proceedings, Part II*, volume 9780 of *LNCS*, pages 383–401. Springer, 2016. doi:10.1007/  
     978-3-319-41540-6\21.

- 800    11 Leonardo Mendonça de Moura and Nikolaj Bjørner. Z3: an efficient SMT solver. In C. R.  
 801    Ramakrishnan and Jakob Rehof, editors, *Tools and Algorithms for the Construction and*  
 802    *Analysis of Systems, 14th International Conference, TACAS 2008. Proceedings*, volume 4963  
 803    of *LNCS*, pages 337–340. Springer, 2008. doi:10.1007/978-3-540-78800-3\\_24.
- 804    12 Vidroha Debroy and W. Eric Wong. Using mutation to automatically suggest fixes for faulty  
 805    programs. In *Third International Conference on Software Testing, Verification and Validation,*  
 806    *ICST 2010*, pages 65–74. IEEE Computer Society, 2010. doi:10.1109/ICST.2010.66.
- 807    13 Aleksandar Dimovski and Danilo Gligoroski. Generating highly nonlinear boolean functions  
 808    using a genetic algorithm. In *6th Int. Conference on Telecommunications in Modern Satellite,*  
 809    *Cable and Broadcasting Service, TELSIKS 2003*, volume 2 of *IEEE*, pages 604–607, 2003.  
 810    doi:10.1109/TELSKS.2003.1246297.
- 811    14 Aleksandar S. Dimovski. Symbolic game semantics for model checking program families. In  
 812    *Model Checking Software - 23nd International Symposium, SPIN 2016, Proceedings*, volume  
 813    9641 of *LNCS*, pages 19–37. Springer, 2016.
- 814    15 Aleksandar S. Dimovski. Lifted static analysis using a binary decision diagram abstract domain.  
 815    In *Proceedings of the 18th ACM SIGPLAN International Conference on GPCE 2019*, pages  
 816    102–114. ACM, 2019. doi:10.1145/3357765.3359518.
- 817    16 Aleksandar S. Dimovski. Ctl\* family-based model checking using variability abstractions  
 818    and modal transition systems. *Int. J. Softw. Tools Technol. Transf.*, 22(1):35–55, 2020.  
 819    doi:10.1007/s10009-019-00528-0.
- 820    17 Aleksandar S. Dimovski. Error invariants for fault localization via abstract interpretation.  
 821    In *Static Analysis - 30th International Symposium, SAS 2023, Proceedings*, volume 14284 of  
 822    *LNCS*, pages 190–211. Springer, 2023. doi:10.1007/978-3-031-44245-2\\_10.
- 823    18 Aleksandar S. Dimovski. Generalized program sketching by abstract interpretation and logical  
 824    abduction. In *Static Analysis - 30th International Symposium, SAS 2023, Proceedings*, volume  
 825    14284 of *LNCS*, pages 212–230. Springer, 2023. doi:10.1007/978-3-031-44245-2\\_11.
- 826    19 Aleksandar S. Dimovski. Quantitative program sketching using decision tree-based lifted  
 827    analysis. *J. Comput. Lang.*, 75:101206, 2023. URL: <https://doi.org/10.1016/j.cola.2023.101206>. doi:10.1016/J.COLA.2023.101206.
- 829    20 Aleksandar S. Dimovski and Sven Apel. Lifted static analysis of dynamic program families  
 830    by abstract interpretation. In *35th European Conference on Object-Oriented Programming,*  
 831    *ECOOP 2021*, volume 194 of *LIPICS*, pages 14:1–14:28. Schloss Dagstuhl - Leibniz-Zentrum  
 832    für Informatik, 2021. doi:10.4230/LIPIcs.ECOOP.2021.14.
- 833    21 Aleksandar S. Dimovski, Sven Apel, and Axel Legay. Program sketching using lifted analysis  
 834    for numerical program families. In *NASA Formal Methods - 13th International Symposium,*  
 835    *NFM 2021, Proceedings*, volume 12673 of *LNCS*, pages 95–112. Springer, 2021. doi:10.1007/  
 836    978-3-030-76384-8\\_7.
- 837    22 Aleksandar S. Dimovski, Sven Apel, and Axel Legay. Several lifted abstract domains for  
 838    static analysis of numerical program families. *Sci. Comput. Program.*, 213:102725, 2022. URL:  
 839    <https://doi.org/10.1016/j.scico.2021.102725>, doi:10.1016/J.SCICO.2021.102725.
- 840    23 Aleksandar S. Dimovski, Claus Brabrand, and Andrzej Wasowski. Finding suitable variability  
 841    abstractions for lifted analysis. *Formal Aspects Comput.*, 31(2):231–259, 2019. doi:10.1007/  
 842    s00165-019-00479-y.
- 843    24 Aleksandar S. Dimovski and Ranko Lazic. Compositional software verification based on game  
 844    semantics and process algebra. *Int. J. Softw. Tools Technol. Transf.*, 9(1):37–51, 2007. URL:  
 845    <https://doi.org/10.1007/s10009-006-0005-y>, doi:10.1007/S10009-006-0005-Y.
- 846    25 Aleksandar S. Dimovski and Andrzej Wasowski. From transition systems to variability models  
 847    and from lifted model checking back to UPPAAL. In *Models, Algorithms, Logics and Tools*,  
 848    volume 10460 of *LNCS*, pages 249–268. Springer, 2017. doi:10.1007/978-3-319-63121-9\_13.
- 849    26 Niklas Eén and Niklas Sörensson. An extensible sat-solver. In *Theory and Applications of*  
 850    *Satisfiability Testing, 6th International Conference, SAT 2003*, volume 2919 of *LNCS*, pages  
 851    502–518. Springer, 2003. doi:10.1007/978-3-540-24605-3\\_37.

- 852   **27** Paul Gazzillo and Robert Grimm. Superc: parsing all of C by taming the preprocessor. In  
853   Jan Vitek, Haibo Lin, and Frank Tip, editors, *ACM SIGPLAN Conference on Programming*  
854   *Language Design and Implementation, PLDI '12, Beijing, China - June 11 - 16, 2012*,  
855   pages 323–334. ACM, 2012. URL: <http://doi.acm.org/10.1145/2254064.2254103>, doi:  
856   10.1145/2254064.2254103.
- 857   **28** Claire Le Goues, ThanhVu Nguyen, Stephanie Forrest, and Westley Weimer. Genprog: A  
858   generic method for automatic software repair. *IEEE Trans. Software Eng.*, 38(1):54–72, 2012.  
859   doi:10.1109/TSE.2011.104.
- 860   **29** Todd L. Graves, Mary Jean Harrold, Jung-Min Kim, Adam A. Porter, and Gregg Rothermel.  
861   An empirical study of regression test selection techniques. *ACM Trans. Softw. Eng. Methodol.*,  
862   10(2):184–208, 2001. doi:10.1145/367008.367020.
- 863   **30** Alexandru F. Iosif-Lazar, Ahmad Salim Al-Sabah, Aleksandar S. Dimovski, Juha Erik Sa-  
864   volainen, Krzysztof Sierszecki, and Andrzej Wasowski. Experiences from designing and  
865   validating a software modernization transformation (E). In *30th IEEE/ACM International*  
866   *Conference on Automated Software Engineering, ASE 2015*, pages 597–607, 2015. URL:  
867   <http://dx.doi.org/10.1109/ASE.2015.84>, doi:10.1109/ASE.2015.84.
- 868   **31** Manu Jose and Rupak Majumdar. Cause clue clauses: error localization using maximum  
869   satisfiability. In *Proceedings of the 32nd ACM SIGPLAN Conference on Programming Language*  
870   *Design and Implementation, PLDI 2011*, pages 437–446. ACM, 2011. doi:10.1145/1993498.  
871   1993550.
- 872   **32** Christian Kästner. *Virtual Separation of Concerns: Toward Preprocessors 2.0*. PhD thesis,  
873   University of Magdeburg, Germany, May 2010.
- 874   **33** Christian Kästner, Sven Apel, Thomas Thüm, and Gunter Saake. Type checking annotation-  
875   based product lines. *ACM Trans. Softw. Eng. Methodol.*, 21(3):14, 2012.
- 876   **34** Christian Kästner, Paolo G. Giarrusso, Tillmann Rendel, Sebastian Erdweg, Klaus Ostermann,  
877   and Thorsten Berger. Variability-aware parsing in the presence of lexical macros and conditional  
878   compilation. In *OOPSLA'11*, pages 805–824. ACM, 2011.
- 879   **35** Christian Kästner, Alexander von Rhein, Sebastian Erdweg, Jonas Pusch, Sven Apel, Tillmann  
880   Rendel, and Klaus Ostermann. Toward variability-aware testing. In *FOSD '12*, pages 1–8,  
881   2012.
- 882   **36** Etienne Kneuss, Manos Koukoutos, and Viktor Kuncak. Deductive program repair. In  
883   *Computer Aided Verification - 27th International Conference, CAV 2015, Proceedings, Part II*,  
884   volume 9207 of *LNCS*, pages 217–233. Springer, 2015. doi:10.1007/978-3-319-21668-3\\_13.
- 885   **37** Robert Könighofer and Roderick Bloem. Repair with on-the-fly program analysis. In *8th*  
886   *International Haifa Verification Conference, HVC 2012*, volume 7857 of *LNCS*, pages 56–71.  
887   Springer, 2012. doi:10.1007/978-3-642-39611-3\\_11.
- 888   **38** Jeff Kramer, Jeff Magee, Morris Sloman, and A. Lister. Conic: An integrated approach to  
889   distributed computer control systems. *IEE Proc.*, 130(1):1–10, 1983.
- 890   **39** Mark H. Liffiton and Jordyn C. Maglalang. A cardinality solver: More expressive constraints  
891   for free - (poster presentation). In *Theory and Applications of Satisfiability Testing - SAT 2012*  
892   - 15th International Conference, Proceedings, volume 7317 of *LNCS*, pages 485–486. Springer,  
893   2012. doi:10.1007/978-3-642-31612-8\\_47.
- 894   **40** Fan Long and Martin C. Rinard. Staged program repair with condition synthesis. In  
895   *Proceedings of the 2015 10th Joint Meeting on ESEC/FSE 2015*, pages 166–178. ACM, 2015.  
896   doi:10.1145/2786805.2786811.
- 897   **41** Sergey Mechtaev, Manh-Dung Nguyen, Yannic Noller, Lars Grunske, and Abhik Roychoudhury.  
898   Semantic program repair using a reference implementation. In *Proceedings of the 40th*  
899   *International Conference on Software Engineering, ICSE 2018*, pages 129–139. ACM, 2018.  
900   doi:10.1145/3180155.3180247.
- 901   **42** Sergey Mechtaev, Jooyong Yi, and Abhik Roychoudhury. Angelix: scalable multiline program  
902   patch synthesis via symbolic analysis. In *Proceedings of the 38th International Conference on*  
903   *Software Engineering, ICSE 2016*, pages 691–701. ACM, 2016. doi:10.1145/2884781.2884807.

- 904   **43** Jan Midgaard, Aleksandar S. Dimovski, Claus Brabrand, and Andrzej Wasowski. Systematic  
 905   derivation of correct variability-aware program analyses. *Sci. Comput. Program.*, 105:145–  
 906   170, 2015. URL: <http://dx.doi.org/10.1016/j.scico.2015.04.005>.  
 907
- 908   **44** Kien-Tuan Ngo, Thu-Trang Nguyen, Son Nguyen, and Hieu Dinh Vo. Variability fault  
 909   localization: a benchmark. In *SPLC '21: 25th ACM International Systems and Software*  
 910   *Product Line Conference, Leicester, United Kingdom, September 6-11, 2021, Volume A*, pages  
 911   120–125. ACM, 2021. doi:10.1145/3461001.3473058.
- 912   **45** Hoang Duong Thien Nguyen, Dawei Qi, Abhik Roychoudhury, and Satish Chandra. Semfix:  
 913   program repair via semantic analysis. In *35th International Conference on Software Engineering,*  
 914   *ICSE '13*, pages 772–781. IEEE Computer Society, 2013. doi:10.1109/ICSE.2013.6606623.
- 915   **46** Thanh-Toan Nguyen, Quang-Trung Ta, and Wei-Ngan Chin. Automatic program repair using  
 916   formal verification and expression templates. In *Verification, Model Checking, and Abstract*  
 917   *Interpretation - 20th International Conference, VMCAI 2019, Proceedings*, volume 11388 of  
 918   *LNCS*, pages 70–91. Springer, 2019. doi:10.1007/978-3-030-11245-5\\_4.
- 919   **47** Thu-Trang Nguyen, Kien-Tuan Ngo, Son Nguyen, and Hieu Dinh Vo. A variability fault  
 920   localization approach for software product lines. *IEEE Trans. Software Eng.*, 48(10):4100–4118,  
 921   2022. doi:10.1109/TSE.2021.3113859.
- 922   **48** Yuhua Qi, Xiaoguang Mao, Yan Lei, Ziying Dai, and Chengsong Wang. The strength of  
 923   random search on automated program repair. In *36th International Conference on Software*  
 924   *Engineering, ICSE '14*, pages 254–265. ACM, 2014. doi:10.1145/2568225.2568254.
- 925   **49** Urmas Repinski, Hanno Hantson, Maksim Jenihhin, Jaan Raik, Raimund Ubar, Giuseppe Di  
 926   Guglielmo, Graziano Pravadelli, and Franco Fummi. Combining dynamic slicing and mutation  
 927   operators for ESL correction. In *17th IEEE European Test Symposium, ETS 2012*, pages 1–6.  
 928   IEEE Computer Society, 2012. doi:10.1109/ETS.2012.6233020.
- 929   **50** Bat-Chen Rothenberg and Orna Grumberg. Sound and complete mutation-based program  
 930   repair. In *FM 2016: Formal Methods - 21st International Symposium, Proceedings*, volume  
 931   9995 of *LNCS*, pages 593–611, 2016. doi:10.1007/978-3-319-48989-6\\_36.
- 932   **51** Bat-Chen Rothenberg and Orna Grumberg. Must fault localization for program repair. In  
 933   *Computer Aided Verification - 32nd International Conference, CAV 2020, Proceedings, Part II*,  
 934   volume 12225 of *LNCS*, pages 658–680. Springer, 2020. doi:10.1007/978-3-030-53291-8\\_  
 935   \_33.
- 936   **52** Armando Solar-Lezama. Program sketching. *STTT*, 15(5-6):475–495, 2013. doi:10.1007/  
 937   s10009-012-0249-7.
- 938   **53** Shin Hwei Tan, Jooyong Yi, Yulis, Sergey Mechtaev, and Abhik Roychoudhury. Codeflaws:  
 939   a programming competition benchmark for evaluating automated program repair tools. In  
 940   *Proceedings of the 39th International Conference on Software Engineering, ICSE 2017 -*  
 941   *Companion Volume*, pages 180–182. IEEE Computer Society, 2017. doi:10.1109/ICSE-C.  
 942   2017.76.
- 943   **54** Thomas Thüm, Ina Schaefer, Martin Hentschel, and Sven Apel. Family-based deductive  
 944   verification of software product lines. In *Generative Programming and Component Engineering,*  
 945   *GPCE'12*, pages 11–20. ACM, 2012. URL: <http://doi.acm.org/10.1145/2371401.2371404>,  
 946   doi:10.1145/2371401.2371404.
- 947   **55** Alexander von Rhein, Jörg Liebig, Andreas Janker, Christian Kästner, and Sven Apel.  
 948   Variability-aware static analysis at scale: An empirical study. *ACM Trans. Softw. Eng.*  
 949   *Methodol.*, 27(4):18:1–18:33, 2018. doi:10.1145/3280986.
- 950   **56** Alexander von Rhein, Thomas Thüm, Ina Schaefer, Jörg Liebig, and Sven Apel. Variability  
 951   encoding: From compile-time to load-time variability. *J. Log. Algebraic Methods Program.*,  
 952   85(1):125–145, 2016. doi:10.1016/j.jlamp.2015.06.007.