Monotonic Path-Specific Effects:

Application to Estimating Educational Returns[*]

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Abstract

Conventional research on educational effects typically employs either a "years of schooling" measure of education, or dichotomizes attainment as a point-in-time treatment. Nevertheless, such a conceptualization of education is at odds with the sequential process by which individuals make educational transitions. In this paper, I propose a causal mediation framework for the study of educational effects. The framework considers the effect of any given educational transition as operating indirectly, via subsequent educational transitions, as well as directly, net of these transitions. I demonstrate that the total effect of education can be additively decomposed into mutually exclusive components that capture these direct and indirect effects. The proposed decomposition has a number of special properties which distinguish it from conventional decompositions of the average treatment effect (ATE) into mutually exclusive path-specific effects (PSEs), properties which facilitate less restrictive identification assumptions as well as identification of all of the causal paths in the decomposition. I introduce a locally semiparametric efficient estimator for estimating the direct and indirect effects, establish rate conditions required of the nuisance functions to attain semiparametric efficiency, and illustrate the proposed framework using data from the NLSY97.

1   Introduction

One of the most resilient social scientific findings across a range of national contexts is the strong association between educational attainment and a variety of life outcomes, such as earnings, health, social capital, and family stability (Hout, 2012; Chetty et al., 2023). Conventionally, researchers have taken one of two approaches to evaluating the social and economic returns to education: the first employs a "years of schooling" measure of educational attainment (Card, 1999, 2001), while the second dichotomizes attainment as a point-in-time treatment. This latter approach has been particularly influential in the study of the impact of postsecondary attainment on earnings, where the treatment considered is often an indicator for whether a high school graduate has attended, or graduated from, college (Brand and Xie, 2010; Carneiro et al., 2011; Zimmerman, 2014; Goodman et al., 2017; Smith et al., 2020; Bleemer, 2022; Mountjoy, 2022).

Despite the important insights this literature has made into establishing the causal effect of educational attainment on important social and economic outcomes, extant work has been inattentive to the sequential process by which people make educational transitions (Mare, 1980).[1] At the end of high school, individuals decide to enroll at college or not. Among college enrollees, only 60% receive a BA within six years of initial college entry (Snyder et al., 2016), with an even loIr proportion for low-income students and students of color (Eller and DiPrete, 2018; Zhou and Pan, 2023). Moreover, amidst higher educational expansion in the US, college graduates must increasingly choose whether to enter the labor market or to enroll in post-graduate education. Increasingly, therefore, educational attainment in the US has become a field of multiple levels with sequential transitions, all of which are independently consequential for individuals' labor market outcomes, and therefore of independent scientific interest for education scholars.

The sequential nature of educational transitions implies that a causal mediation framework can be employed to study the causal paths by which education's "value-added" occurs. Specifically, I can consider the first transition in a sequence of educational levels of interest as a treatment variable, A, and subsequent transitions as mediators that "transmit" the effects of the treatment and of prior transitions, M_k ($1 \leq k \leq K$). For example, if we are interested in the

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[1]I use the term "educational transition" to refer both to vertical transitions (e.g. enrollment at a secondary or tertiary institution), as well as to the attainment of a qualification at that institutional level (e.g. high school graduation or BA completion).

total effect of high school completion on earnings, we may ask to what extent this total effect operates indirectly, through the effect of college attendance and college completion (putative mediators) on earnings, or directly, through alternative causal pathways. The insight that the total causal effect of a education can be decomposed into its direct and indirect effects opens up a range of important research and policy-oriented questions. For example, researchers are often interested in assessing the long-term effects of a policy intervention to induce students to complete high school or to attend college (e.g. Hurwitz and Howell, 2014; Sullivan et al., 2019; Castleman et al., 2020; Bird et al., 2021; Dynarski et al., 2021; Black et al., 2023; Turner and Gurantz, 2024). Tracing to what extent an early-stage educational intervention boosts outcomes such as earnings via its promotion of subsequent educational attainment (its indirect effects), or 'via earnings directly, would enable policy-makers to discern what drives the intervention's value and to hone subsequent policy. Relatedly, if the early intervention's effects are heterogeneous across demographic groups, assessment of the direct and indirect effects of the intervention guide researchers to aspects of the educational experience that inhibit upward mobility. For example, if an intervention to bolster high school completion is weaker for socio-economically disadvantaged than advantaged students, such heterogeneity could result from direct labor market differences in the return to a high school diploma, or because the indirect effects are smaller for disadvantaged students (who may be less likely to attend or complete college after completing high school, as compared with their wealthier peers) Yet, prior empirical approaches not Ill-suited for answering these questions. The "years-of-schooling" approach focuses on the "direct effect" of each additional year of schooling, while the dichotomous approach conflates the direct and indirect effects.[2]

In this article, I introduce a causal mediation framework for analyzing the effects of educational transitions. I make two contributions. First, for the setting of K (≥ 1) monotonic mediators, I develop a general formula that decomposes the total effect of any level of education into $K+1$ monotonic path-specific effects (MPSEs): a direct effect net of K subsequent educational transitions, reflecting the path $A \rightarrow Y$, and K mutually exclusive "continuation" or gross ef-

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[2]A further strand of literature, especially prominent in labor economics, explores labor market returns to horizontal aspects of differentiation within a given level of education (e.g. college selectivity, as well as specific colleges) (e.g. Cohodes and Goodman, 2014; Goodman et al., 2017; Mountjoy and Hickman, 2021; Chetty et al., 2023; Eller, 2023). While my proposed framework prioritizes the effects of different levels of education, I discuss in my concluding remarks how the framework could be extended to accommodate multivariate mediators.

fects, reflecting the paths $A \to M_1 \to Y$, $A \to M_1 \to M_2 \to Y$,... and $A \to M_1 \cdots \to M_K \to Y$. This general decomposition extends previous literature on mediation under monotonicity which has focused exclusively on the case of a single mediator (e.g. Zhou, 2022a). The $K + 1$ monotonic path-specific effects (MPSEs) are all non-parametrically identified under the assumption of sequential ignorability, which allows for the effect of each transition to be confounded by a different set of intermediate covariates. Second, I introduce several estimation strategies for my proposed decomposition, including a simple linear model-based regression-with-residuals (RWR) procedure, and a non-parametric estimation strategy based on the efficient influence functions (EIFs) of the target parameters (see Chernozhukov et al., 2017; Kennedy, 2022).

Within the realm of education research, my work draws on important work by Heckman et al. (2018), who present a similar decomposition of the effect of schooling over the early life course, but differs in two important respects. First, I provide nonparametric definitions, identification results, and estimation strategies for decomposing the total effect of schooling through its direct and indirect components. Second, my decomposition accommodates the presence of a distinct set of observed intermediate confounders for each transition. Although one limitation of my approach is that I assume away the presence of *unobserved* confounders for each transition, I propose a sensitivity analysis that assesses the robustness of the results to unobserved confounding, under a set of simplifying assumptions.

More broadly, my framework speaks to the burgeoning field of causal mediation analysis in the social economic, and health sciences, targeted at assessing the causal pathways by which a treatment affects an outcome. While the majority of potential outcomes approaches to mediation focus on mediation quantities in the single-mediator setting, a growing body of work increasingly examines mediation estimands in settings where there are multiple mediators of interest (Avin et al., 2005; Albert and Nelson, 2011; VanderWeele and Vansteelandt, 2014; Lin and VanderWeele, 2017; Miles et al., 2017; Steen et al., 2017; Vansteelandt and Daniel, 2017; Miles et al., 2020). In particular, in the case of two causally ordered mediators, Daniel et al. (2015) show that the ATE can be decomposed into multiple path-specific effects (PSEs), and outline the assumptions under which some of these effects are identified. Most recently, Zhou (2022b) generalized this framework to the case of K mediators, establishing the set of identifiable PSEs, introducing a battery of regression-based, weighting, and locally semiparametric efficient estimators for the PSEs.

While the field of causal mediation has displayed rapid innovation and development in re-

cent years, the above body of work is implicitly concerned only with settings where all values of a putative mediator are accessible to all individuals, irrespective of their treatment status. I enrich this literature by examining a special empirical setting where mediators are characterized by "monotonicity": that is, where an individual's potential $k + 1$ mediator value is deterministically zero if that individual's kth mediator value is 0. The resultant decomposition of the ATE into MPSEs can both be identified under weaker assumptions than those required for conventional mediation settings, and permit identification of all of the causal paths in question, as opposed to just a strict subset of them. It further facilitates a finer-grained components compared with in traditional mediation analyses. I also contribute to a growing parallel literature that proposes a range of nonparametric, and semi-parametric efficient estimators for alternative mediation estimands, based on the efficient influence functions (EIFs) of the causal quantities of interest (e.g. Miles et al., 2020; Farbmacher et al., 2022; Zhou, 2022b), as well as to related closely-related work that proposes semi-parametric efficient estimators for dynamic treatment effects (Lewis and Syrgkanis, 2020; Viviano and Bradic, 2021; Bodory et al., 2022). Although my empirical motivation is to estimate educational returns, my framework applies widely to a range of demographic and organizational settings characterized by "state dependency" between treatment and mediators. This setting is particularly salient in demographic phenomena, which often involve sequential transitions over the life course. Certain demographic events are rigid in their monotonicity as a result of their definition. For example, researchers may be interested in discerning the degree to which positive effects of marriage on total income are undermined by the negative effects of divorce and separation (and, in turn, their mitigation by re-marriage) (Kenney, 2004; Sweeney and Phillips, 2004). A similar perspective may be taken in a criminal justice context: the total effect of early-stage police contact (such as being searched for contraband) on education and socio-psychological outcomes can be decomposed into path-specific effects via subsequent arrest and incarceration (Weaver and Lerman, 2010; Kirk and Sampson, 2013; Sugie and Turney, 2017). Similarly, the efficacy of pre-trial judge release decisions on long-term recidivism can be discerned via its effects on pre-trial misconduct if released (Arnold et al., 2018; Hull, 2021).

In the following sections, I first introduce the decomposition for the case of a single intermediate educational transition, before discussing the general case of K intermediate transitions and identification under the assumption of sequential ignorability (Section 2). In Section 3, I introduce a semiparametric estimation strategy for estimating the proposed decomposition.

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Section presents simulation evidence for my approach, evaluating the performance of the proposed estimators under different degrees of model misspecification. In Section 5, I illustrate the proposed framework and methods using data from the National Longitudinal Survey of Youth (NLSY97) cohort. Section 6 concludes.

2 Monotonic Path-Specific Effects

2.1 A Single Intermediate Transition

I first consider the case of one intermediate educational transition. Suppressing subscripts, let A denote an indicator for high school graduation (the initial educational transition), M_1 denote an indicator for college attendance (the intermediate transition), and Y a binary or continuous outcome of interest such as earnings. my single-transition decomposition thus assesses the educational sequence $A \to M \to Y$, that is, high school graduation→college attendance→earnings. I thus treat college attendance as a mediator of the total effect of high school graduation on earnings, in relation to which the total effect of high school graduation can be decomposed into an indirect effect (that "flows through" college enrollment), and a direct effect (net of college attendance) . Following Heckman et al. (2018), I refer to this latter term as the "continuation" value of any given educational transition A.

Using potential outcomes notation, and suppressing subscripts i, let $M(a)$ denote an individual's potential value of the mediator if their treatment status Ire set to a, and let $Y(a, m)$ denote that individual's potential outcome if their treatment and mediator statuses Ire set to a and m, respectively. I assume, as I formalize in the following section, that sequential transitions are characterized by monotonicity; here, this means that individuals who do not attend college cannot complete college, or $M(0) = 0$. This sequential nature of educational transitions therefore implies the following potential outcomes: $Y(1)$, $Y(0)$, $Y(1,0)$, $Y(0,0)$, and $Y(1,1)$. Further, by the composition assumption, $Y(0) = Y(0, M(0) = Y(0,0)$, and

$$Y(1) = Y(1,0) + M(1)[Y(1,1) - Y(1,0)].$$

Thus, the individual total effect of A on Y can be decomposed as

$$Y(1) - Y(0) = Y(1,0) - Y(0,0) + M(1)[Y(1,1) - Y(1,0)]. \qquad (1)$$

Since these individual-level quantities are unidentified, I focus on their population-level analogs. Taking the expectation of Equation 1, I obtain the following decomposition of the ATE (see Zhou, 2022a):

$$\begin{aligned}
\text{ATE} &= \mathbb{E}[Y(1) - Y(0)] \\
&= \mathbb{E}[Y(1,0) - Y(0,0)] + \mathbb{E}[M(1)]\mathbb{E}[Y(1,1) - Y(1,0)] + \text{cov}[M(1), Y(1,1) - Y(1,0)] \quad (2) \\
&= \underbrace{\Delta_0}_{A \to Y} + \underbrace{\pi_1 \Delta_1 + \eta_1}_{A \to M \to Y}, \qquad (3)
\end{aligned}$$

where Δ_0 and Δ_1 denote the direct effects of the first and intermediate transitions on the outcome, $A \to Y$ and $M \to Y$, respectively, π_1 denotes the total effect of the first transition on the intermediate transition $A \to M$, and η_1 denotes the covariance between the effect of the initial transition on completion of the second and the effect of the second transition on Y: η_1 is positive if those who would attend college given high school completion (i.e., $M(1) = 1$) benefit more from college attendance in terms of their later earnings (i.e., have a larger $Y(1,1) - Y(1,0)$) than those who do not (i.e., $M(1) = 0$), and negative if the opposite holds. Meanwhile, the composite term $(\pi_1 \Delta_1 + \eta_1)$ captures the average indirect effect of the first transition via the intermediate transition $(A \to M \to Y)$, comprising the sum of the probability of college enrollment if an individual graduated high school multiplied by the direct of college enrollment and the covariance between college enrollment and its direct effect on earnings.

2.2 Generalization to K Intermediate Transitions

I now generalize the approach introduced in the preceding section to the case of K intermediate educational transitions. As previously, I denote the initial educational transition of interest, high school graduation, by A, and use $M_1, \ldots M_K$ to refer to the K subsequent transitions of interest, where I assume that all of $M_1, \ldots, M_K$ are binary and that for any $i < j$, M_i temporally precedes M_j. For instance, we may wish to decompose the total effect of high school completion on earnings via college attendance (M_1), college completion (M_2) and graduate school attendance (M_3). Let an overbar denote a vector of variables, such that $\overline{M}_k = (M_1, M_2, \ldots M_k)$ and $\overline{1}_k = (A = 1, M_1 = 1, \ldots, M_{k-1} = 1)$. Further, let $[K]$ denote the set $\{0, 1, \ldots, K\}$. In addi-

tion, I denote by X a vector of pretreatment confounders of the effect of $(A, \overline{M}_k)$ on (M_{k+1}, Y), and by $\overline{Z}_k = (Z_1, \ldots Z_k)$ a vector of intermediate confounders that may confound the causal effect of M_k on (M_{k+1}, Y). Using the potential outcomes notation, $Y(a, \overline{m}_k)$ thus denotes an individual's potential earnings if they completed, possibly contrary to fact, $k + 1$ sequential educational transitions. Similarly, $M_{k+1}(a, \overline{m}_k)$ denotes an individual's potential value of an educational transition Ire that individual to complete the $k + 1$ prior educational transitions. I invoke the following constraint on units' potential transition values:

Assumption 1. *Monotonicity:* $M_{k+1}(M_k = 0) = 0 \forall k \in [K - 1], M_0 = 0.$

Informally, Assumption 1 (*monotonicity*) states that an individual's potential $k + 1$th transition value is deterministically 0 if that individual does not complete the prior (kth) transition. It is analogous to a one-sided non-compliance assumption within an instrumental variables (IV) framework, which rules out an "always-takers" principal stratum. I can then use this assumption to decompose the ATE of A on Y, which I denote by τ_0. Specifically, let τ_k denote the gross effect of the kth mediator on Y, i.e.,

$$\tau_k = \mathbb{E}[Y(\overline{1}_{k+1}) - Y(\overline{1}_k, 0)],$$

and let Δ_0 denote the direct effect of A on Y, and Δ_k, the direct effect of the kth mediator on Y, i.e.,

$$\Delta_k = \mathbb{E}[Y(\overline{1}_{k+1}, 0) - Y(\overline{1}_k, 0)].$$

To explicate my approach, note that the gross effect of the kth mediator, τ_k, includes not only the direct effect $M_k \rightarrow Y$, net of subsequent educational transitions Δ_k), but also the indirect effects of M_k via subsequent transitions ($M \rightsquigarrow Y$, where a squiggly arrow denotes a combination of multiple paths). This insight motivates us to further decompose τ into its direct and indirect components. Thus, under the composition assumption, τ_k can be decomposed as

$$\tau_k = \Delta_k + \pi_{k+1}\tau_{k+1} + \eta_{k+1}, \tag{4}$$

where

$$\pi_{k+1} = \mathbb{E}[M_{k+1}(\overline{1}_{k+1})],$$

$$\eta_{k+1} = \text{cov}[M_{k+1}(\overline{1}_{k+1}), Y(\overline{1}_{k+2}) - Y(\overline{1}_{k+1}, 0)].$$

For $k = 1, \ldots, K-1$, iteratively substituting equation 4 into the corresponding expression for τ_{k-1} yields

$$\tau_0 = \underbrace{\Delta_0}_{A \to Y} + \sum_{k=1}^{K} \underbrace{(\Pi_{j=1}^{k}\pi_j)\Delta_k + (\Pi_{j=1}^{k-1}\pi_j)\eta_k}_{\theta_k \triangleq A \to M_1 \ldots \to M_k \to Y}, \tag{5}$$

where $\Delta_K = \tau_K$, i.e. Δ_K is a *gross* or composite continuation effect, since this latter path is a composite path in that it contains all residual paths omitted in the decomposition, i.e., through educational transitions subsequent to K (unless he decomposition saturates the total number of transitions available to individuals). Thus, the θ_k terms capture how much of the total effect of high school completion flows through each intermediate transition considered in the decomposition (i.e., via college attendance, via college completion, and via graduate school attendance), while Δ_0 captures that portion of the total effect that operates directly, net of the $K + 1$ transitions considered in the decomposition.

2.3   Identification

To identify the causal effects of interest, I rely on a series of "sequential ignorability" assumptions. While most closely associated with the dynamic treatment effects literature, which rely on observing a complete set of time-varying confounders in order to identify longitudinal treatment effects (see e.g. Lewis and Syrgkanis, 2020; Viviano and Bradic, 2021; Bodory et al., 2022), these assumptions can be transferred into a mediation context, given the fact that the mediators of interest are all causally ordered (since they refer to a series of educational levels). As will be discussed in the following section, sequential ignorability identification assumptions are distinct from - and in fact weaker than - the assumptions typically employed in studies of causal mediation.

Before proceeding, I introduce the following shorthands. Let $M_0 \triangleq A$ and $M_k = \emptyset \forall k < 0$. In order to estimate the decomposition shown in Equation 5, it suffices to identify the expectation of two types of composite counterfactuals ($Y(\overline{1}_k, m_{k+1})$ and $M_{k+1}(\overline{1}_{k+1})$), as well as covariance terms of the form $\text{cov}[M_{k+1}(\overline{1}_{k+1}), Y(\overline{1}_{k+2}) - Y(\overline{1}_{k+1}, 0)] \ \forall k \in [K-1]$. I invoke the following three assumptions:

Assumption 2. *Consistency: for any unit, if $A = a$, $Y = Y(a)$; if $(A, \overline{M}_k) = \{\overline{1}_k, m_k\}$, then*

$Y = Y(\overline{1}_k, m_k) \ \forall k \in [K]$, and if $(A, \overline{M}_k) = \overline{1}_{k+1}$, then $M_{k+1} = M_{k+1}(\overline{1}_{k+1}) \forall m_{k+1} \in \{0, 1\}, \forall k \in [K-1]$.

Assumption 3. *Sequential ignorability:* $(M(1), Y(a)) \perp\!\!\!\perp A | X;\ Y(\overline{1}_k, m_k) \perp\!\!\!\perp \overline{M}_k | X, \overline{Z}_k, \overline{M}_{k-1}$ and $M_{k+1}(\overline{1}_{k+1}) \perp\!\!\!\perp \overline{M}_k | X, \overline{Z}_k, \overline{M}_{k-1}, \forall k \in \{1, \ldots, K\}, \forall m_k \in \{0, 1\}, M_0 \equiv 0$.

Assumption 4. *Positivity:* $p_{A|X}(a|x) > \epsilon > 0,\ p_{M_k|X, A, \overline{Z}_k, \overline{M}_{k-1}}(m_k|x, a, \overline{z}_k, \overline{m}_{k-1}) > \epsilon > 0$ $\forall k \in [K]$.

Assumption 2 (*consistency*) states that a unit's observed outcome equals its potential outcome under a given treatment sequence. Assumption 3 (*sequential ignorability*) is the no unmeasured confounding assumption for the treatment and all mediators. It is considered plausible when sufficient pre-treatment and intermediate covariates $(X, \overline{Z}_K)$ are collected. Finally, Assumption 4 (*positivity*) requires that treatment and mediator assignment is not deterministic. Under Assumptions 2-4, $\mathbb{E}[Y(\overline{1}_k, m_k)]$ and $\mathbb{E}[M_{k+1}(\overline{1}_{k+1})]$ are identified, respectively, as

$$\mathbb{E}[Y(\overline{1}_k, m_k)] = \int_x \int_{\overline{z}_k} \mathbb{E}[Y|x, \overline{z}_k, \overline{1}_k, m_k] \Big[\prod_{j=1}^{k} dP(z_j|x, \overline{z}_{j-1}, \overline{1}_{j-1}) \Big] dP(x) \tag{6}$$

$$\mathbb{E}[M_{k+1}(\overline{1}_{k+1})] = \int_x \int_{\overline{z}_k} \mathbb{E}[M_{k+1}|x, \overline{z}_K, \overline{1}_{k+1}] \Big[\prod_{j=1}^{k} dP(z_j|x, \overline{z}_{j-1}, \overline{1}_{j-1}) \Big] dP(x) \tag{7}$$

For a proof of the above formulas, see Robins (1986). The covariance (η_k) components in the decomposition are then identified as the "residual" terms such as in equation 4, which follows directly from the fact that all other components in these equations are identified. Thus, for $k \in \{1, \ldots K\}$, I can identify η_k as

$$\eta_k = \tau_{k-1} - \Delta_{k-1} - \pi_k \tau_k.$$

2.4 A Comparison with Conventional Mediation Analysis with Multiple Causally Ordered Mediators

my decomposition has an analog in the context of a mediation–based decomposition of the ATE with multiple ordered mediators, but differs from conventional mediation analysis in important ways. To illustrate the differences, consider a binary treatment, A, an outcome of interest,

Y, and a vector of pretreatment covariates, X, and let $M_1, M_2, \ldots M_K$ denote K causally ordered mediators, assuming that for any $i < j, M_i$ precedes M_j, as above. Moreover, let an overbar denote a vector of variables, so that $\bar{M}_k = (M_1, M_2, \ldots M_k), \bar{m}_k = (m_1, m_2, \ldots m_k)$, and $\bar{a}_k = (a_1, a_2, \ldots a_k)$. Using the potential outcomes notation as above, I can define the following expectation of a nested counterfactual,

$$\psi_{a\bar{a}_k} \triangleq \mathbb{E}\left[Y(a, \bar{M}_k(\bar{a}_k))\right],$$

where $\bar{M}_k(\bar{a}_k) \triangleq \left(\bar{M}_{k-1}(\bar{a}_{k-1}), M_k(a_k, \bar{M}_{k-1}(\bar{a}_{k-1}))\right), \forall k \in [K]$. Under Pearl's (2009) non-parametric structural equation model (NPSEM), Zhou (2022b) demonstrates that the ATE of A on Y can be decomposed into $K+1$ identifiable PSEs corresponding to each of the causal paths $A \to Y$ and $A \to M_k \rightsquigarrow Y$ $(k \in [K])$:

$$\text{ATE} = \psi_{\bar{1}} - \psi_{\bar{0}} = \underbrace{\psi_{1,\bar{0}_K,} - \psi_{\bar{0}_{K+1}}}_{A \to Y} + \sum_{k=1}^{K} \underbrace{\left(\psi_{\bar{1}_{k+1},\underline{0}_{k+1}} - \psi_{\bar{1}_k,\underline{0}_{k+1}}\right)}_{A \to M_k \rightsquigarrow Y}. \tag{8}$$

This decomposition holds algebraically when Assumption 1 does not hold (i.e., when the mediators are not monotonic). In contrast, the monotonic characteristic of my proposed decomposition leads to several important differences. First, for general mediation settings, the PSE decomposition is not algebraically unique, and thus the PSEs defined under alternative decompositions will differ if the effects of the treatment and each mediator differs across levels of the other mediators. In fact, depending on the order in which the paths $A \to Y$ and $A \to M_k \rightsquigarrow Y$ are considered, there are $(K+1)!$ identifiable different ways of decomposing the ATE; the decomposition shown in Equation 8 is just one such decomposition. Consider the case of two causally dependent mediators. In this setting, the causal pathway $A \to M_2 \rightsquigarrow Y$ can be defined with respect to four different combinations of treatment and mediator 1 status: under (i) $A = 1$ and $M_1(a) = 1$, (ii) $A = 1$ and $M_1(a) = 0$, (iii) $A = 0$ and $M_1(a) = 1$, or (iv) $A = 0$ and $M_1(a) = 0$. In my setting, however, the decomposition shown is in fact the unique PSE decomposition of the ATE under monotonicity. This is a direct consequence of the monotonicity assumption: under monotonicity, hoIver, $M_2(1, M_1(0)) = M_2(0, M_1(1)) = M_2(0, M_1(0)) = 0$, and thus, only the first definition of the PSE $A \to M_2 \rightsquigarrow Y$ is defined.

Second, for general PSE decompositions of the ATE, the set of identifiable decompositions is merely a small subset of the total number of decompositions that hold algebraically (see Avin

et al., 2005). In particular, the identifiable decomposition does not enable us to disentangle the mediating effects of M_k that are direct (net of subsequent mediators) and indirect (through different combinations of subsequent mediators). This restriction is a result of the fact that a potential outcome is identified (in expectation) only if the value that a mediator M_k takes, i.e., $M_k(a_k)$, is carried over to all future mediators. For example, in the case of two causally dependent mediators, to assess the mediating role of M_1, only the composite path $A \to M_1 \rightsquigarrow Y = (A \to M_1 \to Y) + (A \to M_1 \to M_2 \to Y)$ (as before, the squiggle arrow encompasses all possible causal paths from M_1 to Y). By contrast, mediator monotonicity permits a finer-grained decomposition of the ATE: each PSE is identified, which results from the fact that many non-identifiable paths in the general causal mediation context simply do not exist under monotonicity. In the case of two causally dependent mediators, the causal path $A \to M_2 \to Y$ does not exist, and each of the paths $A \to Y$, $A \to M_1 \to Y$ and $A \to M_1 \to M_2 \to Y$ are identifiable. Figure 1 illustrates the causal pathways defined and identified under my decomposition in the case of two monotonic mediators.

Finally, the sequential ignorability assumption required to identify my decomposition is weaker than the assumptions required to identify the PSE decomposition of the ATE. Specifically, the latter requires Pearl's (2009) non-parametric structural equation model (NPSEM), which stipulates that $(M_{k+1}(a_{k+1}, \bar{m}_k), \dots M_K(a_K, \bar{m}_{K-1}), Y(a_{K+1}, \bar{m}_K)) \perp\!\!\!\perp M_k(a_k, \bar{m}^*_{k-1}) \mid X, A, \bar{M}_{k-1}, \forall k \in [K]$. This assumption, sometimes referred to as the "cross-world" independence assumption, is stronger than the sequential ignorability assumption (3) required to identify the MPSE decomposition, because the former does not allow for confounders of the $M_k \to Y$ effect, be they observed or unobserved. By contrast, the MPSE decomposition identification results accommodate observed intermediate confounding without altering the substance of the decomposition.

[Figure 1 here]

2.5 Extension to Multivalued, Discrete Mediators

The previous sections consider a decomposition of the ATE in the case of binary monotonic mediators (educational transitions educational transitions) but the framework can straightforwardly be extended to accommodate categorical transitions. Such a decomposition is especially appealing when I consider the variegated and complex trajectories that individuals take through

the US postsecondary system, that dichotomizing transitions invariably misses. In particular, in the early 2010s, under 40% of high school graduates immediately enrolled in a four-year college, with around 30% immediately attending a two-year college. Approximately one third of immediate two-year college attendees will then progress to a four-year college; indeed, nearly 50% of BA recipients previously attended a public two-year college in their educational careers. To illustrate the MPSE decomposition for multivalued, discrete mediators in the case of a single mediator, consider high school graduation A, and immediate college enrollment $M \in \{\text{none}, 2\text{yr}, 4\text{yr}\}$ denotes whether an individual did not pursue postsecondary education, or instead attended a 2-year or 4-year college. Assumption 1 is similarly assumed to hold in this context, such that $M(0) = 0$ (high school non-completers cannot pursue *any* form of postsecondary education). Under this assumption, I obtain the following multivariate decomposition:

$$\text{ATE}\mathbb{E}[Y(1) - Y(0)]$$

$$= \mathbb{E}[Y(1,0) - Y(0)]$$

$$+ \mathbb{P}[M(1) = 2\text{yr}]\mathbb{E}[Y(1,2\text{yr}) - Y(1,0)] + \text{cov}\big[\mathbb{I}[M_1(1) = 2\text{yr}], Y(1,2\text{yr}) - Y(1,0)\big] \quad (9)$$

$$+ \mathbb{P}[M(1) = 4\text{yr}]\mathbb{E}[Y(1,4\text{yr}) - Y(1,0)] + \text{cov}\big[\mathbb{I}[M_1(1) = 4\text{yr}], Y(1,4\text{yr}) - Y(1,0)\big]. \quad (10)$$

Each of these quantities can be identified under Assumptions 2-4 above. Further, Equations 9 and 10 can be further decomposed into the direct and gross effects of each of these transitions in a way analogous to Equation 4 (for example, the gross effect of 2-year attendance can be decomposed into its direct effects, net of subsequent transfer to a 4-year college, and its indirect effect via 4-year transfer, which itself can be further decomposed into direct and gross effects).

3   Semiparametric, EIF-Based Estimation

my identification formulae suggest that the proposed decomposition via several approaches, including outcome-based modeling, models for the treatment and mediators via inverse probability weighting, as well as doubly robust approaches. Parametric procedures are attractive because of its conceptual simplicity and ease of implementation: in Appendix A, I show how, under a set of linear models for the outcome and mediators, the θ_k components in Equation 5 can be read off from simple functions of coefficients in these linear models. However, in prac-

tice, when X and $\overline{Z}_K$ are high-dimensional, parametric estimators which require a user-defined specification of the data-generating process may suffer biases resulting from model misspecification. In order to reduce model dependency, in this section I provide a nonparametric estimation approach which draws on a debiased machine learning (DML) approach for estimating Δ_k, π_k and η_k, for all $k \in [K]$ (Chernozhukov et al., 2017). my DML approach is characterized by two components: first, the use of a Neyman orthogonal estimating equation based on the efficient influence function (EIF) for the target parameters, which makes estimates of the parameter 'locally robust' to estimates of the nuisance functions; second, the use of a K-fold cross-fitting algorithm.

Let $O = (X, A, \overline{Z}_K, \overline{M}_K, Y)$ denote the observed data, and $\mathcal{P}$ a nonparametric model over O wherein all laws satisfy the positivity assumption described in Section 2. Before proceeding, I define the following auxiliary functions, as introduced in Section 2: $\psi_{km_k} \triangleq \mathbb{E}[Y(\overline{1}_k, m_k)]$ and $\phi_k \triangleq \mathbb{E}[M_{k+1}(\overline{1}_{k+1})]$, for all $k \in [K]$, $M_0 \triangleq A$. In what follows, I focus on estimation of the former quantity, ψ_{km_k}, but noting that ϕ_k can be estimated in a similar manner. Using the identification formula given in Section 2, and under assumptions 2-4, $\psi_{K^*m_{K^*}}$ can be written in terms of iterated expectations of observed data:

$$\psi_{km_k} = \mathbb{E}_X \mathbb{E}_{Z_1|X,1} \ldots \mathbb{E}_{Z_k|X,a,\overline{Z}_{k-1},\overline{1}_k} \mathbb{E}[Y|X,\overline{Z}_k,\overline{1}_k,m_k].$$

I thus define $\mu_{km_k}^k(X, \bar{Z}_k)$ iteratively as

$$\mu_{km_k}^k(X, \bar{Z}_k) \triangleq \mathbb{E}\left[Y \mid X, \bar{Z}_k, \overline{1}_k, m_k\right]$$
$$\mu_{jm_k}^k(X, \bar{Z}_k) \triangleq \mathbb{E}\left[\mu_{j+1m_k}^k(X, \bar{Z}_{j+1}) \mid X, \bar{Z}_k, \overline{1}_{j+1}\right] \forall j \in [K-1].$$

Further, let $\pi_{km_k}(X, \overline{Z}_k) \triangleq \Pr[M_k = m_k \mid X, \overline{Z}_k, \overline{1}_k] \forall k \in [k]$, and $\pi_{01}(X) \triangleq \Pr[A = 1|X]$. The efficient influence function (EIF) of ψ_{km_k} is closely related to the efficient influence function (EIF) for the g-formula, and can be written as

$$\psi_{km_k}(O) = \sum_{j=0}^{k+1} \varphi_j(O),$$

where

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$$\varphi_0(O) = \mu_0(X) - \psi_{km_k}$$

$$\varphi_j(O) = \frac{A}{\pi_{01}(X)} \left(\prod_{l=1}^{j-1} \frac{M_l}{\pi_{l1}(X, \overline{Z}_l)} \right) \left(\mu_{jm_k}^k \left(X, \overline{Z}_j \right) - \mu_{j-1m_k}^k \left(X, \bar{Z}_{j-1} \right) \right), \quad j \in \{1 \ldots, K\}$$

$$\varphi_{k+1}(O) = \frac{A}{\pi_{01}(X)} \left(\frac{\mathbb{I}(M_k = m_k)}{\pi_{km_k}(X, \overline{Z}_k)} \prod_{k=1}^{k-1} \frac{M_l}{\pi_{l1}(X, \overline{Z}_l)} \right) \left(Y - \mu_{km_k}^k \left(X, \bar{Z}_k \right) \right).$$

For a proof of this formula, see Rotnitzky et al. (2017). The semiparametric efficiency bound for any asymptotically linear estimator of ψ_{km_k} in $\mathcal{P}$ is therefore $\mathbb{E}[\left(\varphi_{km_k}(O) \right)^2]$. The EIF motivates an EIF-based estimator for ψ_{km_k}, obtained by solving the empirical moment condition $\mathbb{P}_n[\varphi_{km_k}(O; \hat{\eta})] = 0$, where $\mathbb{P}_n[\cdot]$ denotes an empirical average, and where $\varphi_{km_k}(O; \hat{\eta})$ denotes the estimated EIF, evaluated using plug-in estimators for the nuisance functions. Specifically,

$$\begin{aligned}
\hat{\psi}_{K^*, m_{K^*}}^{\text{eif}} = \mathbb{P}_n \Bigg[&\frac{A}{\hat{\pi}_{01}(X)} \left(\frac{\mathbb{I}(M_k = m_k)}{\hat{\pi}_{km_k}(X, \overline{Z}_k)} \prod_{k=1}^{k-1} \frac{M_l}{\hat{\pi}_{l1}(X, \overline{Z}_l)} \right) \left(Y - \hat{\mu}_{km_k}^k \left(X, \bar{Z}_k \right) \right) \\
&+ \sum_{j=1}^{k} \frac{A}{\hat{\pi}_{01}(X)} \left(\prod_{l=1}^{j-1} \frac{M_l}{\hat{\pi}_{l1}(X, \overline{Z}_l)} \right) \left(\hat{\mu}_{jm_k}^k \left(X, \overline{Z}_j \right) - \hat{\mu}_{j-1m_k}^k \left(X, \bar{Z}_{j-1} \right) \right) \quad (11) \\
&+ \hat{\mu}_0^k(X) \Bigg].
\end{aligned}$$

A similar EIF-based estimator can be used for ϕ_{K^*}, to estimate the π_k terms in Equation 5, based on the following nuisance functions for estimation (see Appendix F for details):

$$\gamma_{K^*} \left(X, \bar{Z}_{K^*} \right) \triangleq \mathbb{E} \left[M_{K^*+1} \mid X, \bar{Z}_{K^*}, \overline{1}_{K^*+1} \right]$$

$$\gamma_k \left(X, \bar{Z}_k \right) \triangleq \mathbb{E} \left[\gamma_{k+1} \left(X, \bar{Z}_{k+1} \right) \mid X, \bar{Z}_k, \overline{1}_{k+1} \right] \forall k \in [K^* - 1].$$

Following Kennedy (2022, p. 15), I let $\mathbb{IF} : \Psi \to L_2(\mathbb{P})$ denote the operator mapping the functionals $\{\Delta_k, \pi_k, \eta_k\} : \mathcal{P} \to \mathbb{R}, \forall \in [K]$ to their respective influence functions under the nonparametric model $\mathcal{P}$. Because the (Δ_k, τ_k) components of the decomposition are linear in $\psi_{K^* m_{K^*}}$, by linearity of the EIF, $(\mathbb{IF}(\Delta_k), \mathbb{IF}(\tau_k))$ can be expressed as linear combinations of $\varphi_{km_k}(O)$. In particular, $\mathbb{IF}(\tau_k) = \varphi_{k+1,1}(O) - \varphi_{k,0}(O)$ and $\mathbb{IF}(\Delta_k) = \varphi_{k+1,0}(O) - \varphi_{k,0}(O)$. The EIFs of η_k and θ_k, $\forall k \in [K]$ under $\mathcal{P}$ are derived as in Proposition 1:

Proposition 1. *The EIFs of η_k, θ_k $\forall k \in [1, \ldots, K]$ under P are given, respectively, by*

$$\mathbb{IF}(\eta_k) = \mathbb{IF}(\tau_{k-1}) - \mathbb{IF}(\Delta_{k-1}) - \tau_k \mathbb{IF}(\pi_k) - \pi_k \mathbb{IF}(\tau_k),$$

$$\mathbb{IF}(\theta_k) = \mathbb{IF}(\Delta_k) \prod_{j=1}^{k} \pi_j + \Delta_k \sum_{j=1}^{k} \mathbb{IF}(\pi_j) \prod_{l:l \neq j}^{k} \pi_l + \mathbb{IF}(\eta_k) \prod_{j=1}^{k-1} \pi_j + \eta_k \sum_{j=1}^{k-1} \mathbb{IF}(\pi_j) \prod_{\substack{l=1 \\ l \neq j}}^{k-1} \pi_l,$$

for $k \in \{1, \ldots K\}$, with $\theta_0 = \Delta_0$, and where $\mathbb{RIF}(\phi) = \mathbb{IF}(\phi) + \phi$, denotes the recentered EIF of a parameter (about the truth). Their corresponding EIF-based estimators are (see Appendix F for derivations):

$$\hat{\eta}_k^{\text{eif}} = \widehat{\mathbb{RIF}}(\tau_{k-1}) - \widehat{\mathbb{RIF}}(\Delta_{k-1}) - \hat{\tau}_k \widehat{\mathbb{RIF}}(\pi_k) - \hat{\pi}_k \widehat{\mathbb{RIF}}(\tau_k) + \hat{\pi}_k \hat{\tau}_k,$$

$$\hat{\theta}_k^{\text{eif}} = \widehat{\mathbb{RIF}}(\Delta_k) \prod_{j=1}^{k} \hat{\pi}_j + \hat{\Delta}_k \sum_{j=1}^{k} \widehat{\mathbb{RIF}}(\pi_j) \prod_{l:l \neq j}^{k} \hat{\pi}_l + \widehat{\mathbb{RIF}}(\eta_k) \prod_{j=1}^{k-1} \hat{\pi}_j + \hat{\eta}_k \widehat{\mathbb{RIF}}(\pi_j) \prod_{\substack{l=1 \\ l \neq j}}^{k-1} \hat{\pi}_l$$

$$- k \hat{\Delta}_k \prod_{j=1}^{k} \hat{\pi}_j - (k-1) \hat{\eta}_k \prod_{j=1}^{k-1} \hat{\pi}_j.$$

where $\widehat{\mathbb{RIF}}(\phi) = \widehat{\mathbb{IF}}(\phi) + \phi$, and $\widehat{\mathbb{IF}}(\phi)$ denotes the influence function of a parameter evaluated at estimates of its component nuisance functions. When machine learning estimators are used to compute the nuisance functions, on order to ensure the convergence rates outlined in Proposition 2 below, one could assume Donsker-type conditions for the nuisance function estimators, which restricts the set of estimators one can use. Alternatively, sample-splitting can be used, in order to expand the class of estimators that can be used for estimating the nuisance functions. In particular, Chernozhukov et al. (2017) suggest a "cross-fitting" procedure, which comprises the following steps: (1) Randomly split data into J folds: $\{S_1, \ldots S_J\}$; (2) For each fold S_j, use the remaining $(j-1)$ folds (training sample) to fit a flexible machine-learning model for each of the nuisance functions involved in the estimating equations; (3) For each observation in j (estimation sample), use estimates of the above models to construct a set of estimated rEIFs for $\Delta_k \forall k \in \{0, \ldots, K-1\}$, and π_k, τ_k η_k and θ_k $\forall k \in [K]$; (4) Compute an estimate of the decomposition components by averaging the estimated rEIFs across all subsamples S_1 through S_J.

When the nuisance functions are estimated via data-adaptive methods and cross-fitting, the semiparametric efficiency of $(\eta_k^{\text{rEIF}}, \theta_k^{\text{rEIF}})$ is given in the following proposition:

Proposition 2. *Semiparametric efficiency. If, for all $k \in \{1, \ldots K\}$, $(\tau_{k-1}, \Delta_k, \pi_k) < \infty$, then $\hat{\theta}_k^{eif}$ is semiparametric efficient if $\sum_{j=k}^{k+1}\left[\sum_{l=0}^{j} R_n(\hat{\pi}_{l1})R_n(\hat{\mu}_{l0}^j)\right] + \sum_{j=0}^{k-1}\left[R_n(\hat{\pi}_{j1})R_n(\hat{\mu}_{j0}^{k-1}) + R_n(\hat{\pi}_{j1})R_n(\hat{\mu}_{j1}^{k-1})\right] + \sum_{j=0}^{k}\left[\sum_{l=0}^{j} R_n(\hat{\pi}_{l1})R_n(\hat{\gamma}_l^j)\right] = o(n^{-1/2}),$, where $R_n(\cdot)$ denotes a mapping from a nuisance function to its $L_2(P)$ convergence rate, and where $\hat{\mu}_{l0}^{K+1} \triangleq \hat{\mu}_{l1}^K$.*

To obtain some intuition for the result in Proposition 2, I can focus $\theta_1 = \pi_1 \Delta_1 + \eta_1$, i.e., the MPSE through M_1 when $K = 1$. Note that this estimation of $\theta_1 = \pi_1 \Delta_1 + \eta_1$ requires estimation of the following decomposition components $(\pi_1, \Delta_1, \tau_0, \Delta_0, \tau_1)$, for which it suffices to estimate the following quantities: $(\phi_1, \psi_{01}, \psi_{00}, \psi_{10}, \psi_{11})$. Then, in order for $\hat{\theta}_1^{\text{eif}}$ to be semiparametric efficient, I require that the estimators employed for the set $(\phi_1, \psi_{01}, \psi_{00}, \psi_{10}, \psi_{11})$, i.e., $(\hat{\phi}_1^{\text{eif}}, \hat{\psi}_{01}^{\text{eif}}, \hat{\psi}_{00}^{\text{eif}}, \hat{\psi}_{10}^{\text{eif}}, \hat{\psi}_{11}^{\text{eif}})$, are themselves semiparametric efficient. Thus, a sufficient (but not necessary) condition then in order for $\hat{\theta}_1^{\text{eif}}$ to obtain the semiparametric efficiency bound is, if for any two nuisance functions involved in $(\hat{\phi}_1^{\text{eif}}, \hat{\psi}_{01}^{\text{eif}}, \hat{\psi}_{00}^{\text{eif}}, \hat{\psi}_{10}^{\text{eif}}, \hat{\psi}_{11}^{\text{eif}})$, the product of their convergence rates is $o(n^{-1/2})$. In this way, $\hat{\theta}_1^{\text{eif}}$ will obtain the semiparametric efficiency bound if all of its constituent nuisance functions converge at faster than a rate of $n^{-1/4}$ (although it will also obtain the efficiency bound under a wide variety of alternative conditions.

When data-adaptive methods are used to estimated the nuisance functions, inference on all components can be conducted via the variance of the empirical analog of the EIF, i.e. $\mathbb{P}_n[(\hat{\psi}_{km_k}^{\text{EIF}})^2]/n$. For example, inference on (τ_k, Δ_k) can be conducted by estimating $\mathbb{P}_n[(\hat{\psi}_{11}^{\text{EIF}} - \hat{\psi}_{10}^{\text{EIF}})^2]/n$. In Appendix B, I present results from a simulation study that tests the finite-sample performance of the DML estimator (alongside the parametric, RWR estimator described in in Appendix A). When the models are correctly specified, both the parametric and semiparametric approaches perform well; the strong performance of a semiparametric approach compared with a parametric estimation strategy becomes clear under model misspecification.

4   Sensitivity Analyses

While the sequential ignorability assumption facilitates identification of educational effect pathways under a weaker set of conditions than might be typically invoked in mediation settings, it is still strong and fundamentally unverifiable. To assess potential bias of the estimated MPSEs

due to unobserved confounders not picked up in my covariate set $(X, \overline{Z}_K)$, I propose a sensitivity analysis for each of the MPSEs.

Assume first that I have a binary unobserved confounder, U, for the treatment-outcome relationship. Assuming that $\alpha_0 = \mathbb{E}[Y|x, a, U = 1] - \mathbb{E}[Y|x, a, U = 0]$ does not depend on x or a, and further that $\beta_0 = \Pr[U = 1|x, A = 1] - \Pr[U = 1|x, A = 0]$ does depend on x, for $\tau_0 = \mathbb{E}[Y(1) - Y(0)] \triangleq \text{ATE}$, then $\text{bias}(\tau_0) = \alpha\beta$ (VanderWeele and Arah, 2011),

Next, consider an unobserved binary confounder, U_k that affects both M_k and Y for any $k \in \{1, \ldots, K\}$. Then, under a weaker instantiation of Assumption 3 (Sequential Ignorability), i.e.,

$$Y(\overline{1}_k, m_k) \perp\!\!\!\perp (A, \overline{M}_k)|X, A, U_k, \overline{Z}_k, \overline{M}_{k-1} \forall k \in [K], \tag{12}$$

which states that potential outcomes under an arbitrary transition sequence are independent of observed treatment and mediator values conditional on observed confounders $(X, \overline{Z}_k)$ *and unobserved confounders* U_k. Under the following set of assumptions: (Assumption A_k) $\alpha_k = \mathbb{E}[Y|x, \overline{z}_k, \overline{1}_k, m_k, U_k = 1] - \mathbb{E}[Y|x, \overline{z}_k, \overline{1}_k, m_k, U_k = 0]$ does not depend on $(x, \overline{z}_k, \overline{1}_k, m_k)$, and (Assumption B_k), $\beta_k = \Pr[U_k = 1|x, \overline{z}_k, \overline{1}_k, m_k] - \Pr[U_k = 1|x, \overline{z}_k, \overline{1}_k])$ does not depend on $(x, \overline{z}_k)$, I can show that, for any $k \in \{1, \ldots K\}$,

$$\text{bias}(\tau_k) = \alpha_k \beta_k,$$

and, further, that

$$\text{bias}(\Delta_{k-1}) = -\alpha_k \beta_k \pi_k,$$

where $\pi_k = \int_x \int_{\overline{z}_k} \Pr[M_k = 1|x, \overline{z}_k, \overline{1}_k] \prod_{j=1}^k dP(z_j|x, \overline{z}_{j-1}, \overline{1}_{j-1})]dP(x)$, and is estimable from observed using the estimation strategies described previously. A contour plot showing bias-adjusted estimates of Δ_{k^*} and τ_k then enables assessment of how strong the unobserved confounder would need to be to reduce estimates of the direct and gross effects to zero. I illustrate these techniques in my empirical illustration below.

5 Empirical Analysis

To illustrate my approach empirically, I draw on data from the National Longitudinal Survey of Youth 1997 (NLSY97). I parse out the direct effect of high school graduation on adult earnings and its indirect or continuation effects via (i) college attendance, (ii) college graduation, and (iii) graduate school attendance. my analytic sample comprises $N = 8,649$ respondents.

I construct four types of variables: educational transitions, adult earnings, a set of confounders for the effect of high school graduation on subsequent transitions and earnings, and a single set of intermediate confounders for the effect of college completion on subsequent transitions and earnings. my educational transition variables contain a binary treatment denoting whether a respondent had graduated high school by age 22, and three binary mediators denoting whether the respondent had attended a 4-year college by age 22, whether the respondent had received a BA degree by age 29, and whether the respondent had enrolled in a graduate level program by age 29, respectively. I assume that all individuals who make a given educational transition have made all previous educational transitions. Thus, by construction, my coding strategy disallows for cases which violate the monotonicity assumption.[3] my outcome of interest is annual earnings, which I define to be the sum of a respondent's self-reported wage, salary income, and business income. I adjust earnings for inflation to 2023 dollars using the personal consumption expenditures index (PCE), and use the natural logarithm of respondents' average annual earnings at ages 32-26. After dropping respondents with missing earnings information, I accommodate those with zero earnings by adding a small constant of $1,000$ to observed earnings (though in Appendix D, I replicate my main analyses under alternative definitions of earnings).[4]

In an effort to satisfy the sequential ignorability assumption (Assumption 3), I include

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[3]Assuming away cases in which an individual makes a particular educational transition without having made *all* previous transitions serves as a reasonable approximation to reality. In the NLSY97 set of individuals who have non-missing earnings information (i.e., who comprise my analytic sample), 94% of individuals observed to attend graduate school by age 29 also completed a BA by age 29; 93% of respondents who completed a BA by age 29 had attended a 4-year college by age 22 (6% of those who completed a BA by age 29 first attended a 4-year college between ages 23 and 26 inclusive), and 99% of respondents who attended a 4-year college by age 22 had also completed high school.

[4]Age 36 is the oldest age at which I can consistently observe respondents, who Ire between age 13 and 17 in the original 1997 sample.

include a large array of covariates in my models for the effects of completing educational transitions on labor market outcomes (Figure 2 summarizes my assumed data-generating process for this illustration). This set of covariates is more expansive than those used in previous, observational studies of returns to education that employ both administrative and survey data. In particular, in addition to including information on respondent demographics (gender, race, ethnicity, age at 1997), and observed pre-college performance such as overall high school GPA and test score on the Armed Services Vocational Aptitude Battery (ASVAB) (see in particular Scott-Clayton and Wen (2019)), I include detailed information on socioeconomic background (parental education, parental income, parental asset, co-residence with both biological parents, presence of a paternal figure, rural residence, southern residence), an index of substance use, an index of delinquency, whether the respondent had any children by age 18), and peer and school-level characteristics (measures of peers' college expectations and behaviors). Both parental income and parental asset variables are transformed to 2023 dollars. Since my proposed decomposition also facilitates the inclusion of a distinct set of observed intermediate confounders for each transition to adjust for selection processes that may confound the causal effects of each transition on earnings (i.e., the $A-Y$ and M_k-Y relationships, for $k \in \{1, \ldots K\}$, I include two postsecondary characteristics (Z) to adjust for confounders of the effect of BA completion and graduate school attendance on earnings, namely, field of study, and college GPA. Specifically, I use college self-reported major field of study, drawing on the NLSY survey instrument asking respondents about their choice of major in each month in which they Ire enrolled in college, and using a dummy variable to denote whether whether a respondent majored in a STEM or non-STEM field by age 29. Finally, college GPA is measured using the respondent's cumulative GPA from the Post-Secondary Transcript Study. I treat two of the Z_k sets as empty (namely, Z_1 and Z_3), assuming that the effects of the first mediator (college attendance, M_1) on subsequent transitions and adult earnings are unconfounded given background characteristics (X), and that the effects of the third mediator (graduate school attendance, M_3) are unconfounded given background characteristics (X) and postsecondary characteristics (Z). [5]

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[5]To be clear, assuming that Z_1 and Z_3 are empty is not to say that M_1 and M_3 are marginally unconfounded; rather, it means that the set of covariates that confound the effects of M_1 is assumed to be the same as those that confound the effects of A, and that the set of covariates that confound the effects of M_3 are assumed to be the same as those that confound the effects of M_2. This assumption is in part data-driven, given the few variables observed chronologically post high school graduation and pre college attendance.

How convincingly do I satisfy the sequential ignorability assumption? Despite the inclusion of a comprehensive set of background covariates in my models, it is possible that observed variables do not perfectly proxy for all important confounders jointly affecting education and earnings. In particular, researchers often argue that important variables, such as students' innate ability, ambition, and detailed forms of socioeconomic advantage, confound observational estimates of educational returns (e.g. Carneiro et al., 2011). While some research suggests that observational estimates of earnings returns may well capture actual returns to education (Card, 1999), and that the degree of observational bias may be rather small, it is of course impossible to quantify the true extent of the bias in the estimates I produce. Nevertheless, as a step towards this goal, in the following section I implement the sensitivity analysis outlined in Section 4. Under certain simplifying assumptions about the nature of the unobserved confounding, I can produce a set of "bias-corrected" estimates of the components of the MPSE decomposition.

I note that my assumption of ignorability of M_3 without conditioning on intermediate variables Z_3 is perhaps the strongest assumption I make. For example, many individuals take time off to work before enrolling in graduate school, and labor market experience and earnings gained in the interim period between college completion and graduate school enrollment may confound the latter variable's effects on earnings. Nevertheless, including a measure of labor market characteristics for this period is difficult because some respondents enroll directly in graduate school after BA completion. As a consequence, my analyses should be seen as illustrative of my proposed methodology rather than a definitive assessment of the effects in question.

[Figure 2 here]

A large proportion (just under 50%) of respondents are missing information on covariates X and Z. For my main analyses, I impute missing values on these covariates via multiple imputation, but in Appendix C I replicate these analyses restricted to the sample of respondents with complete information, with substantively similar results (for covariate means for each of these analytic samples, see Appendix E). After constructing the analytical sample, I apply both the DML estimator described in Section 3 as well as a parametric, regression-with-residuals (RWR) algorithm (described in Section A) to implement the proposed decomposition. For the DML approach, I estimate all nuisance functions, using a super learner composed of the Lasso and random forest and, following Chernozhukov et al. (2017), use five-fold cross-fitting.

All weights involved in computing the rEIFs are censored at their 1st and 99th percentiles. Appendix J gives further details about the particular models required given my assumed data generation process.

[Table 1 here]

Table 1 (column 2) presents the proportion of individuals who have attained each level of education constructed above. By age 22, a small, but not insignificant, proportion of individuals who begin studying at a high school do not complete their studies (13), and by this same age, just over 40% of individuals have attended a 4-year college. By age 29, 29 of individuals have attained a Bachelor's degree or higher. These estimates of high school completion and BA completion align closely both with those reported in previous studies that employ the NLSY97 (e.g. Scott-Clayton and Wen (2019), as well as with those reported in the Current Population Survey (CPS). Table 1 (columns 3-4) also shows mean log earnings by educational group (column 3), alongside the estimated gap between these means and mean log earnings among high school non-completers (column 4). High school dropouts earn an average of 9.07 log earnings, while groups with higher levels of attainment earn successively more than high school dropouts, though at a decreasing rate. High school graduates earn on average 1.11 log earnings more than high school non-completers, implying an earnings premium in excess of 200% ($\exp(1.11) - 1$), while college goers earn on average 1.5 log earnings more than high school non-completers (or 0.6 log earnings more than high school graduates). At the highest end, graduate school goers earn on average 10.88 log earnings. These educational premia are extremely high, since they reflect both the causal effect of a given educational level as well as the effects of individual, geographic and family factors correlated both with attainment and with adult earnings. To net out these patterns of selection, I can now turn to estimates of the MPSE decomposition, as well as its constituent components.

Figure 3 shows my estimates of the average total effect (ATE) on log earnings and its direct and continuation components under both my DML and RWR procedures. Both procedures return similar estimates, though deviate in the estimated magnitude of MPSE θ_1, and DML estimates come expectedly with a significantly greater amount of precision. The first column shows that the estimated ATE of attending high school on log earnings under DML (RWR) is 0.68 (0.63), which implies an earnings premium of approximately 97%. The next two columns indicate that the vast majority (70% under DML and 75% under RWR) of the ATE operates

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directly, i.e. net of college attendance, BA completion and graduate school attendance (MPSE θ_0, $A \rightarrow Y$). Specifically, high school graduates who do not go on to college can be expected to earn on average 0.47 log earnings more than high school non-completers, an earnings premium in excess of 59%, (the DML and RWR estimates are identical in this case).

Under DML, the continuation effects of high school graduation via college attendance without BA completion (MPSE θ_1, $A \rightarrow M_1 \rightarrow Y$) and via BA completion without graduate school participation (MPSE θ_2, $A \rightarrow M_1 \rightarrow M_2 \rightarrow Y$) both mediate roughly 15% of the ATE, and correspond to an approximate 10% earnings premium. The RWR estimate of θ_1 is notably loIr at 0.03, and is imprecisely estimated. Under both estimation procedures, the continuation effect via graduation ($A \rightarrow M_1 \rightarrow M_2 \rightarrow M_3 \rightarrow Y$) is very small and fails to reach conventional levels of significance. In sum, the total effect of high school graduation on earnings is determined overwhelmingly by the direct effect of high school graduation on earnings.

Table 2 shows DML and RWR estimates of the various components (the direct effects (Δ_k), probabilities (π_k) and covariance terms (η_k)) that constitute the continuation effects (θ_k). Several points are of note. First, the components in the table offer insights into the economic and educational returns at different educational stages. The direct effects of each educational transition (Δ_k) are highly variable: they are largest for high school graduation and for college completion (at 0.47 and 0.46 under DML), and lowest for college attendance and graduate school participation (at 0.2 and 0.12 under DML). The payoff to graduate school attendance could be depressed by the fact that I observe individuals at a maximum age of 36, if graduate school earnings premia materialize only later in the life course. The counterfactual continuation probabilities (π_k) also provide insight into barriers in educational participation. In particular, even under an intervention to complete high school, without further intervention, that individual would only have under a 50% chance of continuing to a 4-year college. Further, under a series of interventions to *both* high school completion and 4-year college attendance, only a very small proportion ($\pi_1 \cdot \pi_2 = 0.24$) would be expected to complete their BA degree. Second, the fine-grained nature of the MPSE decomposition enables us to trace the continuation effects to their constituent components. In particular, while the direct effect of high school completion is comparable to the direct effect of BA graduation on earnings, suggesting an earnings premium of 59% relative to college attendance without completion, the continuation effect via BA completion that it informs (MPSE θ_2, $A \rightarrow M_1 \rightarrow M_2 \rightarrow Y$) only mediates a small amount of the ATE because because θ_2 is approximately (plus the small value of η_2) equal

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to Δ_2 scaled by the product, $\pi_1 \cdot \pi_2 = 0.24$. In words, despite the relatively large effect of BA completion on earnings, given individuals' low counterfactual probability of BA completion, even after completing high school and attending a 4-year college, BA completion is not an important mediating pathway of the total effect of high school completion on earnings. The result is that college attendance without completion mediates high school graduation's earnings effects as much as BA completion, despite the fact that college attendance without completion yields a much smaller earnings return for high school graduates than BA completion does for college enrollees.

Finally, the sum of the estimated direct effects for contiguous transitions can be interpreted as the joint effect of undergoing the transitions under examination. In particular, the sum of the four direct effects under DML is 1.247, which captures the expected earnings return to succesfully completing all four educational transitions considered, compared with high school non-completion. In other words, graduating high school, completing a 4-year college program and subsequently enrolling in a graduate program on average yields an earnings premium in excess of 248%.

How do these estimates tally with previous findings on the labor market returns to education? While previous work does not estimate quantities analogous to the direct and indirect effects of interest (i.e., the θ_k terms), some prior educational returns estimates are closely related to the net effect (τ_k) terms that inform the total, direct and indirect components of the decomposition. My estimate of the net effect of 4-year college enrollment (τ_1) is large, at 54%, but not implausibly so. While Zimmerman (2014) and Smith et al. (2020) recover college earnings returns at around 20by age 30 (exploiting admissions discontinuities in the Florida and Georgia state university systems), both of these studies estimate the earnings premium from attending a less selective 4-year college rather than a community college, for the marginally qualified university attendee. By contrast, τ_1 captures the effect of 4-year college enrollment compared with community college *and* no college enrollment, pooling across the less selective colleges examined in Zimmerman (2014) and Smith et al. (2020) as well as more selective colleges which could have greater earnings effects. Moreover, since τ_1 represents an effect averaged over all individuals, it reflects college return among a broader population than the marginal college-goers in previous studies, for whom economic theories of "positive selection" suggest the effect of college attendance is lowest (Willis and Rosen, 1979; Carneiro et al., 2011).[6] One

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[6]my estimate of Δ_1 (the direct effect of 4-year college attendance on earnings) further tallies

additional point of comparison are instrumental variable (IV) estimates of returns to years of schooling, which are typically estimated in the range of 6% to 12% (Angrist and Krueger, 1991, 1992; Kane and Rouse, 1993; Card, 1994; Ashenfelter and Zimmerman, 1997; Angrist and Chen, 2011) - although one estimate puts the return for women at 18.5% (Butcher and Case, 1994). While my estimate of the overall return to high school graduation (τ_0) could appear large in this light, several factors could explain this discrepancy. In particular, IV estimates of schooling returns capture direct effects of schooling (whereas τ_0 captures both the direct effect of schooling as well as its continuation value via subsequent education. Moreover, the ATE τ_0 captures the effect of multiple years of schooling: the high school graduates and high school non-completers that form the comparison groups differ by multiple years of schooling: in the NLSY97, high school non-completers attained on average 3.7 feIr years of schooling than high school completers.

Of course, an alternative explanation is that OLS estimates are upwardly biased by a large degree of unobserved confounding. In order to assess the robustness of the above findings to potential violations of Assumption 3 (Sequential Ignorability), I implement my sensitivity analysis as discussed in Section 4. Figure 4 displays a set of contour plots, which capture the bias-corrected estimates of the (Δ_k, τ_k) terms under varying degrees of confounding (that is, under different values of α_k and β_k). For example, the level set marked "0" corresponds to values of (α_k, β_k) required in order for the unobserved confounder to fully "explain away" estimates (Δ_k, τ_k) (i.e., to reduce their true values to zero). Importantly, each row corresponds to a different set of (α_k, β_k) terms for a given U, such that the top row corresponds to (α_0, β_0), while the second row corresponds to (α_1, β_1), and so on.

For simplicity, I consider U to be an unmeasured binary confounder that is (marginally) positively associated with each transition $A, M_1, \ldots M_3$ as well as with adult earnings Y. To

with a similar quantity estimated by Scott-Clayton and Wen (2019). On the intensive margin of employment (i.e. dropping respondents with zero observed earnings), the authors estimate a return to college attendance without degree completion of 0.21. While, theoretically, one might expect my estimate - which corresponds to the extensive employment margin (including respondents with zero observed earnings - to be larger, the fact that it is slightly smaller could reflect several factors, including the richer array of pre-college controls I use in my models, model mis-specification resulting from linearities imposed in prior work and, perhaps most importantly, collider-stratification biases induced by conditioning on BA completion in Scott-Clayton and Wen (2019)'s models (biases that are likely reduced by the inclusion of time-varying controls).

benchmark the hypothetical behavior of U, for each plot, I also display the values of (α_k, β_k) that would correspond to a U that behaved similarly to a given confounder that I do observe in the data: an indicator for whether an individual's test score on the ASVAB is above the median. In each plot, I mark this point and label it "Ability". For each plot, I also mark the point on the zero contour that corresponds to $\alpha_k = \beta_k$ (i.e, the point at which the unobserved confounder's association with the treatment and with the outcome are equal, and reduce the true value of the parameter to zero).

I focus on estimates of τ_0 and Δ_0 to assess how robust my primary conclusion - that the ATE of high school completion is overwhelmingly mediated by high school's direct effect on earnings - is to unobserved confounding. Bias-adjusted estimates of the ATE, τ_0 are presented in the top row of Figure 4. Since U is assumed to be positively associated with both A and Y, τ_0 is overestimated, suffering from a bias of $\alpha_0 \beta_0$ (as described in Section 4). my estimate of τ_0 at 0.61 is nevertheless quite robust: if U had similar effects to ability, the effect would be diminuted by 0.05 log points, to 0.56, still implying a high earnings premium to high school completion overall in excess of 75%.

How do my estimates of the direct effect of high school completion Δ_0 (and, in particular, about the proportion of the total effect that is direct) fare under unobserved confounding? The second row of Figure 4 considers bias-adjusted estimates of $\Delta_0 = \theta_0$ under different values of (α_1, β_1), which correspond to the effects of an unobserved confounder U (marginally) positively associated with both M_1 and with Y. As described in Section 4, in this scenario, Δ_0 is affected by a bias of $-\alpha_1 \beta_1 \pi_1$. Importantly, even if the unobserved confounder U is *marginally* positively associated with high school graduation (A), the conditional association between U and A may be zero or even negative since M_1 is a collider of A and U. In this case, $-\alpha_1 \beta_1 \pi_1$ would be positive, implying an overestimation of the direct effect θ_0. On the plot, I show estimates of U if it behaved similarly to the ability variable. Indeed, despite the fact that ability is marginally positively associated with high school completion (top row of Figure 4), its conditional association conditional on college attendance is depressed to zero. Thus, it would take an extreme form of confounding for θ_0 to be largely different from its estimated value of .47. In this way, my primary finding that the ATE of high school graduation is overwhelmingly mediated via its direct effect remains highly robust to patterns of unobserved confounding, under my set of simplifying assumptions.

[Figure 3 here]

[Table 2 here]

[Figure 4 here]

6 Conclusion

In this article, I have developed a causal mediation framework for analyzing education effects on earnings. First, I have demonstrated that the total effect of any level of education can be decomposed into a direct effect, net of K subsequent educational transitions, and K mutually exclusive "continuation" or gross effects: $A \rightarrow M_1 \cdots \rightarrow M_k \rightarrow Y$, are of which all identifiable under the assumption of sequential ignorability, which allows for the effect of each educational transition to be confounded by a different set of variables. Next, I have introduced a non-parametric estimation strategy for my decomposition via the use of data-adaptive methods. I have highlighted how my mediation-based decomposition of the total effect of education compares with conventional mediation analysis with multiple mediators, in particular a decomposition of the total treatment effect into a set of path-specific effects (PSEs) (see Miles et al., 2017; Zhou, 2022b). The monotonic nature of the mediators in my decomposition distinguishes it from conventional decompositions of the average treatment effect (ATE) into mutually exclusive path-specific effects (PSEs), properties which facilitate less restrictive identification assumptions as well as identification of all of the causal paths in the decomposition. In particular, an especially important advantage of my proposed decomposition is that it enables researchers to include a distinct set of intermediate confounders for each mediator, which is not the case in conventional mediation decompositions.

While I have introduced my mediation decomposition in the context of a particular substantive issue - the effect of education via direct and continuation effects - my framework applies more broadly to other substantive settings where mediators are monotonic in form. Applied researchers are often engaged in settings in which mediators are "monotonic" in nature — that is, in which certain mediator values are accessible only to individuals who have received a previous treatment. This setting is particularly salient in demographic phenomena, which often involve sequential transitions over the life course. Certain demographic events are rigid in their

monotonicity as a result of their definition. For example, researchers may be interested in decomposing the total effect of marriage on various outcomes such as earnings or life satisfaction, into its direct and and indirect components via divorce; yet, divorce can only be "attained" among individuals who are already married. Similarly, the effect of parenthood on earnings can be seen as operating directly, through the effect of having a first child net of subsequent children, as well as operating indirectly through the effects of having multiple children, transitions which are clearly monotonic in nature. A similar perspective may be taken to decompose the total effect of criminal offending into a direct effect and an indirect effect via incarceration. As I have demonstrated, such settings enable a wider range of available estimands to be identified, and under weaker assumptions, compared with conventional causal mediation analysis; in particular, an especially important advantage of my proposed decomposition is that it enables researchers to include a distinct set of intermediate confounders for each mediator, which is not the case in conventional mediation decompositions.

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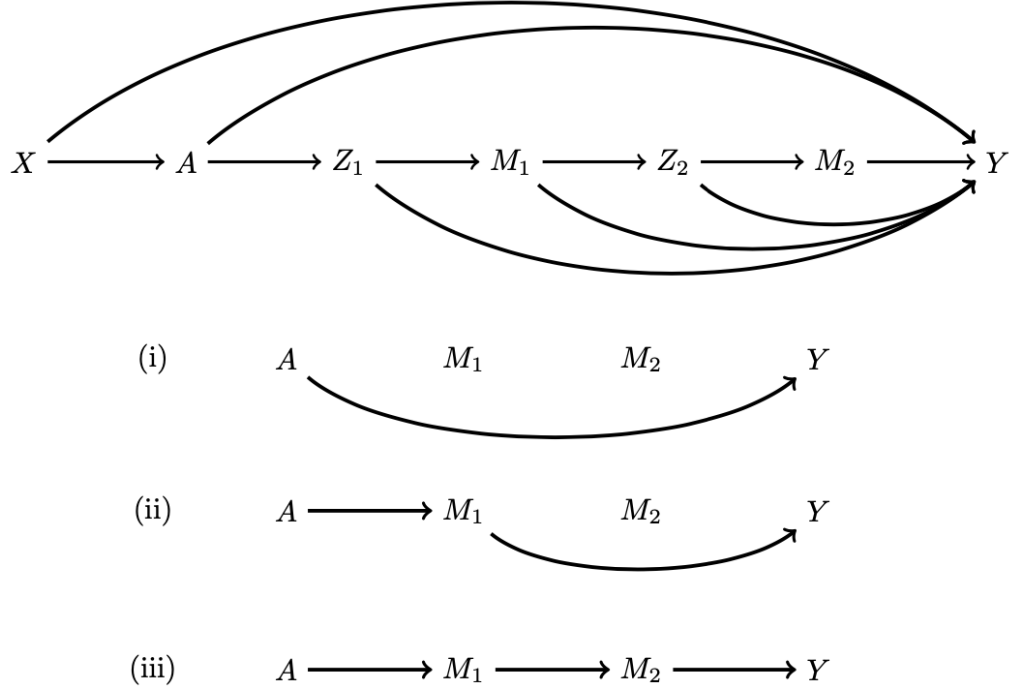
Figure 1: Causal Relationships with Two Monotonic Mediators Shown in a Directed Acyclic Graph (DAG) and the 3 Monotonic Path Specific Effects (MPSEs). A denotes an initial transition of interest, Y an outcome, and M_1 and M_2 are two chronologically-ordered, monotonic mediators. The set (X, Z_1, Z_2) captures pre-treatment as well as intermediate confounders.
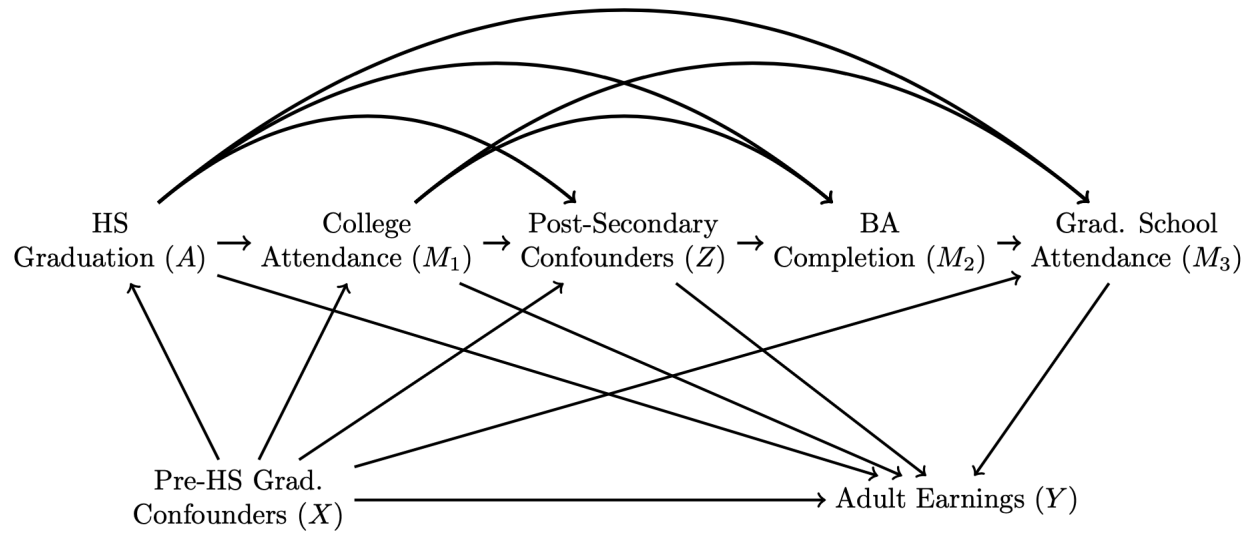
Figure 2: DAG showing the hypothesized causal relationships between high school completion A and adult earnings Y via mediators M_1, M_2 and M_3.
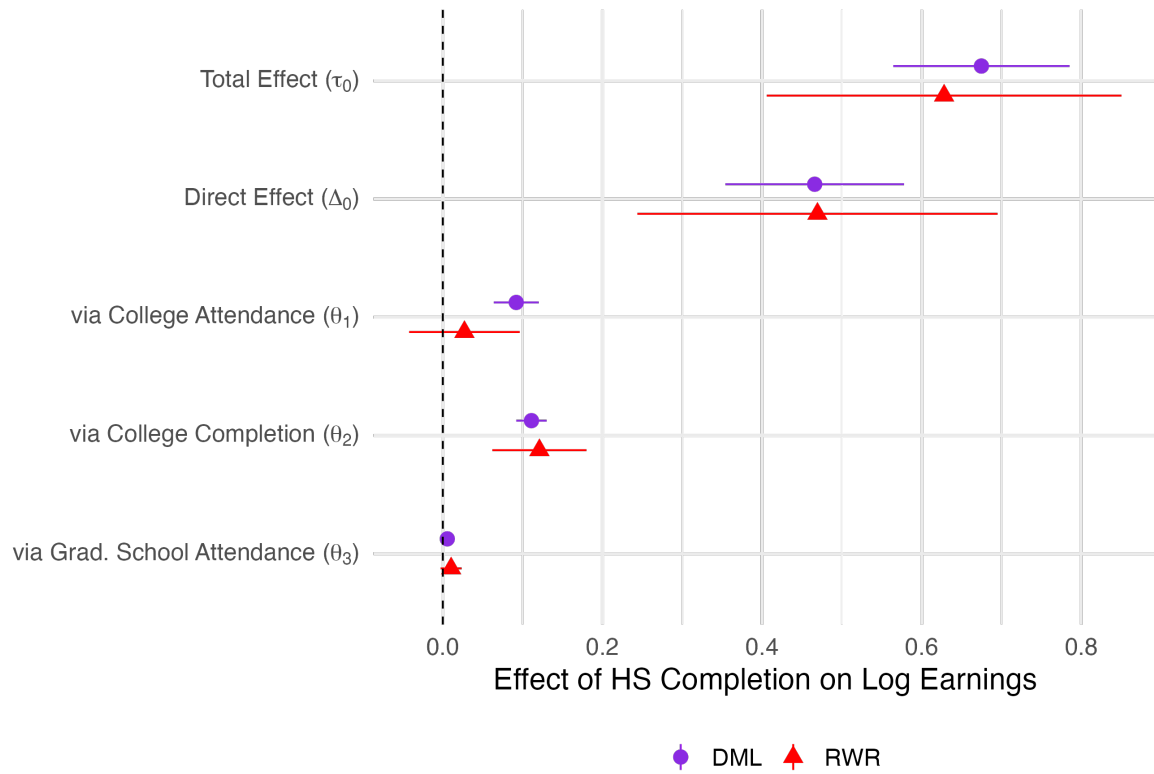
Figure 3: Decomposition of the Average Total Effect (ATE) of High school Graduation on Logged Earnings via Debiased Machine-Learning (DML) and Regression-With-Residuals (RWR).
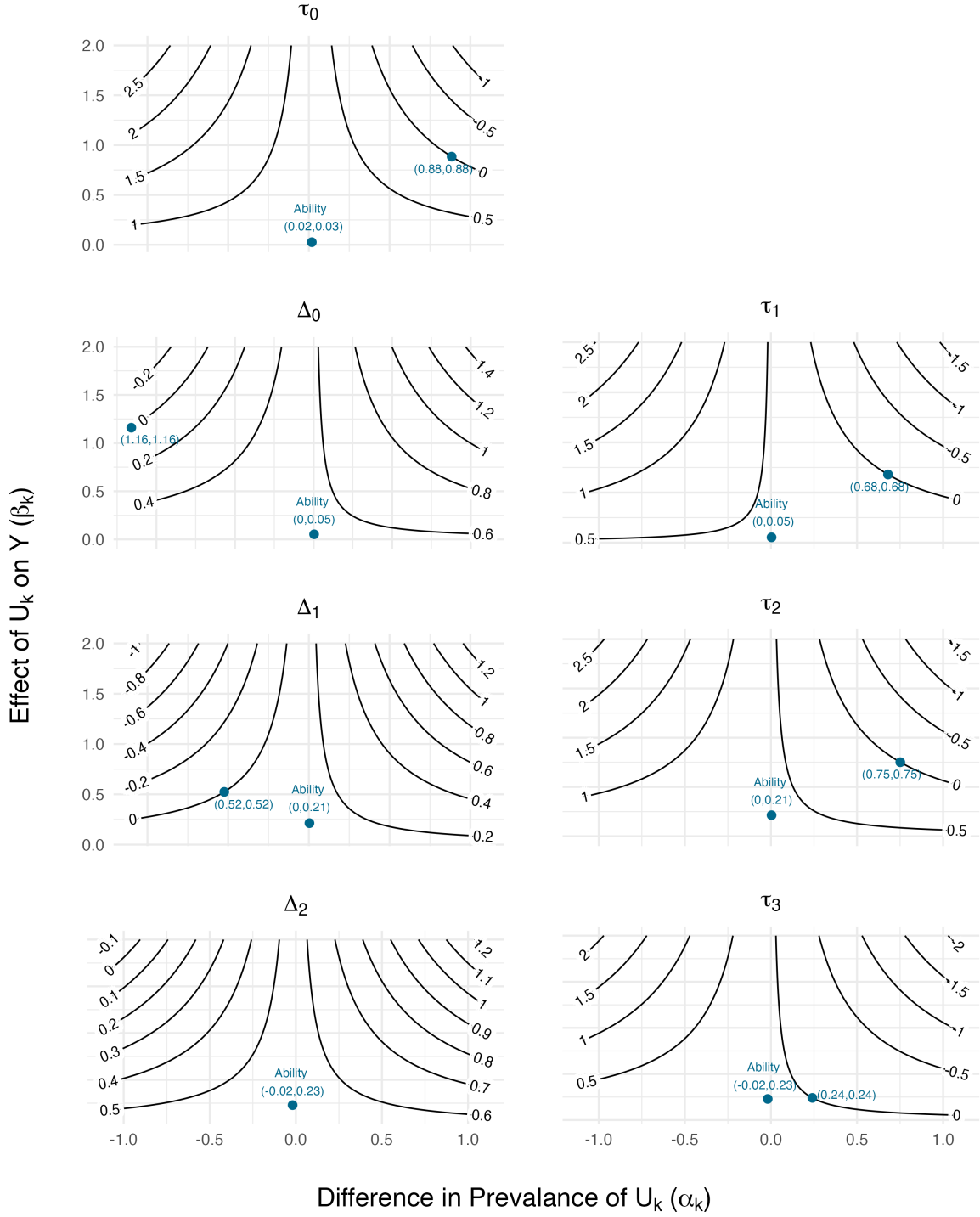
Figure 4: Sensitivity Analysis for the "gross effect" (τ_k) and "direct effect" (Δ_k) terms in decomposition. Each row corresponds to a different set of (α_k, β_k) terms, where $\alpha_k = \mathbb{E}[Y|x, \bar{z}_k, \bar{1}_k, m_k, U_k = 1] - \mathbb{E}[Y|x, \bar{z}_k, \bar{1}_k, m_k, U_k = 0]$ parameterizes the effect of U_k on Y, and $\beta_k = \Pr[U_k = 1|x, \bar{z}_k, \bar{1}_k, m_k] - \Pr[U_k = 1|x, \bar{z}_k, \bar{1}_k])$ parameterizes the effect of M_k on U_k. Each row corresponds to a different set of (α_k, β_k) terms. For example, the top row corresponds to (α_0, β_0), while the second row corresponds to (α_1, β_1), and so on.

Table 1: Means of observed log earnings by educational participation, and earnings gaps (versus high school non-completers).

Group	Population Proportion	Log Earnings	Gap (vs HS Non-Completers)
HS Non-Completers	0.13	9.07 (0.05)	
HS Graduates	0.87	10.18 (0.02)	1.11 (0.05)
College Goers	0.41	10.57 (0.03)	1.5 (0.05)
BA Completers	0.29	10.73 (0.03)	1.66 (0.06)
Grad. School Goers	0.09	10.88 (0.05)	1.81 (0.07)

Note: The category "High School Non-Completers" captures all individuals who attended high school but did not obtain a high school diploma; "High School Graduates" refers to those individuals who graduated high school, regardless of their subsequent educational experiences (i.e., whether or not they proceeded to college); "College Goers" refers to individuals who attended a 4-year college, irrespective of whether they completed their degree; "BA Completers" denotes individuals who completed a Bachelor's degree, while "Grad. School Goers" captures individuals who participated in a graduate-level degree program. A small constant of $1,000$ is added to observed earnings before taking the log. All statistics are computed with a monotonicity assumption imposed on the observed data (i.e. such that all individuals who complete a given educational level are coded as having completed all prior levels). All statistics are calculated using NLSY97 sampling weights, and standard errors are in parentheses.

Table 2: Direct Effects (Δ_k), Probabilities (π_k) and Covariance Terms (η_k) Involved in Decomposition via Debiased Machine-Learning (DML) and Regression-With-Residuals (RWR).

	Δ_0	Δ_1	Δ_2	Δ_3	π_1	π_2	π_3	η_1	η_2	η_3
DML	0.466	0.200	0.461	0.120	0.427	0.555	0.314	0.007	0.004	-0.014
	(0.057)	(0.031)	(0.039)	(0.031)	(0.008)	(0.016)	(0.014)	(0.007)	(0.008)	(0.008)
RWR	0.469	0.116	0.466	0.163	0.374	0.515	0.159	-0.016	0.083	0.028
	(0.115)	(0.093)	(0.097)	(0.114)	(0.015)	(0.077)	(0.022)	(0.007)	(0.015)	(0.051)

Note: The Δ_k parameters capture the average effect of completing the kth mediator *but no subsequent mediator* on earnings, relative to completing the $k - 1$th mediator. For instance, Δ_0 denotes the effect of completing high school (M_1) but not attending college nor, under Assumption 1, completing any subsequent mediators, relative to attending high school but not completing it ($M_0 \equiv A$). The π_k terms capture the average of individuals' counterfactual completion status of the kth mediator under completion of all prior mediators $M_0, \ldots M_{k-1}$. For example, π_1 denotes individuals' average counterfactual college attendance, after - possibly contrary to fact - their completion of high school. Finally, the η_k terms refer to the covariance between individuals' own counterfactual completion status of the kth mediator, and their own "gross" effect of completing the kth mediator on earnings. To recall, the "gross" effect of the kth mediator captures the effect of completing that mediator, relative to completing only the $k - 1$th mediator, irrespective of whether that effect operates directly (net of subsequent mediators) or via subsequent transitions. For example, η_1 denotes the covariance between each individual's counterfactual college attendance status and their gross effect of college attendance on earnings.

A Parametric, regression-with-residuals (RWR) estimation

In this section, I propose a linear regression-with-residuals (RWR) approach for the MPSE decomposition. The approach relies on two steps. The first involves residualizing pre-treatment confounders with respect to their marginal means, and intermediate on causally prior confounders, ie., $X^\perp \triangleq X - \mathbb{E}[X]$, and $Z_k^\perp \triangleq M_{k-1}\big[Z_k - \mathbb{E}[Z_k \mid X, \overline{Z}_{k-1}, M_{k-1} = 1]\big]$. For now, we are agnostic about the functional form used for $\mathbb{E}[Z_k \mid X, \overline{Z}_{k-1}, M_{k-1} = 1]$. The second step involves three sets of models. The first is simply a model for the outcome given pre-treatment covariates and the treatment, namely,

$$\mathbb{E}[Y \mid X, A] = \lambda_0 + \lambda_1 A + \alpha_1^T X^\perp + \alpha_2^T A X^\perp; \tag{13}$$

The second is a set of models for the outcome given covariates, the treatment and M_k for all $k \in [K]$, i.e.,

$$\begin{aligned}
\mathbb{E}[Y|X, \overline{Z}_k, A, \overline{M}_k] =& \beta_{k,0} + c_{k,0} A + \sum_{j=1}^{k} \beta_{k,j} M_j + \eta_{k,1}^\top X^\perp + c_{k,1} A X^\perp + \sum_{j=1}^{k-1} \eta_{k,j}^T M_j X^\perp \\
& + \sum_{j=1}^{k} \gamma_{k,j}^T Z_j^\perp + \sum_{j=1}^{k-1} M_j \sum_{l=1}^{j} \xi_{k,k,l}^\top Z_l^\perp,
\end{aligned} \tag{14}$$

and the third is a set of models for each mediator given covariates, the treatment, conditional on the treatment and all prior mediators, i.e., for all $k \in [K-1]$,

$$\mathbb{E}[M_{k+1} \mid X, \overline{Z}_k, \overline{1}_{k+1}] = \theta_{k,0} + \delta_{k,1}^T X^\perp + \sum_{j=1}^{k} \delta_{k,j+1}^T Z_j^\perp, \tag{15}$$

These models differ from conventional linear regression in that (i) pre-treatment variables are centered around their marginal means, and (ii) post-treatment confounders $Z_k \forall k \in \{1, \dots K\}$ are centered around their conditional means given all antecedent variables. Under Assumptions 2-4, and assuming that the outcome and mediators are linear in pre- and post-treatment confounders, the treatment, and prior mediators, and that all necessary interaction terms have been accounted for, then the ATE τ_0 can be obtained from the linear model $\mathbb{E}[Y \mid X, A]$ as λ_1, and coefficients from the models $\mathbb{E}[Y \mid X, A, \overline{Z}_k, \overline{M}_k]$ and $\mathbb{E}[M_{k+1} \mid X, A, \overline{Z}_k, \overline{M}_k]$ yield estimates of the components of the decomposition as follows:

$$\tau_k = \mathbb{E}[Y(\overline{1}_{k+1}) - Y(\overline{1}_k, 0)] = \beta_{k,k}, \forall k \in \{1, \dots, K\},$$

$$\Delta_k = \mathbb{E}[Y(\overline{1}_{k+1}, \underline{0}_{k+2}) - Y(\overline{1}_k, \underline{0}_{k+1})] = \beta_{k+1,k-1}, \forall k \in \{0, \dots, K-1\},$$

$$\pi_{k+1} = \mathbb{E}[M_{k+1}(\overline{1}_{k+1})] = \theta_{k,0}, \forall k \in \{0, \dots, K-1\}.$$

I state the RWR estimation procedure formally in the following algorithm:

Algorithm 1. *RWR.*

1. For each of the baseline confounders, compute $\hat{X}^\perp = X - \mathbb{P}_n[X]$, where $\mathbb{P}_n[\cdot]$ denotes empirical average.

2. Fit $\hat{\mathbb{E}}[Y \mid X, A]$ using the linear specification shown above; an estimate of τ_0 is given by $\hat{\lambda}_1$.

3. For each set of post-treatment confounders Z_k, $k \in \{1, \dots, K\}$, compute $Z_k^\perp = M_{k-1}\big[Z_k - \mathbb{E}[Z_k \mid X, \overline{Z}_{k-1}, M_{k-1} = 1]\big]$ where an overbar denotes a vector of variables such that $\overline{Z}_k = (Z_1, \dots, Z_k)$, by fitting a regression of Z_k on X and $\overline{Z}_{k-1}$ among units with $M_{k-1} = 1$ and then calculating the residuals.

4. For each $k \in \{1, \dots K\}$:

   (a) compute least squares estimates of equations 14 and 15, using estimates of $X^\perp$ and $Z_k^\perp$.

   (b) compute $\hat{\tau}_k^{\mathrm{RWR}} = \hat{\beta}_{k,k}$, $\hat{\Delta}_{k-1}^{\mathrm{RWR}} = \hat{\beta}_{k,k-1}$, and $\hat{\pi}_k^{\mathrm{RWR}} = \hat{\theta}_{k-1,0}$.

5. Compute the decomposition using $\hat{\tau}_k$, $\hat{\Delta}_k$ and $\hat{\pi}_{k+1}$, and estimating the covariance terms as $\hat{\eta}_k^{\mathrm{RWR}} = \hat{\beta}_{k-1,k} - \hat{\beta}_{k,k-1} - \hat{\beta}_{k,1}\hat{\theta}_{k-1,k}$, and the continuation effects as $\hat{\theta}_k^{\mathrm{RWR}} = (\Pi_{j=1}^k \hat{\pi}_j^{\mathrm{RWR}})\hat{\Delta}_k^{\mathrm{RWR}} + (\Pi_{j=1}^{k-1} \hat{\pi}_j^{\mathrm{RWR}})\hat{\eta}_k^{\mathrm{RWR}}$.

Standard errors and confidence intervals can then be obtained via the non-parametric bootstrap, or by using their asymptotic analytic variance. Specifically, let $\hat{\theta}_k^* \triangleq (\hat{\beta}_{k,0}, \hat{\beta}_{k,1}, \theta_{k,0})$ denote a set of parameters. Under the above models, I have that $\hat{\theta}^* = \{\hat{\lambda}_1, \theta_1^*, \dots \theta_K^*\}$ solves $\mathbb{P}_n[g(O; \hat{\theta}^*)] = 0$, where $g(O; \theta^*)$ is the set of stacked moment conditions with solution $\hat{\theta}^*$. Under standard regularity conditions (Newey and McFadden, 1994), under correct specification of Models 9-11 wherein all residualized quantities are estimated via linear models, the set $\hat{\theta}^*$ is consistent and asymptotically normal, such that $\sqrt{n}(\hat{\theta}^* - \theta^*)$ converges to a mean-zero normal distribution with finite variance $V = G^{-1}\Omega(G^{-1})^\top$, where $\Omega = \mathbb{E}[g(O; \theta^*)g(O; \theta^*)^\top]$, and where $G = \mathbb{E}[\frac{\partial g(O; \theta^*)}{\partial \theta^\top}]$. It follows by a simple application of the Delta Method that the set $\hat{\gamma}_k^* \triangleq (\hat{\tau}_k^{\mathrm{RWR}}, \hat{\Delta}_{k-1}^{\mathrm{RWR}}, \hat{\pi}_k^{\mathrm{RWR}}, \hat{\eta}_k^{\mathrm{RWR}}, \hat{\theta}_k^{\mathrm{RWR}})\forall k \in [K]$ is also consistent and asymptotically normal.

B A Simulation Study

In this section, I evaluate the finite-sample performance of my two estimation procedures via a simulation experiment. Specifically, I compare how the DML estimator proposed in Section 3 (as well as the parametric, RWR estimator described in Appendix A) perform under different degrees of misspecification. Without loss of generality, I focus on the single-mediator setting, and focus on the path-specific effect $A \to M \to Y$. I generate simulations of observed data $O = (X_1, X_2, A, Z, M, Y)$ as follows:

$$U_1, U_2, U_3, U_4 \sim \text{MVN}(0_4, I_4)$$
$$X_1 \sim \text{N}\big((U_1, U_2, U_3, U_4)\beta_{X_1}, 1\big)$$
$$X_2 \sim \text{N}\big((U_1, U_2, U_3, U_4)\beta_{X_2}, 1\big)$$
$$A \sim \text{Bern}\big(\text{logit}^{-1}\big[(1, X)\beta_A\big]\big)$$
$$Z|A = 1 \sim \text{Bern}\big(\text{logit}^{-1}\big[(1, X)\beta_Z\big]\big)$$
$$M|A = 1 \sim \text{Bern}\big(\text{logit}^{-1}\big[(1, X, Z)\beta_M\big]\big) \quad M|A = 0 = 0$$
$$Y \sim \text{N}\big((1, A, X, AZ, AM)\beta_Y, 1\big).$$

The coefficients $(\beta_{X_1}, \beta_{X_2}, \beta_Y)$ are drawn from a $\text{Unif}(-1, 1)$, while the coefficient β_A is drawn from a $\text{Unif}(-0.5, 0.5)$ distribution. In order to test how the DML and RWR methods perform when the relevant models are misspecified, I also construct transformations of the observed covariates (X_i^*, Z_i^*) as follows, employing a similar setup to Kang and Schafer (2007):

$$X_1^* = (\exp(X_1/2) - 1)^2$$
$$X_1^* = X_2/(1 + \exp(X_2)) + 10$$
$$Z^*|A = 1 = (X_1 \cdot Z/25 + 0.6)^3$$

For each simulated dataset, I construct two estimates of the path-specific effect θ_1 ($A \to M \to Y$) by estimating the parameter set $(\tau_0, \Delta_0, \Delta_1, \pi_1)$ via the RWR and DML procedures described in Section 3. Standard errors for the coverage rates are computed via the estimated variance of the estimated EIFs for the DML approach, and via the nonparametric bootstrap with 1000 replications for the RWR procedure.

I run 1000 replications of this DGP and compute the average bias, the root mean square error (RMSE), and the coverage of nominal 95% confidence intervals for sample sizes of 250, 500, and 1000 and using either the "correctly specified" covariates (X_1, X_2, Z) and the "incorrectly specified", transformed versions (X_1^*, X_2^*, Z^*). I calculated the true value of θ_1 by recovering the true values of the parameter set $(\tau_0, \Delta_0, \Delta_1, \pi_1)$ in each Monte Carlo simulation.

Figure 5 presents the results of this simulation experiment. Under correctly specified models,

I observe that the DML and RWR estimators perform similarly, with each displaying low bias, low RMSE and close to nominal coverage at all sample sizes. Under incorrectly specified models, however, the DML and RWR approaches diverge in performance. Whereas the DML estimator under an incorrect feature space performs similarly to when it is used on the correct feature space, the RWR estimator performs much more poorly, displaying a large amount of bias that in fact grows with the sample size, a large RMSE, and coverage rates that are not close to nominal. In short, when the models are correctly specified, both the parametric and semiparametric approaches perform well; the strong performance of a semiparametric approach compared with a parametric estimation strategy becomes clear under model misspecification.
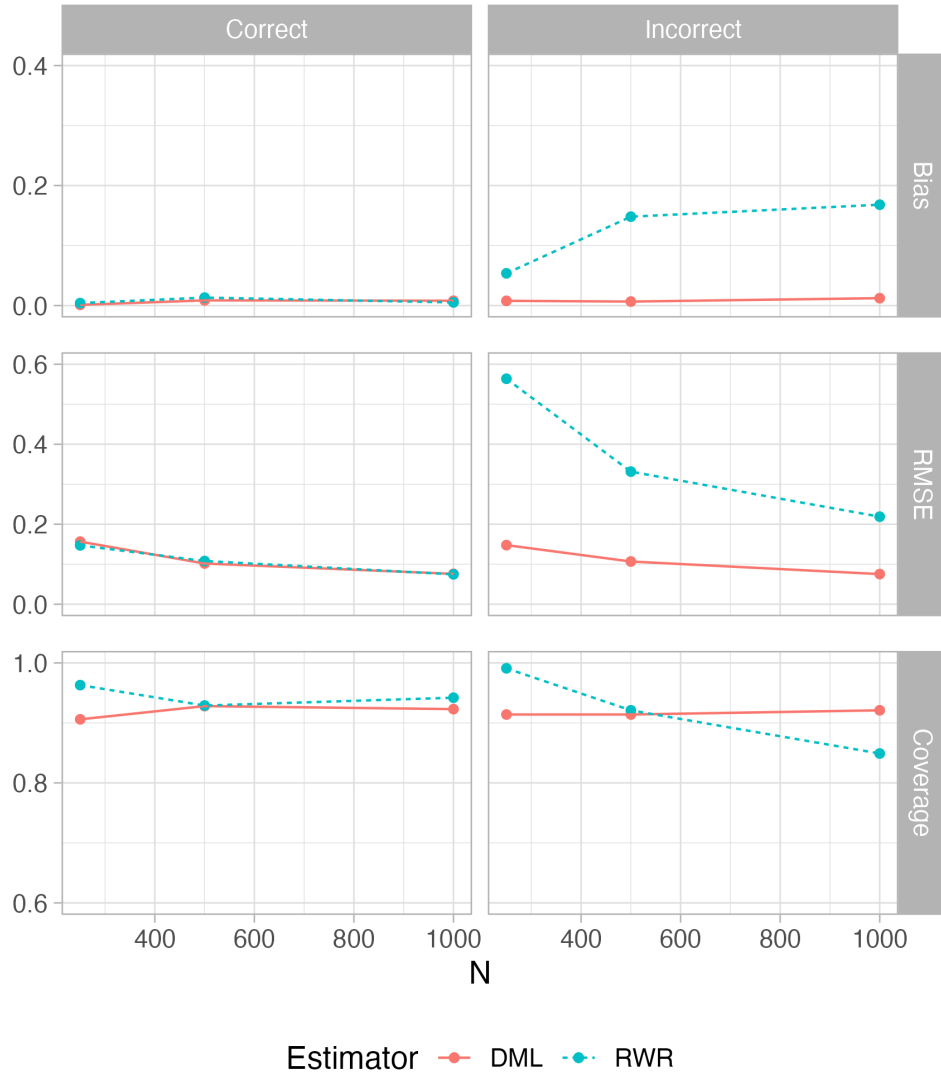
Figure 5: Bias, RMSE, and coverage of DML and RWR estimators for $n = 250, 500, 1000$. The left panel shows the performance of the DML and RWR estimators when the correct feature matrix is supplied to the estimators; the right panel shows the performance of the two estimators when an incorrect feature matrix is supplied to the estimators, as described in the main text.

C Results without imputation of missing covariates

In the main text, I report estimates of the MPSE decomposition for the ATE of high school attendance on logged annual earnings for the full sample of NLSY97 respondents with non-missing educational information and non-missing earnings ($N = 7,305$). A very large number (approximately 50%) of these respondents are missing information on one or more of the covariates (X, Z) used in the models in order to identify the decomposition components, due especially to the large amount of missingness on variables such parental assets (25% of the sample), and on the Armed Services Vocational Aptitude Battery (ASVAB) test (20% of the sample). To assess the sensitivity of my primary conclusions to the use of multiple imputation as opposed to dropping observations with missing values, I replicate my DML and RWR estimates on a non-imputed analytic sample. Figure 6 below shows the results of this exercise. For both estimation procedures, results under multiple imputation and non-imputation are highly similar. As is to be expected, imputation reduces standard errors significant, especially for the parametric, RWR procedure. Further, the greatest variability between imputed and non-imputed results come from effects pertaining to high school completion, perhaps because patterns of missingness are correlated with educational attainment. Despite this, because the total effect τ_0 and direct effect θ_0 are similarly attenuated in the imputed sample, the overall conclusion about the importance of the direct effect in explaining the ATE of high school completion on earnings remains unaffected.
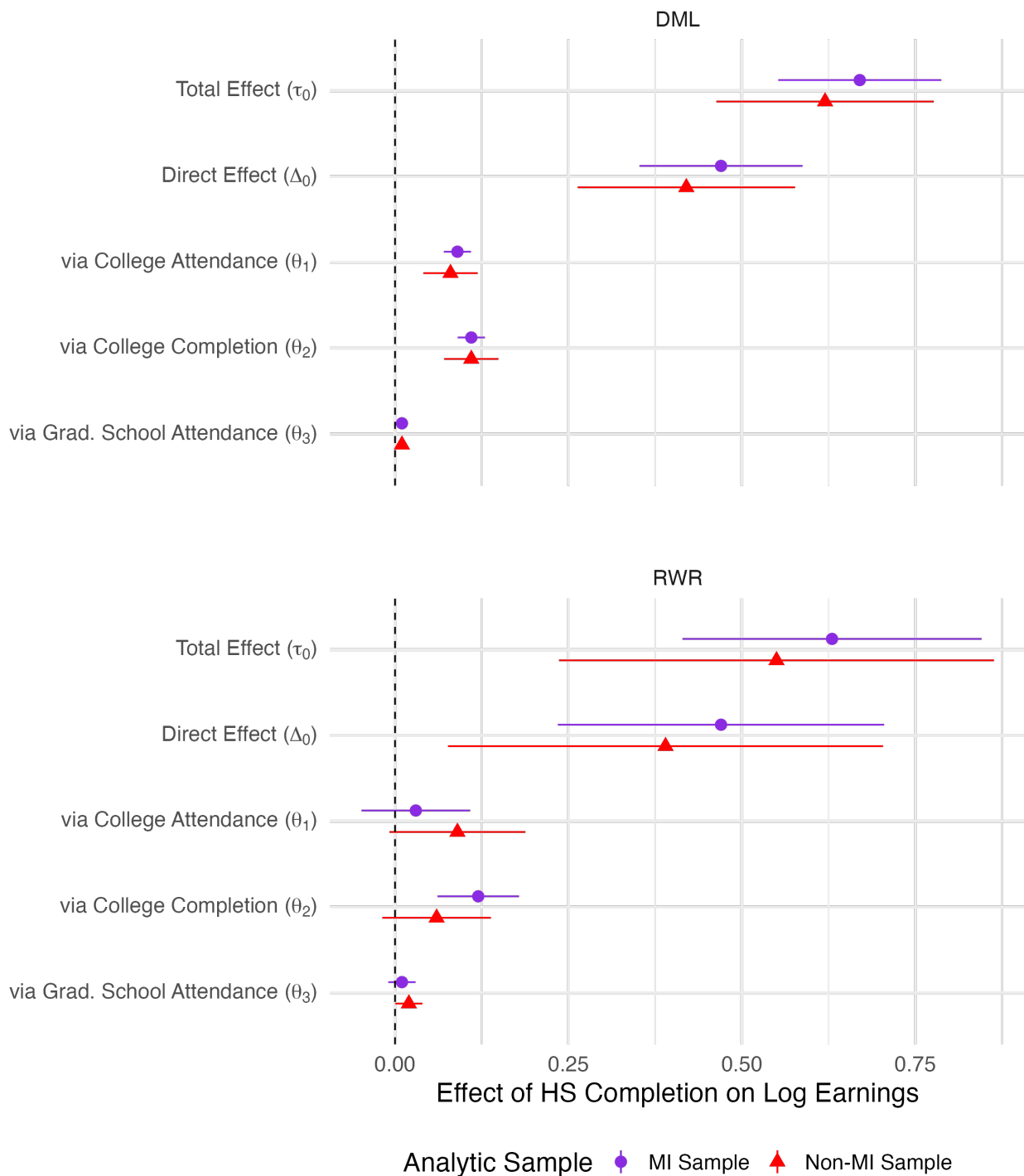
Figure 6: Decomposition of the Average Total Effect (ATE) of High School Graduation on Logged Earnings Under Multiple Imputation (MI) and Under Dropping Observations with Missing (X, Z) Values. Results with multiple imputation (purple lines) are reproduced from the main text ($N = 7,305$); results without multiple imputation (red lines) employ a sample restricted to respondents with observed values for all covariates used ($N = 3,735$).

D   Results under alternative definitions of earnings

In the main text, I report estimates of the MPSE decomposition components for the ATE of high school attendance on logged annual earnings. Logged annual earnings in the main text are defined as the log of observed annual earnings plus a small constant of $1,000$, in order to accommodate respondents with zero observed annual earnings. In order to assess the sensitivity of the reported results to the choice of this constant, I replicate the main analyses under alternative definitions of earnings. Figure 7 reports estimates of the direct and indirect effects under a series of different constants c added to pre-logged annual earnings, for $c \in \{10, 100, 1000\}$, while Figure 8 shows estimates of these direct and indirect effects for observed annual earnings in dollar values. Beginning with Figure 7, I see that, while for the indirect effects θ_1, θ_2, and θ_3, both DML and RWR estimates are quite consistent under these different constants, estimates of the total effect τ_0 as well as the direct effect Δ_0 are quite sensitive to the choice of constant. Specifically, loIr constant values correspond with large increases in the DML estimate of τ_0 from 0.67 ($c = 1000$) to 1.20 ($c = 10$), and of Δ_0 from 0.47 ($c = 1000$) to 0.89 ($c = 10$). This is because individuals with less than a high school degree are more likely than their higher-educated counterparts to have zero or low earnings, making their logged earnings rather sensitive to the choice of constant. Nevertheless, because the total effect τ_0 and direct effect θ_0 are similarly affected by the change in constant value, the importance of the direct effect in explaining the ATE of high school completion is reinforced. In particular, the proportion of the total effect that is direct is estimated to be 70%, 74% and 72% under each of $c = 10, 100, 1000$. Turning next to Figure 7, under DML, estimates of high school completion increases earnings in expectation by roughly $16,500$, corresponding to an earnings return of approximately 53 relative to a baseline of $31,300$ without high school graduation ($\mathbb{E}[Y(0)]$) . Almost half ($\frac{7824}{16454.} \cdot 100 = 47.5\%$) of the total effect is estimated to operate directly, with 29% and 22% mediated via college attendance and college completion, respectively.

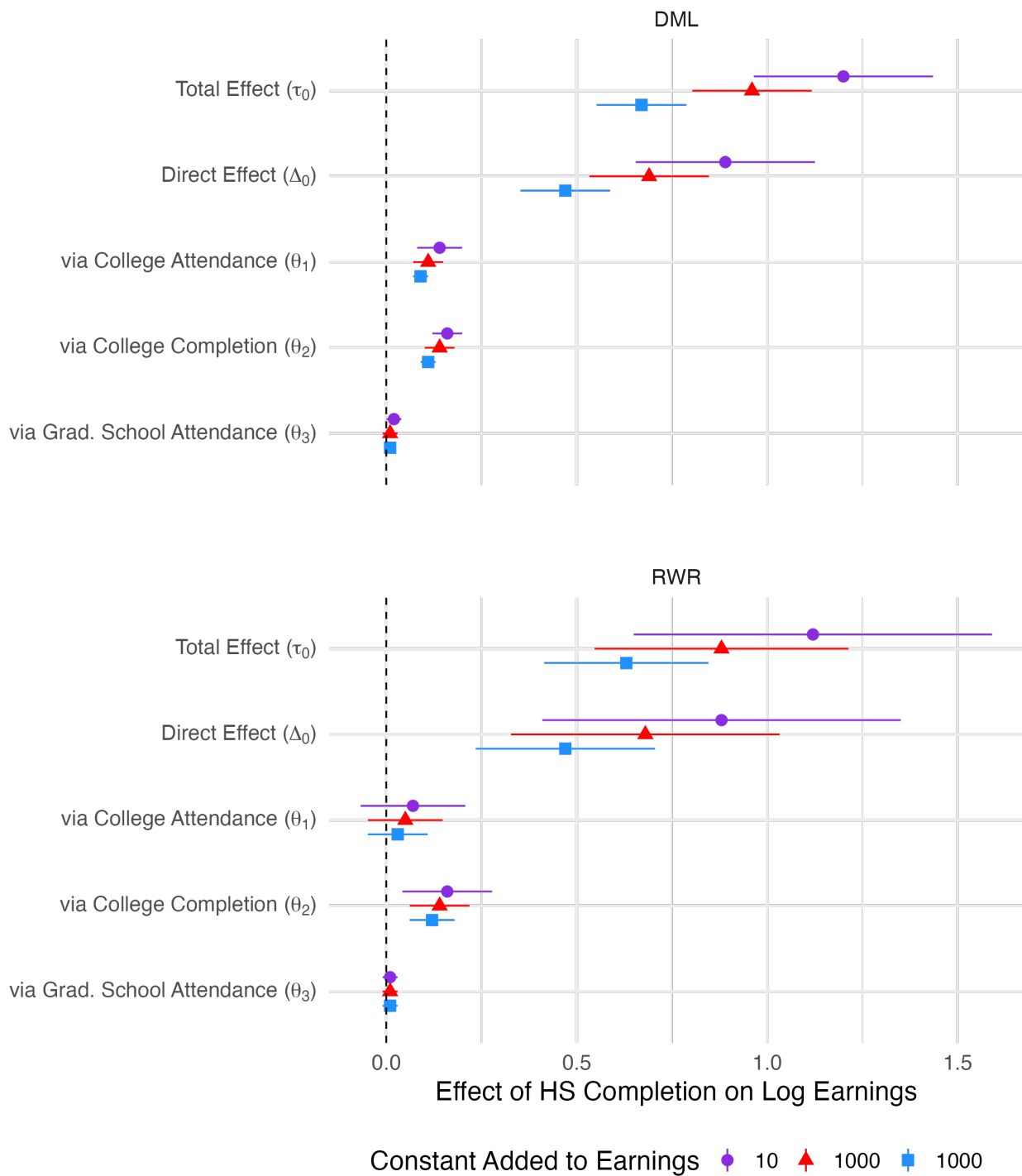Figure 7: Decomposition of the Average Total Effect (ATE) of High School Graduation on Logged Earnings Under Alternative Definitions of Earnings. The figure shows estimates of the total effect (τ_0) as well as the indirect effects $\Delta_0, \Delta_1, \ldots, \ldots, \Delta_K$ when constants of 10, 100 and 1000, respectively, are added to raw annual earnings (in dollar amounts) before taking the log.
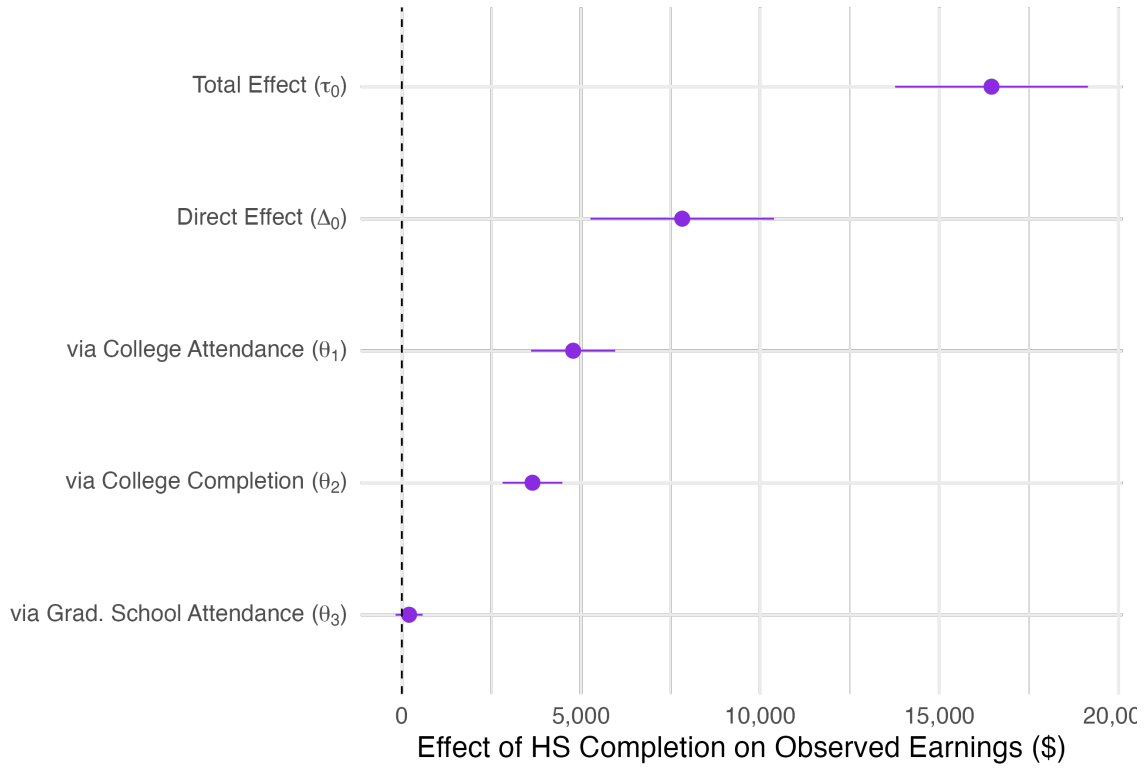
Figure 8: Decomposition of the Average Total Effect (ATE) of High School Graduation on Logged Earnings with Different Definitions of Earnings. The figure shows estimates of the total effect (τ_0) as well as the indirect effects $\Delta_0, \Delta_1, \ldots \ldots, \Delta_K$ when constants of 10, 100 and 1000 are added to raw annual earnings (in dollar amounts) before taking the log.

E   Marginal and conditional means of covariates and earnings

Table 3 shows conditional means of respondent attributes X and Z by for the full (imputed) and restricted (non-imputed) samples, showing first the mean among the full population (of high school goers), and progressively restricting the sample from (i) high school (HS) non-completers, to (ii) HS graduates, to (iii) college attendees and, finally, to (iv) BA completers. Imputed and non-imputed means (shown without and with brackets, respectively) are highly similar across variables. As I progressively restrict the sample to those who attained higher educational levels, I see that variables capturing components of socioeconomic advantage (such as parental income, parental education and household net worth) increase monotonically in value. Background covariates measuring aspects of the school environment (such as peers' college expectations - which is an indicator for whether over 90 of a respondent's peers expected to go to college) behave similarly. I also see that students who progress to higher educational levels have higher levels of pre-college ability: HS non-completers have on average an ASVAB Percentile score of 22.3, compared with only in excess of 70 among BA completers. Similarly, college-goers average high school GPA is approximately .5 higher than high school graduates overall (regardless of whether or not they proceed to college). Nevertheless, the association between high school GPA and attainment declines at higher educational levels: BA completers have only on average a .11 higher a high school GPA than the pooled group of college goers, irrespective of their BA completion status. At this stage, college GPA appears to matter more: college goers overall have on average a college GPA of 2.77, while BA completers' average college GPA is 3.07.

Table 3: Conditional means in background and college-level attributes by sample type (imputed dataset and non-imputed dataset).

	Full Population	HS Non-Completers	HS Graduates	College Goers	BA Completers
Female	0.50 (0.50)	0.44 (0.47)	0.51 (0.51)	0.55 (0.56)	0.57 (0.58)
Black	0.17 (0.16)	0.25 (0.24)	0.16 (0.15)	0.13 (0.10)	0.10 (0.09)
Hispanic	0.14 (0.12)	0.20 (0.18)	0.13 (0.12)	0.09 (0.08)	0.08 (0.07)
Parental Income	80,235 (79,243)	43,308 (43,863)	85,898 (83,865)	110,240 (109,636)	118,026 (114,329)
Parental Education	12.85 (12.84)	11.18 (11.24)	13.11 (13.04)	14.06 (14.13)	14.38 (14.34)
Household Net Worth	175,513 (176,097)	61,997 (62,318)	192,920 (190,959)	272,606 (284,029)	306,177 (307,101)
Lived w Biological Parents	0.52 (0.51)	0.30 (0.28)	0.55 (0.55)	0.67 (0.69)	0.72 (0.73)
Father Figure Present	0.75 (0.75)	0.61 (0.61)	0.77 (0.76)	0.84 (0.83)	0.86 (0.86)
Lived in Rural Area	0.28 (0.30)	0.25 (0.27)	0.28 (0.31)	0.28 (0.32)	0.28 (0.31)
Lived in South	0.36 (0.35)	0.44 (0.42)	0.35 (0.34)	0.33 (0.30)	0.31 (0.29)
Children by 18	0.07 (0.07)	0.21 (0.24)	0.05 (0.05)	0.01 (0.01)	0.01 (0.01)
Substance Abuse Score	1.09 (1.10)	1.32 (1.37)	1.06 (1.07)	0.86 (0.82)	0.82 (0.77)
Delinquency Score	1.38 (1.39)	2.07 (2.13)	1.27 (1.29)	0.91 (0.88)	0.81 (0.79)
Peers' College Expectations	0.56 (0.56)	0.40 (0.38)	0.59 (0.58)	0.69 (0.69)	0.72 (0.71)
Property Stolen at School	0.24 (0.23)	0.28 (0.29)	0.23 (0.22)	0.21 (0.20)	0.20 (0.19)
Threatened at School	0.22 (0.23)	0.30 (0.33)	0.20 (0.21)	0.14 (0.13)	0.13 (0.11)
In a Fight at School	0.16 (0.16)	0.32 (0.34)	0.14 (0.13)	0.07 (0.07)	0.06 (0.06)
ASVAB Percentile	48.36 (48.52)	22.27 (21.77)	52.37 (52.02)	67.22 (69.66)	70.66 (71.82)
High School GPA	2.84 (2.81)	2.12 (2.14)	2.95 (2.90)	3.30 (3.33)	3.41 (3.43)
Stem Major				0.17 (0.19)	0.18 (0.19)
College GPA				2.78 (2.86)	3.07 (3.12)
Earnings ($)	46,505 (45,151)	21,597 (21,566)	46,505 (45,151)	64,984 (64,595)	72,374 (70,212)
Log (Earnings + c)	10.03 (10.06)	9.07 (9.14)	10.03 (10.06)	10.57 (10.65)	10.73 (10.78)

Note: Numbers denote means for the imputed sample (non-imputed sample). Means are adjusted for multiple imputation via Rubin's (1987) method, and all statistics are calculated using NLSY97 sampling weights.

F Proofs and technical details

F.1 EIFs for η_k and θ_k terms (Proposition 1)

Under Assumptions 2-4, the covariance component η_k is identified as $\eta_k = \tau_{k-1} - \Delta_{k-1} - \pi_k \tau_k$. Following (Kennedy, 2022, , p. 15), I let $\mathbb{IF} : \Psi \to L_2(\mathbb{P})$ denote the operator mapping the functionals $\{\Delta_k, \pi_k, \eta_k\} : \mathcal{P} \to \mathbb{R}$, $\forall \in [K]$ to their respective influence functions under the nonparametric model $\mathcal{P}$. First, by linearity of the EIF, $\mathbb{IF}(\eta_k)$ is given by

$$\mathbb{IF}(\eta_k) = \mathbb{IF}(\tau_{k-1}) - \mathbb{IF}(\Delta_{k-1}) - \mathbb{IF}(\pi_k \tau_k).$$

Since $\mathbb{IF}(\pi_k \tau_k)$ can be written as follows $\mathbb{IF}(\pi_k \tau_k) = \tau_k \mathbb{IF}(\pi_k) + \pi_k \mathbb{IF}(\tau_k)$, $\mathbb{IF}(\eta_k)$ can be written as

$$\begin{aligned}
\mathbb{IF}(\eta_k) &= \mathbb{IF}(\tau_{k-1}) - \mathbb{IF}(\Delta_{k-1}) - \big(\tau_k \mathbb{IF}(\pi_k) + \pi_k \mathbb{IF}(\tau_k)\big) \\
&= \mathbb{IF}(\tau_{k-1}) - \mathbb{IF}(\Delta_{k-1}) - \tau_k \mathbb{IF}(\pi_k) - \pi_k \mathbb{IF}(\tau_k).
\end{aligned}$$

Noticing that I can rewrite this expression as

$$\begin{aligned}
\mathbb{IF}(\eta_k) &= \mathbb{R}\mathbb{IF}(\tau_{k-1}) - \tau_{k-1} - \mathbb{R}\mathbb{IF}(\Delta_{k-1}) + \Delta_{k-1} - \tau_k \mathbb{R}\mathbb{IF}(\pi_k) + \tau_k \pi_k - \pi_k \mathbb{R}\mathbb{IF}(\tau_k) + \pi_k \tau_k \\
&= \mathbb{R}\mathbb{IF}(\tau_{k-1}) - \mathbb{R}\mathbb{IF}(\Delta_{k-1}) - \tau_k \mathbb{R}\mathbb{IF}(\pi_k) - \pi_k \mathbb{R}\mathbb{IF}(\tau_k) + \pi_k \tau_k - \eta_k,
\end{aligned}$$

where $\mathbb{R}\mathbb{IF}(\phi) = \mathbb{IF}(\phi) + \phi$, I can obtain the corresponding EIF-based estimator for η_k can obtained by solving the empirical moment condition obtained by setting the average of the above equation equal to 0, and plugging in the set of estimated nuisance functions.

$$\hat{\eta}_k^{\text{eif}} = \mathbb{R}\hat{\mathbb{IF}}(\tau_{k-1}) - \mathbb{R}\hat{\mathbb{IF}}(\Delta_{k-1}) - \tau_k \mathbb{R}\hat{\mathbb{IF}}(\pi_k) - \pi_k \mathbb{R}\hat{\mathbb{IF}}(\tau_k) + \pi_k \tau_k,$$

where $\mathbb{R}\hat{\mathbb{IF}}(\phi) = \hat{\mathbb{IF}}(\phi) + \phi$, and $\hat{\mathbb{IF}}(\phi)$ denotes the influence function of a parameter evaluated at estimates of its component nuisance functions.

Turning next to the influence functions for the continuation effects θ_k, $k \in \{1, \ldots K\}$, following the same logic as the above, I can write the EIF of θ_k, $\mathbb{IF}(\theta_k)$, as

$$\mathbb{IF}(\theta_k) = \mathbb{IF}(\Delta_k) \prod_{j=1}^{k} \pi_j + \Delta_k \sum_{j=1}^{k} \mathbb{IF}(\pi_j) \prod_{l:l \neq j}^{k} \pi_l + \mathbb{IF}(\eta_k) \prod_{j=1}^{k-1} \pi_j + \eta_k \sum_{j=1}^{k-1} \mathbb{IF}(\pi_j) \prod_{l:l \neq j}^{k-1} \pi_l.$$

Rewriting this expression as

$$\mathbb{IF}(\theta_k) = \mathbb{RIF}(\Delta_k) \prod_{j=1}^{k} \pi_j + \Delta_k \sum_{j=1}^{k} \mathbb{RIF}(\pi_j) \prod_{l:l\neq j} \pi_l + \mathbb{RIF}(\eta_k) \prod_{j=1}^{k-1} \pi_j + \eta_k \sum_{j=1}^{k-1} \mathbb{RIF}(\pi_j) \prod_{l:l\neq j} \pi_l$$
$$- k\Delta_k \prod_{j=1}^{k} \pi_j - (k-1)\eta_k \prod_{j=1}^{k-1} \pi_j - \theta_k,$$

I obtain the corresponding EIF-based estimator for θ_k can obtained by solving the empirical moment condition obtained by setting the average of the above equation equal to 0, and plugging in the set of estimated nuisance functions, as:

$$\hat{\theta}_k^{\text{eif}} = \widehat{\mathbb{RIF}}(\Delta_k) \prod_{j=1}^{k} \hat{\pi}_j + \hat{\Delta}_k \sum_{j=1}^{k} \widehat{\mathbb{RIF}}(\pi_j) \prod_{l:l\neq j} \hat{\pi}_l + \widehat{\mathbb{RIF}}(\eta_k) \prod_{j=1}^{k-1} \hat{\pi}_j + \hat{\eta}_k \widehat{\mathbb{RIF}}(\pi_j) \prod_{l:l\neq j} \hat{\pi}_l$$
$$- k\hat{\Delta}_k \prod_{j=1}^{k} \hat{\pi}_j - (k-1)\hat{\eta}_k \prod_{j=1}^{k-1} \hat{\pi}_j.$$

F.2 Semiparametric efficiency (Proposition 2)

In this section, I establish the conditions required for the semiparametric efficiency of all terms featured in the decomposition. Before proceeding, I establish some notational preliminaries. Let $||g|| = (\int g^\top g dP)^{1/2}$ denote the $L_2(P)$ norm, and let $R_n(\,\cdot\,)$ denote a mapping from a nuisance function to its $L_2(P)$ convergence rate. Let $\hat{\varphi}^{\text{EIF}}_{K^* m_{K^*}} = \mathbb{P}_n[m(O; \hat{\eta})]$, where $m(O; \hat{\eta})$ is the quantity inside $\mathbb{P}_n[\cdot]$ in equation 11, and $\hat{\eta} = (\hat{\pi}_0, \ldots, \hat{\pi}_K, \hat{\mu}_0, \ldots, \hat{\mu}_K)$. I have that

$$
\begin{aligned}
\hat{\varphi}^{\text{EIF}}_{K^* m_{K^*}} - \varphi_{K^* m_{K^*}} &= \mathbb{P}_n[m(O; \hat{\eta})] - P[m(O; \eta)] \\
&= \mathbb{P}_n[m(O; \eta)] + \underbrace{P[m(O; \hat{\eta}) - m(O; \eta)]}_{\triangleq R_2(\hat{\eta})} + (\mathbb{P}_n - P)[m(O; \hat{\eta}) - m(O; \eta)],
\end{aligned}
$$

$$(16)$$

where $Pg = \int g dP$ denotes the expectation of function g at the truth, and where $\hat{\eta} = \{\hat{\pi}_{1,1}\}$ The first term in equation 16 is a sample average, and can be analyzed with the central limit theorem. It has an asymptotic variance of $\mathbb{E}\left[(\varphi_{am}(O; \eta))^2\right]$. The last term is an empirical process term that will be $o_p(n^{-1/2})$ if either the nuisance functions fall in a Donsker class or if cross-fitting is used to induce independence between $\hat{\eta}$ and O. Thus, $\hat{\theta}^{\text{EIF}}$ will be asymptotically normal and semiparametric efficient if $R_2(\hat{\eta})$ is $o_p(n^{-1/2})$. To analyze this term, I first note that

$$P[m(O;\eta)] = P\left[\frac{A}{\pi_{01}(X)}\left(\frac{\mathbb{I}(M_{K^*}=m_{K^*})}{\pi_{K^*m_{K^*}}(X,\overline{Z}_{K^*})}\prod_{j=1}^{K^*-1}\frac{M_j}{\pi_{j1}(X,\overline{Z}_j)}\right)(Y - \mu_{K^*m_{K^*}}(X,\bar{Z}_{K^*}))\right.$$

$$\left. + \sum_{k=1}^{K^*}\frac{A}{\pi_{01}(X)}\left(\prod_{j=1}^{k-1}\frac{M_j}{\pi_{j1}(X,\overline{Z}_j)}\right)(\mu_{km_{K^*}}(X,\overline{Z}_k) - \mu_{(k-1)m_{K^*}}(X,\bar{Z}_{k-1})) + \mu_0(X)\right]$$

$$= P\left[\frac{A}{\pi_{01}(X)}\left(\frac{\mathbb{I}(M_{K^*}=m_{K^*})}{\pi_{K^*m_{K^*}}(X,\overline{Z}_{K^*})}\prod_{j=1}^{K^*-1}\frac{M_j}{\pi_{j1}(X,\overline{Z}_j)}\right)\cdot\right.$$

$$\left(\underbrace{\mathbb{E}[Y - \mu_{K^*m_{K^*}}(X,\bar{Z}_{K^*})\,|\,X,\bar{Z}_{K^*},\overline{1}_{K^*},m_{K^*}]}_{=0}\right)$$

$$+ \sum_{k=1}^{K^*}\frac{A}{\pi_{01}(X)}\left(\prod_{j=1}^{k-1}\frac{M_j}{\pi_{j1}(X,\overline{Z}_j)}\right)\cdot$$

$$\left.\left(\underbrace{\mathbb{E}[\mu_{km_{K^*}}(X,\overline{Z}_k) - \mu_{(k-1)m_{K^*}}(X,\bar{Z}_{k-1})\,|\,X,\bar{Z}_{k-1},\overline{1}_k]}_{=0}\right) + \mu_0(X)\right]$$

$$= P\left[\mu_0(X)\right].$$

Plugging this result into $R_2(\hat{\eta})$, I have that

$$R_2(\hat{\eta}) = P[m(O;\hat{\eta}) - m(O;\eta)]$$

$$= P\left[\sum_{k=0}^{K^*}\frac{A}{\hat{\pi}_{01}(X)}\left(\prod_{j=1}^{k-1}\frac{M_j}{\hat{\pi}_{j1}(X,\bar{Z}_j)}\right)\cdot\right.$$

$$\left.\left(\hat{\pi}_k(X,\overline{Z}_j) - \pi_k(X,\overline{Z}_j)\right)\left(\hat{\mu}_{km_{K^*}}(X,\bar{Z}_k) - \mu_{km_{K^*}}(X,\bar{Z}_k)\right)\right]$$

$$= \sum_{k=0}^{K^*}O_p\left(||\hat{\pi}_{k1}(X,\overline{Z}_j) - \pi_{k1}(X,\overline{Z}_j)||\cdot||\hat{\mu}_{km_{K^*}}(X,\bar{Z}_k) - \mu_{km_{K^*}}(X,\bar{Z}_k)||\right),$$

where the last equality results from the positivity assumption that $\hat{\pi}_{k1}(X,\bar{Z}_k)$ is bounded away from zero, for all $k \in [K]$, and from the Cauchy-Schwartz inequality. Then, assuming that the empirical process term is of order $o_p(n^{-1/2})$, I can write $\hat{\varphi}^{\text{EIF}}_{K^*m_{K^*}} - \varphi_{K^*m_{K^*}}$ as

$$\hat{\psi}^{\text{EIF}}_{K^*m_{K^*}} - \psi_{K^*m_{K^*}} = \mathbb{P}_n[m(O;\eta) - \psi_{K^*m_{K^*}}]$$

$$+ \sum_{k=0}^{K^*} O_p \big(||\hat{\pi}_{k1}(X, \overline{Z}_j) - \pi_{k1}(X, \overline{Z}_j)|| \big) \cdot O_p \big(||\hat{\mu}_{km_{K^*}}(X, \bar{Z}_k) - \mu_{km_{K^*}}(X, \bar{Z}_k)|| \big)$$

$$+ o_p(n^{-1/2}).$$

Thus, letting $R_n(K^*, m_{K^*}) \triangleq \sum_{k=0}^{K^*} R_n(\hat{\pi}_k) R_n(\hat{\mu}_{km_{K^*}}^{K^*})$, $\hat{\varphi}_{K^* m_{K^*}}^{\mathrm{rEIF}}$ is consistent if $R_n^*(K^*, m_{K^*}) = o(1)$ and it is semiparametric efficient if $R_n^*(K^*, m_{K^*}) = o(n^{-1/2})$. Clearly, then, $\hat{\Delta}_k^{\mathrm{rEIF}}$ is consistent if $\sum_{j=k}^{k+1} R_n^*(j, 0) = o(1)$ and it is semiparametric efficient if $\sum_{j=k}^{k+1} R_n^*(j, 0) = o(n^{-1/2})$. Similarly, $\hat{\tau}_k^{\mathrm{rEIF}}$ is consistent if $\sum_{j=0}^{1} R_n^*(k, j) = o(1)$ and it is semiparametric efficient if $\sum_{j=0}^{1} R_n^*(k, j) = o(n^{-1/2})$.

Turning next to to $\phi_{K^*} \triangleq \mathbb{E}[M_{K^*+1}(\overline{1}_{K^*+1})]$, for all $K^* \in [K-1]$, I can similarly define $\gamma_{K^*}(X, \bar{Z}_{K^*})$ iteratively as

$$\gamma_{K^*}(X, \bar{Z}_{K^*}) \triangleq \mathbb{E}\left[M_{K^*+1} \mid X, \bar{Z}_{K^*}, \overline{1}_{K^*+1}\right]$$
$$\gamma_k(X, \bar{Z}_k) \triangleq \mathbb{E}\left[\gamma_{k+1}(X, \bar{Z}_{k+1}) \mid X, \bar{Z}_k, \overline{1}_{k+1}\right] \forall k \in [K^*-1].$$

Similarly to the previous case, the EIF of ϕ_{K^*} is equal to

$$\varphi_{K^*}(O) = \frac{A}{\pi_{01}(X)} \left(\prod_{j=1}^{K^*} \frac{M_j}{\pi_{j1}(X, \overline{Z}_j)} \right) \left(Y - \gamma_{K^*}(X, \bar{Z}_{K^*}) \right).$$
$$+ \sum_{k=0}^{K^*} \frac{A}{\pi_{01}(X)} \left(\prod_{j=1}^{k-1} \frac{M_j}{\pi_{j1}(X, \overline{Z}_j)} \right) \left(\gamma_k(X, \overline{Z}_k) - \gamma_{k-1}(X, \bar{Z}_{k-1}) \right)$$
$$+ \gamma_0(X) - \phi_{K^*}.$$

Following similar arguments to the above, I have that

$$\hat{\phi}_{K^*}^{\mathrm{EIF}} - \phi_{K^*} = \mathbb{P}_n[m_2(O; \eta) - \phi_{K^*}]$$
$$+ \sum_{k=0}^{K^*} O_p \big(||\hat{\pi}_{k1}(X, \overline{Z}_j) - \pi_{k1}(X, \overline{Z}_j)|| \big) \cdot O_p \big(||\hat{\gamma}_k(X, \bar{Z}_k) - \gamma_k(X, \bar{Z}_k)|| \big)$$
$$+ o_p(n^{-1/2}),$$

where $m_2(O; \hat{\eta}) = \varphi_{K^*} + \phi_{K^*}$. Thus, letting $R_n^*(K^*, \gamma) \triangleq \sum_{k=0}^{K^*} R_n(\hat{\pi}_k) R_n(\hat{\gamma}_k)$, $\hat{\phi}_{K^*}^{\mathrm{EIF}}$ is consistent if $R_n^*(K^*, \gamma) = o(1)$ and it is semiparametric efficient if $R_n^*(K^*, \gamma) = o(n^{-1/2})$. This result implies that if all nuisance functions are consistently estimated and converge at faster than $n^{1/4}$ rates, then $\hat{\phi}_{K^*}^{\mathrm{EIF}}$ is semiparametric efficient. I first establish the following lemma:

Lemma S: Let X_n and Y_n denote two convergent sequences, where $X_n = O_p(n^{-1/2})$ and $Y_n = o_p(n^{-1/2})$. Then, (a) $X_n Y_n = o_p(n^{-1/2})$, and (b) $X_n X_n = o_p(n^{-1/2})$.

Proof. (a) $X_n = O_p(n^{-1/2}) = n^{-1/2}O_p(1) = o_p(1)$. Thus, $X_n Y_n = o_p(1)o_p(n^{-1/2}) = o_p(n^{-1/2})$.
(b) $X_n X_n = O_p(n^{-1/2})O_p(n^{-1/2}) = O_p(n^{-1}) = n^{-1/2}O_p(n^{-1/2}) = n^{-1/2}o_p(1)$ (by (a)). Thus, $X_n X_n = o_p(n^{-1/2})$. □

Next, I establish rate conditions for the semiparametric efficiency of $\eta_k = \tau_{k-1} - \Delta_{k-1} - \pi_k \tau_k$. I can analyze the asymptotic behavior of $\hat\eta_k = \hat\tau_{k-1} - \hat\Delta_{k-1} - \hat\pi_k \hat\tau_k$ via a distributional expansion of each plug-in estimator:

$$
\begin{aligned}
\hat\eta_k &= \hat\tau_{k-1} - \hat\Delta_{k-1} - \hat\pi_k \hat\tau_k \\
&= (\tau_{k-1} + \mathbb{P}_n[\tau_{k-1}^{\text{EIF}}] + \tau_{k-1}^{\text{EP}} + \tau_{k-1}^{\text{R2}}) - (\Delta_{k-1} + \mathbb{P}_n[\Delta_{k-1}^{\text{EIF}}] + \Delta_{k-1}^{\text{EP}} + \Delta_{k-1}^{\text{R2}}) \\
&\quad - [\pi_k + \mathbb{P}_n[\pi_k^{\text{EIF}}] + \pi_k^{\text{EP}} + \pi_k^{\text{R2}}][\tau_k + \mathbb{P}_n[\tau_k^{\text{EIF}}] + \tau_k^{\text{EP}} + \tau_k^{\text{R2}}] \\
&= (\tau_{k-1} + \mathbb{P}_n[\tau_{k-1}^{\text{EIF}}] + o_p(n^{-1/2}) + \tau_{k-1}^{\text{R2}}) - (\Delta_{k-1} + \mathbb{P}_n[\Delta_{k-1}^{\text{EIF}}] + o_p(n^{-1/2}) + \Delta_{k-1}^{\text{R2}}) \\
&\quad - [\pi_k + \mathbb{P}_n[\pi_k^{\text{EIF}}] + o_p(n^{-1/2}) + \pi_k^{\text{R2}}][\tau_k + \tau_k^{\text{EIF}} + o_p(n^{-1/2}) + \tau_k^{\text{R2}}] \\
&= [\tau_{k-1} - \Delta_{k-1} - \pi_k \tau_k] + \mathbb{P}_n[(\tau_{k-1} - \Delta_{k-1} - \pi_k \tau_k)^{\text{EIF}}] \\
&\quad + \tau_{k-1}^{\text{R2}} + \Delta_{k-1}^{\text{R2}} + \pi_k^{\text{R2}} + \tau_k^{\text{R2}} + O_p(n^{-1/2})O_p(n^{-1/2}) + O_p(n^{-1/2})o_p(n^{-1/2}) + o_p(n^{-1}) + o_p(n^{-1/2}) \quad \text{(by Pro} \\
&= [\tau_{k-1} - \Delta_{k-1} - \pi_k \tau_k] + \mathbb{P}_n[(\tau_{k-1} - \Delta_{k-1} - \pi_k \tau_k)^{\text{EIF}}] \\
&\quad + \tau_{k-1}^{\text{R2}} + \Delta_{k-1}^{\text{R2}} + \pi_k^{\text{R2}} + \tau_k^{\text{R2}} + o_p(n^{-1/2}) \quad \text{(by Lemma S} F.2\text{).}
\end{aligned}
$$

Thus, for any $k \in \{1, \ldots, K\}$, $\hat\eta_k = \hat\tau_{k-1} - \hat\Delta_{k-1} - \hat\pi_k \hat\tau_k$ is semiparametric efficient if $\tau_{k-1}^{\text{R2}} + \Delta_{k-1}^{\text{R2}} + \pi_k^{\text{R2}} + \tau_k^{\text{R2}} = o_p(n^{-1/2})$.

For the continuation terms $(\theta_k = (\Pi_{j=1}^{k}\pi_j)\Delta_k + (\Pi_{j=1}^{k-1}\pi_j)\eta_k)$, I proceed by induction. Let $\hat\Delta_* = \Delta_* + \mathbb{P}_n[\Delta_*^{\text{EIF}}] + \Delta_*^{\text{EP}} + \Delta_*^{\text{R2}}$ and $\hat\eta_* = \eta_* + \mathbb{P}_n[\eta_*^{\text{EIF}}] + \eta_*^{\text{EP}} + \eta_*^{\text{R2}}$ be asymptotically linear, where $* \in \{1, \ldots K\}$. For $k = 1$, I can asymptotically expand $\hat\pi_1 \hat\Delta_*$ as

$$
\begin{aligned}
\hat\pi_1 \hat\Delta_* &= (\pi_1 + \mathbb{P}_n[\pi_1^{\text{EIF}}] + \pi_1^{\text{EP}} + \pi_1^{\text{R2}})(\Delta_* + \mathbb{P}_n[\Delta_*^{\text{EIF}}] + \Delta_*^{\text{EP}} + \Delta_*^{\text{R2}}) \\
&= (\pi_1 + \mathbb{P}_n[\pi_1^{\text{EIF}}] + o_p(n^{-1/2}) + \pi_1^{\text{R2}})(\Delta_* + \mathbb{P}_n[\Delta_*^{\text{EIF}}] + o_p(n^{-1/2}) + \Delta_*^{\text{R2}}) \\
&= \pi_1 \Delta_* + \mathbb{P}_n[\pi_1^{\text{EIF}}\Delta_* + \Delta_*^{\text{EIF}}\pi_1] + \pi_1^{\text{R2}} + \Delta_*^{\text{R2}} + o_p(n^{-1/2}) \quad \text{(by Lemma S} F.2\text{)} \\
&= \sum_{j=1}^{k} \pi_j \Delta_* + \mathbb{P}_n[(\sum_{j=1}^{k}\pi_j\Delta_*)^{\text{EIF}}] + \Delta_*^{\text{R2}} + \sum_{j=1}^{k}\pi_j^{\text{R2}} + o_p(n^{-1/2}) \quad \text{(by Proposition 1),}
\end{aligned}
$$

and expand $(\Pi_{j=1}^{k-1}\pi_j)\eta_*$, similarly, as

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$$\sum_{j=1}^{k-1} \hat{\pi}_j \hat{\eta}_* = \eta_* + \mathbb{P}_n[\eta_*^{\text{EIF}}] + \tau_{*-1}^{\text{R2}} + \Delta_{*-1}^{\text{R2}} + \pi_*^{\text{R2}} + \tau_*^{\text{R2}} + o_p(n^{-1/2})$$

$$= \sum_{j=1}^{k-1} \pi_j \eta_* + \mathbb{P}_n[(\sum_{j=1}^{k-1} \pi_j \eta_*)^{\text{EIF}}]$$

$$+ \tau_{*-1}^{\text{R2}} + \Delta_{*-1}^{\text{R2}} + \pi_*^{\text{R2}} + \tau_*^{\text{R2}} + \sum_{j \in \{1...,k^*-1\}: j \neq *} \pi_j^{\text{R2}} + o_p(n^{-1/2}) \quad \text{(following a similar logic to above).}$$

Now, assume that, for $k^* \in \{1, \ldots, K\}$, $(\Pi_{j=1}^{k^*} \hat{\pi}_j)\hat{\Delta}_* = \Delta_k \prod_{j=1}^{k^*} \pi_j + \mathbb{P}_n[(\Delta_* \prod_{j=1}^{k^*} \pi_j)^{\text{EIF}}] + \Delta_*^{\text{R2}} + \sum_{j=1}^{k^*} \pi_j^{\text{R2}} + o_p(n^{-1/2})$ and, further, that $(\Pi_{j=1}^{k^*-1} \hat{\pi}_j)\hat{\eta}_* = \Pi_{j=1}^{k^*-1} \pi_j \eta_* + \mathbb{P}_n[(\Pi_{j=1}^{k^*-1} \pi_j \eta_*)^{\text{EIF}}] + \tau_{*-1}^{\text{R2}} + \Delta_{*-1}^{\text{R2}} + \pi_*^{\text{R2}} + \tau_*^{\text{R2}} + \sum_{j \in \{1...,k^*-1\}: j \neq *} \pi_j^{\text{R2}} + o_p(n^{-1/2})$. Then, by induction, I have that

$$(\Pi_{j=1}^{k^*+1} \hat{\pi})\hat{\Delta}_* = \left[(\pi_{k^*+1} + \mathbb{P}_n[\pi_{k^*+1}^{\text{EIF}}] + o_p(n^{-1/2}) + \pi_{k^*+1}^{\text{R2}}) \right]$$

$$\left[\Delta_* \prod_{j=1}^{k^*} \pi_j + \mathbb{P}_n[(\Delta_* \sum_{j=1}^{k^*} \pi_j)^{\text{EIF}}] + \Delta_*^{\text{R2}} + \sum_{j=1}^{k^*} \pi_j^{\text{R2}} + o_p(n^{-1/2}) \right]$$

$$= \Delta_* \prod_{j=1}^{k^*+1} \pi_j + \mathbb{P}_n[(\Delta_* \prod_{j=1}^{k^*+1} \pi_j)^{\text{EIF}}] + \Delta_*^{\text{R2}}$$

$$+ \sum_{j=1}^{k^*+1} \pi_j^{\text{R2}} + O_p(n^{-1/2})O_p(n^{-1/2})$$

$$+ O_p(n^{-1/2})o_p(n^{-1/2}) + o_p(n^{-1}) + o_p(n^{-1/2})$$

$$= \Delta_* \sum_{j=1}^{k^*+1} \pi_j + \mathbb{P}_n[(\Delta_* \prod_{j=1}^{k^*+1} \pi_j)^{\text{EIF}}] + \Delta_*^{\text{R2}} + \sum_{j=1}^{k^*+1} \pi_j^{\text{R2}} + o_p(n^{-1/2}),$$

and that

$$(\Pi_{j=1}^{k^*} \hat{\pi}_j)\hat{\Delta}_* = \left[(\pi_{k^*+1} + \mathbb{P}_n[\pi_{k^*+1}^{\text{EIF}}] + o_p(n^{-1/2}) + \pi_{k^*+1}^{\text{R2}}) \right]$$

$$\left[\Pi_{j=1}^{k^*-1} \pi_j \eta_* + \mathbb{P}_n[(\Pi_{j=1}^{k^*-1} \pi_j \eta_*)^{\text{EIF}}] + \tau_{*-1}^{\text{R2}} + \Delta_{*-1}^{\text{R2}} + \pi_*^{\text{R2}} + \tau_*^{\text{R2}} + \sum_{j \in \{1...,k^*-1\}: j \neq *} \pi_j + o_p(n^{-1/2}) \right]$$

$$= \Pi_{j=1}^{k^*} \pi_j \eta_* + \mathbb{P}_n[(\Pi_{j=1}^{k^*} \pi_j \eta_*)^{\text{EIF}}] + \tau_{*-1}^{\text{R2}} + \Delta_{*-1}^{\text{R2}} + \pi_*^{\text{R2}} + \tau_*^{\text{R2}} + \sum_{j \in \{1...,k^*\}: j \neq *} \pi_j^{\text{R2}} + o_p(n^{-1/2})$$

It follows that, for any $k \in \{1, \ldots, K\}$,

$$(\Pi_{j=1}^k \hat{\pi}_j)\hat{\Delta}_k = \Delta_k \sum_{j=1}^k \pi_j + \Pi_{j=1}^{k-1} \pi_j \eta_k + \mathbb{P}_n[(\Delta_k \sum_{j=1}^k \pi_j + \Pi_{j=1}^{k-1} \pi_j \eta_k)^{\text{EIF}}] + \sum_{j=k-1}^k \Delta_j^{\text{R2}} + \sum_{j=k-1}^k \tau_j^{\text{R2}} + \sum_{j=1}^k \pi_j^{\text{R2}}.$$

,

Thus, $\hat{\theta}_k = (\Pi_{j=1}^k \hat{\pi}_j)\hat{\Delta}_k + (\Pi_{j=1}^{k-1} \hat{\pi}_j)\hat{\eta}_k$ is semiparametric efficient if $\sum_{j=k-1}^k \Delta_j^{\text{R2}} + \sum_{j=k-1}^k \tau_j^{\text{R2}} + \sum_{j=1}^k \pi_j^{\text{R2}} = o_p(n^{-1/2})$.

G Derivation of RWR procedures

For simplicity, throughout the following I let $Z_0 = X$ and $M_0 = A$. I assume the following linear specification of the outcome model:

$$\mathbb{E}[Y \mid \overline{Z}_k, A, \overline{M}_k] = \beta_{k,0} + c_{k,0}A + \sum_{j=1}^{k} \beta_{k,j}M_j + \eta_{k,1}^\top X^\perp + c_{k,1}AX^\perp + \sum_{j=1}^{k-1} \eta_{k,j}^T M_j X^\perp + \sum_{j=1}^{k} \gamma_{k,j}^T Z_j^\perp + \sum_{j=1}^{k-1} M_j \sum_{l=}^{j}$$

$$(17)$$

where $Z_k^\perp = Z_k - \mathbb{E}[Z_k|\overline{Z}_{k-1}, M_{k-1} = 1_{k-1}]$, $\forall k \in [0, \ldots, K]$. In the following derivations, I use the fact that, $\forall k \in \{1, \ldots, K\}$,

$$\int z_k^\perp dP(z_k|\overline{z}_{k-1}, m_{k-1} = 1)$$
$$= \mathbb{E}[Z_k - \mathbb{E}[Z_k|\overline{z}_{k-1}, m_{k-1} = 1]|\overline{z}_{k-1}, m_{k-1} = 1]$$
$$= 0.$$

Letting $X = Z_0$, the above also implies that $\int z_0^\perp dP(z_0) = \mathbb{E}[Z_0 - \mathbb{E}[Z_0]] = 0$. Under sequential ignorability and assuming linearity of the outcome with respect to all antecedent variables, I have that

$$\Delta_{k-1} = \int \mathbb{E}[Y|\overline{M}_{k-1} = \overline{1}_{k-1}, \overline{z}_k, M_k = 0] \prod_{j=0}^{k} dP(z_j|\overline{z}_{j-1}, m_{j-1} = 1)$$

$$- \int \mathbb{E}[Y|\overline{M}_{k-2} = \overline{1}_{k-2}, \overline{z}_k, M_{k-1} = 0] \prod_{j=1}^{k} dP(z_j|\overline{z}_{j-1}, m_{j-1} = 1)$$

$$= \int \left[\beta_{k,0} + c_{k,0} + \sum_{j=1}^{k-1} \beta_{k,j} + \eta_{k,1}^\top X^\perp + c_{k,1}X^\perp + \sum_{j=1}^{k-2} \eta_{k,j}^T X^\perp + \sum_{j=1}^{k} \gamma_{k,j}^T Z_j^\perp + \sum_{j=1}^{k-2}\sum_{l=1}^{j} \xi_{k,k,l}^\top Z_l^\perp) \right.$$

$$\left. - (\beta_{k,0} + c_{k,0} + \sum_{j=1}^{k-2} \beta_{k,j} M_j + \eta_{k,1}^\top X^\perp + c_{k,1}AX^\perp + \sum_{j=1}^{k-2} \eta_{k,j}^T X^\perp + \sum_{j=1}^{k} \gamma_{k,j}^T Z_j^\perp + \sum_{j=1}^{k-3}\sum_{l=1}^{j} \xi_{k,k,l}^\top Z_l^\perp) \right]$$

$$\prod_{j=0}^{k} dP(z_j|\overline{z}_{j-1}, m_{j-1} = 1)$$

$$= \beta_{k,k-1}.$$

Further, for $\tau_k \forall k \in \{1, \ldots, K\}$ I have that

$$\tau_k = \int \mathbb{E}[Y|A=1, \overline{M}_k = \overline{1}_k, \overline{z}_k] \prod_{j=0}^{k} dP(z_j|\overline{z}_{j-1}, m_{j-1} = 1)$$

$$- \int \mathbb{E}[Y|A=1, \overline{M}_{k-1} = \overline{1}_{k-1}, \overline{z}_k, M_k = 0] \prod_{j=0}^{k} dP(z_j|\overline{z}_{j-1}, m_{j-1} = 1)$$

$$= \int \left[(\beta_{k,0} + c_{k,0} + \sum_{j=1}^{k} \beta_{k,j} + \eta_{k,1}^{\top} X^{\perp} + c_{k,1} X^{\perp} + \sum_{j=1}^{k-1} \eta_{k,j}^{T} X^{\perp} + \sum_{j=1}^{k} \gamma_{k,j}^{T} Z_j^{\perp} + \sum_{j=1}^{k-1} \sum_{l=1}^{j} \xi_{k,j,l}^{\top} Z_l^{\perp}) \right.$$

$$\left. - (\beta_{k,0} + c_{k,0} + \sum_{j=1}^{k-1} \beta_{k,j} + \eta_{k,1}^{\top} X^{\perp} + c_{k,1} X^{\perp} + \sum_{j=1}^{k-2} \eta_{k,j}^{T} X^{\perp} + \sum_{j=1}^{k} \gamma_{k,j}^{T} Z_j^{\perp} + \sum_{j=1}^{k-2} \sum_{l=1}^{j} \xi_{k,j,l}^{\top} Z_l^{\perp}) \right]$$

$$\prod_{j=0}^{k} dP(z_j|\overline{z}_{j-1}, m_{j-1} = 1)$$

$$= \beta_{k,k}.$$

Finally, I assume that

$$\mathbb{E}[M_{k+1} \mid A=1, \overline{Z}_k, \overline{M}_k = \overline{1}_k] = \theta_0 + \sum_{k=0}^{k} \delta_{k+1}^{T} Z_k^{\perp} \quad .$$

Then:

$$\mathbb{E}[M(\overline{1}_{k+1})] = \theta_0 + \int \left[\sum_{k=0}^{k} \delta_{k+1}^{T} Z_k^{\perp} \prod_{j=0}^{k} dP(z_j|\overline{z}_{j-1}, m_{j-1} = 1) \right]$$

$$= \theta_0.$$

H Further details about simulation study

For each component involved in the decomposition, I construct a Neyman-orthogonal "signal" using its EIF. The recentered EIFs for each component are shown below:

$$M^*(1) = \gamma_1(X) + \frac{\mathbb{I}(A = 1)}{\pi_0(X, 1)}(M - \gamma_1(X)),$$

$$Y^*(a) = \mu_0(X, a) + \frac{\mathbb{I}(A = a)}{\pi_0(X, a)}(Y - \mu_0(X, a)), \text{ for } a \in \{0, 1\}$$

$$Y^*(1, m_1) = \nu_1(X, m) + \frac{\mathbb{I}(A = 1)\mathbb{I}(M = m)}{\pi_0(X, 1)\pi_1(X, m_1)}(Y - \mu_1(X, Z, m))$$

$$+ \frac{\mathbb{I}(A = 1)}{\pi_0(X, 1)}(\mu_1(X, Z, m) - \nu_1(X, m)), \text{ for } m \in \{0, 1\}$$

where

$$\pi_0(X, a) \triangleq \Pr[A = a \mid X]$$

$$\pi_1(X, m_1) \triangleq \Pr[M = m \mid X, A = 1]$$

$$\gamma_1(X) \triangleq \mathbb{E}[M \mid X, A = 1]$$

$$\mu_0(X, a) \triangleq \mathbb{E}[Y|X, A = a]$$

$$\mu_1(X, Z, m) \triangleq \mathbb{E}[Y|X, A = 1, Z, M = m]$$

$$\nu_1(X, m) \triangleq \mathbb{E}[\mu_1(X, Z, m)|X, A = 1].$$

I Derivation of bias formulae for sensitivity analysis

In this section, I derive the bias formulae for the set (τ_k, Δ_k) for all $k \in [K]$, where K denotes the number of mediators considered in the decomposition, under a sequence of simplifying assumptions. Assume first that I have a binary unobserved confounder, U, for the treatment-outcome relationship. Assuming that $\alpha_0 = \mathbb{E}[Y|x, a, U = 1] - \mathbb{E}[Y|x, a, U = 0]$ does not depend on x or a, and further that $\beta_0 = \Pr[U = 1|x, A = 1] - \Pr[U = 1|x, A = 0]$ does depend on x, for $\tau_0 = \mathbb{E}[Y(1) - Y(0)] \triangleq$ ATE, I then have that bias$(\tau_0) = \alpha\beta$ (VanderWeele and Arah, 2011).

Next, consider an unobserved confounder, U_k that affects both M_k and Y for any $k \in \{1, \ldots, K\}$. Then, under a weaker iteration of Assumption 3 (Sequential Ignorability), i.e., $Y(\overline{1}_k, m_k) \perp\!\!\!\perp (A, \overline{M}_k)|X, A, U_k, \overline{Z}_k, \overline{M}_{k-1} \forall k \in [K], \mathbb{E}[Y(\overline{1}_k, m_k)]$ is identified as

$$\mathbb{E}[Y(\overline{1}_k, m_k)] = \int_x \int_{\overline{z}_k} \mathbb{E}[Y|x, \overline{z}_k, \overline{1}_k, m_k, u_k] \Big[dP(u_k|x, \overline{z}_k, \overline{1}_{k-1}) \prod_{j=1}^{k} dP(z_j|x, \overline{z}_{j-1}, \overline{1}_{j-1})\Big] dP(x).$$

By contrast, under Assumption 3, my estimator of $\mathbb{E}[Y(\overline{1}_k, m_k)]$, $\tilde{\mathbb{E}}[Y(\overline{1}_k, m_k)]$, converges to

$$\tilde{\mathbb{E}}[Y(\overline{1}_k, m_k)] = \int_x \int_{\overline{z}_k} \mathbb{E}[Y|x, \overline{z}_k, \overline{1}_k, m_k, u_k] \Big[dP(u_k|x, \overline{z}_k, \overline{1}_k) \prod_{j=1}^{k} dP(z_j|x, \overline{z}_{j-1}, \overline{1}_{j-1})\Big] dP(x).$$

I invoke the following three assumptions: (Assumption A_k) $\alpha_k = \mathbb{E}[Y|x, \overline{z}_k, \overline{1}_k, m_k, U_k = 1] - \mathbb{E}[Y|x, \overline{z}_k, \overline{1}_k, m_k, U_k = 0]$ does not depend on $(x, \overline{z}_k, \overline{1}_k, m_k)$; (Assumption B_k) $\beta_k = \Pr[U_k = 1|x, \overline{z}_k, \overline{1}_k, m_k] - \Pr[U_k = 1|x, \overline{z}_k, \overline{1}_k])$ does not depend on $(x, \overline{z}_k)$; Assumption (C) U_k is binary. Taking the difference between the quantities in the above two equations thus gives that, for any $m_k \in \{0, 1\}$, for any $k \in [K]$, I have that

$$\text{bias}(\tilde{\mathbb{E}}[Y(\overline{1}_k, m_k)]) = \int \Big(\mathbb{E}[Y|x, \overline{z}_k, \overline{1}_k, m_k, U_k = 1] - \mathbb{E}[Y|x, \overline{z}_k, \overline{1}_k, m_k, U_k = 0]\Big) \cdot$$

$$\Big(\Pr[U_k = 1|x, \overline{z}_k, \overline{1}_k, m_k] - \Pr[U_k = 1|x, \overline{z}_k, \overline{1}_k]\Big) \prod_{j=1}^{k} dP(z_j|x, \overline{z}_{j-1}, \overline{1}_{j-1})\Big] dP(x). \tag{18}$$

Consider first bias$(\Delta_{k-1}) = \text{bias}(\tilde{\mathbb{E}}[Y(\overline{1}_k, 0) - Y(\overline{1}_{k-1}, 0)])$. Under mediator monotonicity (Assumption 1), I immediately have that $\Pr[U_k = 1|x, \overline{z}_k, \overline{1}_k, m_k] - \Pr[U_k = 1|x, \overline{z}_k, \overline{1}_k])$, and thus that bias$(\Delta_{k-1}) = \text{bias}(\tilde{\mathbb{E}}[Y(\overline{1}_k, 0)])$, which can be written as

$$\text{bias}(\tilde{\mathbb{E}}[Y(\overline{1}_k, 0)]) = \int \Big(\mathbb{E}[Y|x, \overline{z}_k, \overline{1}_k, 0, U_k = 1] - \mathbb{E}[Y|x, \overline{z}_k, \overline{1}_k, 0, U_k = 0]\Big)$$

$$\Big(\text{Pr}[U_k = 1|x, \overline{z}_k, \overline{1}_k, 0] - \text{Pr}[U_k = 1|x, \overline{z}_k, \overline{1}_k]\Big) \prod_{j=1}^{k} dP(z_j|x, \overline{z}_{j-1}, \overline{1}_{j-1})\Big] dP(x)$$

$$= \int \Big(\mathbb{E}[Y|x, \overline{z}_k, \overline{1}_k, 0, U_k = 1] - \mathbb{E}[Y|x, \overline{z}_k, \overline{1}_k, 0, U_k = 0]\Big) \cdot$$

$$\Big(\text{Pr}[U_k = 1|x, \overline{z}_k, \overline{1}_k, 0] - \big(\text{Pr}[U_k = 1|x, \overline{z}_k, \overline{1}_{k+1}]\text{Pr}[M_k = 1|x, \overline{z}_k, \overline{1}_k]$$

$$+ \text{Pr}[U_k = 1|x, \overline{z}_k, \overline{1}_k, 0] - \text{Pr}[U_k = 1|x, \overline{z}_k, \overline{1}_k, 0]\text{Pr}[M_k = 1|x, \overline{z}_k, \overline{1}_k]\big)\Big)$$

$$\prod_{j=1}^{k} dP(z_j|x, \overline{z}_{j-1}, \overline{1}_{j-1})\Big] dP(x)$$

$$= - \int \Big(\mathbb{E}[Y|x, \overline{z}_k, \overline{1}_k, 0 U_k = 1] - \mathbb{E}[Y|x, \overline{z}_k, \overline{1}_k, 0_k, U_k = 0]\Big) \cdot$$

$$\Big(\big(\text{Pr}[U_k = 1|x, \overline{z}_k, \overline{1}_{k+1}] - \text{Pr}[U_k = 1|x, \overline{z}_k, \overline{1}_k, 0]\big)\text{Pr}[M_k = 1|x, \overline{z}_k, \overline{1}_k]\Big)$$

$$\prod_{j=1}^{k} dP(z_j|x, \overline{z}_{j-1}, \overline{1}_{j-1})\Big] dP(x).$$

Next, applying assumptions A_k and B_k, I can write

$$\text{bias}(\tilde{\mathbb{E}}[Y(\overline{1}_{k+1}, 0)]) = -\alpha_k \beta_k \int_x \int_{\overline{z}_k} \text{Pr}[M_k = 1|x, \overline{z}_k, \overline{1}_k] \prod_{j=1}^{k} dP(z_j|x, \overline{z}_{j-1}, \overline{1}_{j-1})\Big] dP(x).$$

Second, to compute $\text{bias}(\tau_k) = \text{bias}(\mathbb{E}[Y(\overline{1}_{k+1}) - Y(\overline{1}_k, 0)])$ for any $k \in \{1, \dots K\}$, beginning with Equation 18 and applying assumptions A_k and B_k once again, I have that

$$\text{bias}(\tau_k) = \alpha_k \beta_k.$$

J Description of EIFs used in empirical illustration

For each component involved in the MPSE, I construct a Neyman-orthogonal "signal" using its EIF, whose exact form depends on whether each set of intermediate confounders is empty or not. Figure 2 in the main text shows a potential data-generating process for the direct and indirect (continuation) effects of high school graduation on adult earnings, via three transitions: college attendance (M_1), BA completion (M_2), and graduate school attendance (M_3). I assume that a set of pre-college characteristics serve as confounders for the $A - (M_1, M_2, M_3, Y)$ relationships, and that a set of post-secondary confounders Z confound the $M_2 - (M_3, Y)$ relationships.

Under these assumptions for the various sets of confounders, my MPSE decomposition implies that, in the case of the four transitions (one treatment and three mediators), it suffices to estimate the following three sets of parameters: (i) four direct effects $\Delta_k, k \in [0 \ldots, 3]$, where $\Delta_3 = \tau_3$, (ii) four gross effects $\tau_k, k \in [0 \ldots, 3]$, where $\tau_0 = \text{ATE}$, (iii) three mediator terms, $\pi_k, k \in [1 \ldots, 3]$. All components in the three-mediator decomposition can then be estimated as functions of these parameters. For each of these target parameters, I construct a Neyman-orthogonal signal using its efficient influence function. Because of my assumed data-generating process, which maintains that there is only a single set of intermediate confounders (as opposed to a separate set of confounders for each mediator), the EIF for each estimand involved in the decomposition simplifies somewhat. Specifically, the recentered EIFs for each component in the decomposition are shown below:

$$M_1^*(1) = \gamma_1(X) + \frac{\mathbb{I}(A=1)}{\pi_0(X,1)}(M_1 - \gamma_1(X)),$$

$$M_2^*(1,1) = \gamma_2(X) + \frac{\mathbb{I}(A=1)\mathbb{I}(M_1=1)}{\pi_0(X,1)\pi_1(X,1)}(M_2 - \gamma_2(X)),$$

$$\begin{aligned}
M_3^*(1,1,1) = {} & \mathbb{E}[\gamma_3(X,Z)|X, A=1, M_1=1] \\
& + \frac{\mathbb{I}(A=1)\mathbb{I}(M_1=1)}{\pi_0(X,1)\pi_1(X,1)}(\gamma_3(X,Z) - \mathbb{E}[\gamma_3(X,Z)|X, A=1, M_1=1]) \\
& + \frac{\mathbb{I}(A=1)\mathbb{I}(M_1=1)\mathbb{I}(M_2=1)}{\pi_0(X,1)\pi_1(X,1)\pi_2(X,Z,1)}(M_3 - \gamma_3(X,Z)),
\end{aligned}$$

$$Y^*(a) = \mu_0(X,a) + \frac{\mathbb{I}(A=a)}{\pi_0(X,a)}(Y - \mu_0(X,a)), \text{ for } a \in \{0,1\}$$

$$Y^*(1,m_1) = \mu_1(X,m_1) + \frac{\mathbb{I}(A=1)\mathbb{I}(M_1=m_1)}{\pi_0(X,1)\pi_1(X,m_1)}(Y - \mu_1(X,m_1)), \text{ for } m_1 \in \{0,1\}$$

$$\begin{aligned}
Y^*(1,1,m_2) = {} & \mathbb{E}[\mu_2(X,Z,m_2)|X, A=1, M_1=1] \\
& + \frac{\mathbb{I}(A=1)\mathbb{I}(M_1=1)}{\pi_0(X,1)\pi_1(X,1)}(\mu_2(X,Z,m_2) - \mathbb{E}[\mu_2(X,Z,m_2)|X, A=1, M_1=1])
\end{aligned}$$

$$+ \frac{\mathbb{I}(A = 1)\mathbb{I}(M_1 = 1)\mathbb{I}(M_2 = m_2)}{\pi_0(X, 1)\pi_1(X, 1)\pi_2(X, Z, m_2)}(Y - \mu_2(X, Z, m_2)), \text{ for } m_2 \in \{0, 1\}$$

$$Y^*(1, 1, 1, m_3) = \mathbb{E}[\mu_3(X, Z, m_3)|X, A = 1, M_1 = 1]$$

$$+ \frac{\mathbb{I}(A = 1)\mathbb{I}(M_1 = 1)}{\pi_0(X, 1)\pi_1(X, 1)}(\mu_3(X, Z, m_3) - \mathbb{E}[\mu_3(X, Z, m_3)|X, A = 1, M_1 = 1])$$

$$+ \frac{\mathbb{I}(A = 1)\mathbb{I}(M_1 = 1)\mathbb{I}(M_2 = 1)\mathbb{I}(M_3 = m_3)}{\pi_0(X, 1)\pi_1(X, 1)\pi_2(X, Z, 1)\pi_3(X, Z, m_3)}(Y - \mu_3(X, Z, m_3)) \text{ for } m_3 \in \{0, 1\},$$

where

$$\pi_0(X, a) \triangleq \Pr[A = a \mid X]$$

$$\pi_1(X, m_1) \triangleq \Pr[M_1 = m_1 \mid X, A = 1]$$

$$\pi_2(X, Z, m_2) \triangleq \Pr[M_2 = m_2 \mid X, A = 1, M_1 = 1, Z]$$

$$\pi_3(X, Z, m_3) \triangleq \Pr[M_3 = m_3 \mid X, A = 1, M_1 = 1, Z, M_2 = 1]$$

$$\gamma_1(X) \triangleq \mathbb{E}[M_1 \mid X, A = 1]$$

$$\gamma_2(X) \triangleq \mathbb{E}[M_2 \mid X, A = 1, M_1 = 1]$$

$$\gamma_3(X, Z) \triangleq \mathbb{E}[M_3 \mid X, A = 1, M_1 = 1, Z, M_2 = 1]$$

$$\mu_0(X, a) \triangleq \mathbb{E}[Y|X, A = a]$$

$$\mu_1(X, m_1) \triangleq \mathbb{E}[Y|X, A = 1, M_1 = m_1]$$

$$\mu_2(X, Z, m_2) \triangleq \mathbb{E}[Y|X, A = 1, M_1 = 1, Z, M_2 = m_2]$$

$$\mu_3(X, Z, m_3) \triangleq \mathbb{E}[Y|X, A = 1, M_1 = 1, Z, M_2 = 1, M_3 = m_3].$$