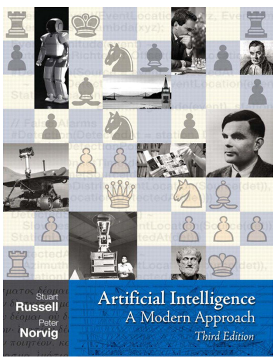
TDT4171 Artificial Intelligence Methods

Exercise 2

January 28, 2020

- **Delivery deadline: February 12, 2020** by 23:59 sharp. No late delivery will be graded! Deadline extensions will only be considered for extraordinary situations such as family or health-related circumstances. However, these circumstances must be documented, e.g., with a doctor's note("legeerklæring"). Having a lot of work in other classes is not a legitimate excuse for late delivery. Also, happy events such as birthdays, weddings, and vacations are not valid excuses as they are typically known well in advance, so you will need to plan your work (and delivery) schedule around them.
- Required reading for this assignment: Chapter 15 (the parts in the curriculum)
- Deliver your solution on *Blackboard*. Please upload your report as a **PDF file**. For the programming part, deliver the source code alongside the pdf file. Please **do not** put the source code into an archive. Just upload the code alongside the pdf file. In other words, upload pdf + source code.
- Students can NOT work in groups. Each student can only submit solution individually.
- This homework counts for 4% of the final grade.
- Cribbing("koking") from other students is not accepted, and if detected, will lead to immediate failure of the course. The consequence will apply to both the source and the one cribbing.
- For help and questions related to the assignment, ask the student assistants during the assignment hours. The assignment hours can be found on the assignment page on Blackboard. If the student assistants are not able to help you or you have other types of inquiries, an email can be sent to Yan (yanzhe.bekkemoen@ntnu.no).
- All references in this text refer to this version of the book, *Artificial Intelligence: A Modern Approach, Third Edition*, "Blue version".



Exercise

There is only one exercise in this assignment. You are going to implement the FORWARD-BACKWARD algorithm for Hidden Markov (HMM) models, and try it out on the *"Umbrella-world"* described in Figure 15.2 (p. 569) in the book. Your code is a part of your delivery, so *please put some effort in making your code as readable as possible.* It is completely up to you which programming language you want to use to implement the FORWARD-BACKWARD algorithm, but it might be a good tip to select a programming language, which offers access to powerful matrix operation (like PYTHON for instance). Recall that all required calculations can be performed as matrix operations in a HMM.

The Numpy library for Python provides a set of matrix algebra routines and types that you can use for this assignment. Using a matrix type, multiplications between matrix objects acts as ordinary matrix multiplication, and thus you don't need to reimplement matrix multiplication just to solve this assignment. See `https://docs.scipy.org/doc/numpy/reference/generated/numpy.matrix.html` for help on how to use the Numpy matrix type. Numpy: `http://www.numpy.org`

The FORWARD-BACKWARD is described in (Figure 15.4, page 576) in the book, but the description is maybe a bit superficial. It might be beneficial to consider the lecture notes from Chapter 15, too (downloadable from Blackboard).

Make sure to read the whole exercise before you start coding.

Part A

Describe the *"Umbrella domain"* as an HMM:

- What is the set of unobserved variable(s) for a given time-slice t (denoted $\mathbf{X}_t$ in the book)?
- What is the set of observable variable(s) for a given time-slice t (denoted $\mathbf{E}_t$ in the book)?
- Present the *dynamic model* $\mathbf{P}(\mathbf{X}_t|\mathbf{X}_{t-1})$ and the *observation model* $\mathbf{P}(\mathbf{E}_t|\mathbf{X}_t)$ as matrices.
- Which assumptions are encoded in this model? (*Hint:* Read page 568). Are the assumptions reasonable for this particular domain?

Part B

Implement filtering using the FORWARD operation (Equation 15.5 and Equation 15.12). Note that this can be done with simple matrix operations in the HMM.

- Verify your implementation by calculating $\mathbf{P}(\mathbf{X}_2|\mathbf{e}_{1:2})$, where $\mathbf{e}_{1:2}$ is the evidence that the Umbrella was used both on day 1 and day 2. The desired result is (confer the slides from the lecture available on Blackboard) that the probability of rain at day 2 (after the observations) is 0.883.

- Use your program to calculate the probability of rain at day 5 given the sequence of observations $\mathbf{e}_{1:5} = \{\texttt{Umbrella}_1 = \texttt{true}, \texttt{Umbrella}_2 = \texttt{true}, \texttt{Umbrella}_3 = \texttt{false}, \texttt{Umbrella}_4 = \texttt{true}, \texttt{Umbrella}_5 = \texttt{true}\}$. Document your answer by showing all *normalized* forward messages (in the book the un-normalized forward messages are denoted $\mathbf{f}_{1:k}$ for $k = 1, 2, \ldots, 5$).

Part C

Implement smoothing using the FORWARD-BACKWARD algorithm (Figure 15.4, page 576). Note that also this can be done with simple matrix operations.

- Verify your implementation by calculating $\mathbf{P}(\mathbf{X}_1|\mathbf{e}_{1:2})$ where $\mathbf{e}_{1:2}$ is the evidence that the umbrella was used the first two days (as in Part (b) of the assignment). The desired result is $\mathbf{P}(\mathbf{X}_1|\mathbf{e}_{1:2}) = \langle 0.883, 0.117\rangle$.
- Use your FORWARD-BACKWARD algorithm to calculate the probability of rain at day 1 given the sequence of observations $\mathbf{e}_{1:5} = \{\texttt{Umbrella}_1 = \texttt{true}, \texttt{Umbrella}_2 = \texttt{true}, \texttt{Umbrella}_3 = \texttt{false}, \texttt{Umbrella}_4 = \texttt{true}, \texttt{Umbrella}_5 = \texttt{true}\}$, i.e., $\mathbf{P}(\mathbf{X}_1|\mathbf{e}_{1:5})$. Document your answer by showing all backward messages ($\mathbf{b}_{k+1:t}$ for $k = 1, 2, \ldots, 5$).

Part D

Make sure your code is readable (add comments where appropriate), and deliver it as part of your solution. Delivery procedure is described in the header of this assignment.