Assignment 2
TDT4171 Artificial Intelligence Methods

ALEKSANDER ELVEBAKK

February 12, 2020

Contents

1 Part A- describe the umbrella domain as a HMM

1.1 Unobserved variables

The unobserved variables at time-slice t ($\mathbf{X}_t$) is Rain_t. That is- whether its actually raining.

1.2 Observed variables

The set of observable variables at time-slice t, ($\mathbf{E}_t$) Umbrella_t. That is whether the boss carries an umbrella to work that day.

1.3 Dynamic model, observation model and assumptions

When describing the umbrella world as a (first-order) Markov process, we assume the future is conditionally independent of the past. We also assume that the model is stationary, meaning that it changes the same from day to day. From this we draw the following **Markov assumption**

$$\mathbf{P}(\mathbf{X}_t|\mathbf{X}_{0:t-1}) = \mathbf{P}(\mathbf{X}_t|\mathbf{X}_{t-1})$$

We also make the following **sensor Markov assumption**

$$\mathbf{P}(\mathbf{E}_t|\mathbf{X}_{0:t}, \mathbf{E}_{0:t-1}) = \mathbf{P}(\mathbf{E}_t|\mathbf{X}_t)$$

With the prior probability distribution at time 0, $\mathbf{P}(\mathbf{X}_0)$ we have a specification of the complete joint distribution over all the variables.

$$\mathbf{P}(\mathbf{X}_{0:t}, \mathbf{E}_{1:t}) = \mathbf{P}(\mathbf{X}_0)\prod_{i=1}^{t}\mathbf{P}(\mathbf{X}_i|\mathbf{X}_{i-1})\mathbf{P}(\mathbf{E}_i|\mathbf{X}_i)$$

Since both $\mathbf{X}_t$ and $\mathbf{E}_t$ can take two values TRUE or FALSE (hereby denoted t and f), the transition model (the dynamic model) and the sensor model (the observation model) are two 2x2 matrices.

$$\mathbf{P}_k = \pi_t \mathbf{P}_{k-1}$$

$$\pi_t = \begin{bmatrix} \mathbf{P}(\mathbf{X_i} = t|\mathbf{X}_{i-1} = t) & \mathbf{P}(\mathbf{X_i} = f|\mathbf{X}_{i-1} = t) \\ \mathbf{P}(\mathbf{X_i} = t|\mathbf{X}_{i-1} = f) & \mathbf{P}(\mathbf{X_i} = f|\mathbf{X}_{i-1} = f) \end{bmatrix} = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$$

We know the evidence variable so for $\mathbf{E}_i = \text{true}$

$$\pi_s = \begin{bmatrix} \mathbf{P}(\mathbf{E_i} = t|\mathbf{X}_i = t) & 0 \\ 0 & \mathbf{P}(\mathbf{E_i} = t|\mathbf{X}_i = f) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.2 \end{bmatrix}$$

and for $\mathbf{E}_i = \text{false}$ we have

$$\pi_s = \begin{bmatrix} \mathbf{P}(\mathbf{E_i} = f|\mathbf{X}_i = t) & 0 \\ 0 & \mathbf{P}(\mathbf{E_i} = f|\mathbf{X}_i = f) \end{bmatrix} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.8 \end{bmatrix}$$

These models are never completely accurate, but in many settings, they are good/sufficient to capture the broad image.

2 Part B

```
aleksander@Aleks-Ubuntu:~/Documents/AI2/Ex2$ python3.7 f_b_ai2.py

Day:  0
Probabilities:
 [0.5 0.5]

Day:  1
Probabilities:
 [0.81818182 0.18181818]

Day:  2
Probabilities:
 [0.88335704 0.11664296]

Day:  3
Probabilities:
 [0.19066794 0.80933206]

Day:  4
Probabilities:
 [0.730794 0.269206]

Day:  5
Probabilities:
 [0.86733889 0.13266111]
```

Figure 1: Only forward calculations

3 part C

```
aleksander@Aleks-Ubuntu:~/Documents/AI2/Ex2$ python3.7 f_b_ai2.py

Printing only forward:


Day:  0
Probabilities:
 [0.5 0.5]

Day:  1
Probabilities:
 [0.81818182 0.18181818]

Day:  2
Probabilities:
 [0.88335704 0.11664296]

Day:  3
Probabilities:
 [0.19066794 0.80933206]

Day:  4
Probabilities:
 [0.730794 0.269206]

Day:  5
Probabilities:
 [0.86733889 0.13266111]


Posterior at time step 5                        [0.86733889 0.13266111]
backwardsMessage at time step  5                [1. 1.]
smoothed probabilities at time step 5           [0.86733889 0.13266111]

Posterior at time step 4                        [0.730794 0.269206]
backwardsMessage at time step  4                [0.62727273 0.37272727]
smoothed probabilities at time step 4           [0.82041905 0.17958095]

Posterior at time step 3                        [0.19066794 0.80933206]
backwardsMessage at time step  3                [0.65334282 0.34665718]
smoothed probabilities at time step 3           [0.30748358 0.69251642]

Posterior at time step 2                        [0.88335704 0.11664296]
backwardsMessage at time step  2                [0.37626718 0.62373282]
smoothed probabilities at time step 2           [0.82041905 0.17958095]

Posterior at time step 1                        [0.81818182 0.18181818]
backwardsMessage at time step  1                [0.5923176 0.4076824]
smoothed probabilities at time step 1           [0.86733889 0.13266111]

Posterior at time step 0                        [0.5 0.5]
backwardsMessage at time step  0                [0.64693556 0.35306444]
smoothed probabilities at time step 0           [0.64693556 0.35306444]
```

Figure 2: forward and backward