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In[2582]:= Clear["Global`*"]
Needs["Notation`"]
Needs["PlotLegends`"]
Needs["CustomTicks`"]
Msun = 2 × 1033;
Mdotsol =  $\left( \frac{Msun}{3.15 \times 10^7} \right)$ 

G = 6.67 × 10-8;
c = 3 × 1010;
σ = 5.67 × 10-5;
kb = 1.38 × 10-16;
mp = 1.67 × 10-24;
me = 9.11 × 10-27;
h = 6.63 × 10-27

Symbolize[  $\dot{M}$  ]

Symbolize[  $\hat{\kappa}$  ]

Symbolize[  $M_7$  ]

Symbolize[  $\alpha_{0.3}$  ]

Symbolize[  $\dot{m}$  ]

Symbolize[  $\dot{M}_{\text{Edd}}$  ]

Symbolize[  $L_{\text{Edd}}$  ]

Symbolize[  $R_s$  ]

Symbolize[  $\epsilon_{0.1}$  ]

Symbolize[  $q_{-3}$  ]

Symbolize[  $f_{-2}$  ]

Symbolize[  $\alpha_{-1}$  ]

Symbolize[  $\dot{m}_{-1}$  ]

fiducial = {α0.3 → 1, μ0 → 0.615, μe → 0.875, ε0.1 → 1,  $\dot{m}$  → 0.1, κ0 → 1, ft → 3 / 8, b → 1}

M = 107 Msun M7;

κ = μe 0.4 κ0;
α = 0.3 α0.3;
m = μ0 mp;

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(*Get rid of explicit opacity dependence in the Eddington
Luminosity. This is for consistency with Goodman 2003 Goodman &
Tan 2004 -- i.e. to get a consistent opacity dependence in the . Not
to mention that the expression which I had for the,
central temperature implicitly sets the opacity to be the electron scattering in  $\dot{M}$ *)

$$L_{\text{Edd}} = 4 \pi G \frac{M}{0.4 \mu e} c ;$$



$$\dot{M}_{\text{Edd}} = \frac{L_{\text{Edd}}}{c^2 \epsilon_{0.1} 0.1} ;$$



$$\dot{M} = \dot{m} \dot{M}_{\text{Edd}} ;$$



$$R_s = 2 G \frac{M}{c^2} ;$$



$$R = 10^3 R_s r3 ;$$



$$Q = G \frac{M}{R^3} ;$$



$$T_{\text{eff}} = \left( \frac{3}{8 \pi \sigma} \frac{G M \dot{M}}{R^3} \right)^{1/4} // \text{PowerExpand} // \text{Simplify} ;$$


Print["Teff= ", Teff]

(*Tc=8 104  $\mu_0^{1/5}$   $\mu e^{-1/5}$   $r3^{-9/10}$   $M_7^{-1/5}$   $\alpha_{0.3}^{-1/5}$   $f_T^{1/5}$   $\left(\frac{\dot{m}}{\epsilon_{0.1}}\right)^{2/5} \hat{\kappa}^{1/5} ; *$ )


$$(* \sqrt{k b \frac{T_c}{\mu_0 \text{ mp}}} *)$$



$$T_c = \left( (\kappa m) / (16 \pi^2 \alpha \beta^{b-1} \sigma k b) \right)^{1/5} \dot{M}^{2/5} Q^{3/10} f_T^{1/5} // \text{PowerExpand}[\#] \& ;$$


Print["Tc= ", Tc]

(*Get different dependences on  $\beta$  and on  $f_T$  from 2 seemingly equivalent expressions*)


$$\Sigma = \frac{2}{\kappa f_T} \frac{T_c^4}{T_{\text{eff}}^4} // \text{PowerExpand}[\#(*, \text{Assumptions} \rightarrow$$


$$\{\beta > 0, b > 0, M_7 > 0, r3 > 0, \mu_0 > 0, \mu e > 0, \dot{m} > 0, \epsilon_{0.1} > 0, f_T > 0, \alpha_{0.3} > 0, \kappa_0 > 0\} *)] \& // \text{Simplify} ;$$



$$\Sigma_2 = \frac{\dot{M}}{3 \pi 0.3 \alpha_{0.3} \beta^b \left( \frac{1}{\beta} k b \frac{T_c}{m} \right)} \left( G \frac{M}{R^3} \right)^{1/2} ;$$


Print[" $\Sigma =$  ",  $\Sigma$ ]

(*H=  $\left( \frac{1}{R_s} \frac{(\beta k b T_c / m)^{(1/2)}}{Q^{(1/2)}} \right) // \text{PowerExpand}[\#,$ 

$$\text{Assumptions} \rightarrow \{\beta > 0, b > 0, M_7 > 0, r3 > 0, \mu_0 > 0, \mu e > 0, \dot{m} > 0, \epsilon_{0.1} > 0, f_T > 0, \alpha_{0.3} > 0, \kappa_0 > 0\} \& ; *)$$



$$H = \left( \frac{(\beta k b T_c / m)^{(1/2)}}{Q^{(1/2)}} /. \beta \rightarrow 1 \right) // \text{PowerExpand}[\#(*, \text{Assumptions} \rightarrow$$


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{β>0,b>0, M7>0, r3>0, μ0>0, μe>0, ṁ>0, ε0.1>0, ft>0, α0.3>0, κ0>0}*)] & // Simplify;

Hi = 10 Rs κ0  $\frac{\dot{m}}{\epsilon_{0.1}}$  ft;

Print[" $\frac{H}{R_s}$  (inner region)= ",  $\frac{Hi}{R_s}$ ]

Print[" $\frac{H}{R_s}$  (middle region)= ",  $\frac{H}{R_s}$ ]

(*Teff (0.4 μe  $\hat{\kappa}_2^{\Sigma} f_T$ )1/4 //
PowerExpand[#, Assumptions→{M7>0, r3>0, μ0>0, μe>0, ṁ>0, ε0.1>0, fT>0, α0.3>0}]&*)

(*β= (3  $\frac{c}{8 \sigma}$  ( $\frac{kb}{\mu_0 \text{ mp}}$ )1/2  $\Sigma \frac{\sqrt{Q}}{T_C^{7/2}}$ )2 //
Simplify[#, Assumptions→{M7>0, r3>0, μ0>0, α0.3>0, μe>0, ṁ>0,  $\hat{\kappa}$ >0, ε0.1>0, fT>0}]&
 $\Sigma \sqrt{Q} / \left( 2 \sqrt{\beta kb \frac{T_C}{\mu_0 \text{ mp}}} \right)$  //
Simplify[#, Assumptions→{M7>0, r3>0, μ0>0, α0.3>0, μe>0, ṁ>0,  $\hat{\kappa}$ >0, ε0.1>0, fT>0}]&*)

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In[2630]:=

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βeqn =  $\frac{\sqrt{\beta}}{1-\beta}$  ==  $\left( 3 \frac{c}{8 \sigma} \left( \frac{kb}{\mu_0 \text{ mp}} \right)^{1/2} \Sigma \frac{\sqrt{Q}}{T_C^{7/2}} \right)$  // PowerExpand // Simplify

inner = r3 /. ((Solve[βeqn /. {β → 0.5}, r3]) /. b → 1);
inner1 = inner[[1]] /. fiducial;
H1 = H /. fiducial
Hi1 = Hi /. fiducial

H2 = If[r3 < inner1, Log[10,  $\frac{Hi1}{R}$ ], Log[10,  $\frac{H1}{R}$ ]]

(*ContourPlot[(Log[10,  $\frac{Hi1}{R}$ ]) /. {r3→10x, M7→10y}), {x, -1, 1}, {y, -2, 2},
FrameLabel→{"Log[r3]", "Log[M7]"}, ContourShading→False, ContourLabels→True ]*)
ContourPlot[(H2 /. {r3 → 10x, M7 → 10y}) /. {r3 → 10x, M7 → 10y}, {x, -1, 1}, {y, -2, 2},
FrameLabel → {"Log[r3]", "Log[M7]"}, ContourShading → False, ContourLabels → True ]

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In[2637]:=

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Teff1 = Teff /. fiducial;
Tc1 = Tc /. fiducial;
Σ1 = Σ /. fiducial;
Q1 = Q /. fiducial;

Σ1 /. {r3 → 0.1, M7 → 100}
Σ1 /. {r3 → 1, M7 → 0.01}
Q1 /. {r3 → 0.1, M7 → 0.01}
Q1 /. {r3 → 1, M7 → 100}

Needs["PlotLegends`"]
ΣMax = Log[10, Σ1] /. {r3 → 0.1, M7 → 100};
ΣMin = Log[10, Σ1] /. {r3 → 1, M7 → 0.01};
QMax = Log[10, Q1] /. {r3 → 0.1, M7 → 0.01};
QMin = Log[10, Q1] /. {r3 → 1, M7 → 100};
TMin = Log[10, Teff1] /. {r3 → 1, M7 → 100};
TMax = Log[10, Teff1] /. {r3 → 0.1, M7 → 0.01};

ΣLegend = Graphics[
  Legend[Function[{x}, ColorData["Rainbow"][x]], 50, NumberForm[ΣMin, 2] // ToString,
    NumberForm[ΣMax, 2] // ToString, LegendShadow → False, LegendBorderSpace → 2]];
QLegend = Graphics[Legend[Function[{x}, ColorData["Rainbow"][x]], 50,
  NumberForm[QMin, 2] // ToString, NumberForm[QMax, 2] // ToString,
  LegendShadow → False, LegendBorderSpace → 2]];
TeffLegend = Graphics[Legend[Function[{x}, ColorData["Rainbow"][x]], 50,
  NumberForm[TMin, 2] // ToString, NumberForm[TMax, 2] // ToString,
  LegendShadow → False, LegendBorderSpace → 2]];

SetOptions[ContourPlot, ImageSize → Medium];
rainbow = Function[{x}, ColorData["Rainbow"][x]]
GraphicsRow[
  GraphicsRow /@ {{ContourPlot[Log[10, Σ1] /. {r3 → 10x, M7 → 10y}, {x, -1, 0}, {y, -2, 2},
    PlotLabel → "Σ Plot", FrameLabel → {"Log[r3]", "Log[M7"]},
    ColorFunction → rainbow, ContourShading → False, ContourLabels → True}},
  {ContourPlot[Log[10, Q1] /. {r3 → 10x, M7 → 10y}, {x, -1, 0}, {y, -2, 2},
    PlotLabel → "Q Plot", FrameLabel → {"Log[r3]", "Log[M7"]},
    ColorFunction → rainbow, ContourShading → False, ContourLabels → True}},
  {ContourPlot[Log[10, Teff1] /. {r3 → 10x, M7 → 10y}, {x, -1, 0},
    {y, -2, 2}, PlotLabel → "Teff Plot", FrameLabel → {"Log[r3]", "Log[M7"]},
    ColorFunction → rainbow, ContourShading → False, ContourLabels → True}}}

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In[2658]:=

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(*Note we are ignoring the order unity term  $\lambda$  and also
terms with  $\delta r_i/r_H$ . We also roughly equate the term  $(\delta x_i/r_s)$  with 0;
Trying to reproduce results from Haiman, Koscis, Loeb 2012*)
Clear[rs2]

kmg = 23 ( $\alpha_{-1}$ )1/2 ( $\dot{m}_{-1}$ )-3/8  $M_7$ -3/4 ( $q_{-3}$ )5/8 (rs2)-7/8;
a = 0.465 ( $\alpha_{-1}$ ) ( $f_{-2}$ )-5/4 ( $q_{-3}$ )-13/12 ( $M_7$ )-1/4 (rs2)5/8;
kmos = 0.97 ( $\alpha_{-1}$ )-2/11 ( $\dot{m}_{-1}$ )-1 ( $M_7$ )1/22 ( $f_{-2}$ )5/22 ( $q_{-3}$ )5/11
(rs2)39/44 (Max[ProductLog[0, -a] // Abs, ProductLog[-1, -a] // Abs])13/11;

kmou = 1.3 ( $\alpha_{-1}$ )-2 ( $\dot{m}_{-1}$ )-1 ( $M_7$ )1/2 ( $f_{-2}$ )5/2 ( $q_{-3}$ )5/2 (rs2)-1/4  $\left(1 + \left(10^{-3} \frac{q_{-3}}{3}\right)^{1/9}\right)^{-115/24}$ ;

kmou = kmou /. { $\alpha_{-1} \rightarrow 1$ ,  $\dot{m}_{-1} \rightarrow 1$ ,  $f_{-2} \rightarrow 1$ };
kmos = kmos /. { $\alpha_{-1} \rightarrow 1$ ,  $\dot{m}_{-1} \rightarrow 1$ ,  $f_{-2} \rightarrow 1$ };
kmg = kmg /. { $\alpha_{-1} \rightarrow 1$ ,  $\dot{m}_{-1} \rightarrow 1$ ,  $f_{-2} \rightarrow 1$ };

rs2 = rs2 /. Solve[Pd == 2  $\frac{\pi}{\sqrt{G \frac{M}{(rs2 \ 100 \ R_s)^3}}}$   $\frac{1}{(3600 \times 24)}$ , rs2][[1]];

(*kmos=kmos/.{M7→102}
kmou=kmou/.{M7→102}
kmg=kmg/.{M7→102}*)

sub = { $M_7 \rightarrow 1$ , Pd → 101p,  $q_{-3} \rightarrow 10^{1q+3}$ };
ContourPlot[Min[kmg /. sub,
kmou /. sub,
kmos /. sub], {lp, 0, Log[10, 3000]}, {lq, -3, -1},
FrameTicks → {LogTicks, LogTicks, LogTicks, LogTicks},
ColorFunctionScaling → False, PlotRange → All, PlotRange → All,
Contours → {2, 10, 20, 50, 100, 150}, ContourLabels → True, ContourShading → None]

sub = { $M_7 \rightarrow 10^{-2}$ , Pd → 101p,  $q_{-3} \rightarrow 10^{1q+3}$ };
ContourPlot[Min[kmg /. sub,
kmou /. sub,
kmos /. sub], {lp, -2, Log[10, 30]}, {lq, -3, -1},
FrameTicks → {LogTicks, LogTicks, LogTicks, LogTicks},
ColorFunctionScaling → False, PlotRange → All, PlotRange → All,
Contours → {2, 10, 20, 50, 100, 250, 500}, ContourLabels → True, ContourShading → None]

RegionPlot[{(kmou /. sub) == Min[kmg /. sub,
kmou /. sub, kmos /. sub], (kmg /. sub) == Min[kmg /. sub,
kmou /. sub, kmos /. sub]}, {lp, -3, Log[10, 30]},
{lq, -3, -0.75}, FrameTicks → {LogTicks, LogTicks, LogTicks, LogTicks}]

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In[2672]:= Clear[rs2]

fiducial = {α0.3 → 1, μ0 → 0.615, μe → 0.875, ε0.1 → 1, ṁ → 0.1, κ0 → 1, ft → 3/8, b → 1};
fiducial2 = {α-1 → 3, ṁ-1 → 1, f-2 → 1, q-3 → 10};

kmg = 23 (α-1)1/2 (ṁ-1)-3/8 M7-3/4 (q-3)5/8 (rs2)-7/8;
a = 0.465 (α-1) (f-2)-5/4 (q-3)-13/12 (M7)-1/4 (rs2)5/8;
kmos = 0.97 (α-1)-2/11 (ṁ-1)-1 (M7)1/22 (f-2)5/22 (q-3)5/11
      (rs2)39/44 (Max[ProductLog[0, -a] // Abs, ProductLog[-1, -a] // Abs])13/11;
kmou = 1.3 (α-1)-2 (ṁ-1)-1 (M7)1/2 (f-2)5/2 (q-3)5/2 (rs2)-1/4  $\left(1 + \left(10^{-3} \frac{q_{-3}}{3}\right)^{1/9}\right)^{-115/24}$ ;

{kmou, kmos, kmg} = {kmou, kmos, kmg} /. fiducial2;

ks = Min[kmou, kmos, kmg];

Teff1 = Teff /. fiducial /. r3 → rs2 / 20;
Σ1 = Σ /. fiducial /. r3 → rs2 / 20;

(*TcMin=Log[10, Tc2] /. {r3→10, M7→100};
TcMax=Log[10, Tc2] /. {r3→0.1, M7→0.01};
TeffMin=Log[10, Teff2] /. {r3→10, M7→100};
TeffMax=Log[10, Teff2] /. {r3→0.1, M7→0.01};*)

(*TcLegend=Graphics[Legend[rainbow, 50, TcMin//ToString,
  TcMax//ToString, LegendShadow→False, LegendBorderSpace→2]];
TeffLegend=Graphics[Legend[rainbow, 50, TeffMin//ToString,
  TeffMax//ToString, LegendShadow→False, LegendBorderSpace→2]];*)

(*ColorFunction→Function[{x}, ColorData["Rainbow"][x]]*)

(*c1=ContourPlot[{Log[10,Teff1]}/.{r3→10x,M7→10y}, {x, -1, 1},
  {y,-2, 2}, ContourShading→False, ContourStyle→Directive[Blue],
  PlotLabel→"Q Contour Plot", FrameLabel→{"Log[r3"],"Log[M7"]}, Contours→contours]*)
kbdry = ContourPlot[ks /. {rs2 → 10x, M7 → 10y}, {x, 0, 2}, {y, -2, 2},
  ContourShading → None, Contours → {1}, ContourStyle → Directive[Red]];

cteff1 = ContourPlot[{Log[10, (1 + ks)1/4]} /. {rs2 → 10x, M7 → 10y}, {x, 0, 1}, {y, -2, 2},
  PlotLabel → "Perturbed Teff", FrameLabel → {"Log[rs2]", "Log[M7"]}, ContourShading → None,
  ContourLabels → True, RegionFunction → Function[{x, y}, (ks /. {rs2 → 10x, M7 → 10y}) > 1]];
cteff2 = ContourPlot[{Log[10, Teff1 (1 + ks)1/4]} /. {rs2 → 10x, M7 → 10y},
  {x, 0, 1}, {y, -2, 2}, PlotLabel → "Perturbed Teff",
  FrameLabel → {"Log[rs2]", "Log[M7"]}, ContourShading → None, ContourLabels → True,
  RegionFunction → Function[{x, y}, (ks /. {rs2 → 10x, M7 → 10y}) > 1]];

cΣ1 = ContourPlot[{Log[10, (1 + ks)3/5]} /. {rs2 → 10x, M7 → 10y}, {x, 0, 1}, {y, -2, 2},
  PlotLabel → "Perturbed Σ", FrameLabel → {"Log[rs2]", "Log[M7"]}, ContourShading → None,

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ContourLabels → True, RegionFunction → Function[{x, y}, (ks /. {rs2 → 10x, M7 → 10y}) > 1]];
cΣ2 = ContourPlot[{Log[10, Σ1 (1 + ks)3/5]} /. {rs2 → 10x, M7 → 10y}, {x, 0, 1}, {y, -2, 2},
  PlotLabel → "Perturbed Σ", FrameLabel → {"Log[rs2]", "Log[M7"]}, ContourShading → None,
  ContourLabels → True, RegionFunction → Function[{x, y}, (ks /. {rs2 → 10x, M7 → 10y}) > 1]];

βtest =
Hold[ $\left( \beta /. \text{Solve}\left[\frac{\sqrt{\beta}}{1 - \beta} = \frac{3.94 r_3^{21/20}}{M_7^{1/10}} (1 + ks)^{-4/5} /. \{r_3 \rightarrow rs2 / 20\} /. \{rs2 \rightarrow 10^x, M_7 \rightarrow 10^y\}, \right.$ 
 $\left. \beta \right) [[1]] < 0.5$ 
]

RegionPlot[ReleaseHold[βtest], {x, 0, 1}, {y, -2, 2}, MaxRecursion → 1]

cH = ContourPlot[{Log[10, (1 + ks)  $\frac{H_i}{R}$  /. fiducial /. r3 → (rs2 / 20) ]} /. {rs2 → 10x, M7 → 10y},
  {x, 0, 1}, {y, -2, 1}, PlotLabel → "Perturbed H",
  FrameLabel → {"Log[rs2]", "Log[M7"]}, ContourShading → None, ContourLabels → True,
  RegionFunction → Function[{x, y}, (ks /. {rs2 → 10x, M7 → 10y}) > 1]];

cH2 = ContourPlot[{Log[10, (1 + ks)  $\frac{H}{R}$  /. fiducial /. r3 → (rs2 / 20) ]} /.
  {rs2 → 10x, M7 → 10y}, {x, 0, 1}, {y, 1, 2}, PlotLabel → "Perturbed H",
  FrameLabel → {"Log[rs2]", "Log[M7"]}, ContourShading → None, ContourLabels → True,
  RegionFunction → Function[{x, y}, (ks /. {rs2 → 10x, M7 → 10y}) > 1]];

Show[cteff1, kbdry]
Show[cteff2, kbdry]
Show[cΣ1, kbdry]
Show[cΣ2, kbdry]
Show[cH, kbdry]
Show[cH2, kbdry]
(*c2=ContourPlot[{Log[10,Tc2]} /. {r3→10x,M7→10y}, {x, -1, 1},
  {y,-2, 2}, PlotLabel→"Perturbed Tc", FrameLabel→{"Log[r3"],"Log[M7"]},
  ContourShading→None, ContourLabels→True,ColorFunction→rainbow]*)

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In[2701]:=
(*fiducial2={rs2→20r3, α-1→3, m-1→1, f-2→1};
test={r3→0.05, M7→1, q-3→100};

q=q-310-3;

rs=R;
rh=( $\frac{a}{3}$ )1/3 rs;

Ωs= $\sqrt{G \frac{M}{rs^3}}$ ;
r=rs+y rh;
Ω= $\sqrt{G \frac{M}{r^3}}$ ;
ri=rs+rh;
Ω0= $\sqrt{G \frac{M}{ri^3}}$ ;
Δ=r-rs;
Δi=ri-rs;
lower= $\frac{ri}{rs}$ ;

upper= $\frac{r}{rs}$ 

ζr=Function[y, NIntegrate[x-7/6 (1-x-3/2)-1/6 (x-1)-10/3, {x, lower/.test, upper/.test}]]
ψ= $\frac{2}{3}$  Beta[ $\frac{\Omega s - \Omega}{\Omega s}$ ,  $\frac{5}{26}$ ,  $\frac{7}{13}$ ] -  $\frac{2}{3}$  Beta[ $\frac{\Omega s - \Omega_0}{\Omega s}$ ,  $\frac{5}{26}$ ,  $\frac{7}{13}$ ];
(*Scale height in units of Schwarzschild*)
Hpert=9.5 (α-1)-2/11 (M7)1/22 (f-2)5/22 (q-3)5/11 (rs2)39/44 ( $\frac{\Omega s - \Omega}{\Omega s}$ )5/26 ( $\frac{r}{rs}$ )5/26 ψ2/11/.fiducial2/.test;

Hpert2=0.065 (α-1)-2 (M7)1/2 (f-2)5/2 rs2-1/4 (r/rs)-5/24/.fiducial2/.test;
(*Flux perturbation in unsaturated regime*)
Fpert=
9.8 1014 (α-1)-2/11 (M7)-21/22 (f-2)5/22 (q-3)5/11 (rs2)-93/44 ( $\frac{\Omega s - \Omega}{\Omega s}$ )5/26 ( $\frac{r}{rs}$ )-73/26 ψ2/11/.fiducial2/.
test;
(*Flux perturbation in saturated regime*)
Fpert2=6.8 1012 (α-1)-2 (M7)-1/2 (f-2)5/2 (q-3)10/3 (rs2)-13/4 ( $\frac{r}{rs}$ )-77/24 ( $\frac{\Delta}{rs}$ )-5/2/.fiducial2/.test;

LogPlot[{2 Hpert,  $\frac{r-rs}{R_s}$ /.test}, {y, 1, 10}]
p1=LogPlot[{Fpert r2/.test}, {y, 1, 10}]
SetDirectory[NotebookDirectory[]]
perturbed=Import["perturbed.csv"]
perturbed[[All, 1]]= $\frac{\text{perturbed}[[All, 1]] - (rs/R_s)}{rh/R_s}$ /.test;
p2=ListLogPlot[perturbed, PlotStyle→Directive[Red]];
Show[p2, p1]

Fpert2=6.8 1012 (α-1)-2 (M7)-1/2 (f-2)5/2 (q-3)10/3 (rs2)-13/4 ( $\frac{r}{rs}$ )-77/24 ( $\frac{\Delta}{rs}$ )-5/2/.fiducial2/.test
Fpert2=Fpert2/.y→perturbed[[All, 1]]

(*x-7/6 (1-x-3/2)-1/6 (x-1)-10/3, *)*)

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In[2702]:=

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(*rs=(Rs/2)100 rs2;

$$\Omega s = \sqrt{G \frac{M}{rs^3}}$$

fiducial={α0.3→1, μ0→0.615, μe→0.875, ε0.1→1, ṁ→0.1, κ0→1, ft→3/8, b→1}
beqn/.{b→1, β→0.5, ft→ $\frac{3}{4}$ , r3→300}

$$\left( \left( \frac{\sqrt{\beta}}{1-\beta} \right) = (8\pi)^{4/5} c \frac{(\kappa b / (\mu_0 \text{ mp}))^{2/5}}{3^{9/10} (\alpha \sigma)^{1/10} \kappa^{9/10}} \frac{Q^{9/20}}{(\dot{M} Q)^{4/5} (1)^{4/5}} \right) // \text{PowerExpand} /. \text{fiducial} *)$$


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