

```

Clear["Global`*"]
<< Notation`
(*Physical constants in cgs units*)
G =  $6.67 \times 10^{-8}$ ; (*Newton's constant in cgs*)
c =  $3 \times 10^{10}$ ; (*Speed of light in cgs*)
 $\kappa r$  =  $1.6 \times 10^{24}$ ;
kb =  $1.38 \times 10^{-16}$ ;
mp =  $1.67 \times 10^{-24}$ ;
(*Stefan-Boltzmann constant in cgs*)
 $\sigma$  =  $5.67 \times 10^{-5}$ ;
h =  $6.63 \times 10^{-27}$ ;
Msun =  $2 \times 10^{33}$ ;
(*Special symbols*)
Symbolize[  $M_7$  ]
Symbolize[  $\epsilon_{0.1}$  ]
Symbolize[  $\hat{\kappa}$  ]
Symbolize[  $\alpha_{0.3}$  ]

```

```

γ = 4. / 3.;
κr = 1.6 × 1024;
M7 = 1;
M = 107 Msun M7;
Rs = 2 G  $\frac{M}{c^2}$ ;
r3 = Range[0.1, 2, 0.1]
R = r3 1000 Rs;
Ω =  $\sqrt{G \frac{M}{R^3}}$ ;
μ0 = 0.615;
μe = 1;
κ̂ = 1;
kes = 0.4 μe;
 $\sqrt{140 c \frac{\kappa r}{\gamma \sigma kes}}$ 
fT = 3 / 8;
α0.3 = 1;
ε0.1 = 1;
LEdd = 4 π G  $\frac{M_7}{kes \hat{\kappa}}$  c 107 Msun;
 $\dot{M}_{Edd} = \frac{L_{Edd}}{c^2 \epsilon_{0.1} 0.1}$ ;
 $\dot{m} = 0.1$ ;
Ṁ = ṁ * ṀEdd;
Print["Ṁ= ", Ṁ]
(*Constant out front is probably slightly
difference if the μe dependence is not include*)
 $\Sigma = \frac{169123 \mu_0^{4/5} \left(\frac{\dot{m}}{\epsilon_{0.1}}\right)^{3/5}}{\mu e^{4/5} \hat{\kappa}^{6/5} \left(\frac{r3^3 f_T}{M_7}\right)^{1/5} (\alpha_{0.3})^{4/5}}$ ;
Print["Σ= ", Σ]

v =  $\frac{\dot{M}}{3 \pi \Sigma}$ ;
Print["v= ", v]
Export["Parameters",
Table[{Round[Σ[[i]], 1], 0.1, Round[1000 r3[[i]], 1]}, {i, 1, Σ // Length}], "Table"]

{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1., 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.}

4.71405 × 1020

```

$$\dot{M} = 1.39696 \times 10^{24}$$

$$\Sigma = \{139475., 92019.2, 72147.9, 60710., 53102.4, 47599.9, 43394.8, 40053.7, 37320.8, 35034.5, 33087.3, 31404.2, 29931.6, 28629.9, 27468.9, 26425.6, 25481.6, 24622.5, 23836.6, 23114.2\}$$

$$\nu = \{1.06271 \times 10^{18}, 1.61077 \times 10^{18}, 2.05442 \times 10^{18}, 2.44148 \times 10^{18}, 2.79125 \times 10^{18}, 3.11392 \times 10^{18}, 3.41567 \times 10^{18}, 3.70059 \times 10^{18}, 3.97157 \times 10^{18}, 4.23074 \times 10^{18}, 4.47974 \times 10^{18}, 4.71982 \times 10^{18}, 4.95202 \times 10^{18}, 5.17718 \times 10^{18}, 5.396 \times 10^{18}, 5.60904 \times 10^{18}, 5.81683 \times 10^{18}, 6.01978 \times 10^{18}, 6.21826 \times 10^{18}, 6.41261 \times 10^{18}\}$$

Parameters

$$\kappa_s = \text{Table}\left[4.7 \times 10^{20} \Omega[[i]] \text{Tp}^{-15/4}, \{i, 1, \Sigma // \text{Length}\}\right];$$

$$(*\kappa_s = 1.83 \cdot 10^9 \Omega(\text{Tp})^{-9/4} *)$$

$$\epsilon_s = \frac{\kappa_s}{\kappa_s + \kappa_{es}};$$

$$\Xi = \frac{0.873 \epsilon_s^{-1/6}}{1 - 0.127 \epsilon_s^{5/6}} \frac{1}{1 + (\epsilon_s^{-1} - 1)^{2/3}};$$

$$r1 = \text{Table}\left[\text{FindRoot}\left[\frac{9}{8} \nu[[i]] \Sigma[[i]] \Omega[[i]]^2 = \Xi[[i]] \sigma \text{Tp}^4, \right.$$

$$\left. \left\{ \text{Tp}1, \left( \frac{9}{8} \nu[[i]] \Sigma[[i]] \frac{\Omega[[i]]^2}{\sigma} \right)^{0.25} \right\} \right], \{i, 1, \text{Length}[\Sigma]\};$$

$$\text{Tp} = \text{Tp}1 / . r1$$

$$\kappa_s = \text{Table}\left[4.7 \times 10^{20} \Omega[[i]] \text{Tp}[[i]]^{-15/4}, \{i, 1, \Sigma // \text{Length}\}\right];$$

$$(*\kappa_s = 1.83 \cdot 10^9 \Omega(\text{Tp})^{-9/4} *)$$

$$\epsilon_s = \frac{\kappa_s}{\kappa_s + \kappa_{es}};$$

$$x = \left(1 + \frac{1}{\epsilon_s}\right)^{-1};$$

$$\xi = h \frac{\nu1}{k_b \text{Tp}};$$

$$f = \xi^{-3} (1 - e^{-\xi});$$

$$K_v = -\frac{1}{3} + \frac{2^{1/3}}{3 \left( -2 + 27 f x^2 + \sqrt{-4 + (-2 + 27 f x^2)^2} \right)^{1/3}} + \frac{\left( -2 + 27 f x^2 + \sqrt{-4 + (-2 + 27 f x^2)^2} \right)^{1/3}}{3 \times 2^{1/3}};$$

$$\epsilon_v = \left(1 + \frac{1}{K_v}\right)^{-1};$$

(\*Peak frequency. Note this is approximate, as it is technically only applicable in for a blackbody. \*)

$$\nu_{\text{max}} = 2.82 k_b \frac{\text{Tp}}{h};$$

FindRoot::lstol :

The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances. >>

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General::stop :

Further output of FindRoot::lstol will be suppressed during this calculation. >>

{26071., 13883.7, 9771.02, 7670.37, 6380.59, 5500.91, 4858.7, 4367.03, 3977.15, 3659.54, 3395.2, 3171.36, 2979.06, 2811.87, 2664.98, 2534.79, 2418.49, 2313.9, 2219.27, 2133.18}

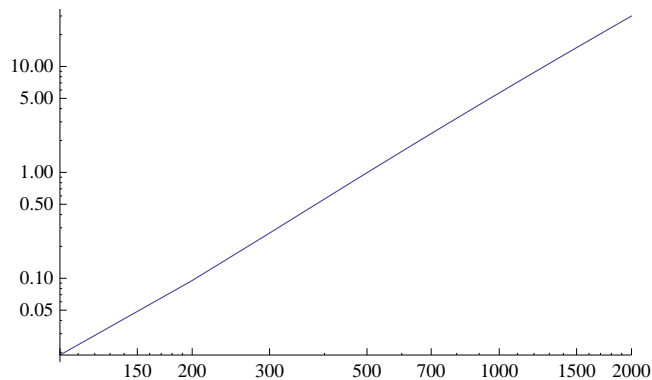
```

ρp = Table[
$$\frac{3 c \Omega[[i]]^2}{4 \gamma \sigma T_p[[i]]^4 K_v[[i]] \kappa \epsilon s^2 (1 + K_v[[i]])}$$
 /. v1 → vmax[[i]], {i, 1, Σ // Length}];

p1 = Transpose[
$$\left\{ \frac{R}{R_s}, \left( \rho_p k_b \frac{T_p}{\mu_0 m_p} \right) / \left( 4 \sigma \frac{T_p^4}{3 c} \right) \right\}] //$$


ListLogLogPlot[#, PlotRange → {{100, 2000}, Automatic}, Joined → True] &
(*p2=Transpose[
$$\left\{ \frac{R}{R_s}, 4 \sigma \frac{T_p^4}{3 c} \right\}] // ListLinePlot[#, PlotStyle → Directive[Red]] &;
Show[p1, p2] *)$$

```



```

Σtmp = Σ
Σ = Σ[[1 ;; 4]];
Ωtmp = Ω
Ω = Ω[[1 ;; 4]];
νtmp = ν
ν = ν[[1 ;; 4]];
Tp = Tp[[1 ;; 4]];
Kv = Kv[[1 ;; 4]];
εv = εv[[1 ;; 4]];

Tpblack =  $\left(\frac{9}{8} \nu \Sigma \frac{\Omega^2}{\sigma}\right)^{1/4}$ ;

(*Peak frequency. Note this is approximate,
as it is technically only applicable in for a blackbody. *)
νmax = 2.82 kb  $\frac{T_p}{h}$ ;

B1 =  $\frac{(2 h \nu^3 / c^2)}{E^{(h \nu) / (k_b T_{pblack})} - 1}$ ;

B2 =  $\frac{(2 h \nu^3 / c^2)}{E^{(h \nu) / (k_b T_p)} - 1}$ ;

Table[LogLogPlot[{B1[[i]] ν1, 2  $\frac{\epsilon v[[i]]^{1/2}}{1 + \epsilon v[[i]]^{1/2}}$  B2[[i]] ν1 // Re},
  {ν1, 0.001 νmax[[i]], 10 νmax[[i]]}, ImageSize → Medium,
  AxesLabel → {"ν [Hz]", "Fν [erg cm-2 s-1"]}, {i, 1, Σ // Length}]

ν = νtmp;
Ω = Ωtmp;
Σ = Σtmp;

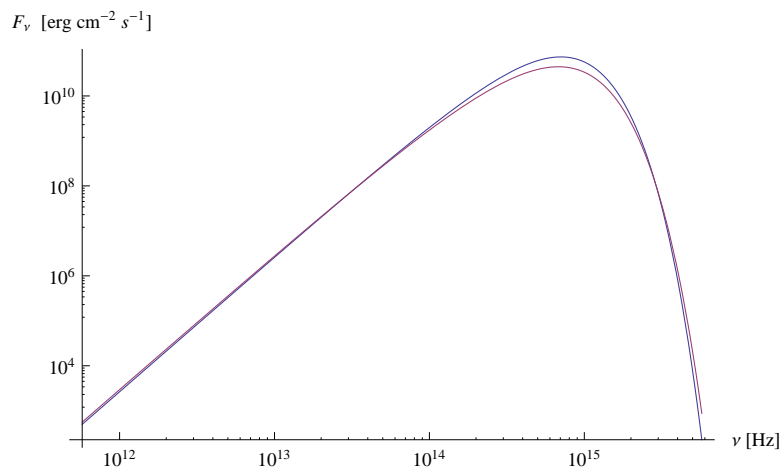
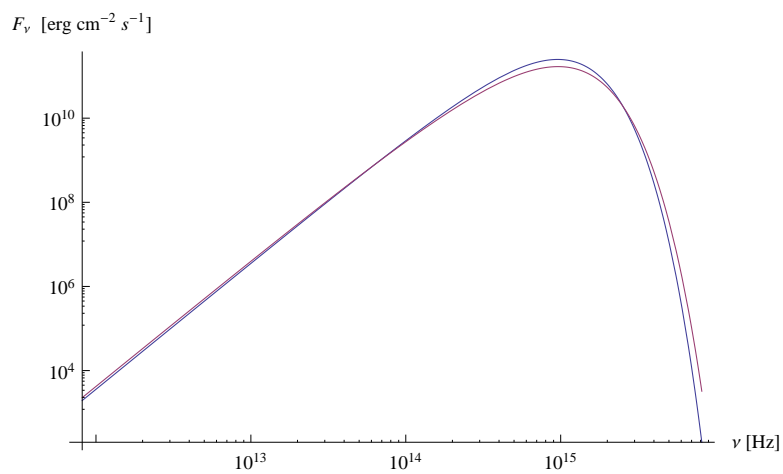
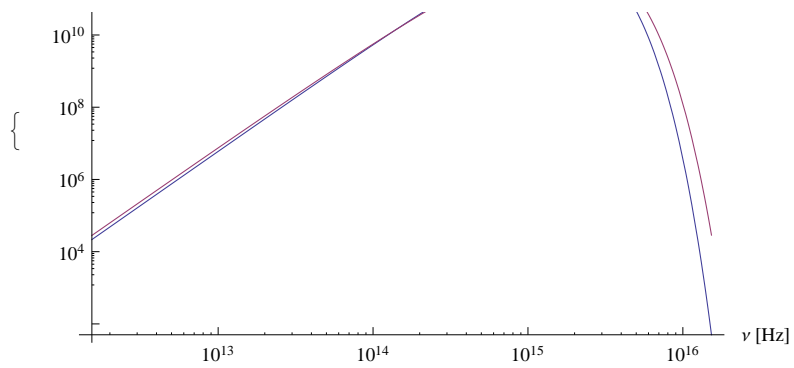
{139475., 92019.2, 72147.9, 60710., 53102.4, 47599.9, 43394.8, 40053.7, 37320.8, 35034.5,
 33087.3, 31404.2, 29931.6, 28629.9, 27468.9, 26425.6, 25481.6, 24622.5, 23836.6, 23114.2}

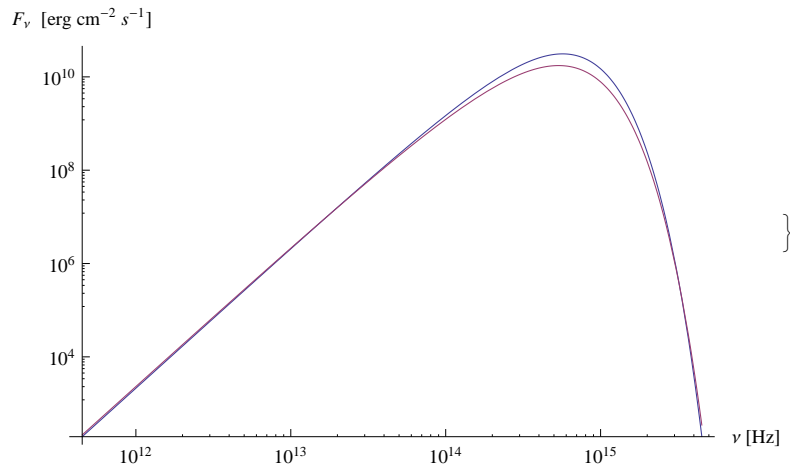
{7.15588 × 10-6, 2.52999 × 10-6, 1.37715 × 10-6, 8.94485 × 10-7, 6.40041 × 10-7,
 4.86896 × 10-7, 3.86381 × 10-7, 3.16248 × 10-7, 2.65033 × 10-7, 2.26289 × 10-7,
 1.96143 × 10-7, 1.72144 × 10-7, 1.52668 × 10-7, 1.36606 × 10-7, 1.23176 × 10-7,
 1.11811 × 10-7, 1.02091 × 10-7, 9.37031 × 10-8, 8.64037 × 10-8, 8.00052 × 10-8}

{1.06271 × 1018, 1.61077 × 1018, 2.05442 × 1018, 2.44148 × 1018, 2.79125 × 1018,
 3.11392 × 1018, 3.41567 × 1018, 3.70059 × 1018, 3.97157 × 1018, 4.23074 × 1018,
 4.47974 × 1018, 4.71982 × 1018, 4.95202 × 1018, 5.17718 × 1018, 5.396 × 1018,
 5.60904 × 1018, 5.81683 × 1018, 6.01978 × 1018, 6.21826 × 1018, 6.41261 × 1018}

Fν [erg cm-2 s-1]

```





```

Σtmp = Σ
Σ = Σ[[6 ;;]];
Ωtmp = Ω
Ω = Ω[[6 ;;]];
νtmp = ν
ν = ν[[6 ;;]];

Tpblack =  $\left(\frac{9}{8} \nu \Sigma \frac{\Omega^2}{\sigma}\right)^{1/4}$ ;

κs =  $1.83 \times 10^9 \Omega (Tp1)^{-9/4}$ ;
εs =  $\frac{\kappa s}{\kappa s + \kappa es}$ ;

Ξ =  $\frac{0.873 \epsilon s^{-1/6}}{1 - 0.127 \epsilon s^{5/6}} \frac{1}{1 + (\epsilon s^{-1} - 1)^{2/3}}$ ;

r1 = Table[FindRoot[ $\frac{9}{8} \nu[[i]] \Sigma[[i]] \Omega[[i]]^2 = \Xi[[i]] \sigma Tp1^4$ ,
  {Tp1,  $\left(\frac{9}{8} \nu[[i]] \Sigma[[i]] \frac{\Omega[[i]]^2}{\sigma}\right)^{0.25}$ }], {i, 1, Length[Σ]};

Tp = Tp1 /. r1
κs = Table[ $1.83 \times 10^9 \Omega[[i]] Tp[[i]]^{-9/4}$ , {i, 1, Σ // Length}];
(*κs=1.83 10^9 Ω (Tp)^{-9/4}*)
εs =  $\frac{\kappa s}{\kappa s + \kappa es}$ ;

```

$$x = \left(1 + \frac{1}{\epsilon s}\right)^{-1};$$

$$\xi = h \frac{v1}{k b T p};$$

$$f = \xi^{-3} (1 - e^{-\xi});$$

$$K v = -\frac{1}{3} + \frac{2^{1/3}}{3 \left( -2 + 27 f x^2 + \sqrt{-4 + (-2 + 27 f x^2)^2} \right)^{1/3}} + \frac{\left( -2 + 27 f x^2 + \sqrt{-4 + (-2 + 27 f x^2)^2} \right)^{1/3}}{3 \times 2^{1/3}};$$

$$\epsilon v = \left(1 + \frac{1}{K v}\right)^{-1};$$

(\*Peak frequency\*)

$$v_{\max} = 2.82 k b \frac{T p}{h};$$

(\*Peak frequency. Note this is approximate,  
as it is technically only applicable in for a blackbody. \*)

$$v_{\max} = 2.82 k b \frac{T p}{h};$$

$$B1 = \frac{(2 h v1^3 / c^2)}{E^{(h v1) / (k b T p_{\text{black}})} - 1};$$

$$B2 = \frac{(2 h v1^3 / c^2)}{E^{(h v1) / (k b T p)} - 1};$$

```
Table[LogLogPlot[{B1[[i]], 2 \frac{\epsilon v[[i]]^{1/2}}{1 + \epsilon v[[i]]^{1/2}} B2[[i]] // Re},
  {v1, 0.001 v_{\max}[[i]], 10 v_{\max}[[i]]}, ImageSize -> Medium,
  AxesLabel -> {"v [Hz]", "F_v [erg cm^{-2} s^{-1}"]}, {i, 1, \Sigma // Length}]
```

```
v = vtmp;
Ω = Ωtmp;
Σ = Σtmp;
```

```
{139475., 92019.2, 72147.9, 60710., 53102.4, 47599.9, 43394.8, 40053.7, 37320.8, 35034.5,
 33087.3, 31404.2, 29931.6, 28629.9, 27468.9, 26425.6, 25481.6, 24622.5, 23836.6, 23114.2}
```

```
{7.15588 × 10^{-6}, 2.52999 × 10^{-6}, 1.37715 × 10^{-6}, 8.94485 × 10^{-7}, 6.40041 × 10^{-7},
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```

```
{1.06271 × 10^{18}, 1.61077 × 10^{18}, 2.05442 × 10^{18}, 2.44148 × 10^{18}, 2.79125 × 10^{18},
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 5.60904 × 10^{18}, 5.81683 × 10^{18}, 6.01978 × 10^{18}, 6.21826 × 10^{18}, 6.41261 × 10^{18}}
```

FindRoot::lstol:

The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances. >>



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The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances. >>

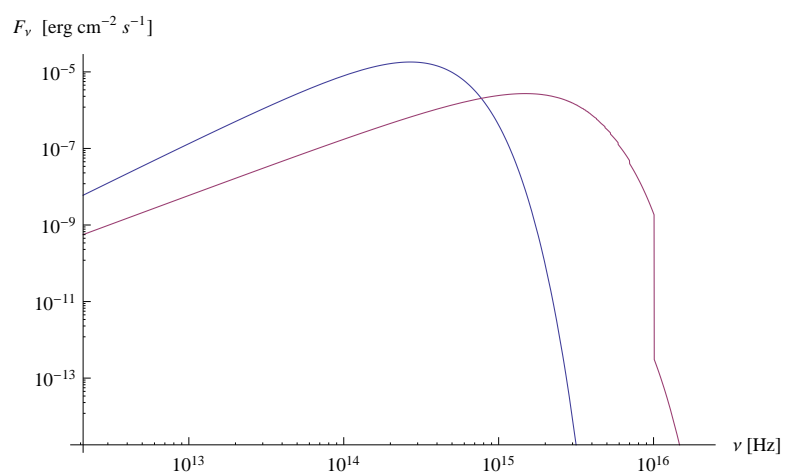
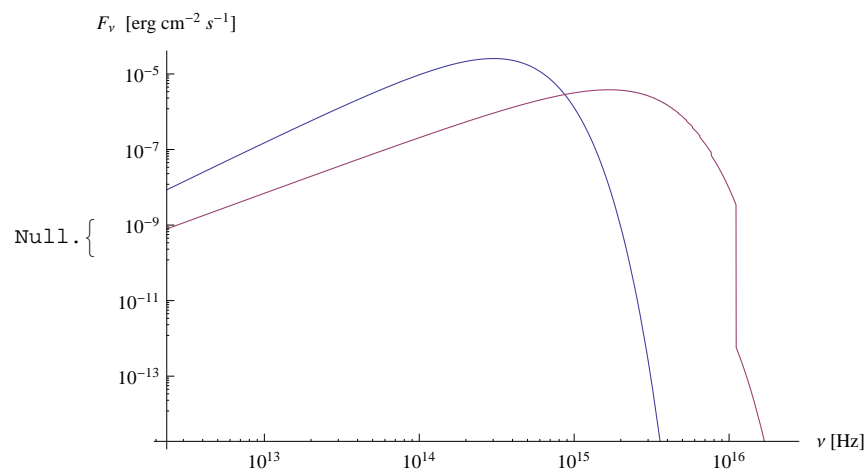
FindRoot::lstol:

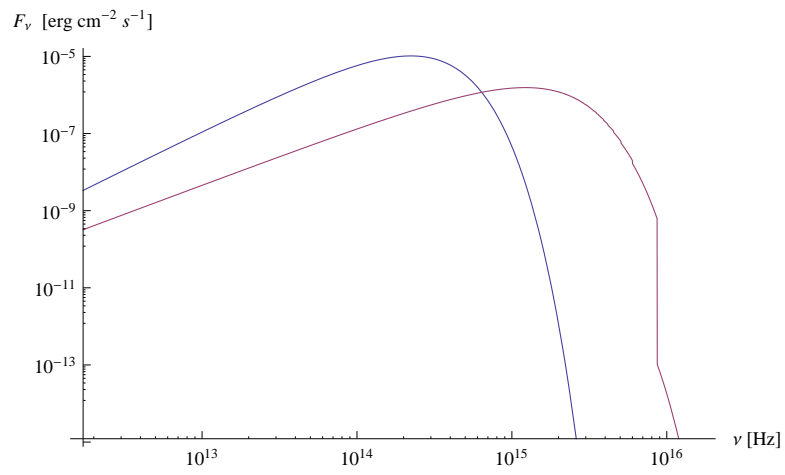
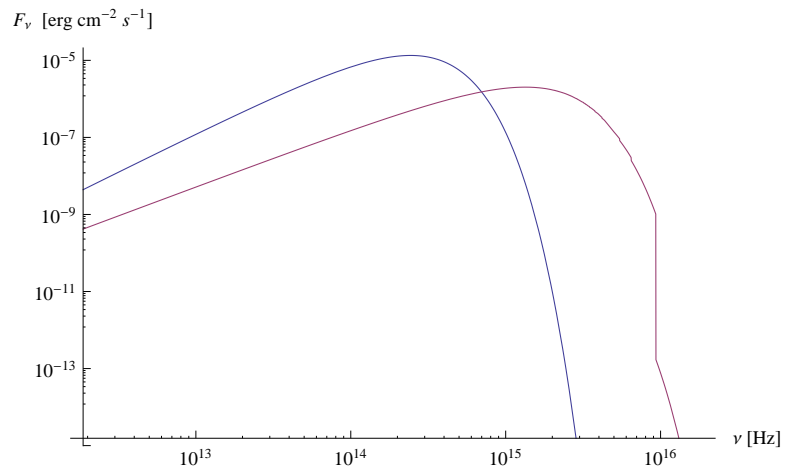
The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances. >>

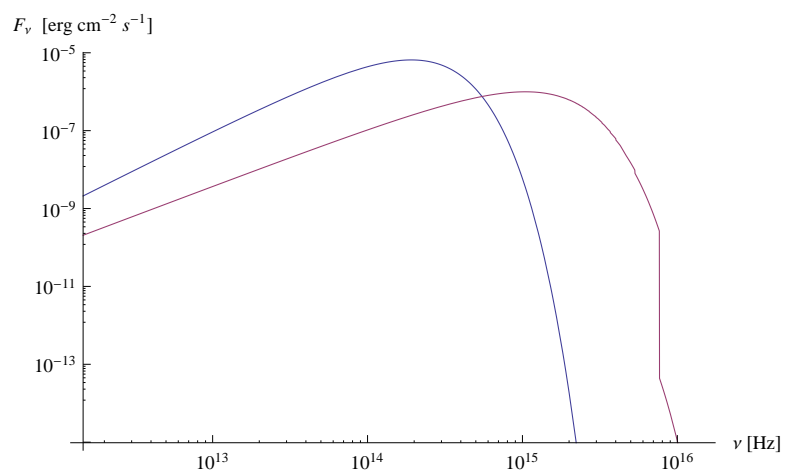
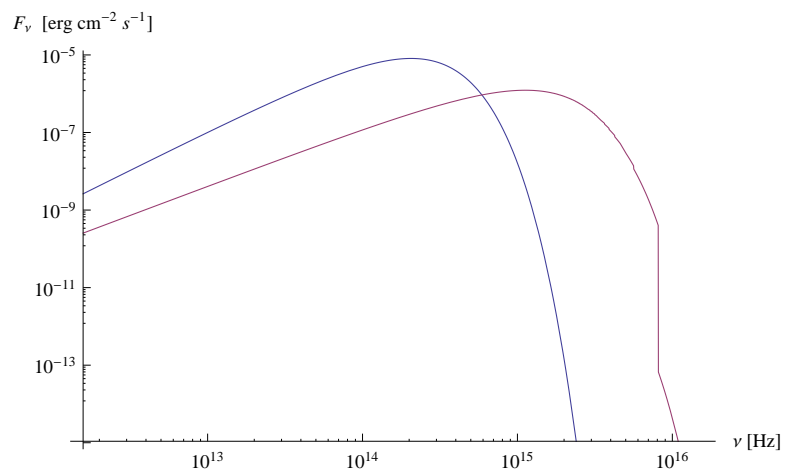
General::stop:

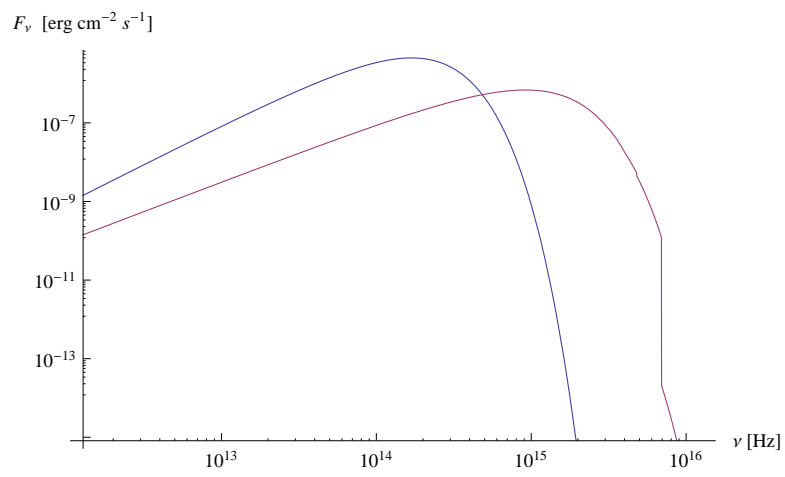
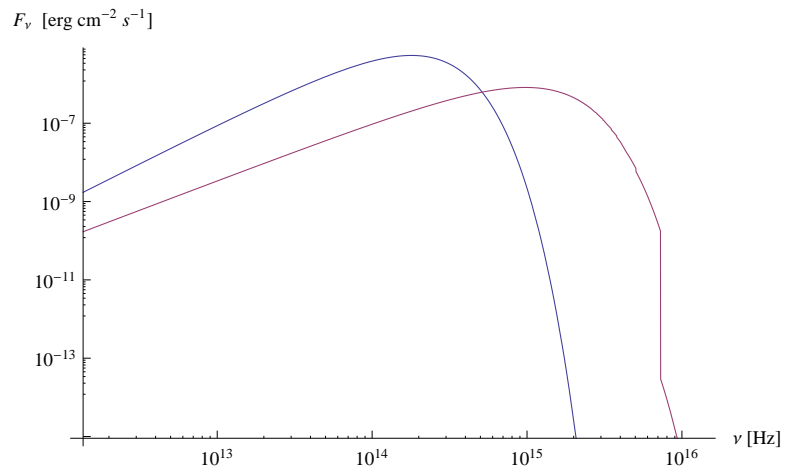
Further output of FindRoot::lstol will be suppressed during this calculation. >>

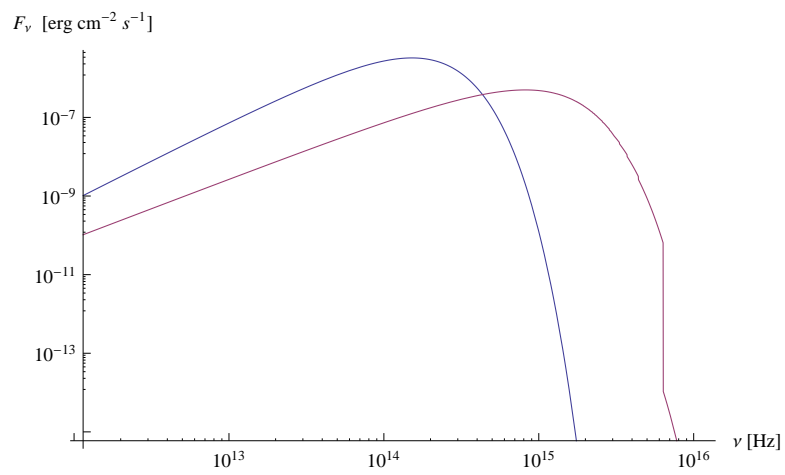
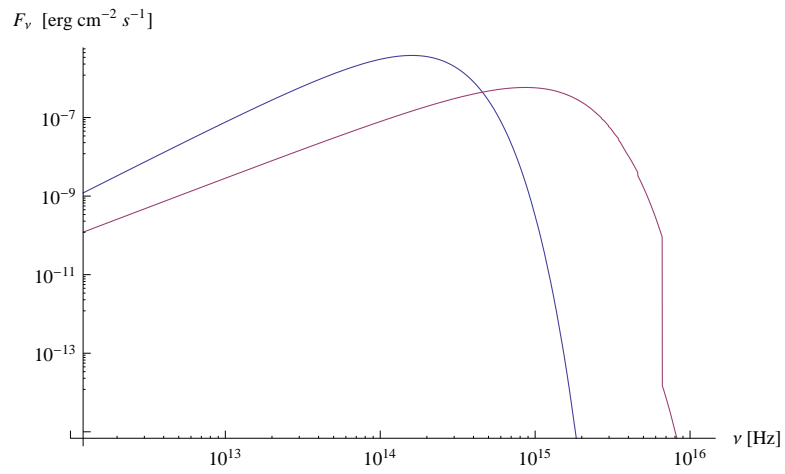
```
{39920.4, 35383.6, 31872.6, 29066., 26765.5, 24841.7, 23206.4,
 21797.3, 20569., 19487.9, 18528., 17669.5, 16896.5, 16196.5, 15559.2}
```

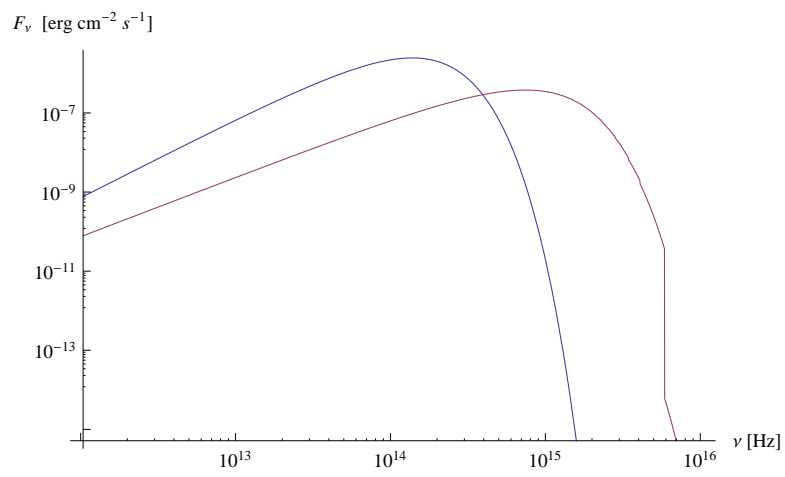
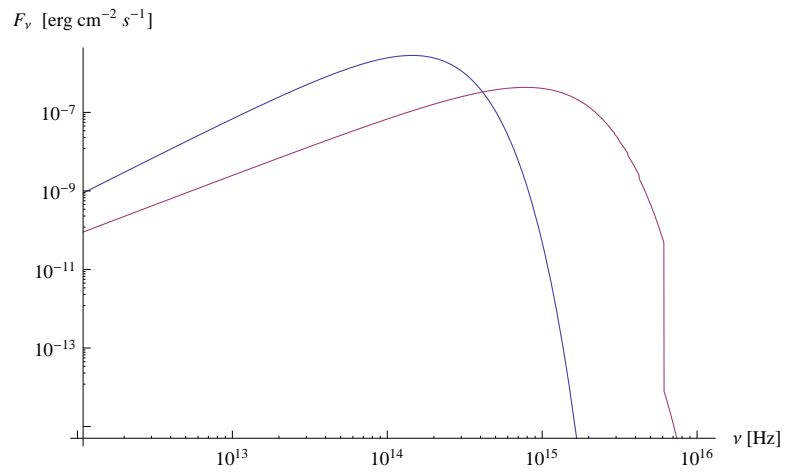


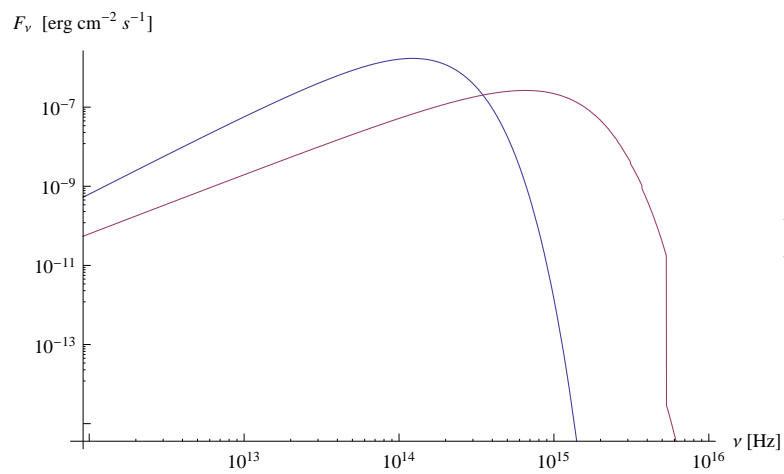
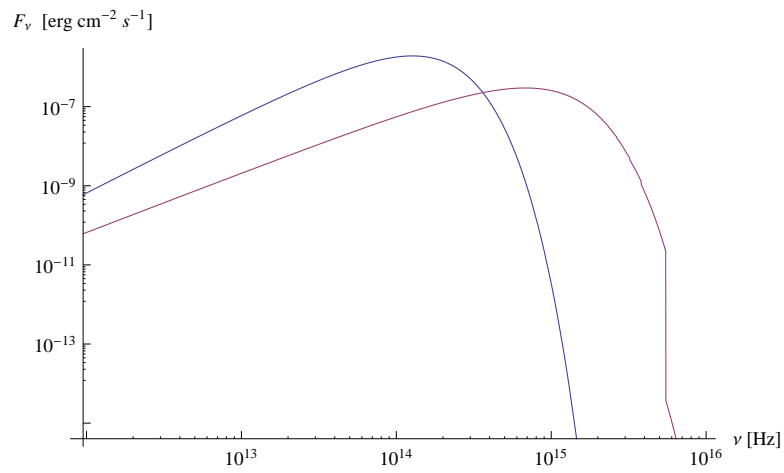
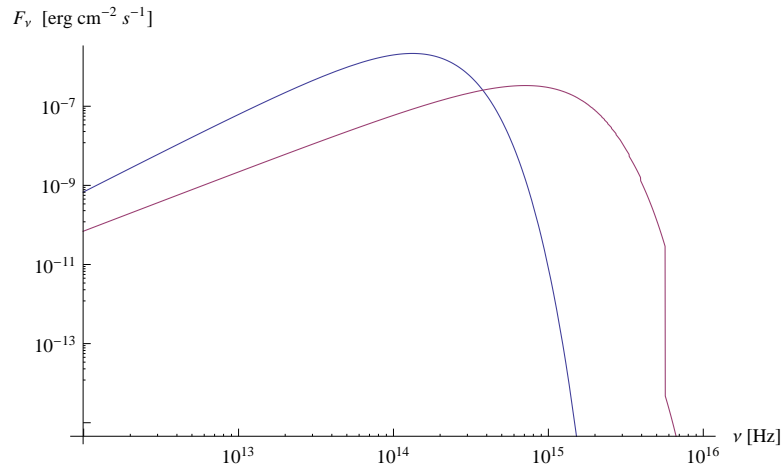














```
Module[{ξ, Ξ, εs, Kv, f, ε, p1, p2},
  Kv = (y /. (Solve[y^2 (1 + y) == (1 - 1/εs)^-2 f, y][[1]]));
  ε = (1 + Kv^-1)^-1;
  f = ξ^-3 (1 - e^-ξ);
  p1 = Plot[15/π^4 NIntegrate[2 (e^(1/2)/(1 + e^(1/2))) (e^-ξ/f), {ξ, 0, ∞}], {εs, 0, 1}, AxesOrigin -> {0, 0}];
  p2 = Plot[0.873 εs^-1/6 / (1 - 0.127 εs^(5/6) / (1 + (εs^-1 - 1)^(2/3))), {εs, 0, 1}, PlotStyle -> Directive[Red]];
  Show[p1, p2]
]
```

$$(*\Xi = \frac{15}{\pi^4} \int_0^\infty 2 \frac{e^{1/2}}{1 + e^{1/2}} e^{-\xi} \frac{d\xi}{f} *)$$

$$(*2 \frac{e^{1/2}}{1 + e^{1/2}} \frac{e^{-\xi}}{f} // \text{Simplify} // \text{TraditionalForm} *)$$

