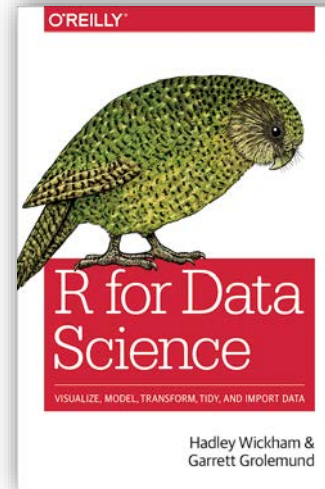
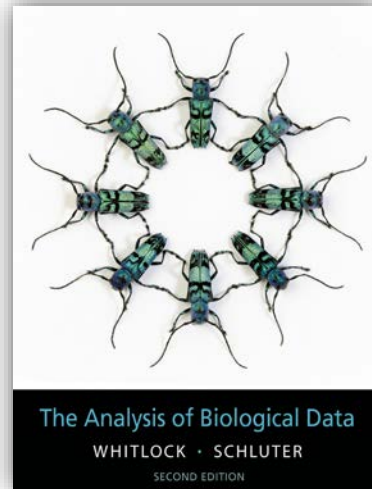


Data Science in Bioinformatics

Palle Villesen & Thomas Bataillon



Outline for week 11

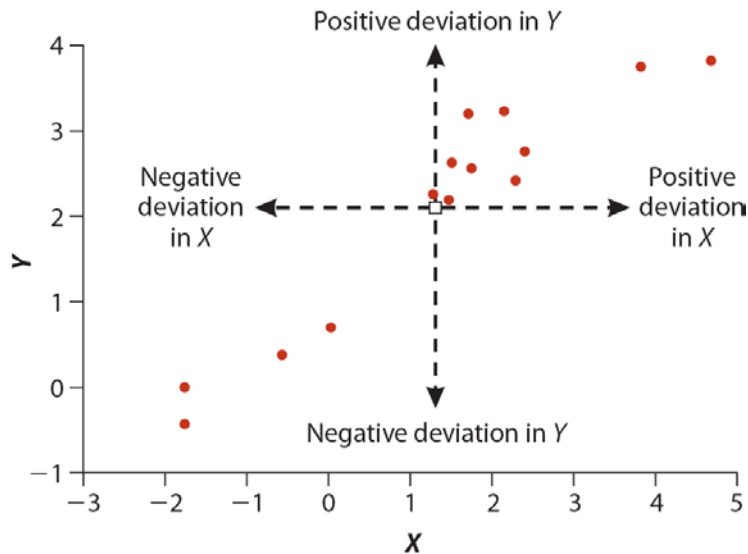
- Chapter 16+17
 - Open discussion & Exercises
- Thursday
 - Who will be responsible

Correlation and regression

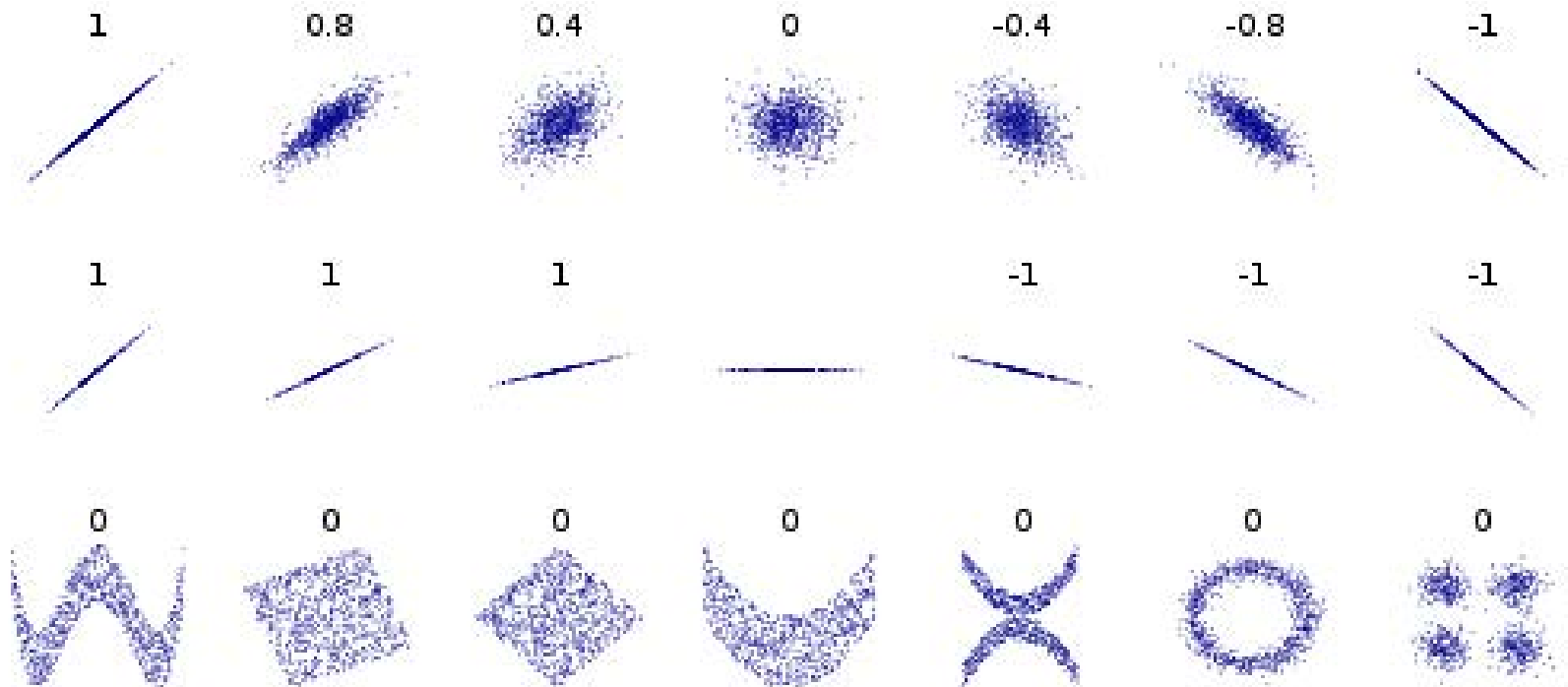
- Is two variables independent or not?
- How much of the variation in Y is explained by the variation in X?
- What is the standard error?
- How strong is the effect?
- How do we test it?

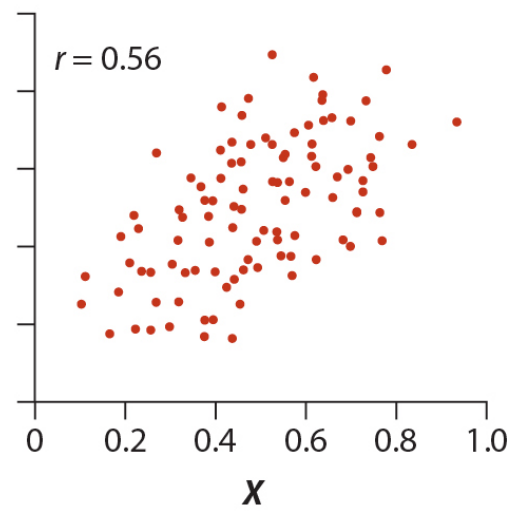
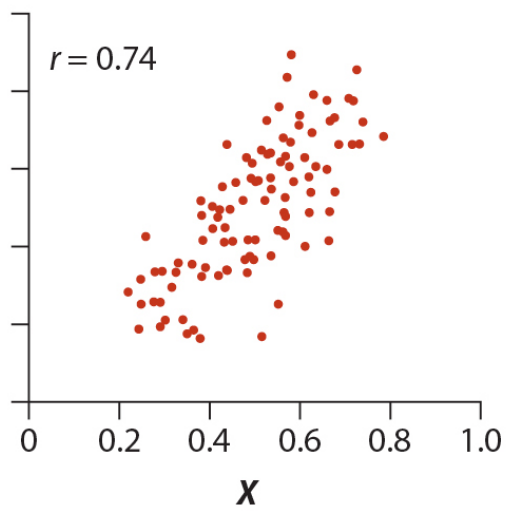
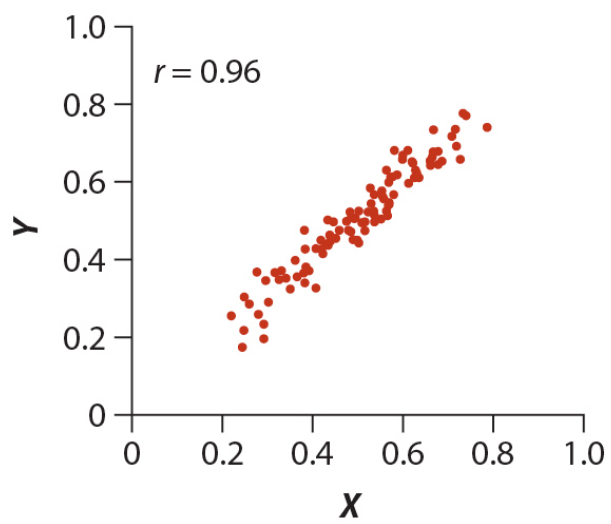
Pearson correlation

$$r_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}},$$



Correlation





Sample size and correlation

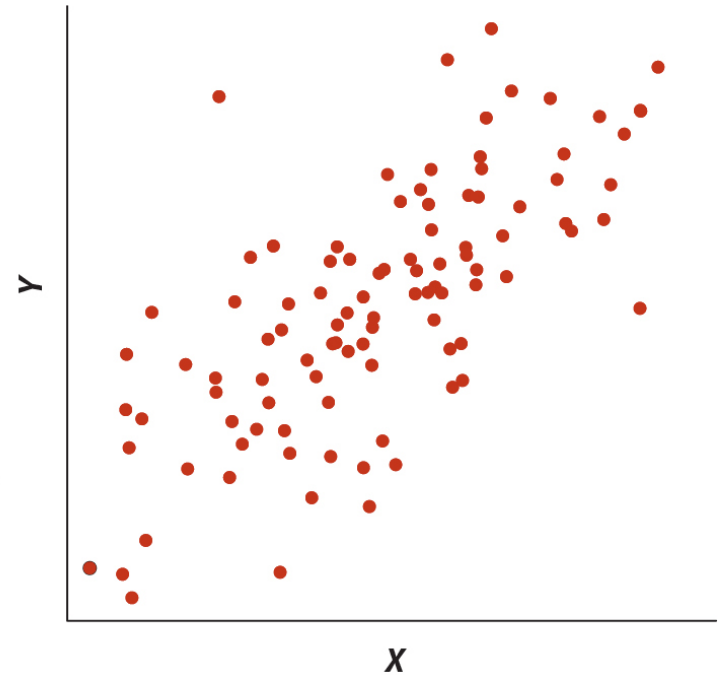
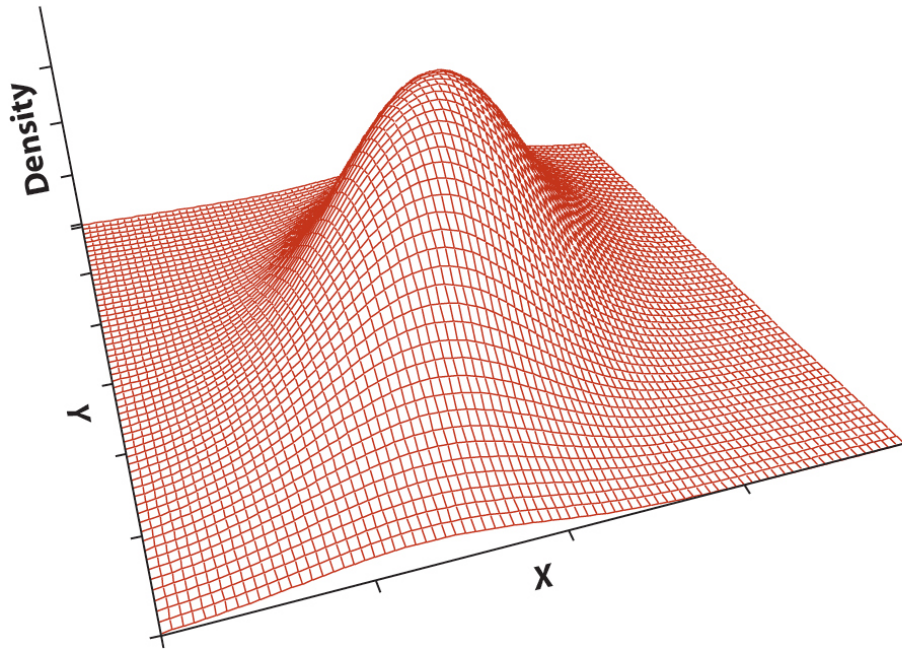
- $SE_r = \sqrt{\frac{(1-r^2)}{n-2}}$
- **Confidence interval**
- Correlation coefficient is not normally distributed
- We convert it to “z” that is normal – calculates 95% CI and converts back from z to r
- Or we bootstrap the confidence interval

Testing correlation

- H_0 : the population correlation is 0
- H_A : the population correlation is not 0
- $t = r/SE$, Use t distribution with $n-2$ df
- `cor.test`
- **Alternative way**
- Permutation test
 - Sample shuffle x and y , calculate r
 - How many are more extreme than observed r ?

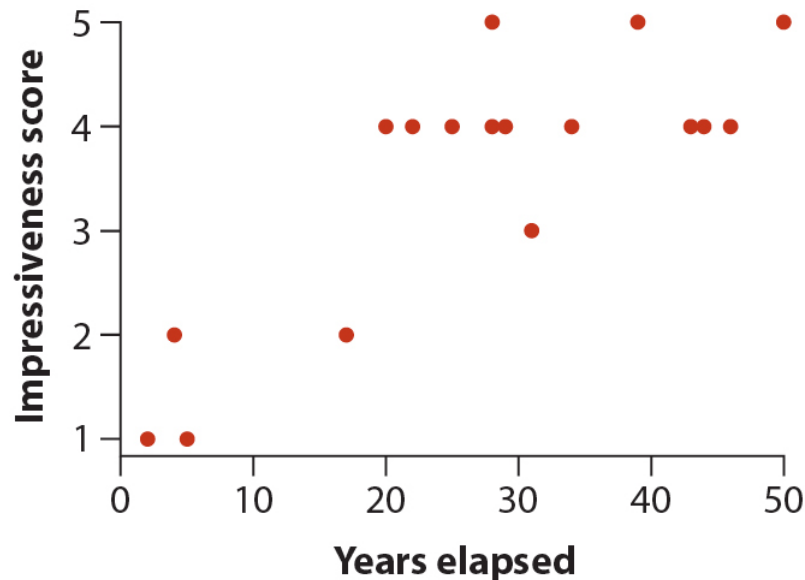
Assumptions

bivariate normal distribution

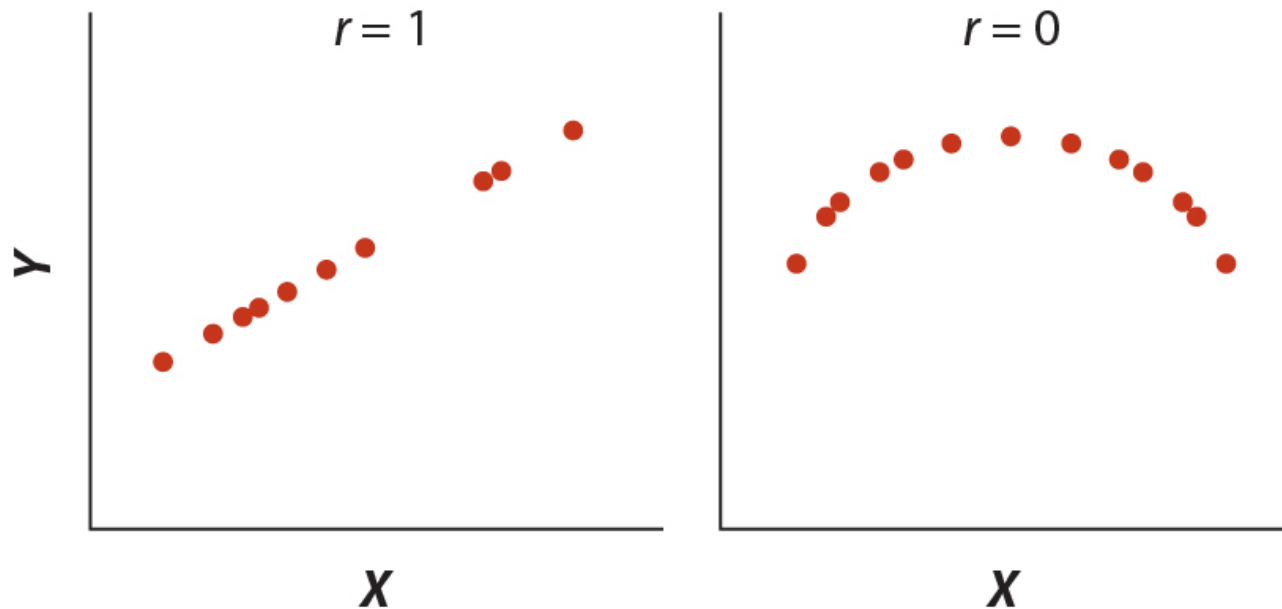


Other stuff

- Rank correlation
 - No assumption on distribution of x and y
 - It ranks all data and works on ranks



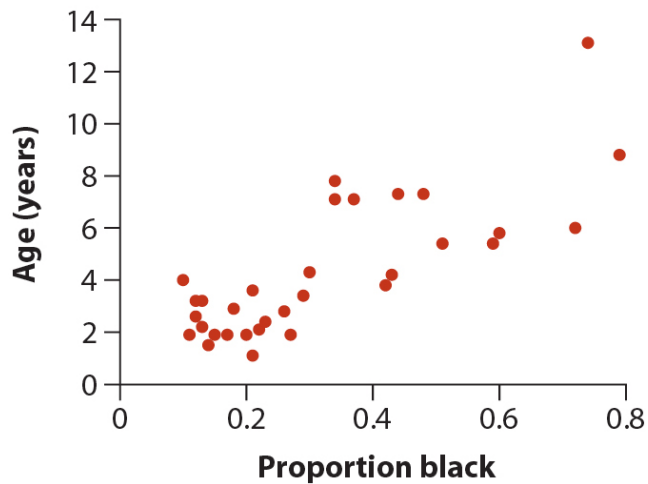
Low correlation?



Linear regression

- $Y = \beta_0 + \beta_1 X \dots$
- $Y = \beta_0 + \beta_1 X + \varepsilon$
- It is a **prediction** method
- Also called statistical learning (or machine learning)
 - Prediction and/or inference
- Supervised learning (we need known data to build the model)

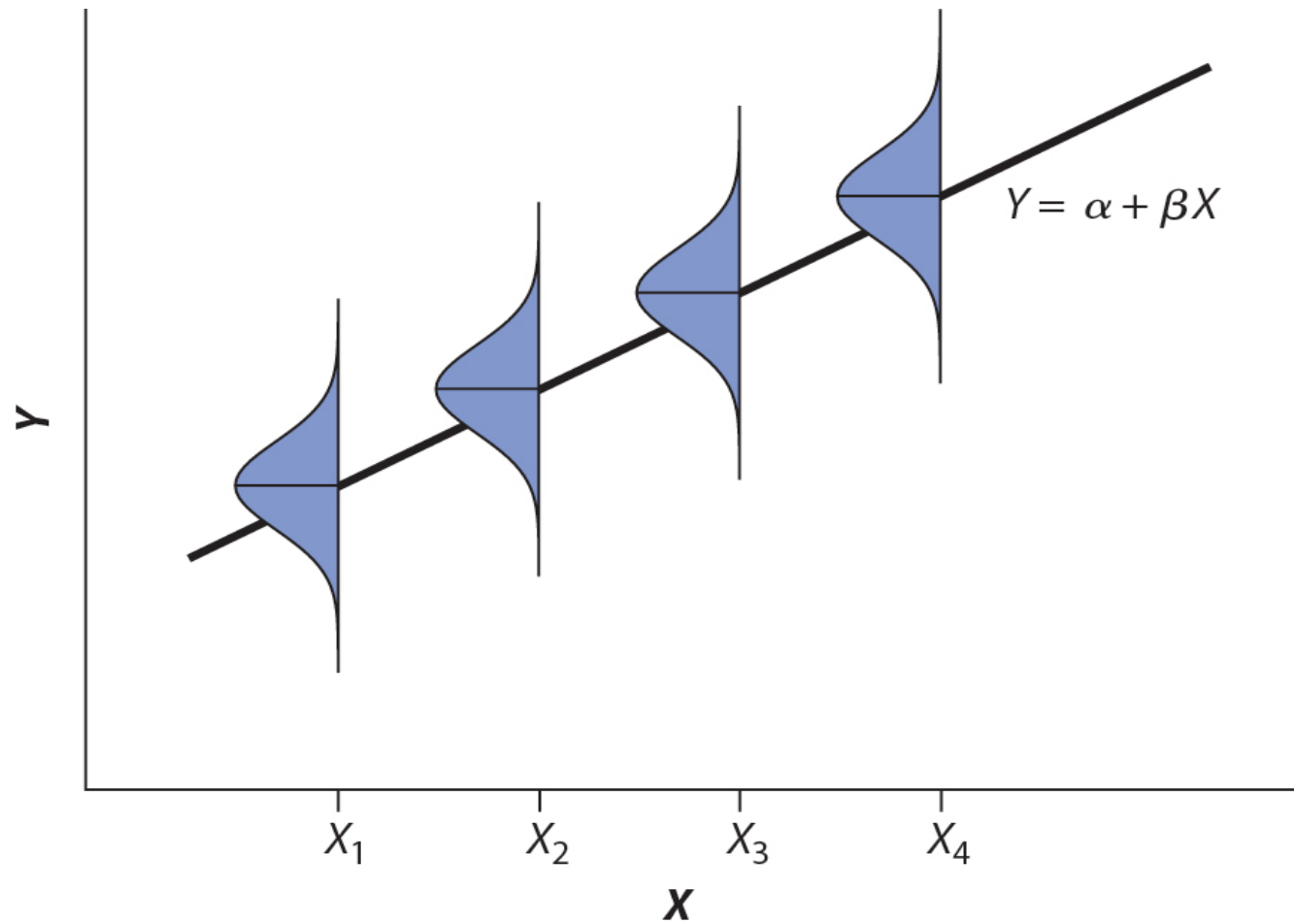
Predict lion age from nose



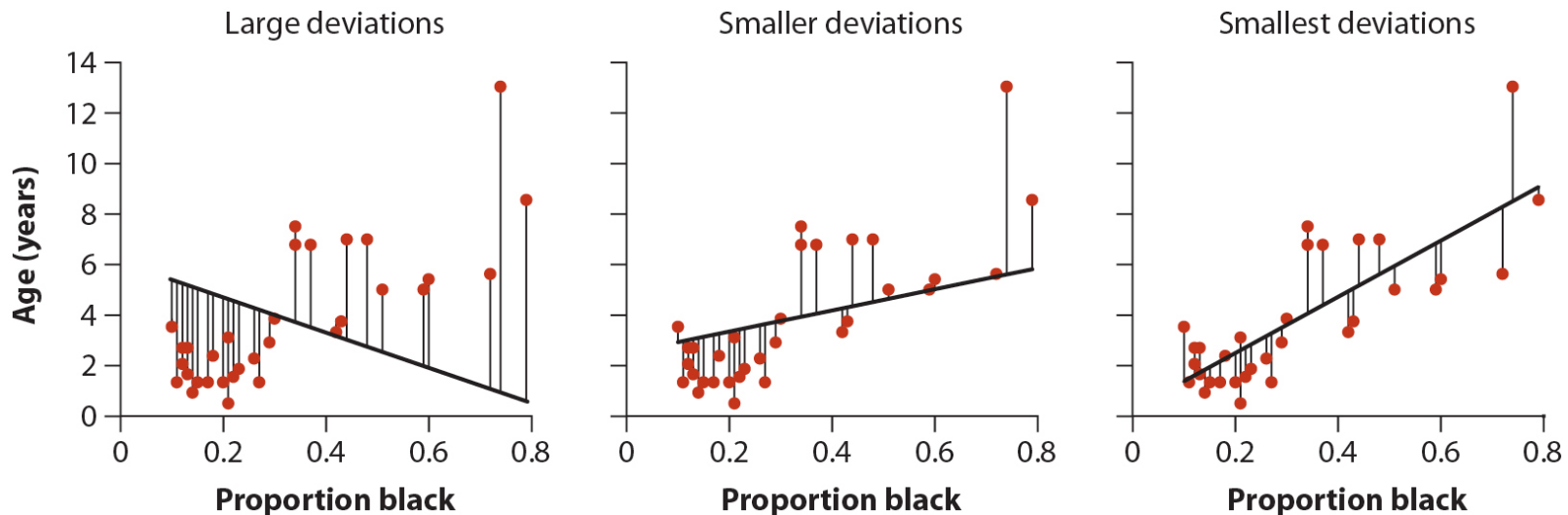
Essentials of regression

- Assumptions
- Tests & Measure of Fit
 - Parametric
 - Resampling methods
- Validation / Inspection
 - Visual
 - Test on residuals
 - Influence of outliers, colinearity

Assumptions...

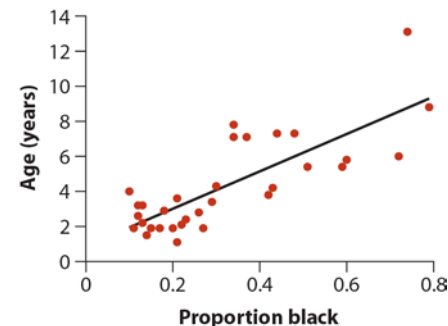


A visual on least squares...



$$\begin{aligned}\hat{\alpha} &= \bar{y} - \hat{\beta} \bar{x}, \\ \hat{\beta} &= \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} \\ &= \frac{\text{Cov}(x, y)}{\text{Var}(x)} \\ &= r_{xy} \frac{s_y}{s_x}.\end{aligned}$$

minimum prediction error



The model for lions

- $\text{Age} = 0.88 + 10.65 * \text{proportion black}$
- Standard error of slope
- Confidence interval for the slope
- Also possible by bootstrapping
- CI for slope: $7.56 < \text{slope} < 13.73$

What can we do

- We have shown that nose patterns change with age (inference)
- We can estimate how much it changes
- We can calculate how much of the variation is explained by this linear relation ship (R^2)
- We can predict the age of lions

$$R^2 = \text{SS}(\text{regression}) / \text{SS}(\text{total})$$

$$\text{TSS} = \sum_{i=1}^n (y_i - \bar{y})^2$$

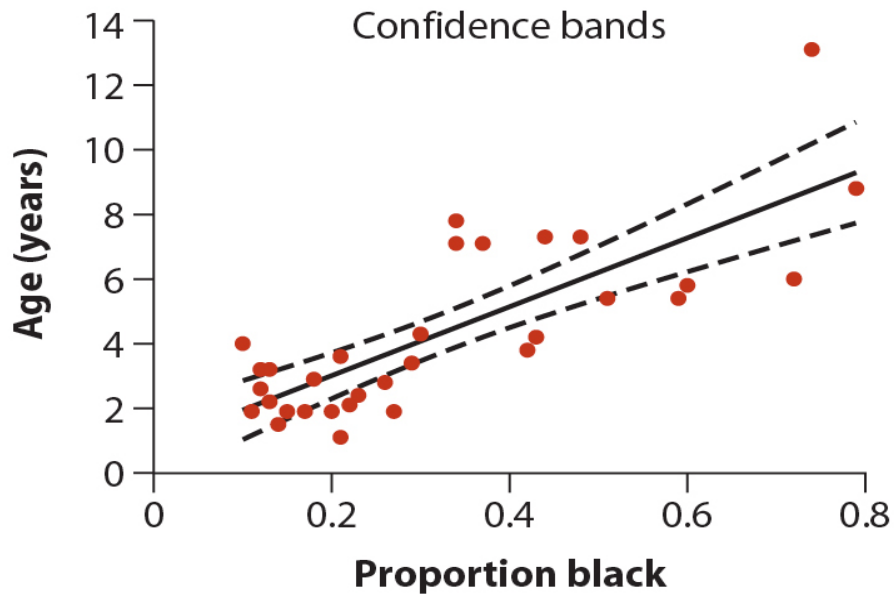
$$\text{ESS} = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2.$$

$$RSS = \sum_{i=1}^n (\varepsilon_i)^2 = \sum_{i=1}^n (y_i - (\alpha + \beta x_i))^2$$

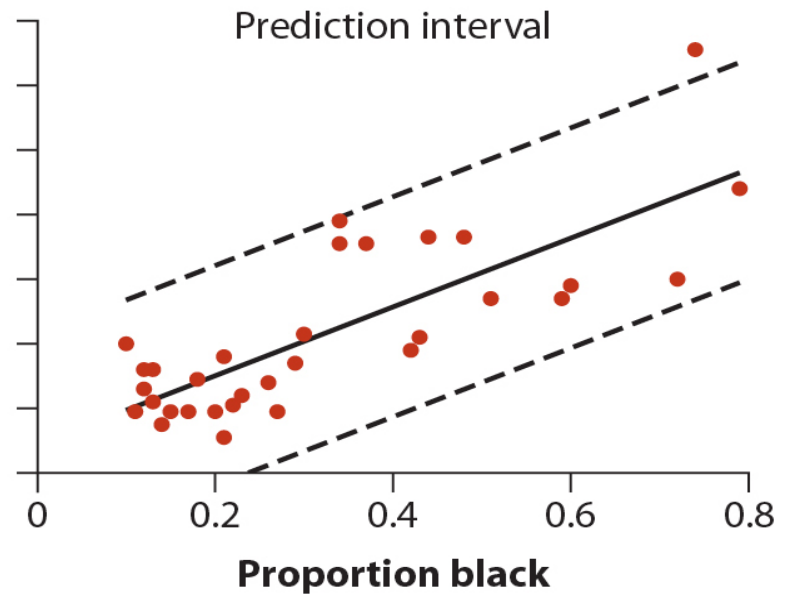
Testing the slope

- H_0 : slope = 0
- H_A : slope $\neq 0$
- T.test – but for any practical purposes R will do an ANOVA test
- Permutation test? How?

Predictions

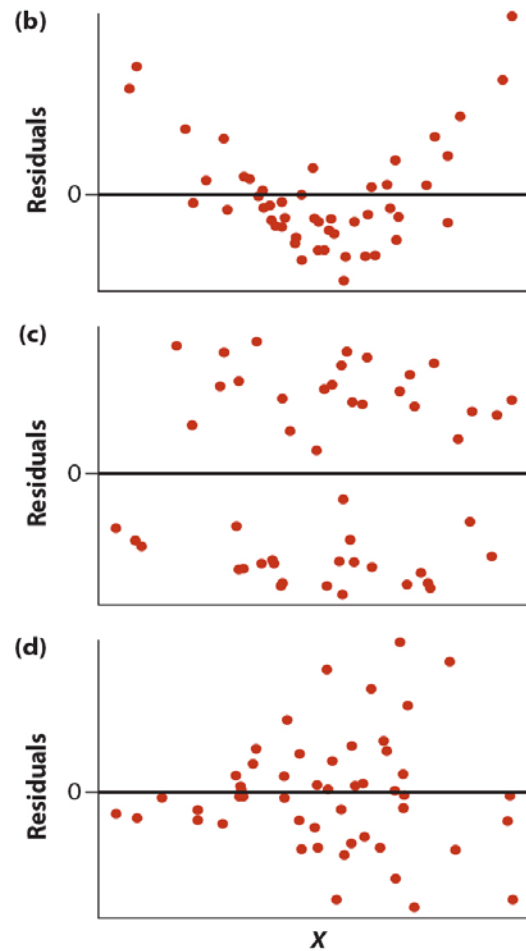


Precision of the predicted **mean**

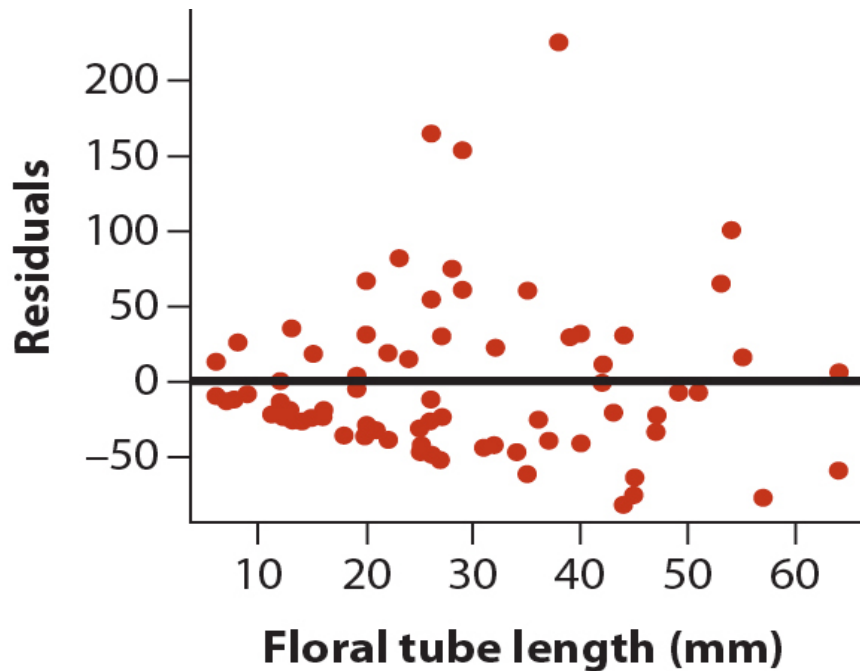


Precision of the predicted **value**

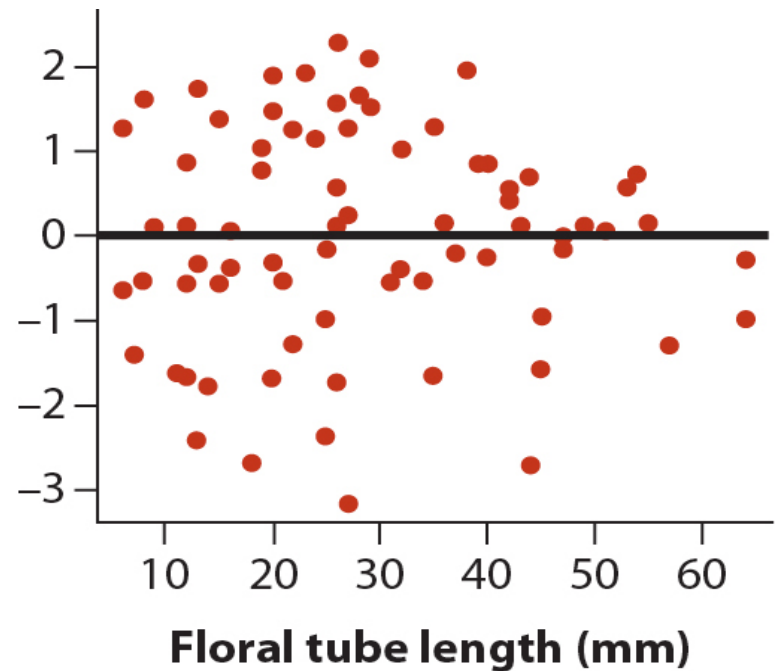
Residual plots are essential



Inspecting residuals



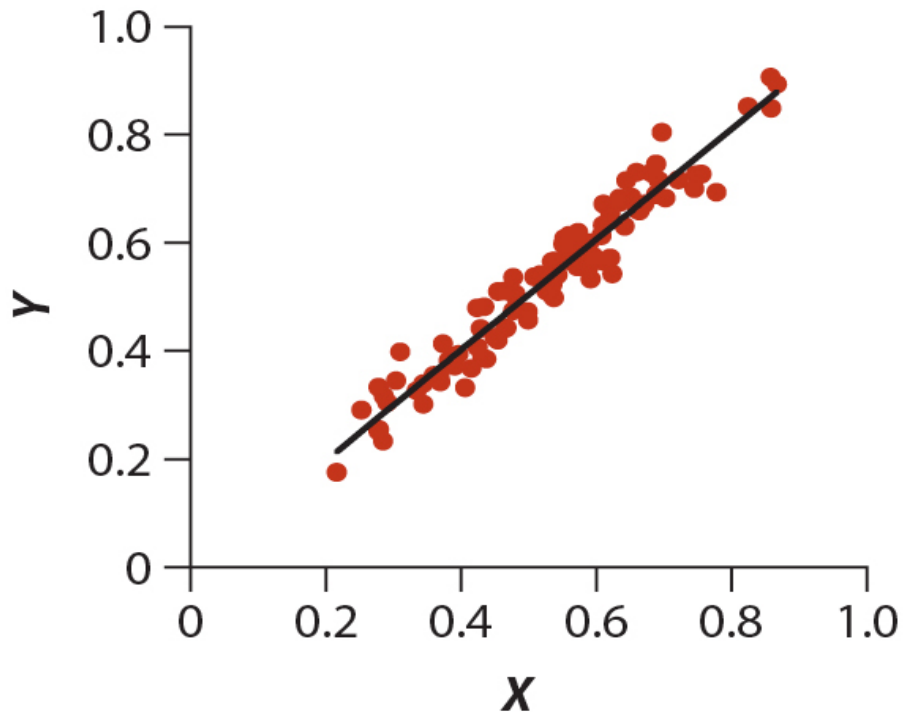
$$y = a + bx$$



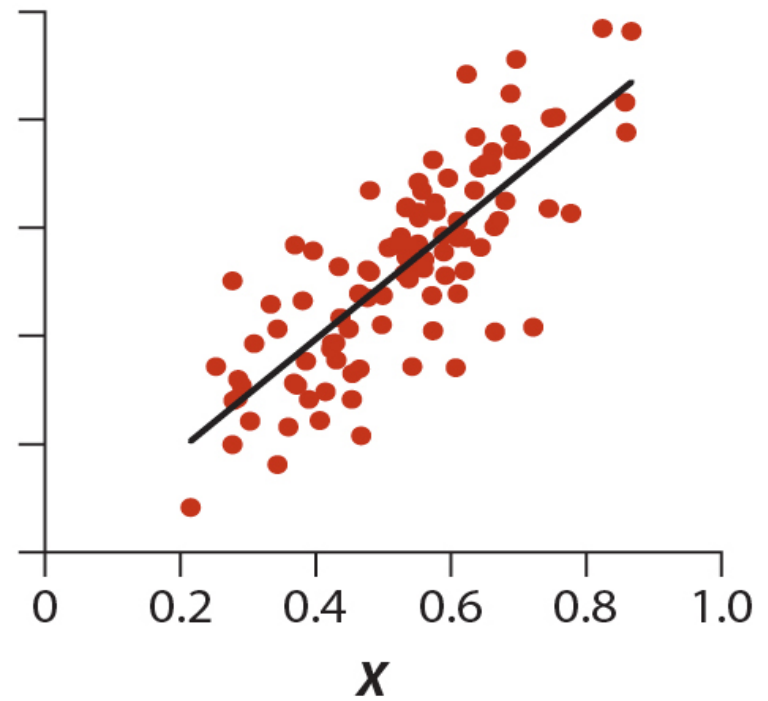
$$\sqrt{y} = a + bx$$

Measurement errors on Y...

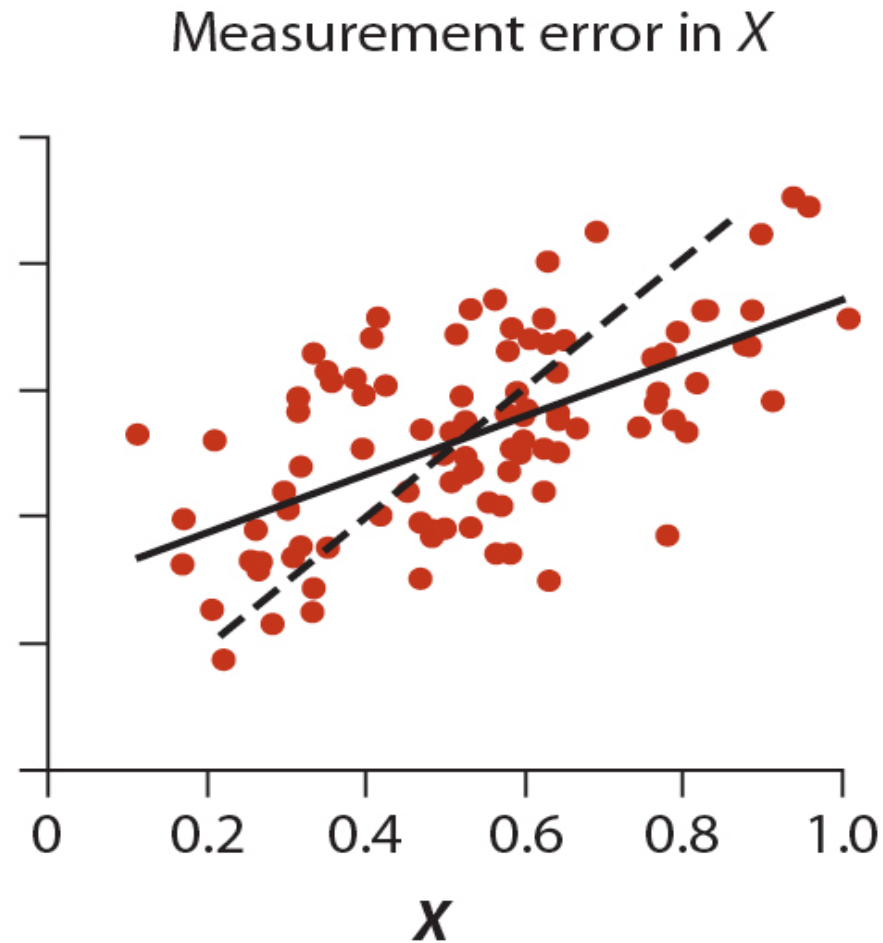
No measurement error



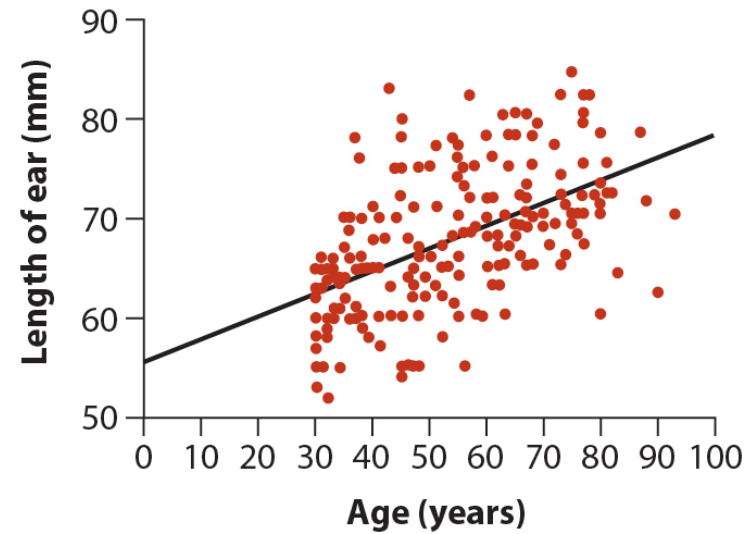
Measurement error in Y



Measurement errors on X



Extrapolation

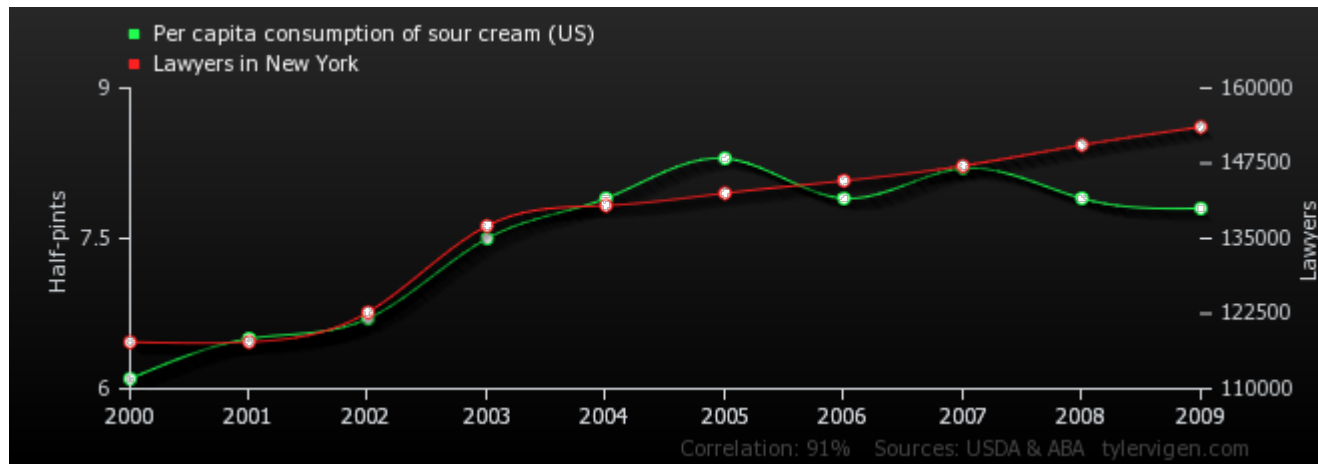


How are they connected?

- What is the difference between correlation and regression?
- What are similarities?
- Sanity checks?
- Transformations?

What does it mean?

- Correlation \neq causation



Exercises

- Distribution of levels of gene expression in Orangutans
- Is evolution (dN/dS) related to gene expression?
- We will summarize all genes pr. Chromosome
- Chromosomes with high gene expression
- Chromosome with high dN/dS
- What is dN/dS ?
- What do you expect? Why?

Pet project – guns and USA

- I have taken data on gun ownership pr. State and combined with death rate (by firearms)
- Also data on countries in the world
- Are these two variables correlated?
- Is it significant?
- How strong is the effect? (slope)
- What is R^2 ?