# Methods 3: Multilevel Statistical Modeling and Machine Learning

## Week 3: *Generalized linear mixed effects models*

September 28, 2021

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# Messages

- Practical exercise due 23.59 tomorrow
- Make sure to add your GitHub repository – a few are still missing: https://cryptpad.fr/pad/#/2/pad/edit/U21qNTbLgfkRiGZU1bnmDE2o/
- Remember, Class 2 (10-12) will be in 1453-116 tomorrow

# *RECAP on pooling*

## SLEEP STUDY EXAMPLE

https://psyteachr.github.io/stat-models-v1/introducing-linear-mixed-effects-models.html

# Learning goals and outline – *Linear Mixed Effects Models (LMM)*

1) Why can it be a good idea to do mixed effects modelling?

2) Understanding the basics of multilevel modelling

- also known as linear mixed effects modelling

3) Appreciating the difference between the different levels of effects

–or *random* and *fixed* effects, as they are also called

# Pooling - summary

- Complete pooling
  - ignoring a categorical predictor (e.g. *subject*)
- No pooling
  - model each level of the categorical predictor separately
- Partial pooling
  - we model both an average and each level of the categorical predictor (e.g. *subject*)

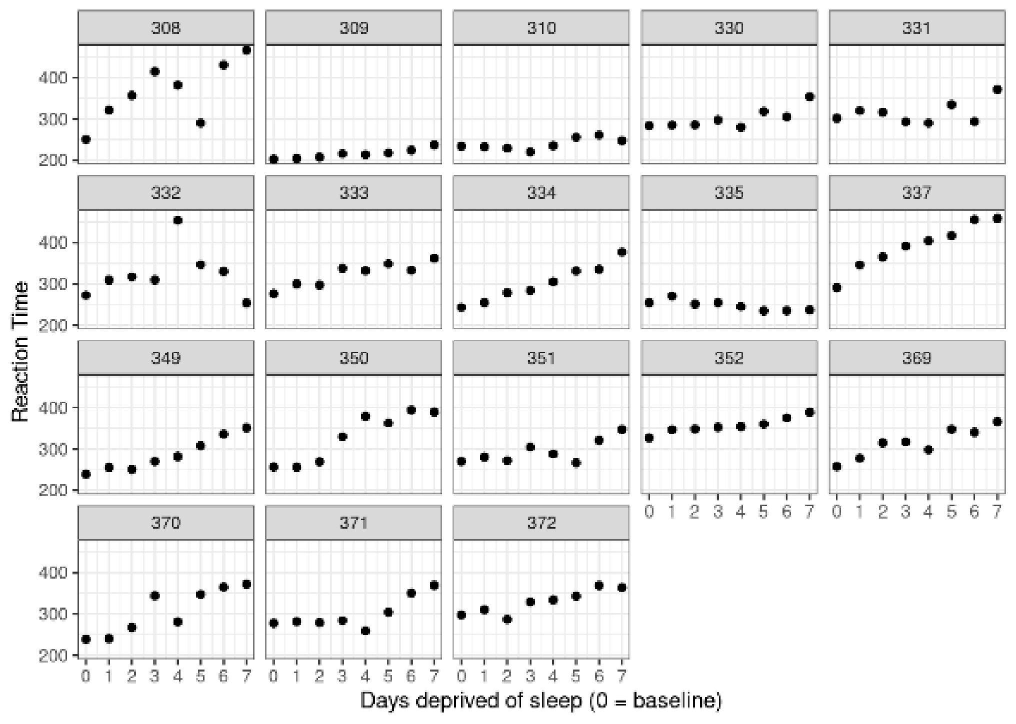
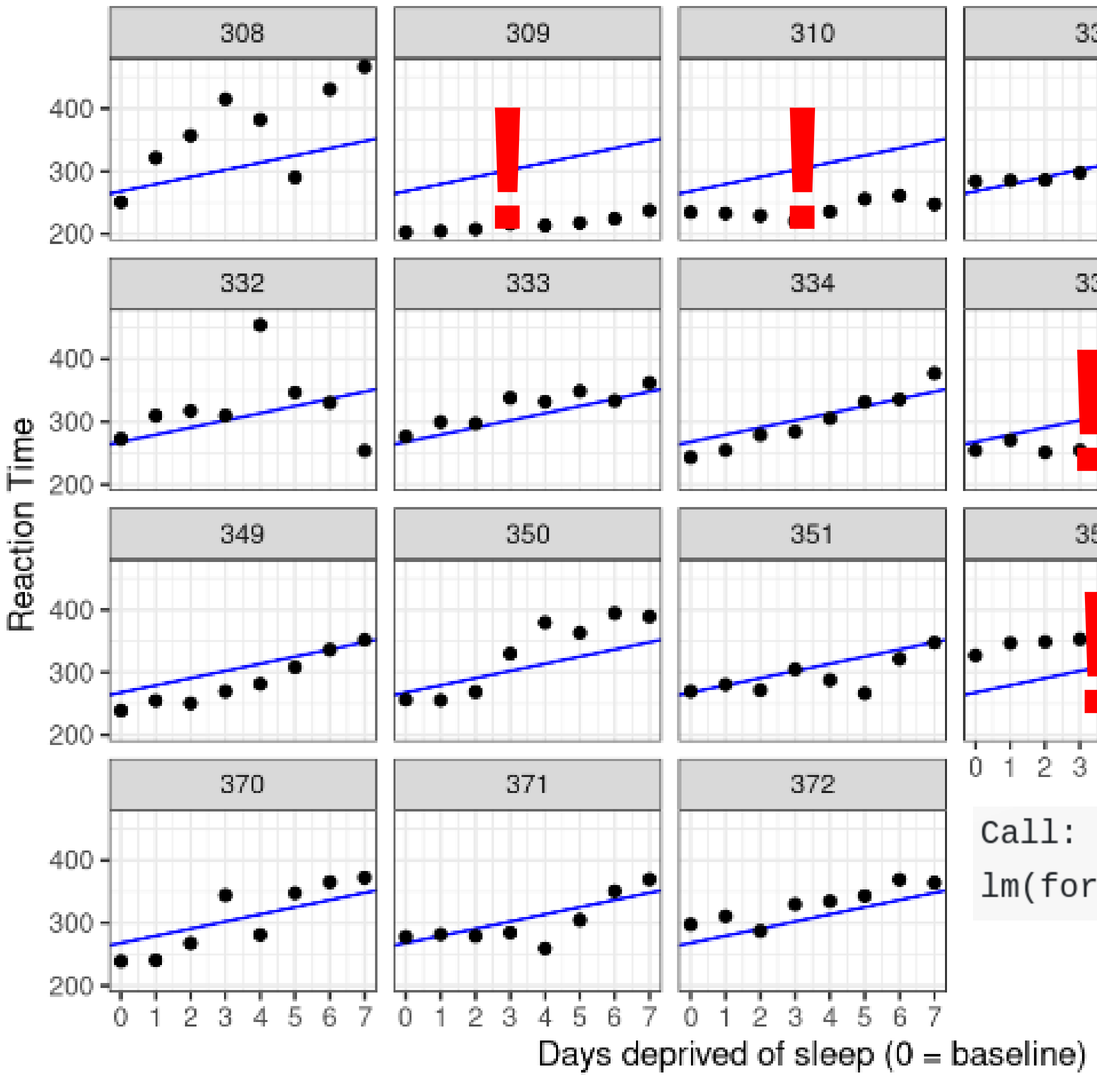



Figure 5.3: *Data from Belenky et al. (2003), showing reaction time at baseline (0) and after each day of sleep deprivation.*



(Barr, University of Glasgow)

# COMPLETE POOLING




```
Coefficients:
                Estimate
(Intercept)      267.967
days_deprived     11.435
```

```
Call:
lm(formula = Reaction ~ days_deprived, data = sleep2)
```

**There are some bad fits**

(Barr, University of Glasgow)

```r
lm(formula = Reaction ~ days_deprived + Subject + days_deprived:Subject,
    data = sleep2)
```

```
## Coefficients:
##                         Estimate
## (Intercept)             288.2175
## days_deprived            21.6905
## Subject309              -87.9262
## Subject310              -62.2856
## Subject330              -14.9533
## Subject331                9.9658
## Subject332               27.8157
```

... and the remaining 12 subjects

```
## days_deprived:Subject309 -17.3334
## days_deprived:Subject310 -17.7915
## days_deprived:Subject330 -13.6849
## days_deprived:Subject331 -16.8231
## days_deprived:Subject332 -19.2947
## days_deprived:Subject333 -10.8151
```

... and the remaining 12 subjects

**NO POOLING**





(Barr, University of Glasgow)

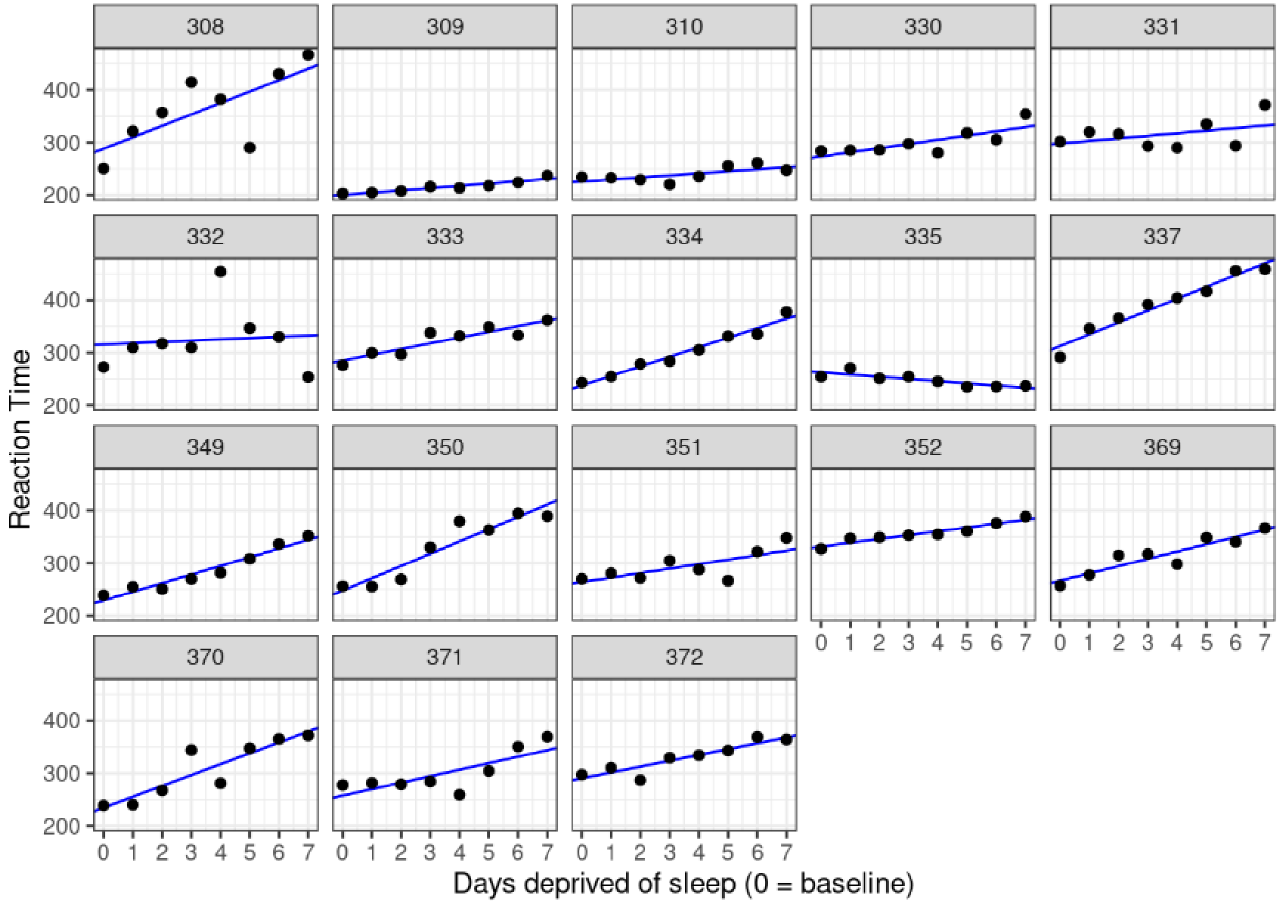



Figure 5.5: Data plotted against fits from the no-pooling approach.

# NO POOLING

Good fits now:

What are the limits of this mo



 (Barr, University of Glasgow)

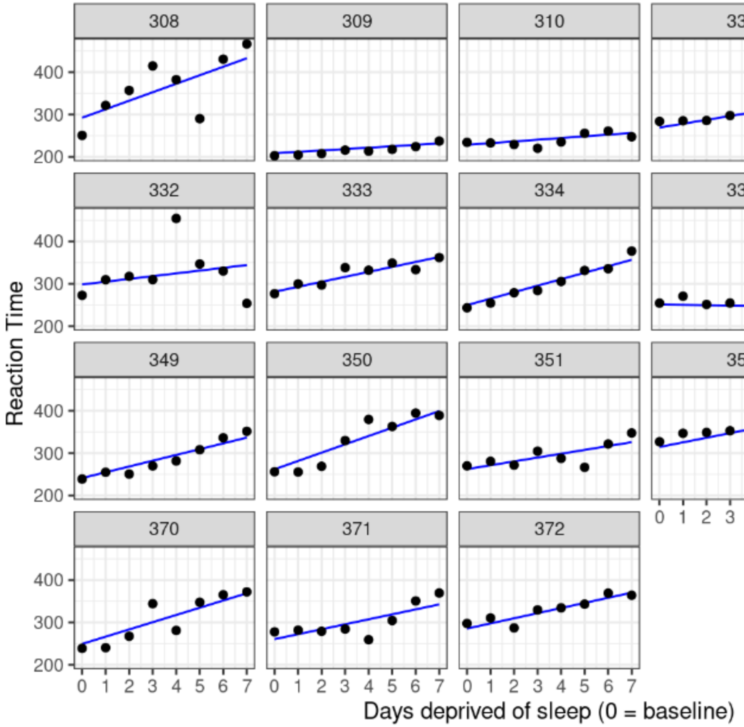



Figure 5.6: Data plotted against predictions from a partial pooling approach.

# PARTIAL POOLING

```
Fixed effects:
               Estimate Std. Error t value
(Intercept)     267.967      8.266  32.418
days_deprived    11.435      1.845   6.197
```

```
ranef(pp_mod)[["Subject"]]
```

| | (Intercept) | days_deprived |
|---|---|---|
| 308 | 24.4992891 | 8.6020000 |
| 309 | -59.3723102 | -8.1277534 |
| 310 | -39.4762764 | -7.4292365 |
| 330 | 1.3500428 | -2.3845976 |

```
Linear mixed model fit by REML ['lmerMod']
Formula: Reaction ~ days_deprived + (days_deprived | Subject)
   Data: sleep2
```





(Barr, University of Glasgow)

# No pooling vs partial pooling

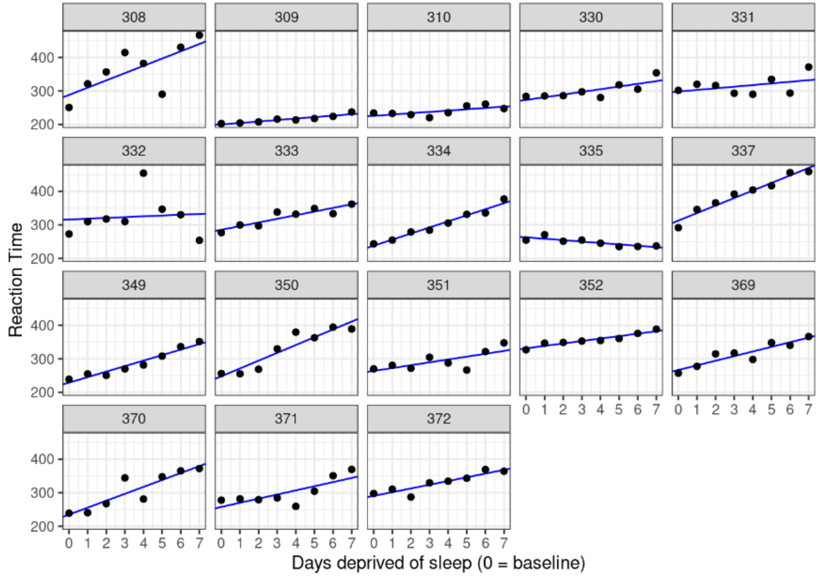



Figure 5.5: Data plotted against fits from the no-pooling approach.

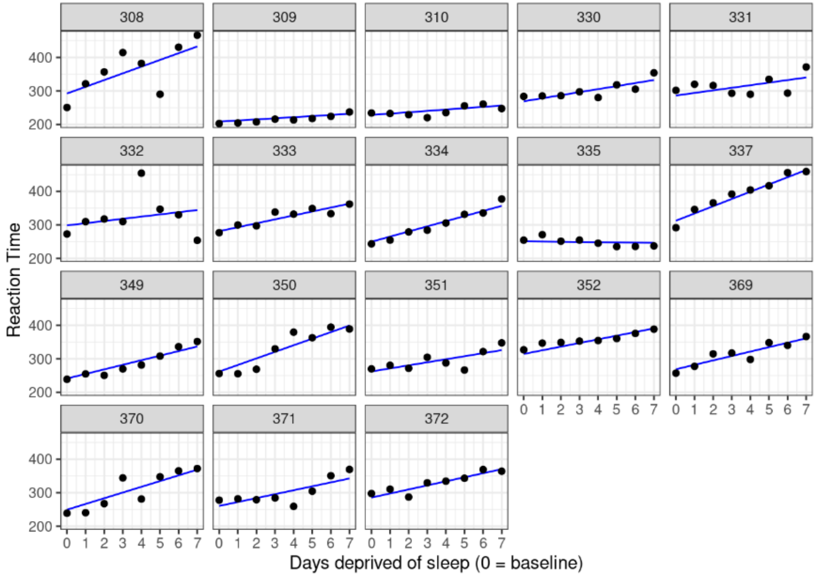



Figure 5.6: Data plotted against predictions from a partial pooling approach.

Both model the individual variance – but only one is generalisable outside the s





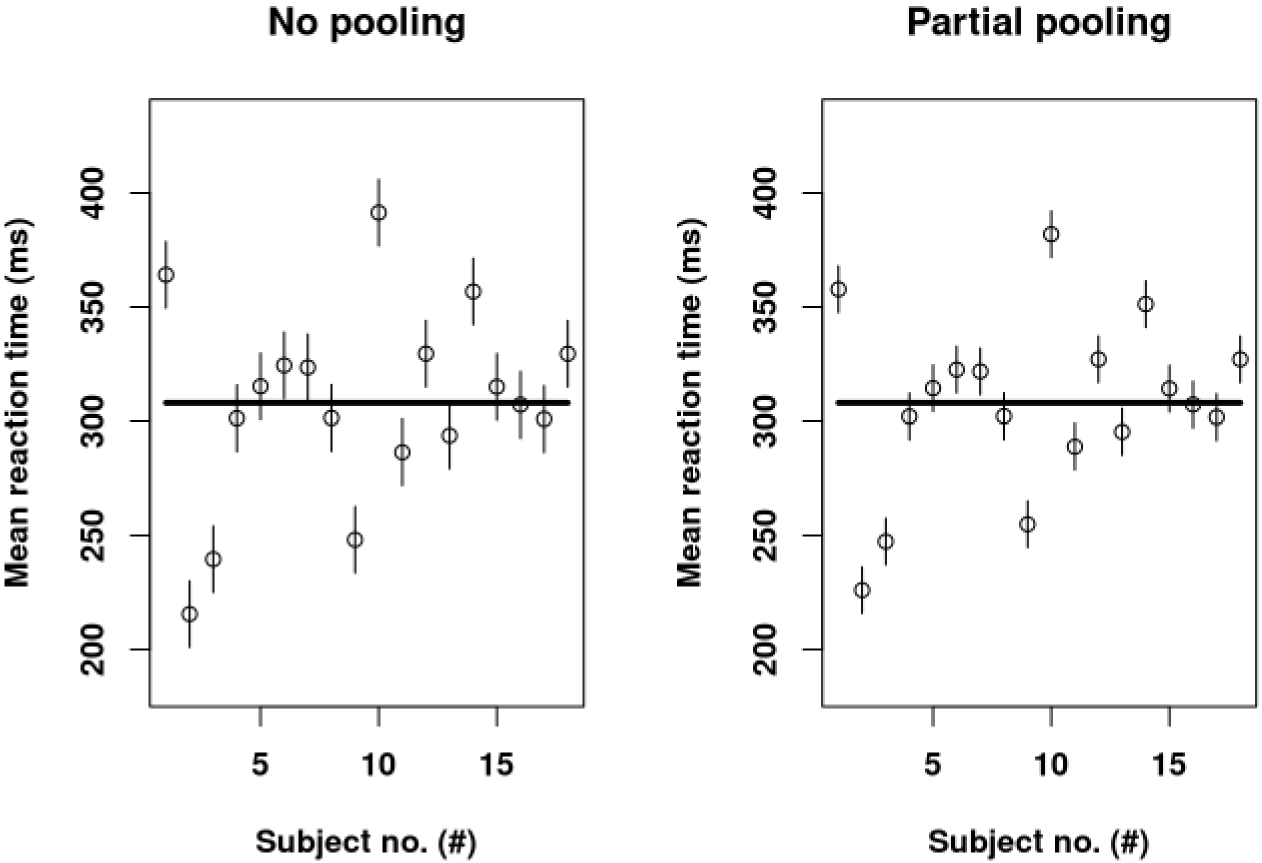
(Barr, University of Glasgow)

# Partial pooling

(Gelman and Hill, 2006 (12.1) )

$$\hat{\alpha}_j^{multilevel} \approx \frac{\frac{n_j}{\sigma_y^2}\bar{y}_j + \frac{1}{\sigma_\alpha^2}\bar{y}_{all}}{\frac{n_j}{\sigma_y^2} + \frac{1}{\sigma_\alpha^2}}$$

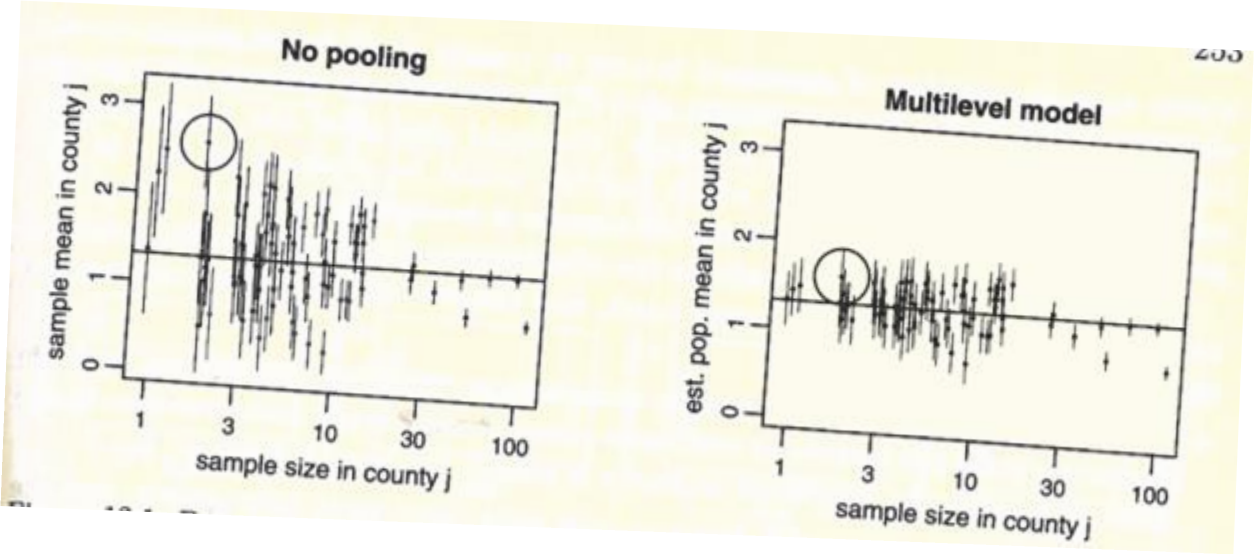
another scary looking th




What is the advant

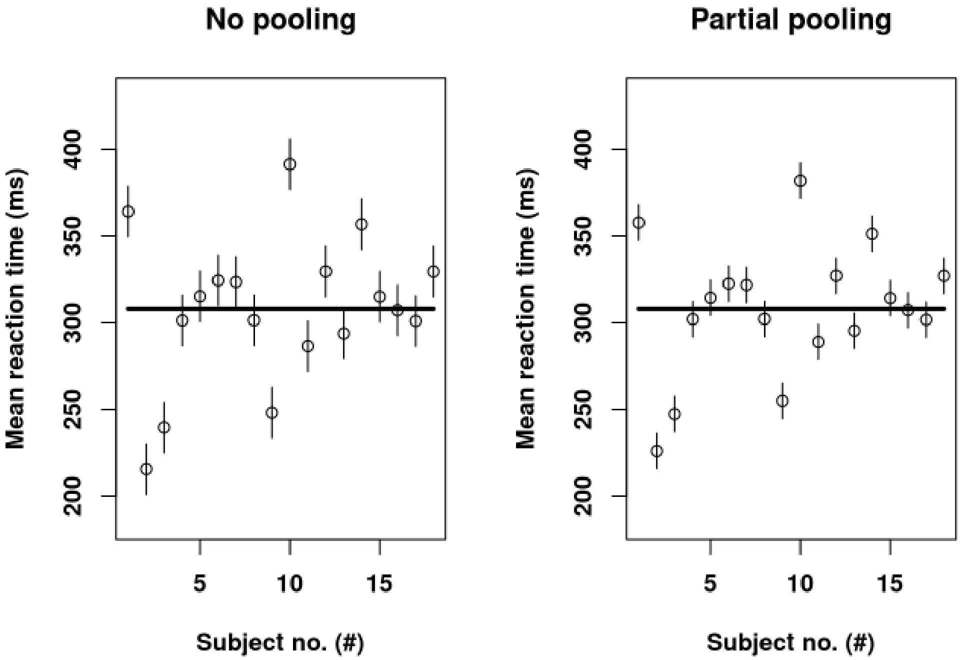
# Now with different sample sizes

What is the advan

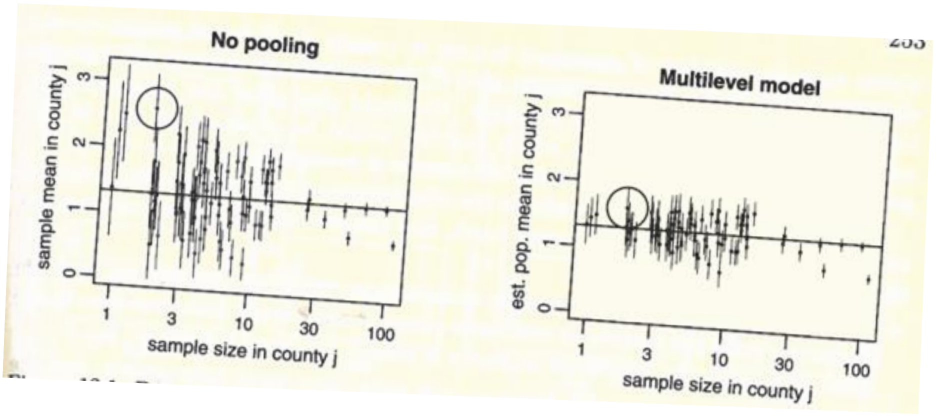



(Gelman and Hill, 2006)

## Same *n* for each subject




## Different *n* for each county

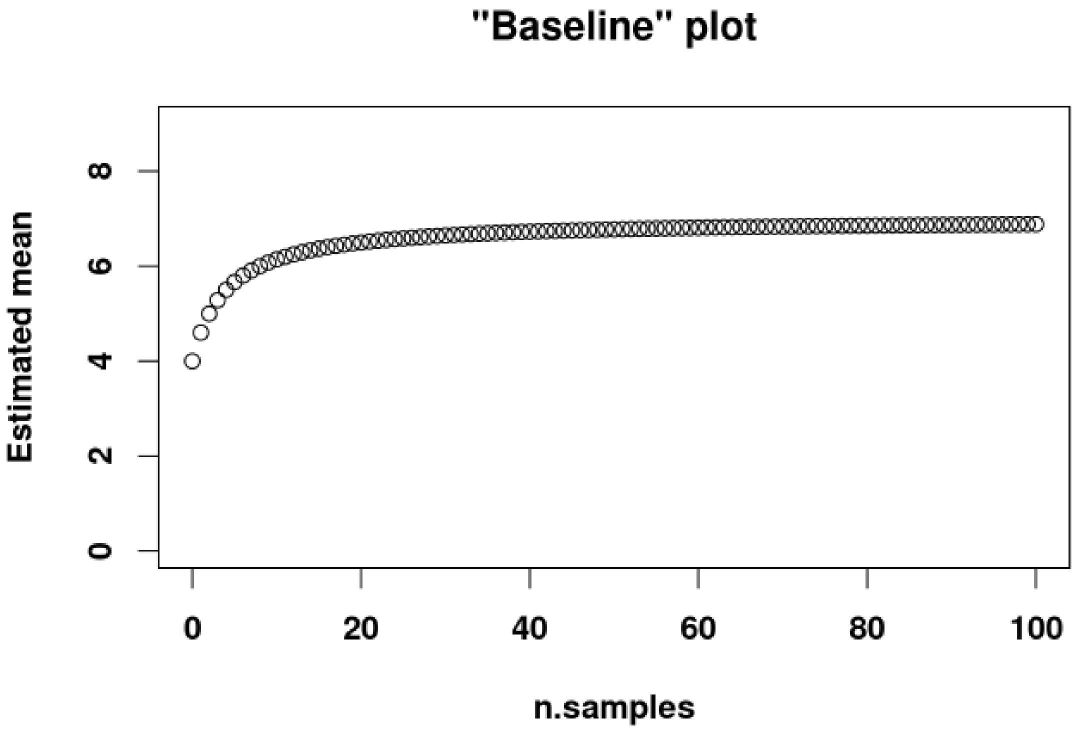



(Gelman and Hill, 2006)

```r
estimate.multilevel.mean <- function(n.j, sigma.y, sigma.mean, y.j, y.al
l)
{
    alpha <- ((n.j / sigma.y^2) * y.j + (1 / sigma.mean^2) * y.all) /
               ( (n.j / sigma.y^2) + (1 / sigma.mean^2))
    return(alpha)
}
```

```r
## "baseline"

sigma.y <- 3
y.j <- 7
sigma.mean <- 1.5
y.all <- 4
ns <- 0:100
```

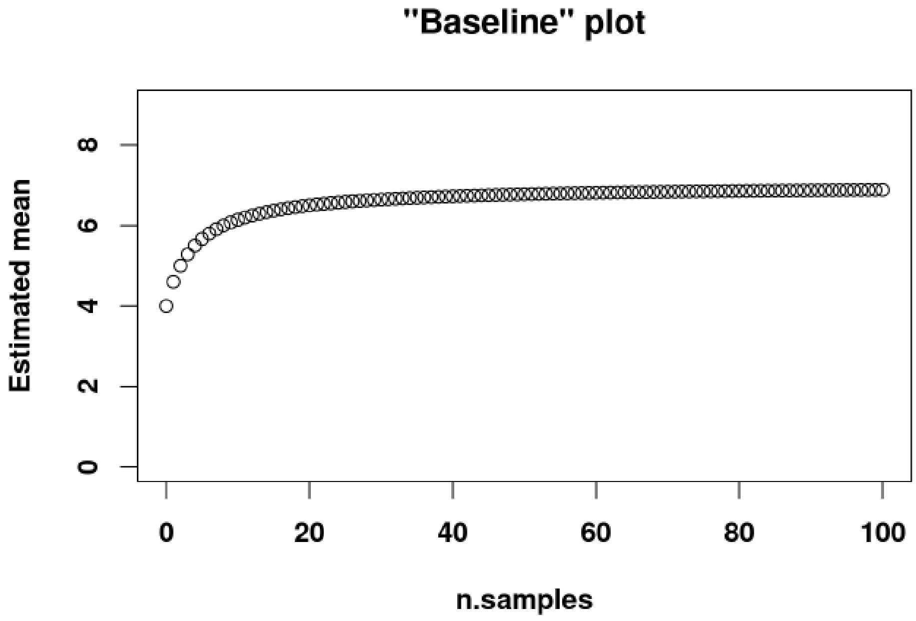


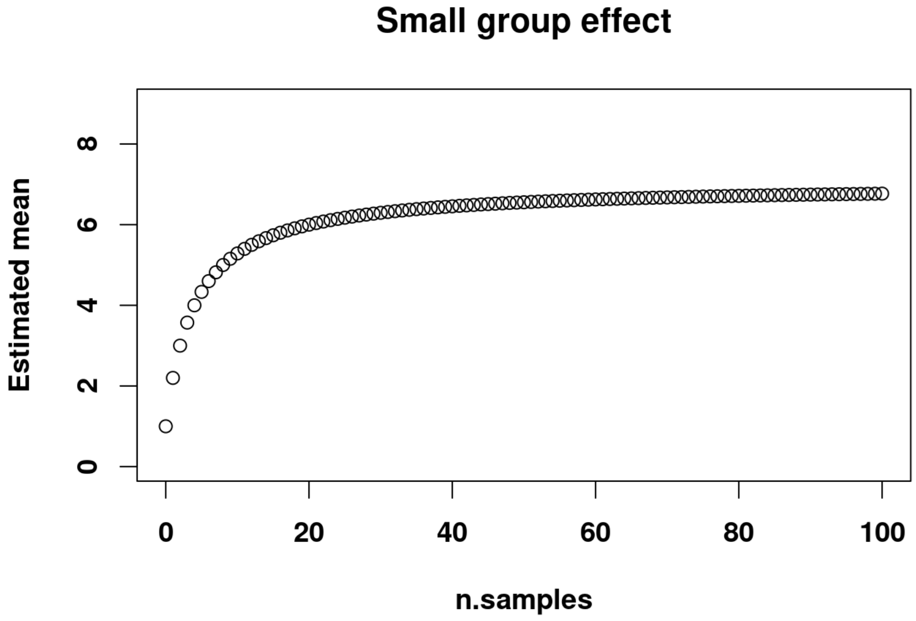
```r
## "baseline"

sigma.y <- 3
y.j <- 7
sigma.mean <- 1.5
y.all <- 4
ns <- 0:100
```

```r
## small group effect

sigma.y <- 3
y.j <- 7
sigma.mean <- 1.5
y.all <- 1
ns <- 0:100
```



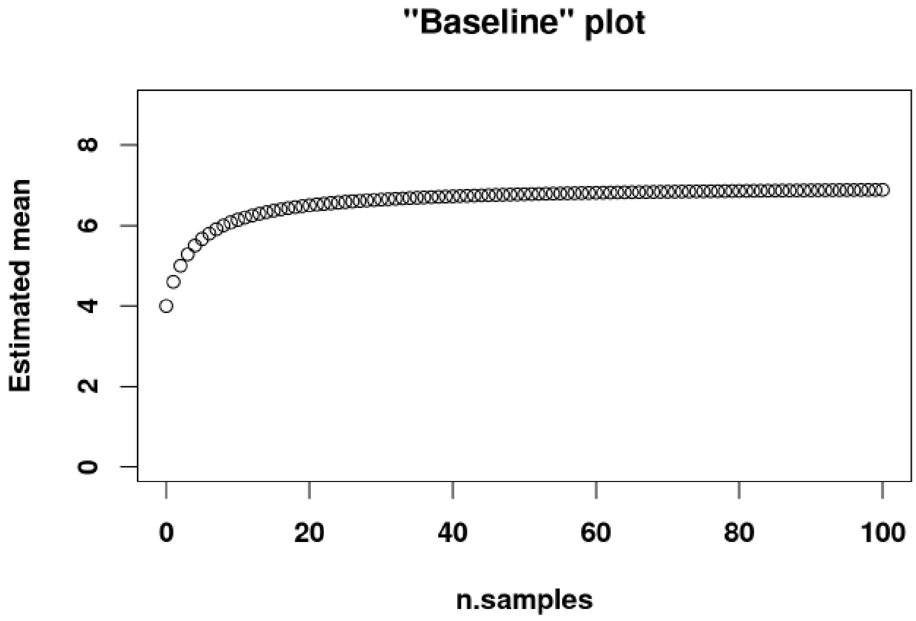



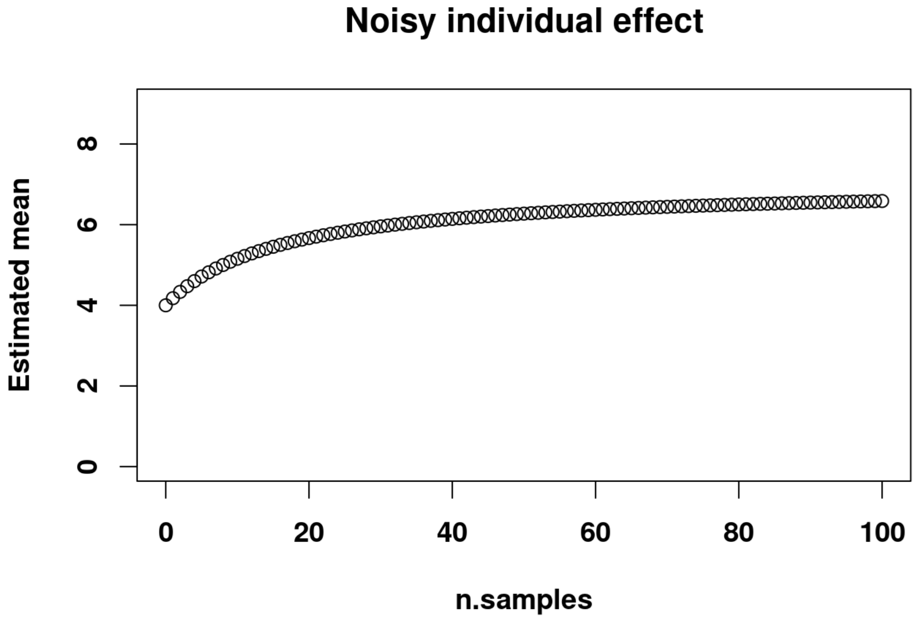
```r
## "baseline"

sigma.y <- 3
y.j <- 7
sigma.mean <- 1.5
y.all <- 4
ns <- 0:100
```

```r
## noisy individual effect

sigma.y <- 6
y.j <- 7
sigma.mean <- 1.5
y.all <- 4
ns <- 0:100
```



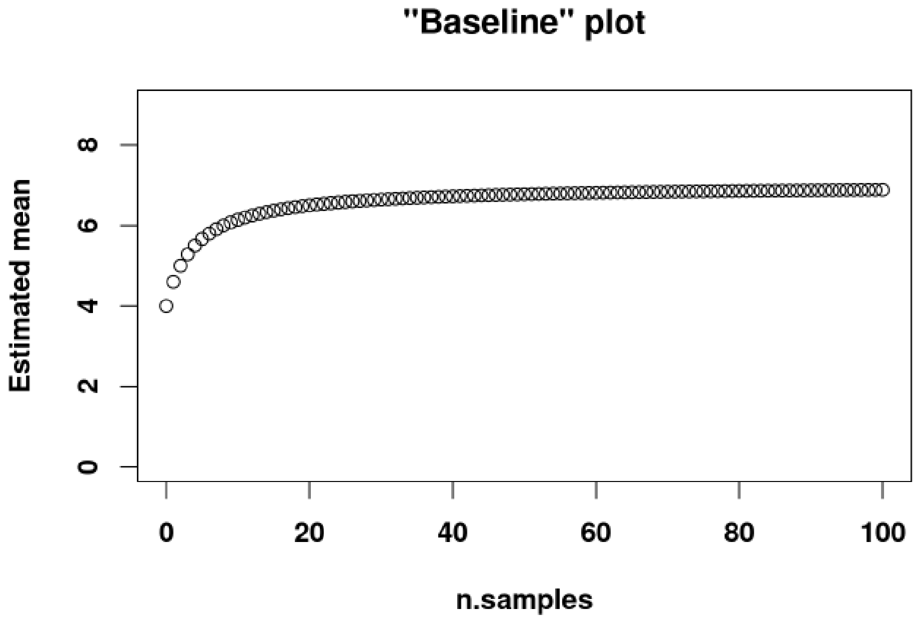



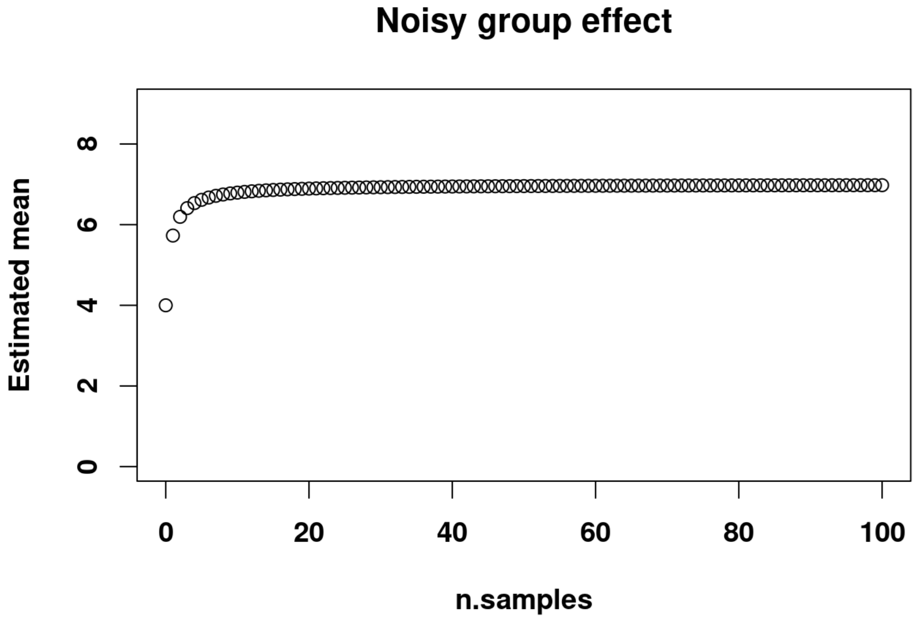
```r
## "baseline"

sigma.y <- 3
y.j <- 7
sigma.mean <- 1.5
y.all <- 4
ns <- 0:100
```

```r
## noisy group effect

sigma.y <- 3
y.j <- 7
sigma.mean <- 3.5
y.all <- 4
ns <- 0:100
```



# Motivation for multilevel modelling:

We want to use all the information in the data while fulfilling the assumptions necessary for the residuals

# We can add:

Without letting small or uncertain samples unduly affect our group estimate

# Now on to generalized linear mixed models ...

# Did you learn?

*Linear Mixed Effects Models (LMM)*

1) Why can it be a good idea to do mixed effects modelling?

2) Understanding the basics of multilevel modelling

- also known as linear mixed effects modelling

3) Appreciating the difference between the different levels of effects

–or *random* and *fixed* effects, as they are also called

# ... but let’s do a recap of the generalized linear model first

# Learning goals

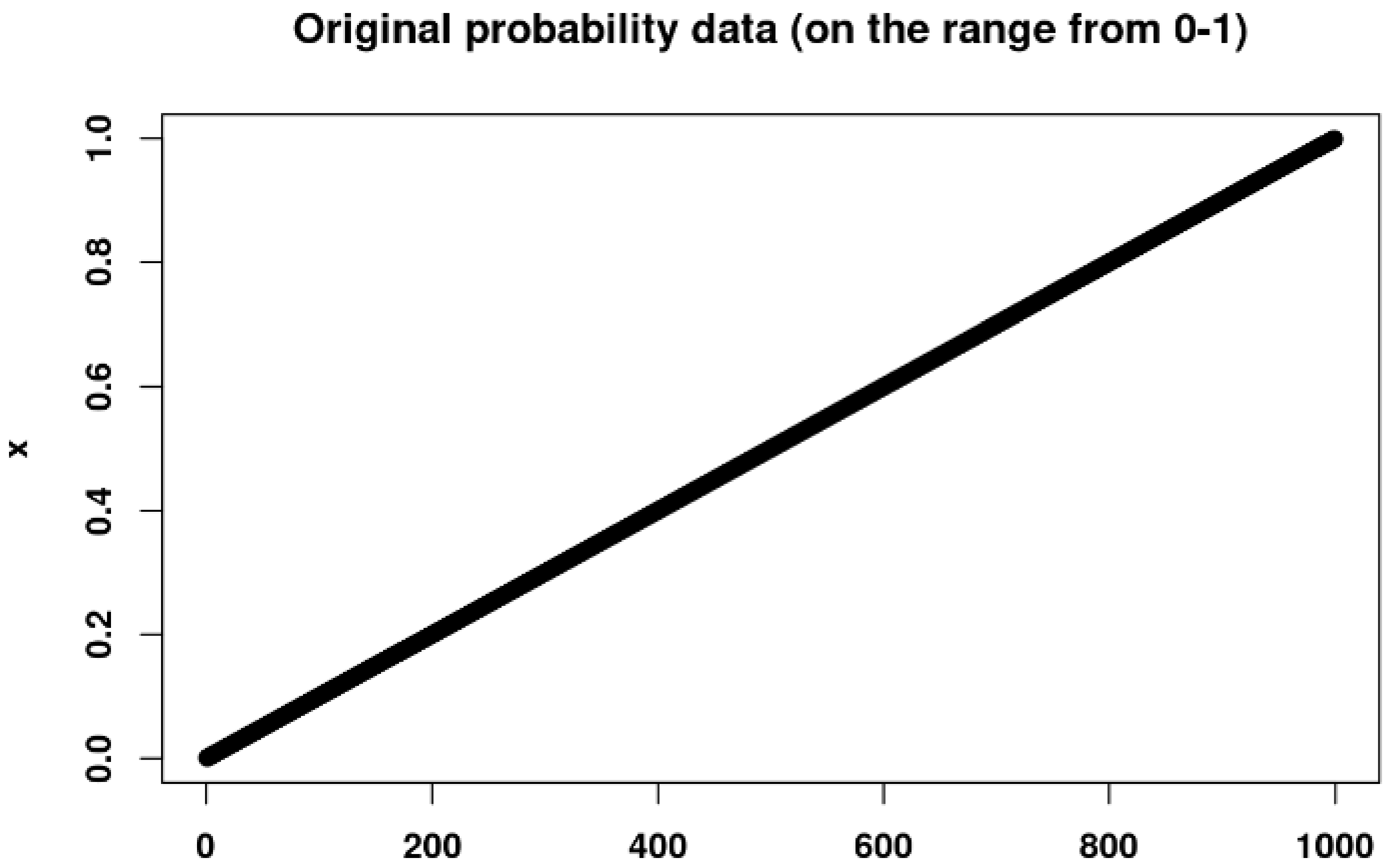
*Generalized Linear Mixed Effects Models (GLMM)*

1) Understanding that we can extend the scope of our multilevel modelling by using appropriate link functions and data distributions

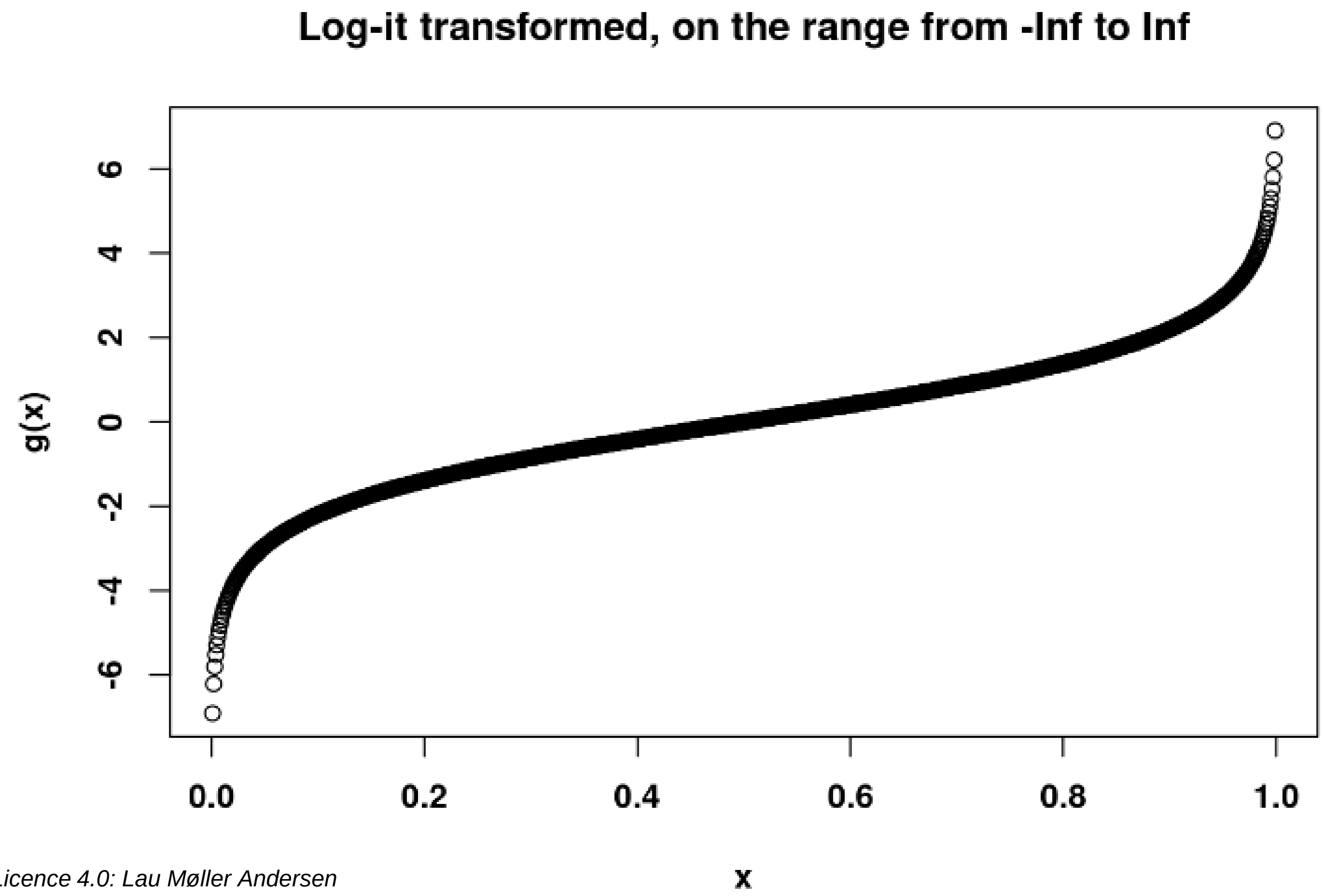
2) Understanding the multilevel equivalent of the GLM


Breaking all promises and goin

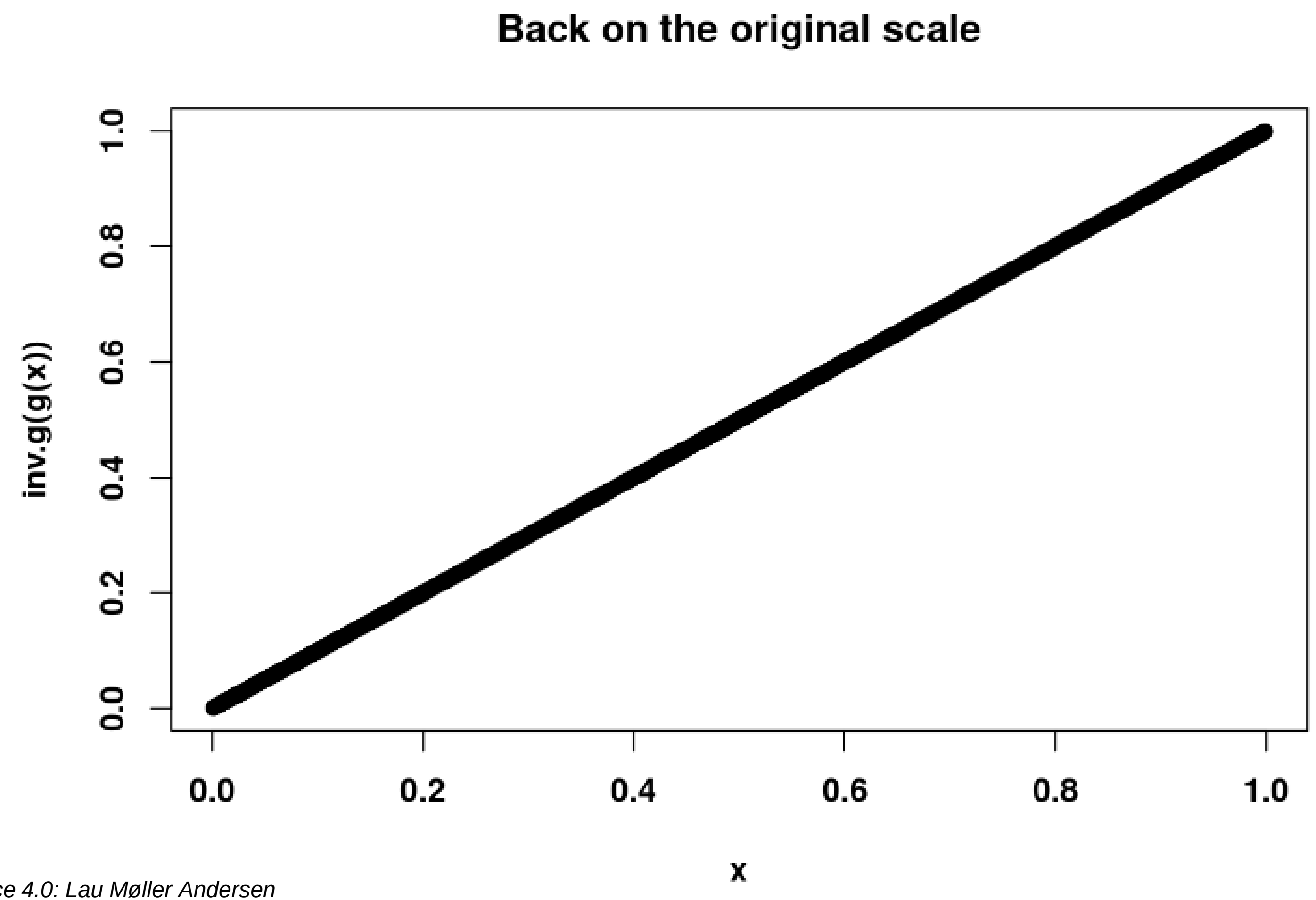
```r
x <- seq(0.001, 0.999, 0.001)
plot(x, main='Original probability data (on the range from 0-1)')
```



```r
plot(x, g(x), main='Log-it transformed, on the range from -Inf to Inf')
```



```r
plot(x, inv.g(g(x)), main='Back on the original scale')
```



# These are the fitted values

```
y.hat <- inv.g(X %*% beta.hat)
print(head(y.hat))
```

```
##                         [,1]
## Mazda RX4          0.8172115
## Mazda RX4 Wag      0.6157283
## Datsun 710         0.9373069
## Hornet 4 Drive     0.2897304
## Hornet Sportabout  0.1415972
## Valiant            0.1320944
```

```
print(head(y.hat - logistic.model$fitted.values))
```

```
##                   [,1]
## Mazda RX4            0
## Mazda RX4 Wag        0
## Datsun 710           0
## Hornet 4 Drive       0
## Hornet Sportabout    0
## Valiant              0
```

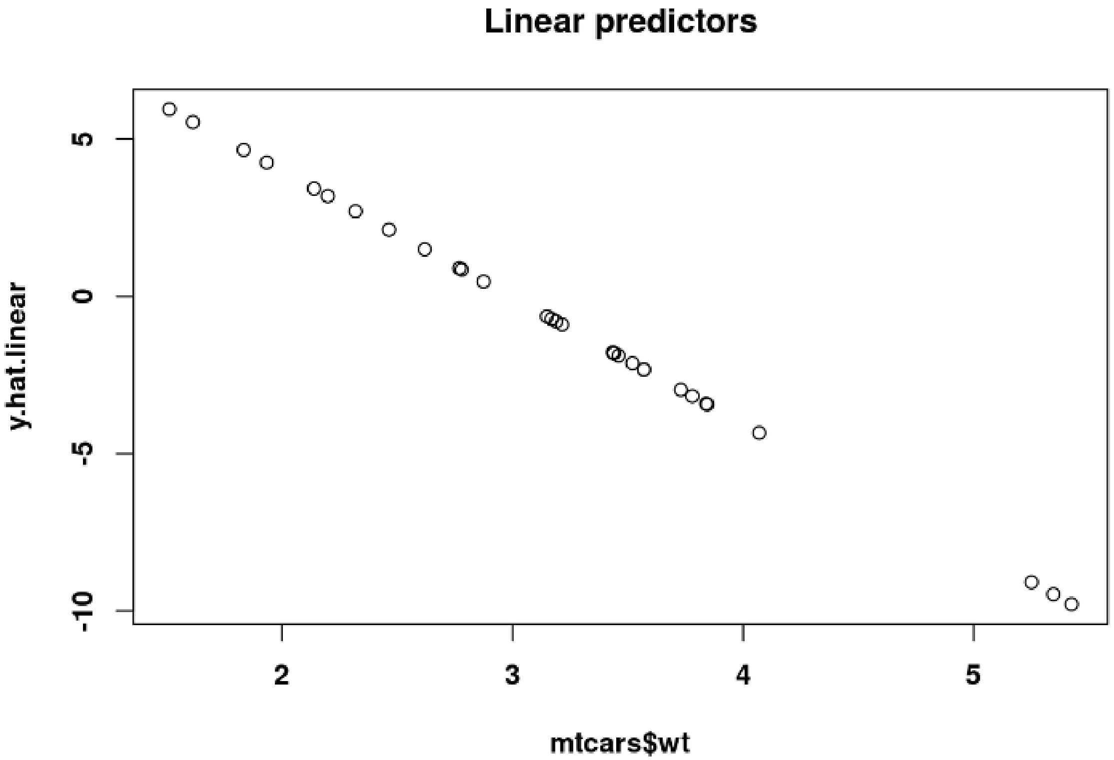
# These are the linear predictors

```
y.hat.linear <- X %*% beta.hat

print(head(y.hat.linear - logistic.model$linear.predictors))
```

```
##                    [,1]
## Mazda RX4             0
## Mazda RX4 Wag         0
## Datsun 710            0
## Hornet 4 Drive        0
## Hornet Sportabout     0
## Valiant               0
```

# Looks like a “normal” linear regression



# Some link functions

**Usage**

```
family(object, ...)

binomial(link = "logit")
gaussian(link = "identity")
Gamma(link = "inverse")
inverse.gaussian(link = "1/mu^2")
poisson(link = "log")
quasi(link = "identity", variance = "constant")
quasibinomial(link = "logit")
quasipoisson(link = "log")
```

# Important difference from the general linear model

We also make maximum likelihood estimates for

# Did you learn?

*Generalized Linear Mixed Effects Models (GLMM)*

1) Understanding that we can extend the scope of our multilevel modelling by using appropriate link functions and data distributions

2) Understanding the multilevel equivalent of the GLM

# References


- Gelman, A., Hill, J., 2006. Data Analysis Using Regression and Multilevel/Hierarchical Models. Cambridge University Press.