Mode coupling at the transition region and the validity of line-tied boundary conditions

Alex Prokopyszyn, Alan Hood, Andrew Wright

University of St Andrews

Solar & Magnetospheric Theory Group
University of St Andrews

Science & Technology Facilities Council

Aims

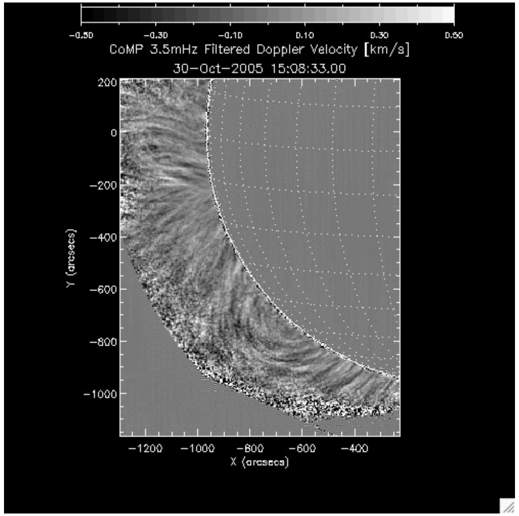
- Show why Fast / Alfvén waves couple at the TR
- Test the validity of line-tied BCs
- Show they can be invalid for highly phase-mixed waves

Structure

- **Background**
- Model 1:
  - Line-tied, pulse
- Model 2:
  - Line-tied, normal mode
- Model 3:
  - Chromosphere, normal mode
- Summary and conclusions

Why study MHD waves?

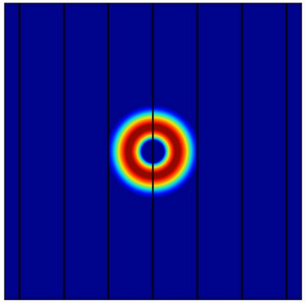
- Ubiquitous
- Coronal heating
- Seismology



Fast vs. Alfvén waves

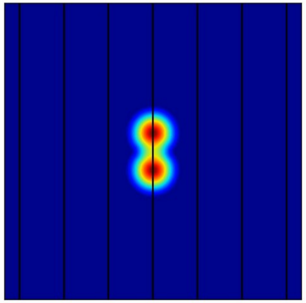
Fast waves:

- Propagate isotropically
- $\frac{\omega^2}{v_A^2} = k_x^2 + k_y^2 + k_z^2$



Alfvén waves:

- Propagate parallel to $\mathbf{B}_0$
- $\frac{\omega^2}{v_A^2} = k_{\|}^2$



Mode conversion

- Can occur via non-linear effects see e.g. verwichte, high beta to low beta see e.g. mclaughlin, gradients in background in Alfven speed $\rightarrow$ resonant absorption

Mode conversion at the TR

- Studied extensively by Halberstadt & Goedbloed analytically closed loops
- Arregui 2003 studied numerically
- Cally & Hansen (2011); Hansen & Cally (2012) suggest that mode conversion from fast waves to Alfv´en waves at the transition region enables sufficient energy flux to enter the corona

Line-tied boundary conditions

- Add VAL
- $\boldsymbol{u} = 0$
- Usually valid
- Might naively expect the only thing that matters is the lengthscale in the radial direction

Normal mode

- $f(\mathbf{r},t) = f_0(\mathbf{r})\exp(i\omega t)$
- Add Morton power spectrum

Model and Equations

- Background quantities:

$$\rho = \rho_0 \qquad \boldsymbol{B_0} = B_0 \hat{\boldsymbol{B}}_0$$

- Perturbations:

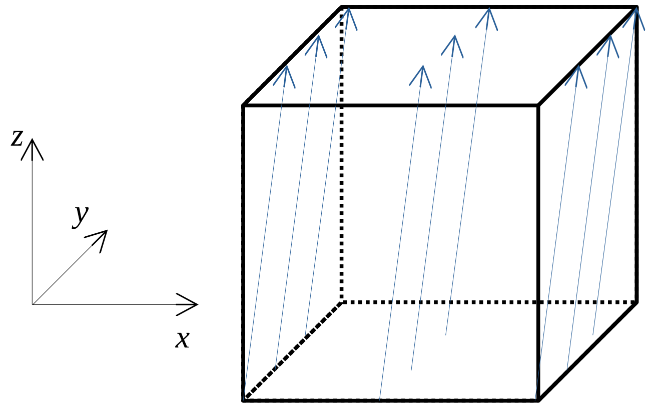
$$\boldsymbol{u} = u_x \hat{\boldsymbol{x}} + u_\perp \hat{\perp} \qquad \boldsymbol{b} = b_x \hat{\boldsymbol{x}} + b_\perp \hat{\perp} + b_\parallel \hat{\boldsymbol{B}}_0$$

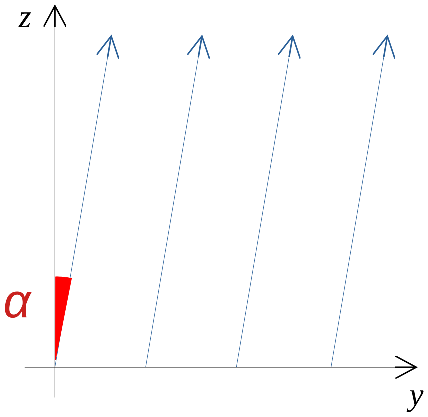
- Unit vectors:

$$\hat{\perp} = \cos(\alpha)\hat{\boldsymbol{y}} - \sin(\alpha)\hat{\boldsymbol{z}} \qquad \hat{\boldsymbol{B}}_0 = \sin(\alpha)\hat{\boldsymbol{y}} + \cos(\alpha)\hat{\boldsymbol{z}}$$

- Equations:

$$\rho_0 \frac{\partial \boldsymbol{u}}{\partial t} = \boldsymbol{j} \times \boldsymbol{B_0} \qquad \frac{\partial \boldsymbol{b}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B_0})$$





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Numerical scheme

- Leapfrog algorithm
- Based on Rickard and Wright (1994)
- Finite-difference
- Staggered grid
- Second-order accurate

Initial / boundary conditions

- Assume that

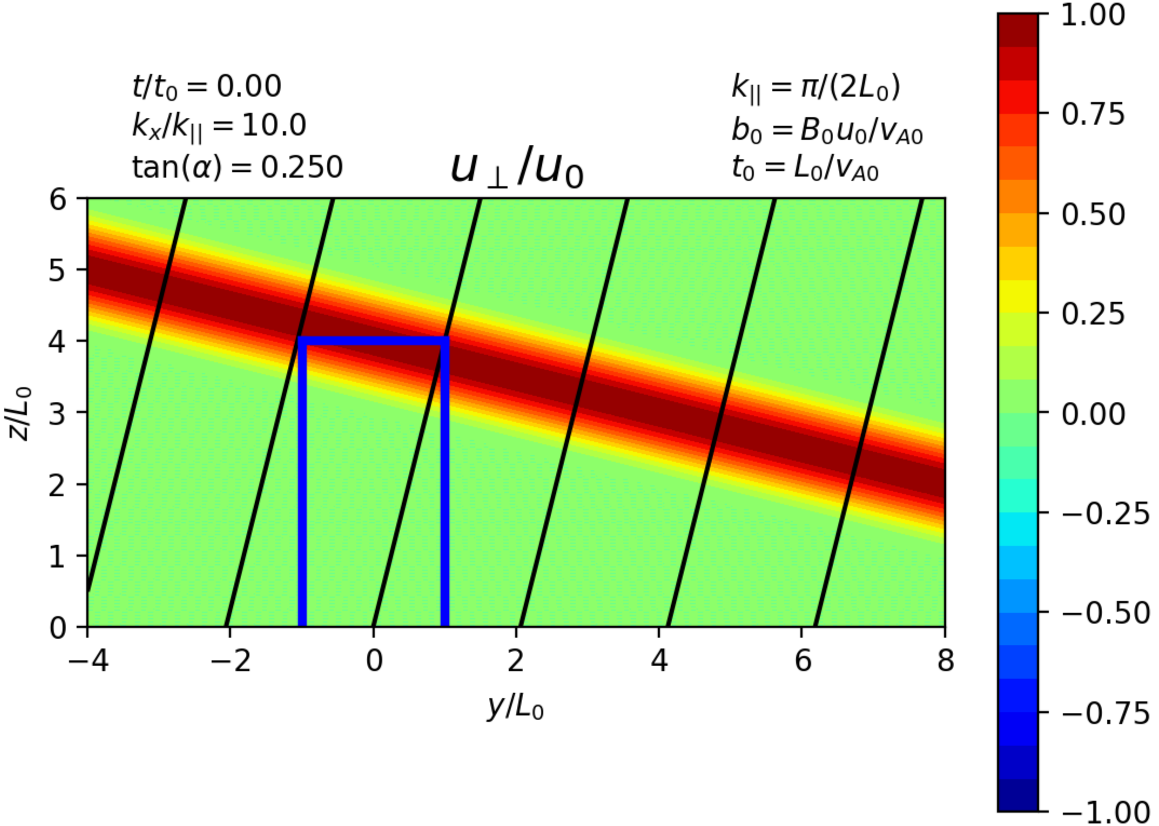
$$u_x, b_x \propto \sin\left(k_x x\right)$$

$$u_\perp, u_\perp, b_\parallel \propto \cos\left(k_x x\right)$$

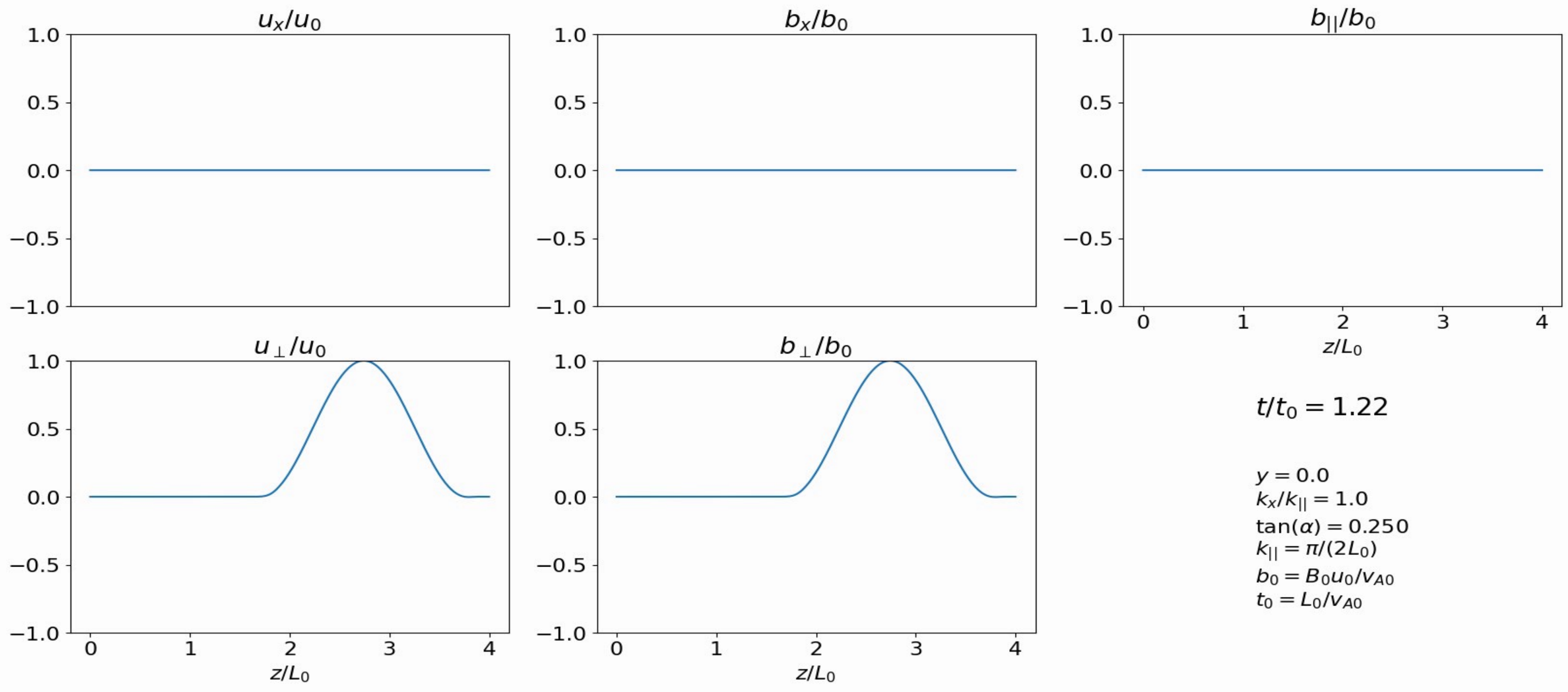
- Initial conditions:

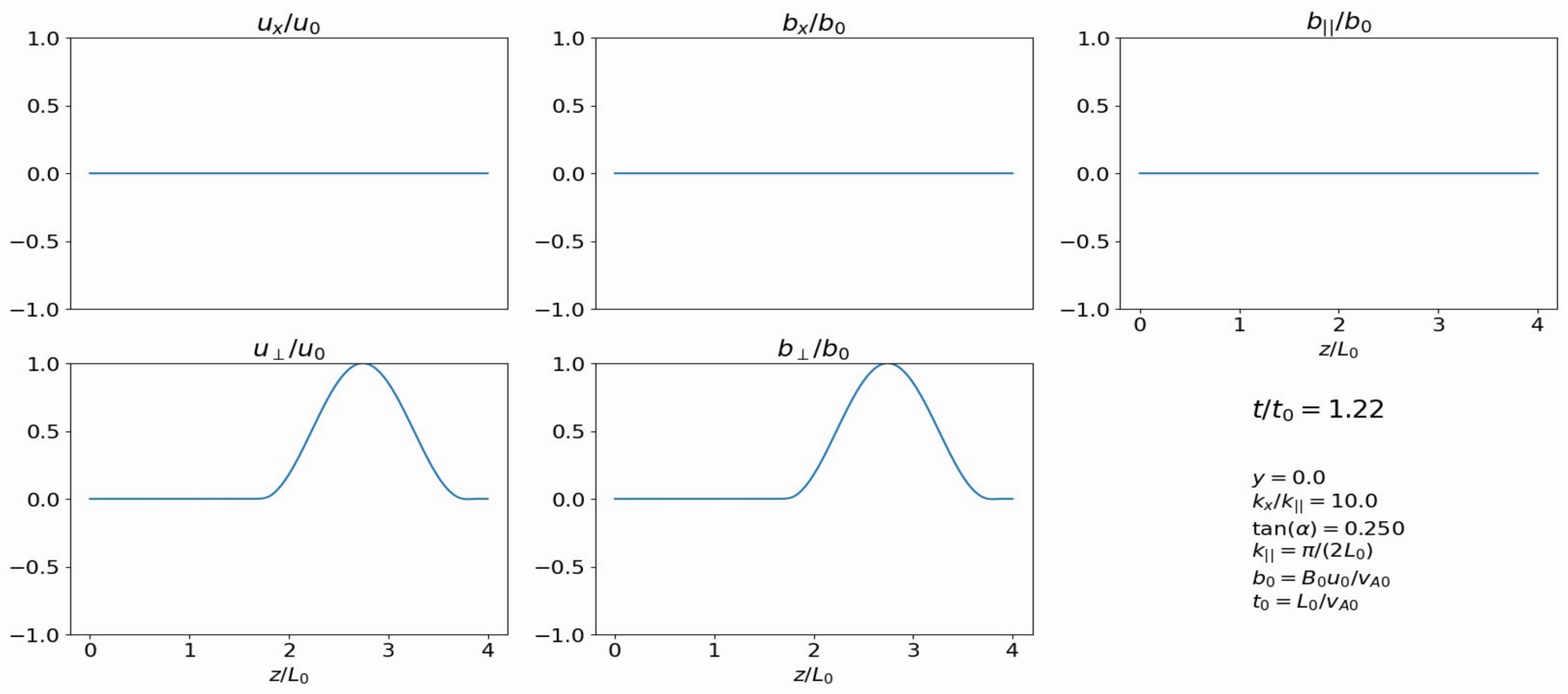
$$\frac{u_\perp}{u_0} = \frac{b_\perp}{b_0} = \begin{cases} \cos^2\theta & \text{if} \quad |\theta| \leq \pi/2 \\ 0 & \text{if} \quad |\theta| > \pi/2 \end{cases}$$

$$\theta = k_\parallel\left(y \sin\alpha + \left(z + 3L_0\right)\cos\alpha + v_A t\right)$$



- Add video with ux blob


u_x/u_0
b_x/b_0
$b_{||}/b_0$
$u_\perp/u_0$
$b_\perp/b_0$
z/L_0
$t/t_0 = 1.22$
$y = 0.0$
$k_x/k_{||} = 1.0$
$\tan(\alpha) = 0.250$
$k_{||} = \pi/(2L_0)$
$b_0 = B_0 u_0/v_{A0}$
$t_0 = L_0/v_{A0}$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Why does the coupling occur?

Summary

At the solar surface:

- Alfvén waves couple to fast waves
- Change polarisation
- If

$$k_x^2 > k_{||}^2 - k_y^2$$

then evanescent boundary layers form

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Normal mode solution

- Assume

$$u_x,\, u_\perp,\, b_x,\, b_\perp,\, b_\parallel \propto \exp\left[i\left(k_x x + k_y y + \omega t\right)\right]$$

- Impose incident Alfvén wave

$$\frac{u_\perp}{u_0} = \frac{b_\perp}{b_0} = \exp\left[i\left(k_x x + k_\parallel s\right) + \omega t\right]$$

- Calculate unique reflected Alfvén and fast wave which ensures ***u* = 0**

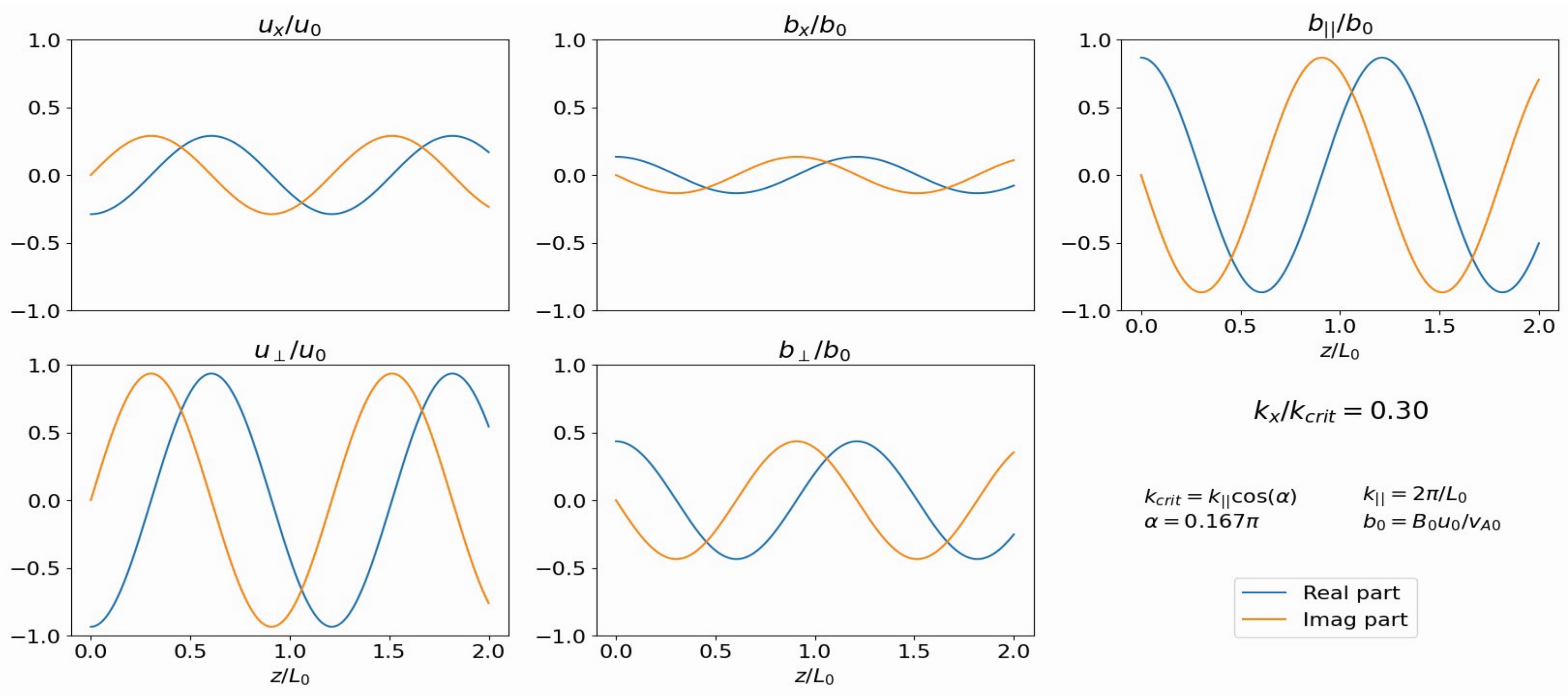
Incident wave

Reflected Alfvén wave

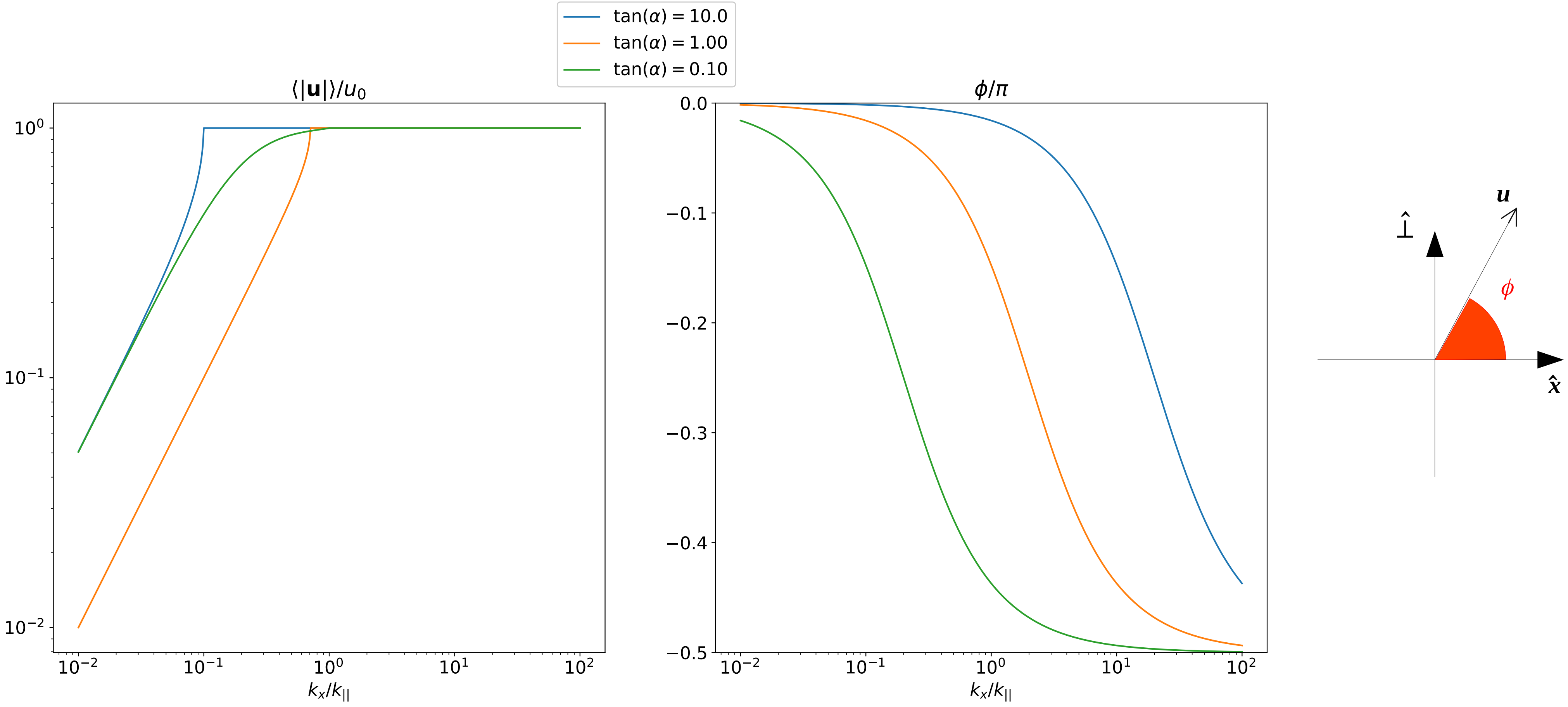
Reflected Fast wave

Full solution

Reflected fast wave



Reflected Alfvén wave



Summary

- Fast wave energy $\rightarrow$ 0 as $k_x \rightarrow \infty$
- Change in polarisation $\rightarrow$ 0 as $k_x \rightarrow \infty$
- $\therefore$ Boundary layers have a minimal impact on resonance absorption

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- Omega ^ 2 / vA^2 = kx ^ 2 + ky ^2 + kz ^ 2
- Therefore, assumign line-tied boundary conditions forces the waves in the chromospehre to be chromospheric.

Model

- Va = (va+ z > 0 and va- z < 0)
- Implies k_par- >> k_par+
- Implies k_crit- >> k_crit+
- Impose continuity of u and b

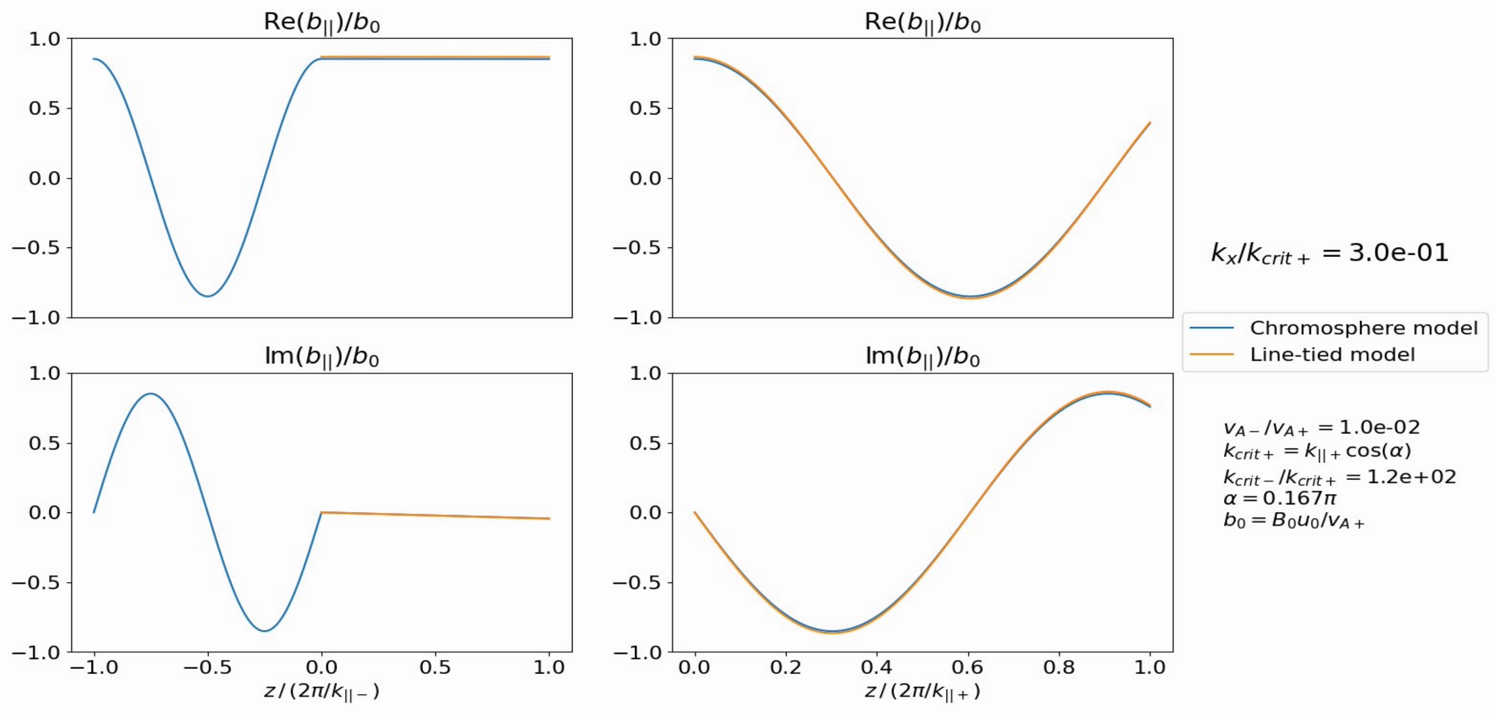
Incident wave

Reflected + Transmitted Alfvén wave

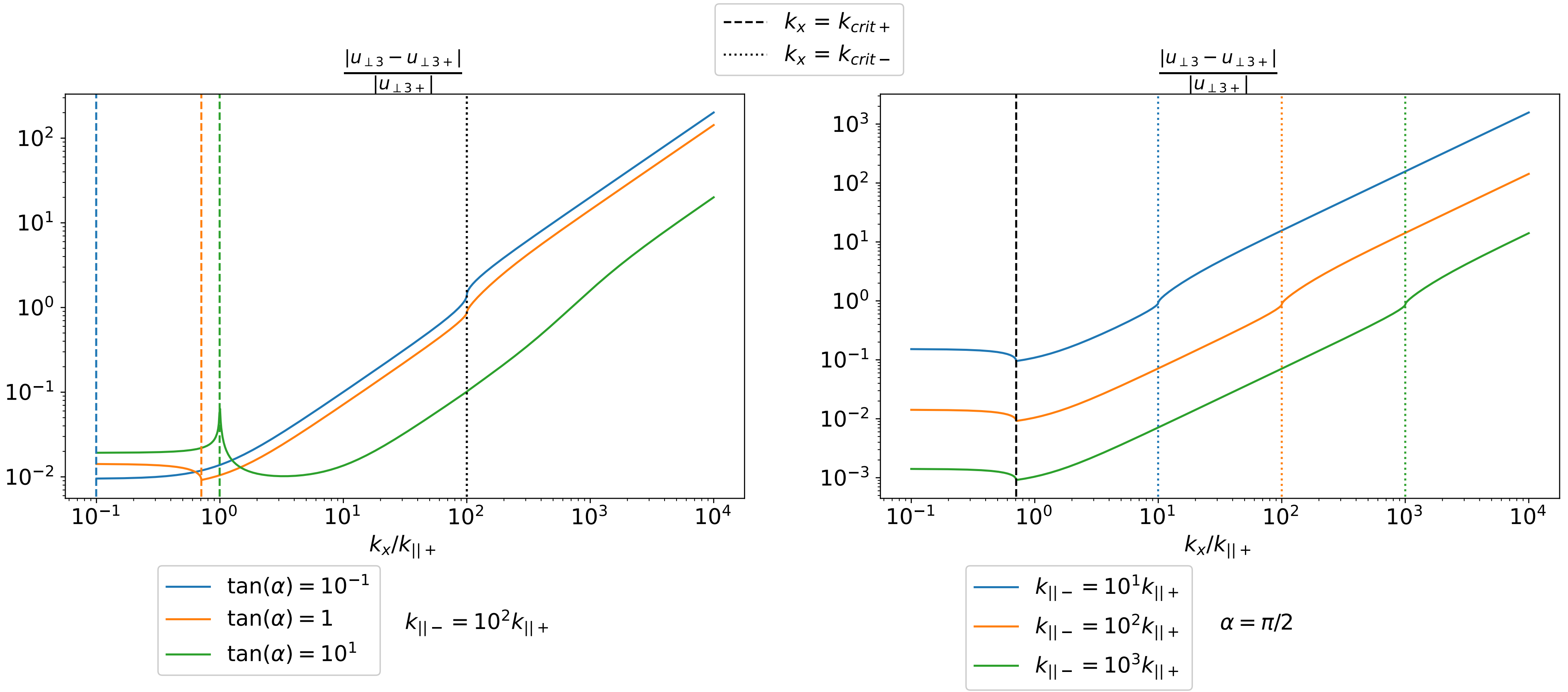
Reflected + Transmitted Fast wave

Full solution

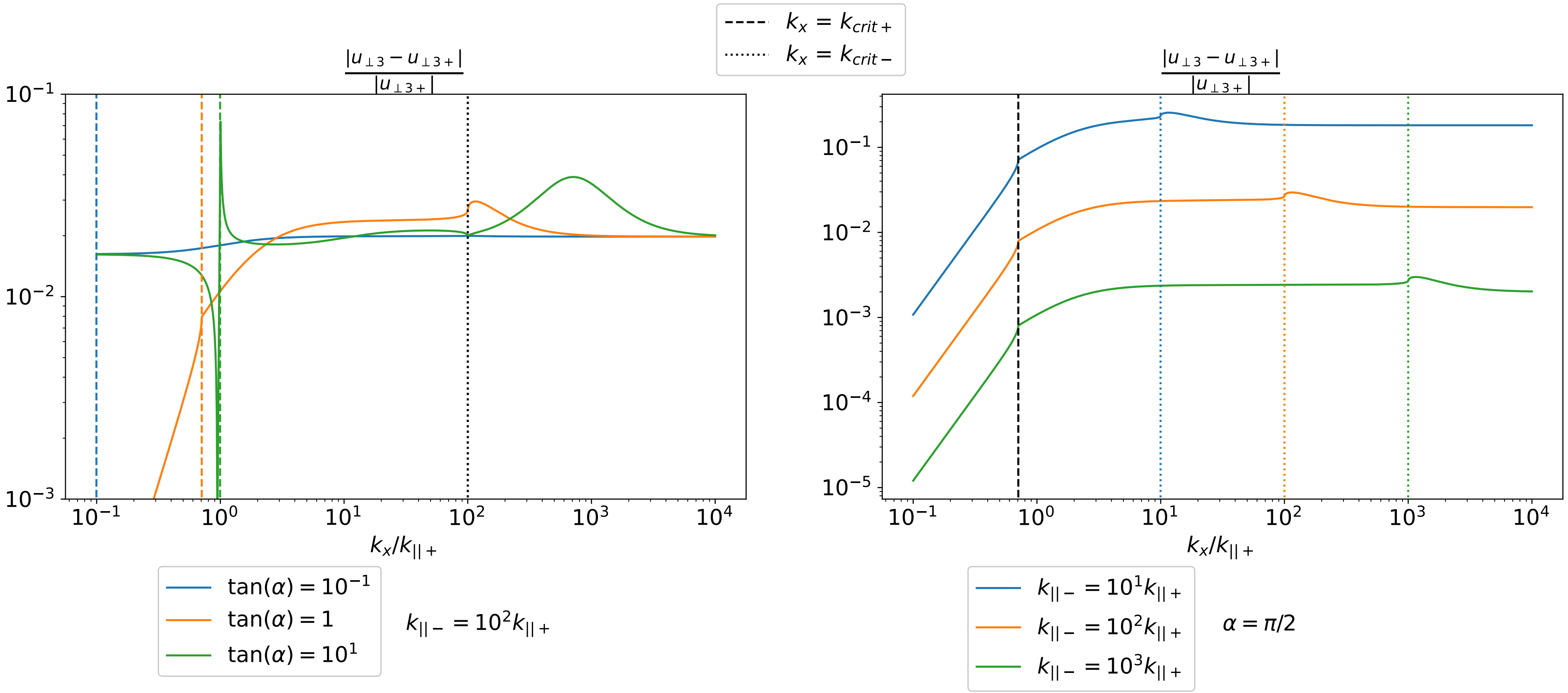
Chromosphere vs. Line-tied model



Fast wave error



Alfvén wave error



Summary + conclusions

- Alfvén waves couple to fast waves at TR
- Alfén waves change polarsation when they reflect
- Line-tied BC’s are usually a good approximation
- However, they generate non-physically large BL’s if kx >~ k_{||-}