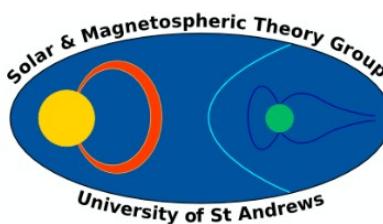


# Mode coupling at the transition region and the validity of line-tied boundary conditions

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Science & Technology  
Facilities Council

# Aims

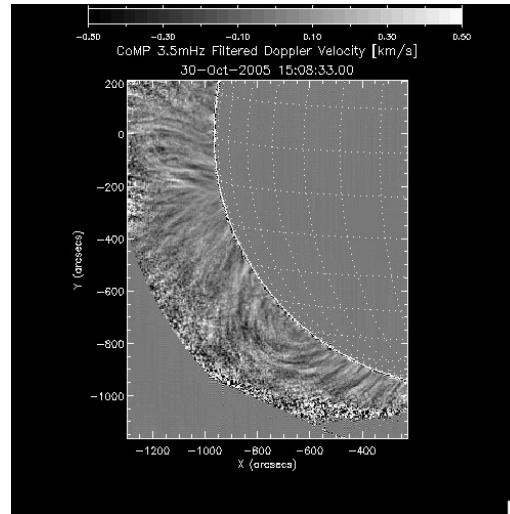
- Show why Fast / Alfvén waves couple at the TR
- Test the validity of line-tied BCs
- Show they can be invalid for highly phase-mixed waves

# Structure

- **Background**
- Model 1:
  - Line-tied, pulse
- Model 2:
  - Line-tied, normal mode
- Model 3:
  - Chromosphere, normal mode
- Summary and conclusions

# Why study MHD waves?

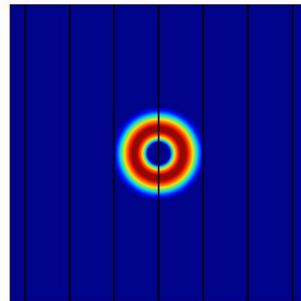
- Ubiquitous
- Coronal heating
- Seismology



# Fast vs. Alfvén waves

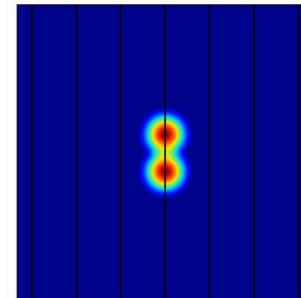
Fast waves:

- Propagate isotropically
- $\frac{\omega^2}{v_A^2} = k_x^2 + k_y^2 + k_z^2$



Alfvén waves:

- Propagate parallel to  $\mathbf{B}_0$
- $\frac{\omega^2}{v_A^2} = k_{\parallel}^2$



# Mode conversion

- Can occur via non-linear effects see e.g. verwichte, high beta to low beta see e.g. mclaughlin, gradients in background in Alven speed → resonant absorption

# Mode conversion at the TR

- Studied extensively by Halberstadt & Goedbloed analytically closed loops
- Arregui 2003 studied numerically
- Cally & Hansen (2011); Hansen & Cally (2012) suggest that mode conversion from fast waves to Alfvén waves at the transition region enables sufficient energy flux to enter the corona

# Line-tied boundary conditions

- Add VAL
- $\mathbf{u} = 0$
- Usually valid
- Might naively expect the only thing that matters is the lengthscale in the radial direction

# Normal mode

- $f(\mathbf{r}, t) = f_0(\mathbf{r}) \exp(i\omega t)$
- Add Morton power spectrum

# Model and Equations

- Background quantities:

$$\rho = \rho_0$$

$$\mathbf{B}_0 = B_0 \hat{\mathbf{B}}_0$$

- Perturbations:

$$\mathbf{u} = u_x \hat{\mathbf{x}} + u_{\perp} \hat{\perp}$$

$$\mathbf{b} = b_x \hat{\mathbf{x}} + b_{\perp} \hat{\perp} + b_{\parallel} \hat{\mathbf{B}}_0$$

- Unit vectors:

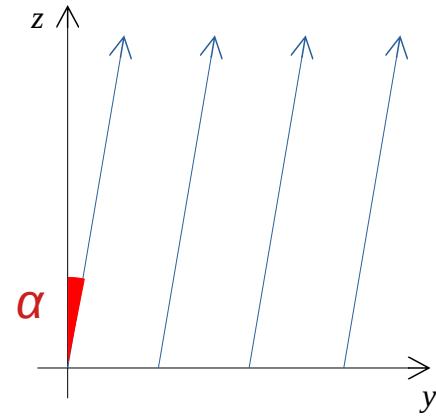
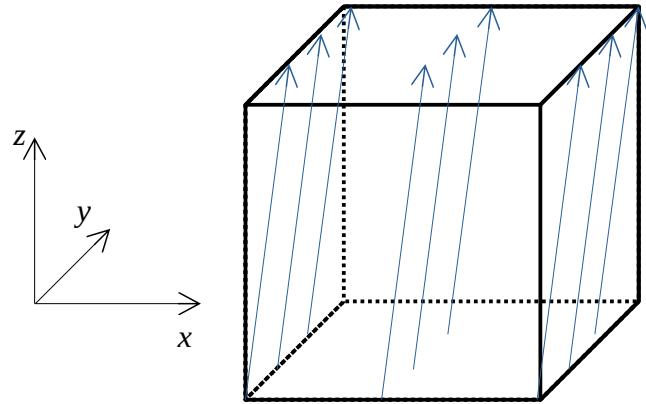
$$\hat{\perp} = \cos(\alpha) \hat{\mathbf{y}} - \sin(\alpha) \hat{\mathbf{z}}$$

$$\hat{\mathbf{B}}_0 = \sin(\alpha) \hat{\mathbf{y}} + \cos(\alpha) \hat{\mathbf{z}}$$

- Equations:

$$\rho_0 \frac{\partial \mathbf{u}}{\partial t} = \mathbf{j} \times \mathbf{B}_0$$

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}_0)$$



# Structure

- Background
- **Model 1:**
  - Line-tied, pulse
- Model 2:
  - Line-tied, normal mode
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# Numerical scheme

- Leapfrog algorithm
- Based on Rickard and Wright (1994)
- Finite-difference
- Staggered grid
- Second-order accurate

# Initial / boundary conditions

- Assume that

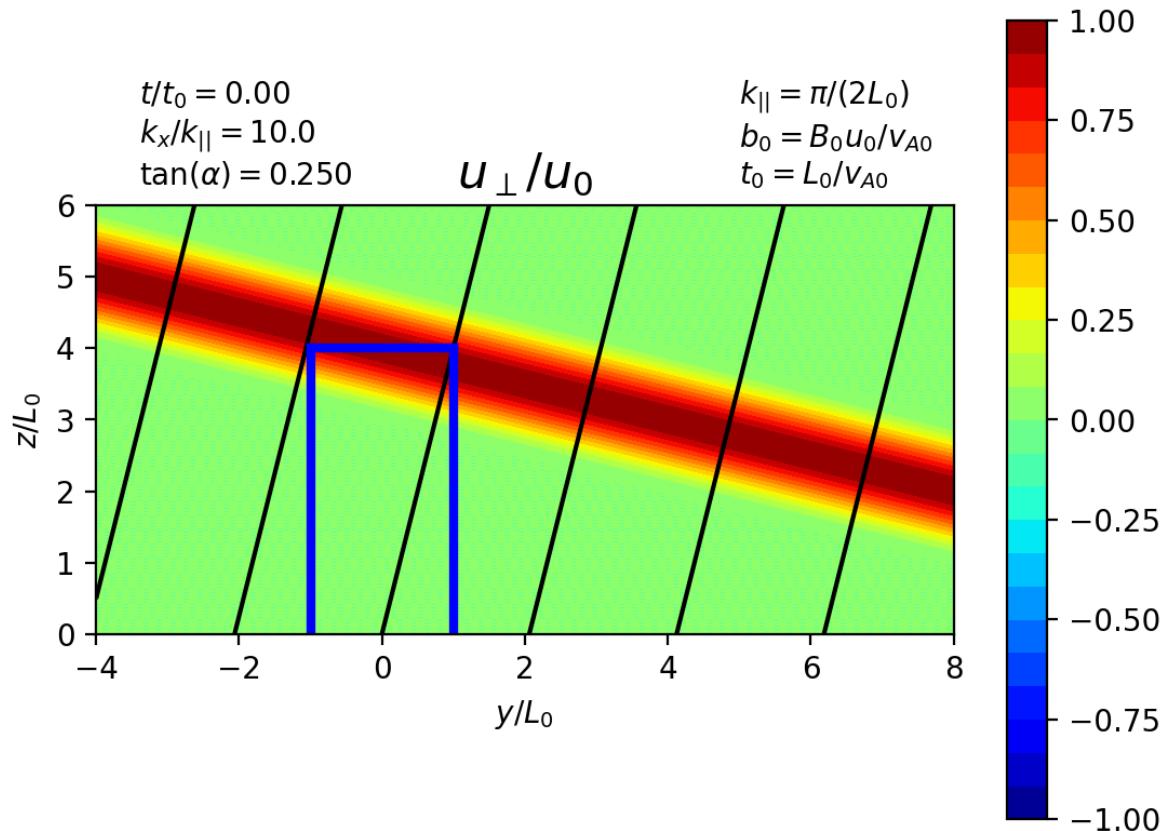
$$u_x, b_x \propto \sin(k_x x)$$

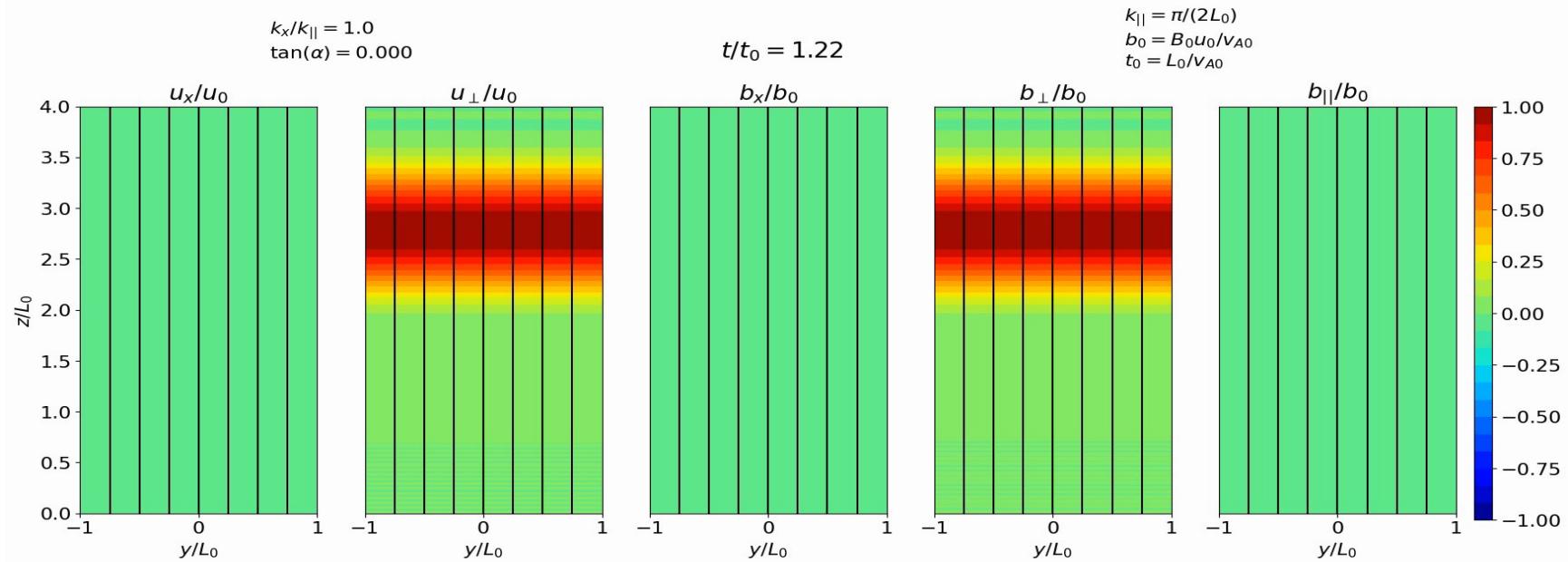
$$u_{\perp}, u_{\parallel}, b_{\parallel} \propto \cos(k_x x)$$

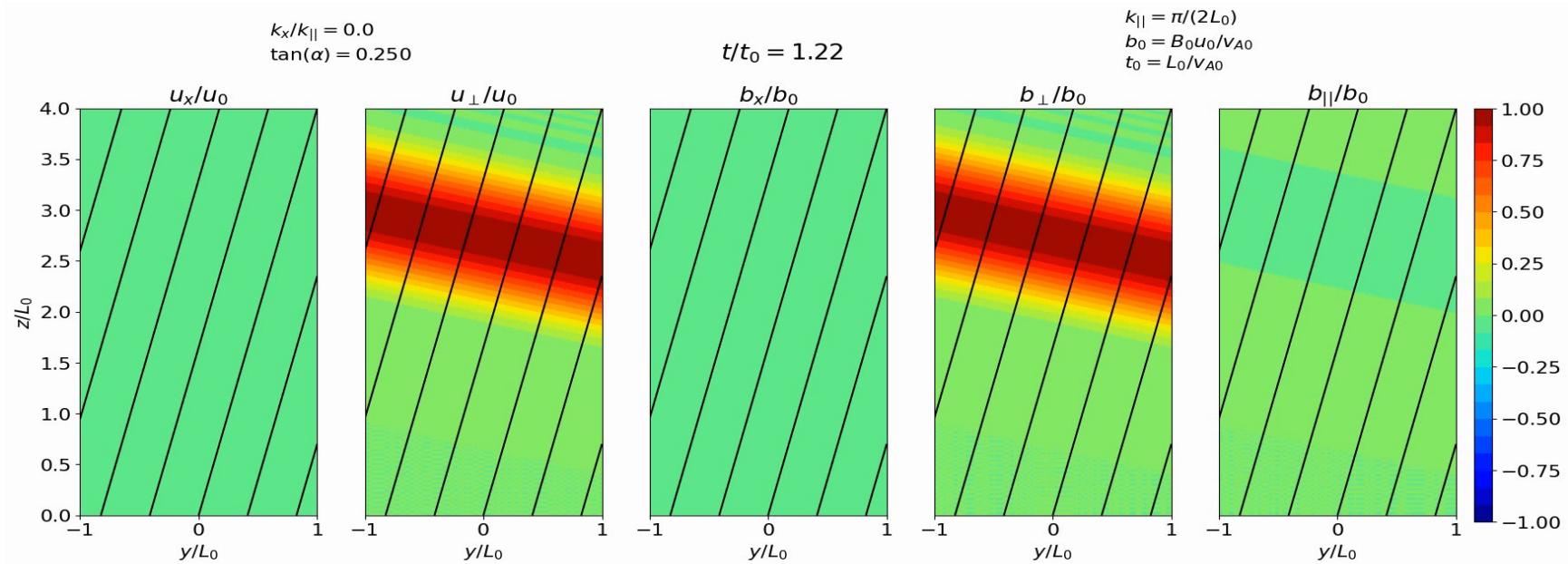
- Initial conditions:

$$\frac{u_{\perp}}{u_0} = \frac{b_{\perp}}{b_0} = \begin{cases} \cos^2 \theta & \text{if } |\theta| \leq \pi/2 \\ 0 & \text{if } |\theta| > \pi/2 \end{cases}$$

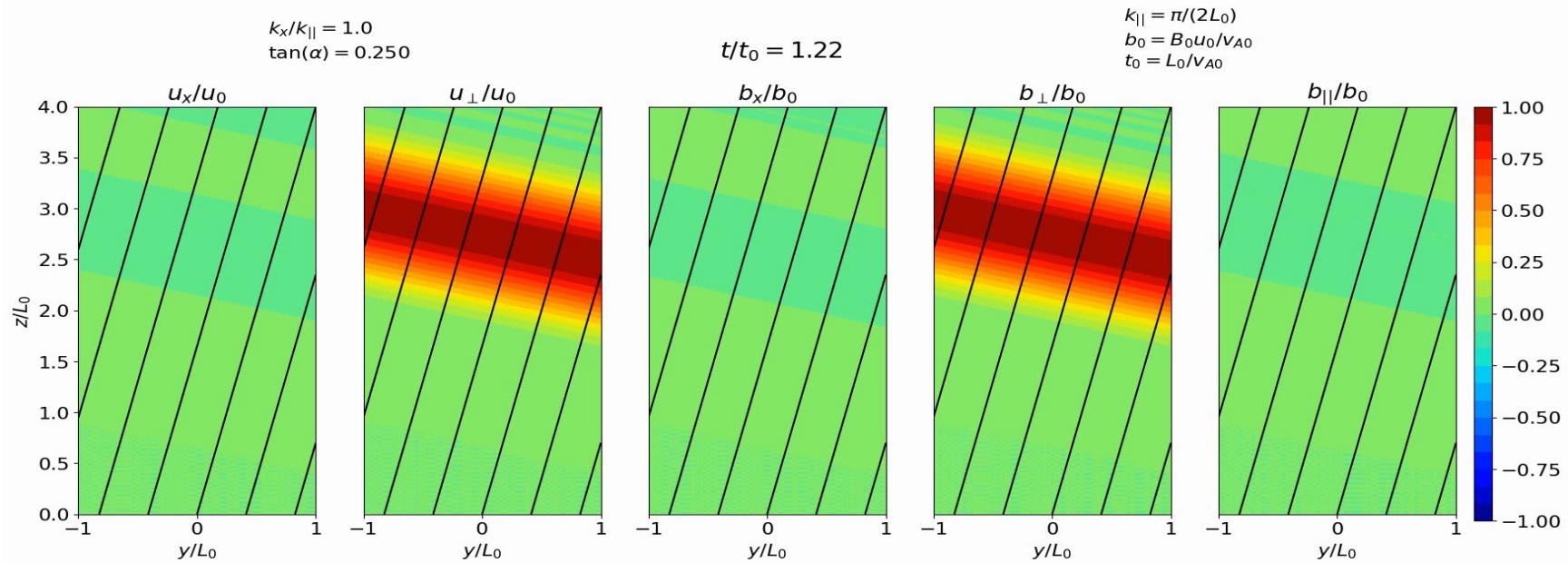
$$\theta = k_{\parallel} (y \sin \alpha + (z + 3L_0) \cos \alpha + v_A t)$$

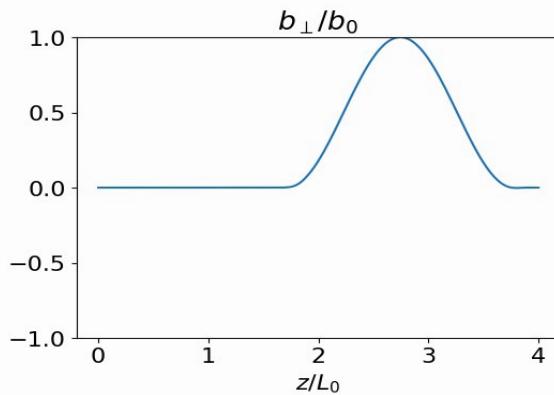
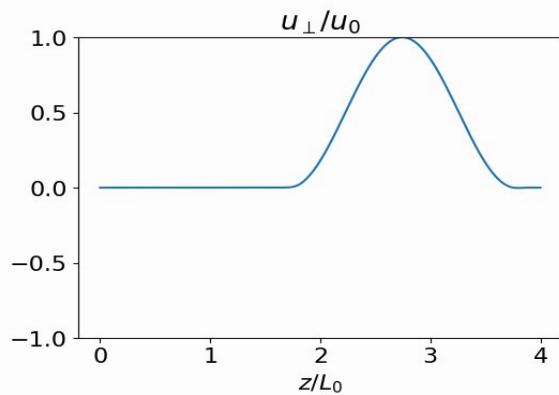
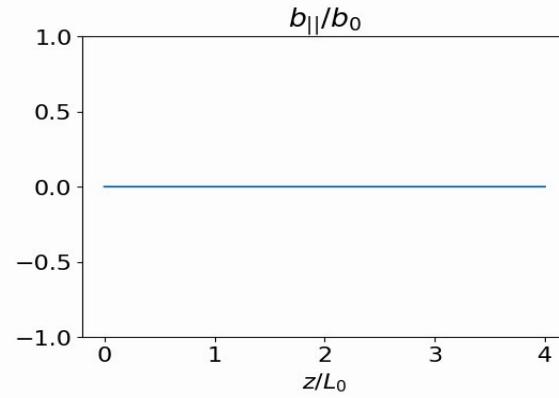
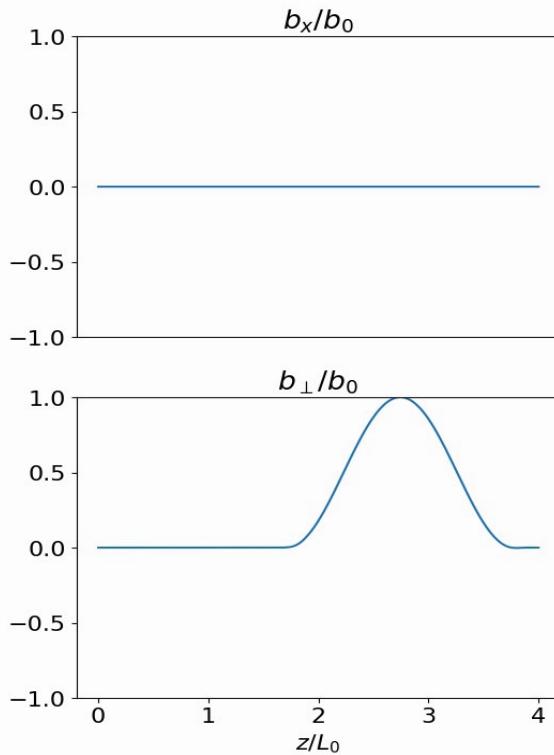
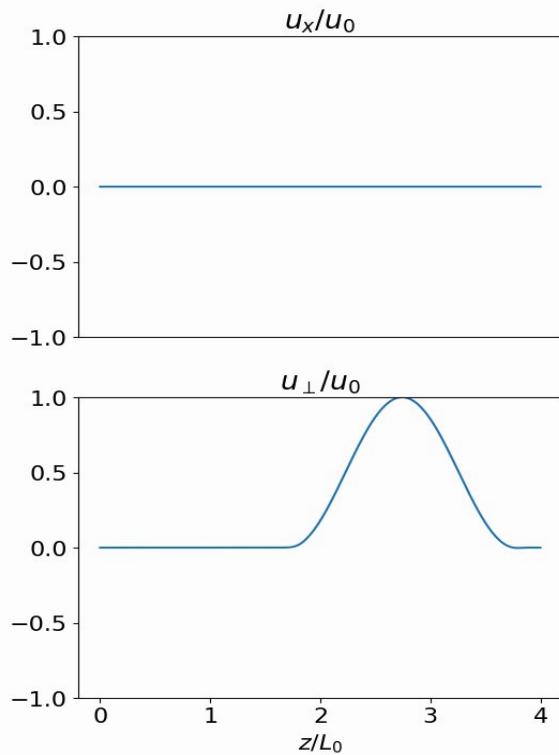






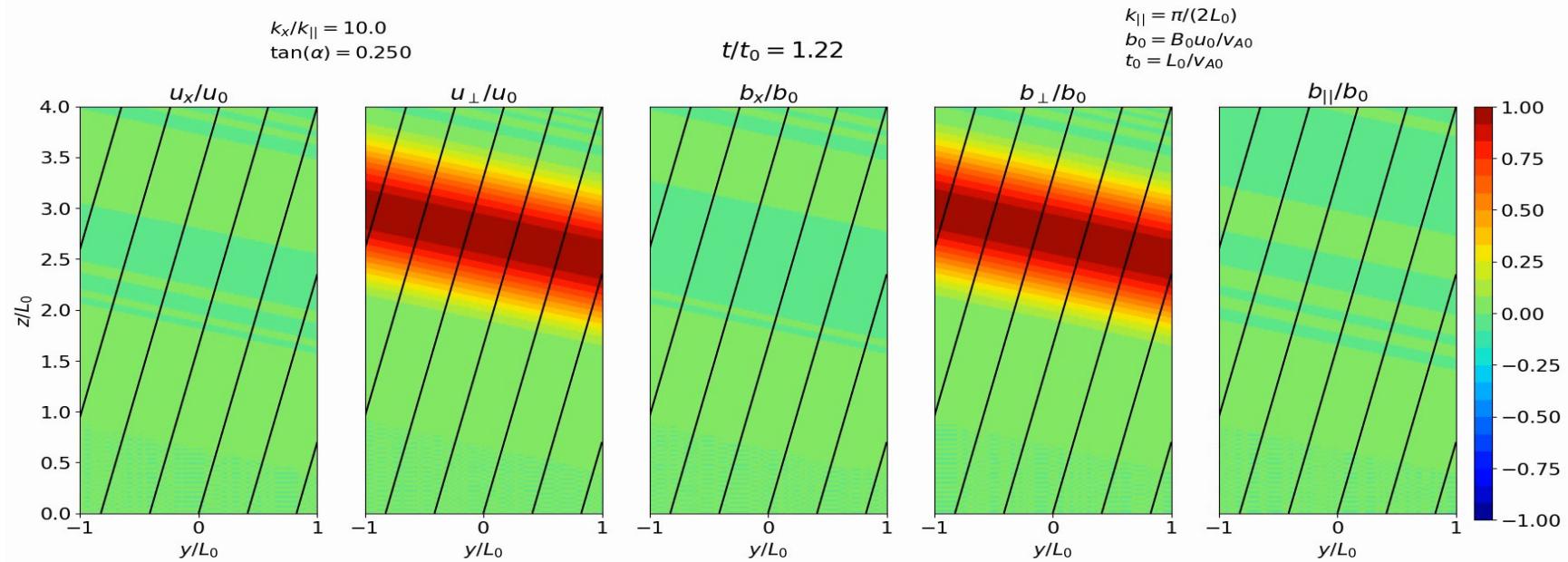
- Add video with ux blob

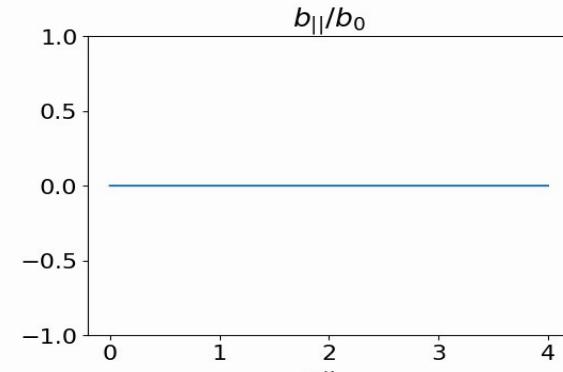
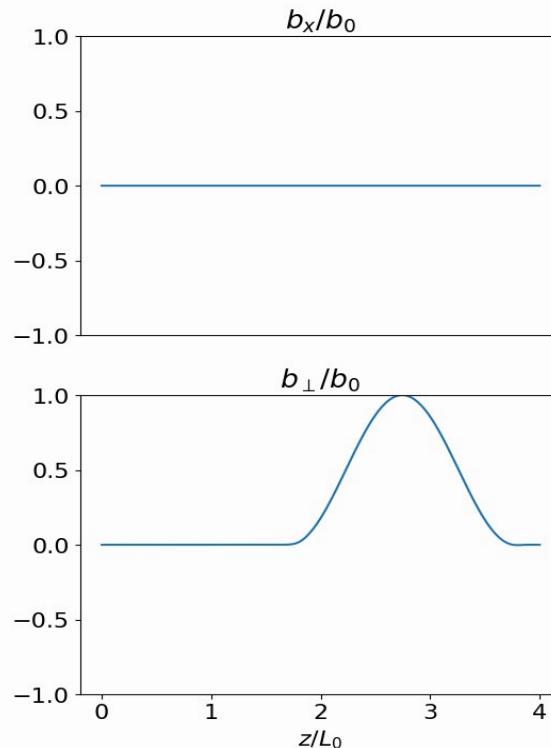
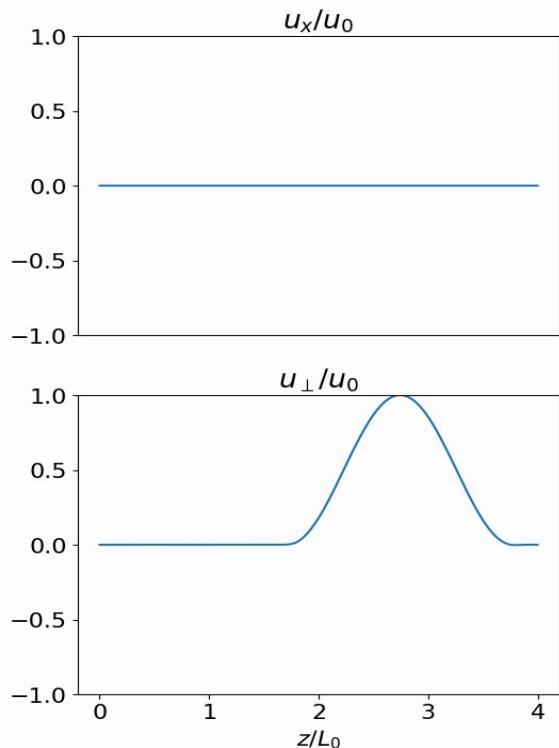




$t/t_0 = 1.22$

$y = 0.0$   
 $k_x/k_{||} = 1.0$   
 $\tan(\alpha) = 0.250$   
 $k_{||} = \pi/(2L_0)$   
 $b_0 = B_0 u_0 / v_{A0}$   
 $t_0 = L_0 / v_{A0}$





$t/t_0 = 1.22$

$y = 0.0$   
 $k_x/k_{||} = 10.0$   
 $\tan(\alpha) = 0.250$   
 $k_{||} = \pi/(2L_0)$   
 $b_0 = B_0 u_0 / v_{A0}$   
 $t_0 = L_0 / v_{A0}$

# Why does the coupling occur?

# Summary

At the solar surface:

- Alfvén waves couple to fast waves
- Change polarisation
- If

$$k_x^2 > k_{\parallel}^2 - k_y^2$$

then evanescent boundary layers form

# Structure

- Background
- Model 1:
  - Line-tied, pulse
- **Model 2:**
  - **Line-tied, normal mode**
- Model 3:
  - Chromosphere, normal mode
- Summary and conclusions

# Normal mode solution

- Assume

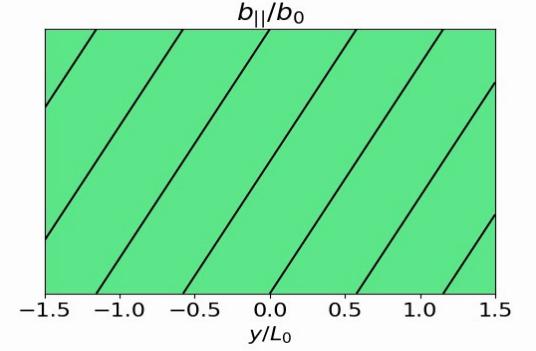
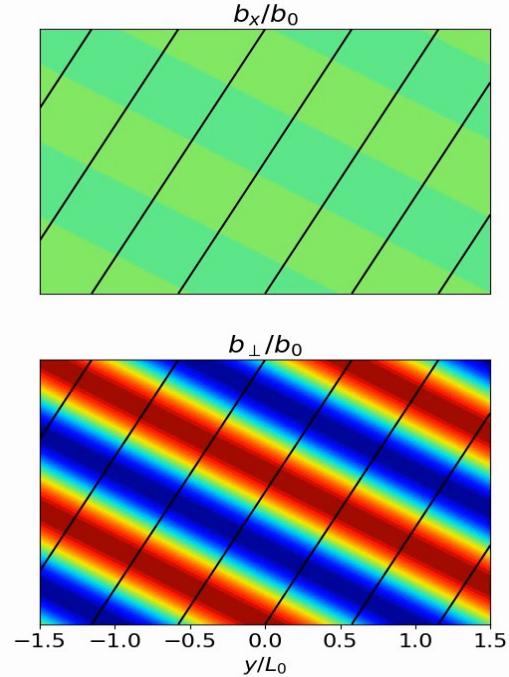
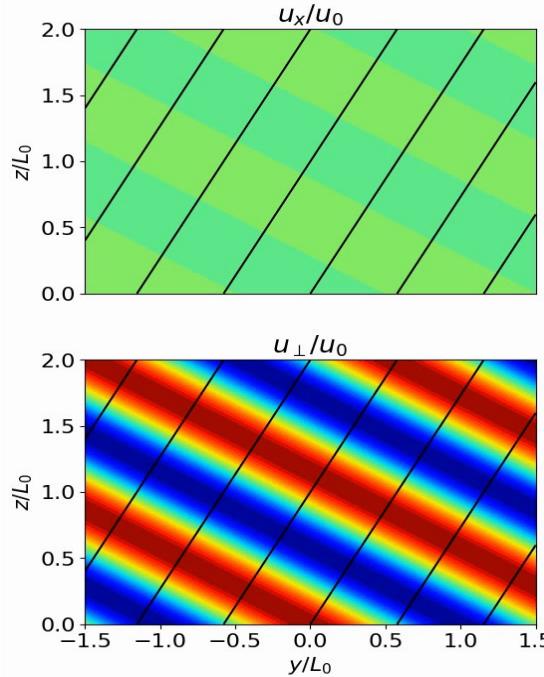
$$u_x, u_\perp, b_x, b_\perp, b_\parallel \propto \exp[i(k_x x + k_y y + \omega t)]$$

- Impose incident Alfvén wave

$$\frac{u_\perp}{u_0} = \frac{b_\perp}{b_0} = \exp[i(k_x x + k_\parallel s) + \omega t]$$

- Calculate unique reflected Alfvén and fast wave which ensures  $\mathbf{u} = \mathbf{0}$

# Incident wave

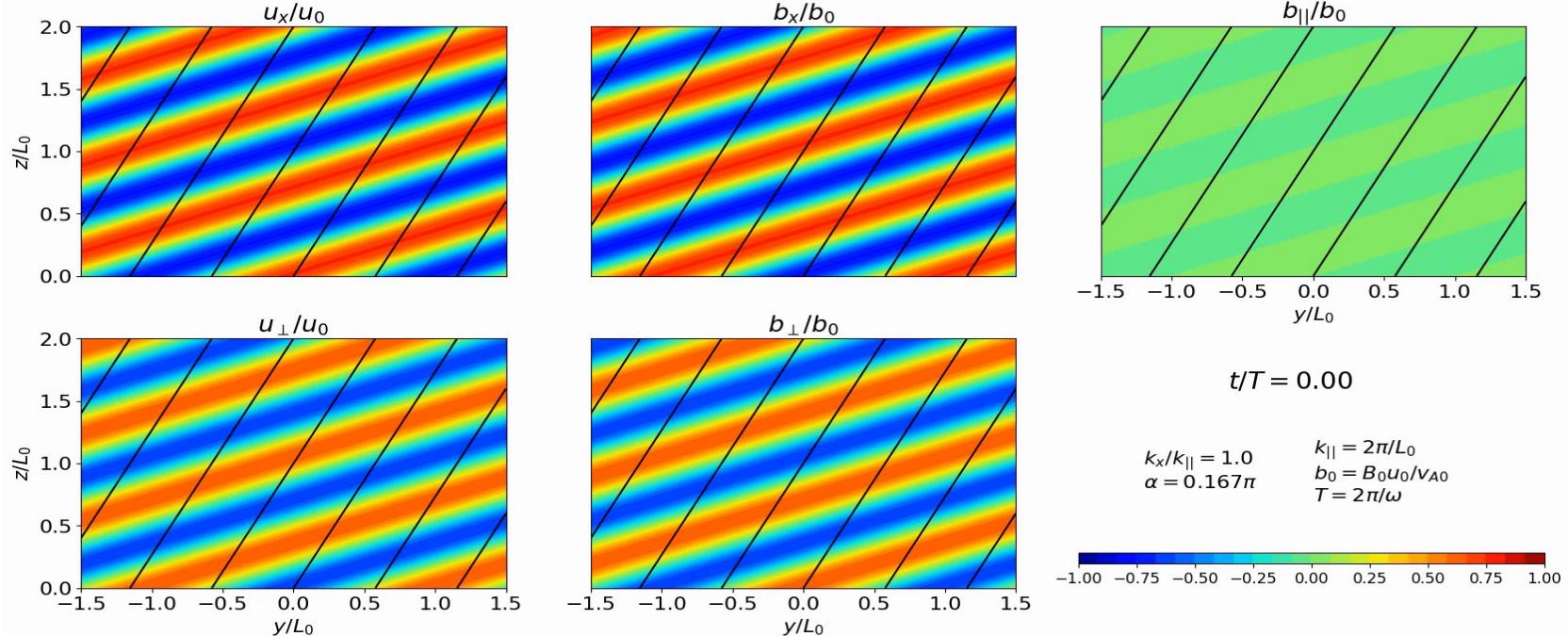


$t/T = 0.00$

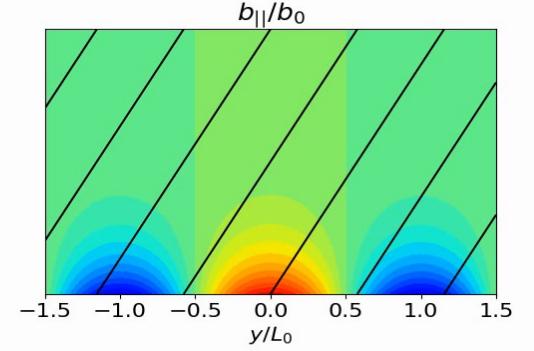
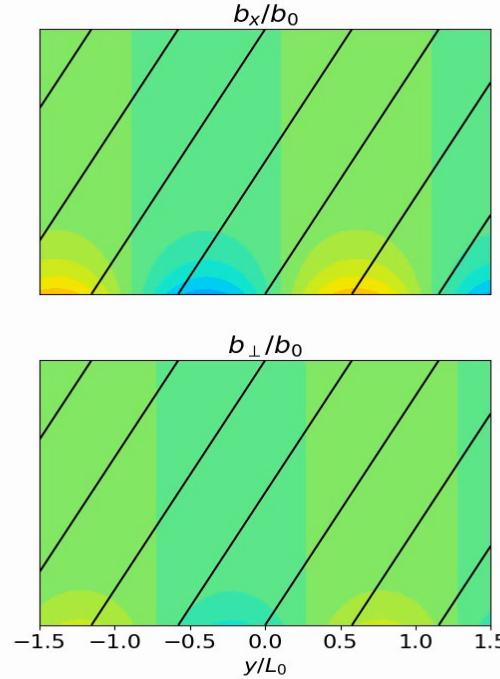
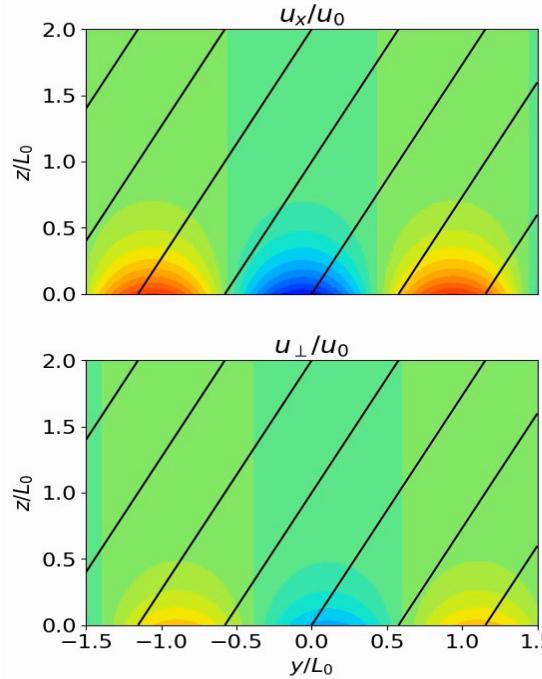
$$\begin{aligned} k_x/k_{||} &= 1.0 \\ \alpha &= 0.167\pi \end{aligned}$$
$$k_{||} = 2\pi/L_0$$
$$b_0 = B_0 u_0 / v_{A0}$$
$$T = 2\pi/\omega$$



# Reflected Alfvén wave



# Reflected Fast wave



$t/T = 0.00$

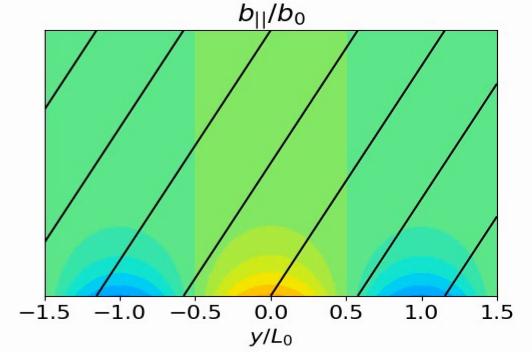
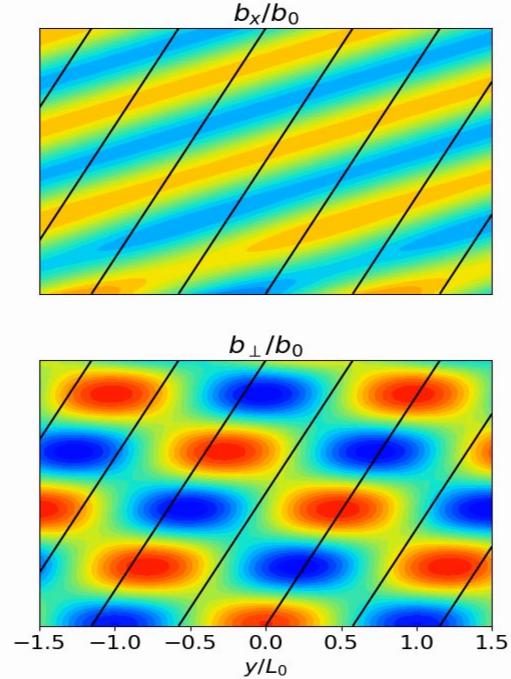
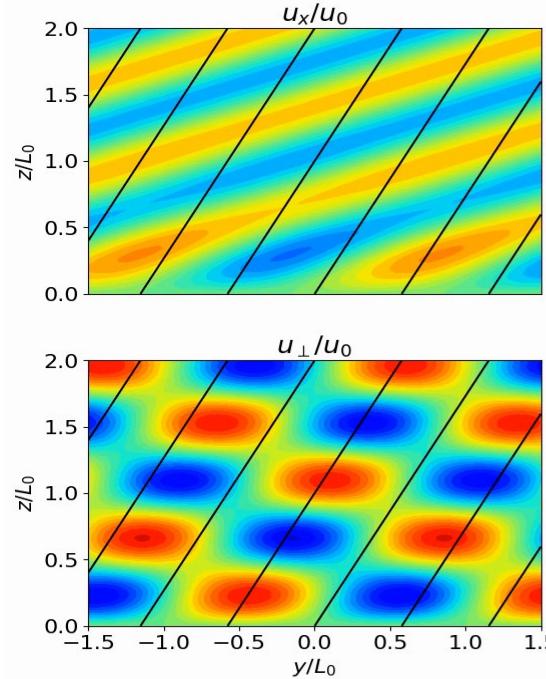
$k_x/k_{||} = 1.0$   
 $\alpha = 0.167\pi$

$k_{||} = 2\pi/L_0$   
 $b_0 = B_0 u_0 / v_{A0}$   
 $T = 2\pi/\omega$



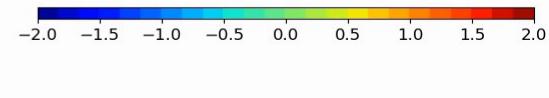
A horizontal color bar with a gradient from blue (-1.00) to red (1.00), with numerical labels at -1.00, -0.75, -0.50, -0.25, 0.00, 0.25, 0.50, 0.75, and 1.00.

# Full solution

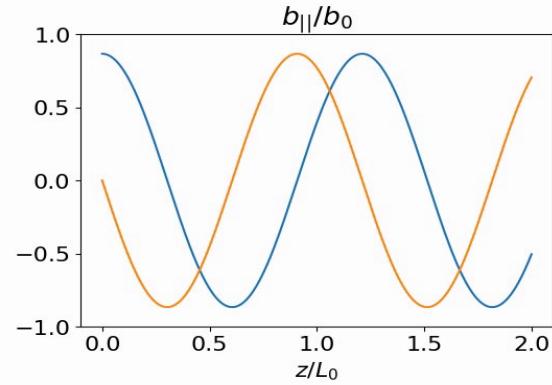
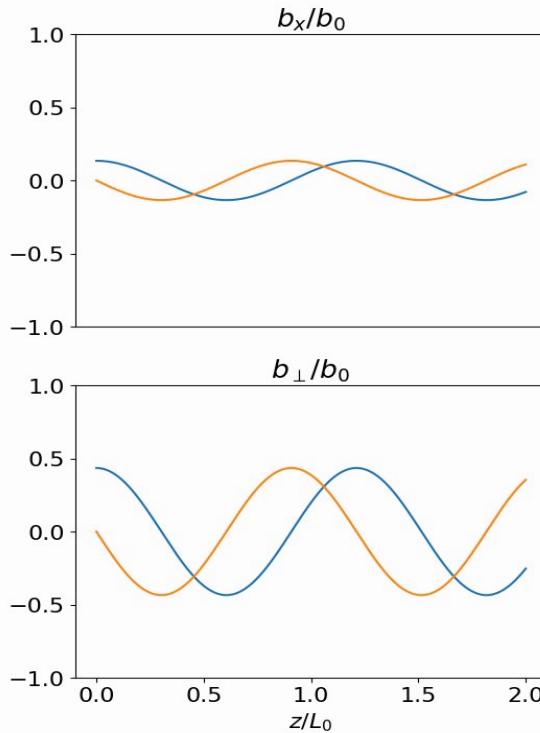
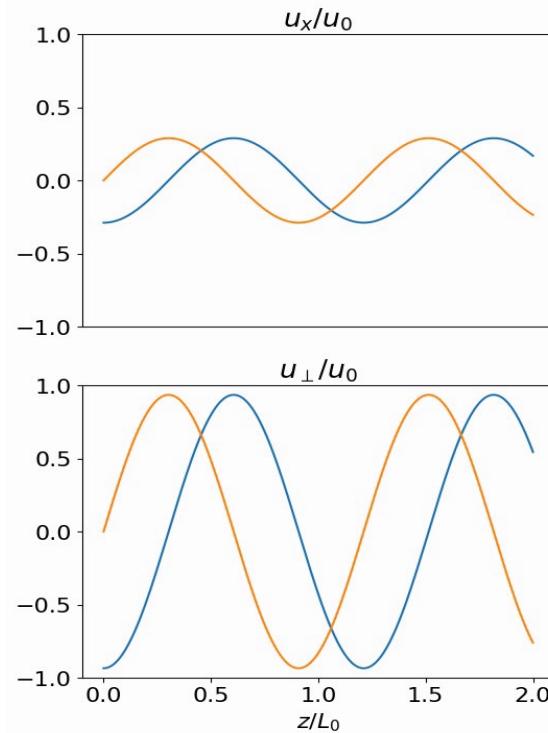


$t/T = 0.00$

$$\begin{aligned} k_x/k_{||} &= 1.0 & k_{||} &= 2\pi/L_0 \\ \alpha &= 0.167\pi & b_0 &= B_0 u_0 / v_{A0} \\ T &= 2\pi/\omega & & \end{aligned}$$



# Reflected fast wave

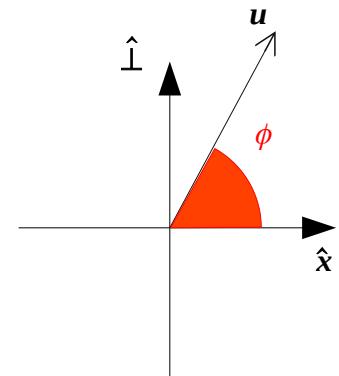
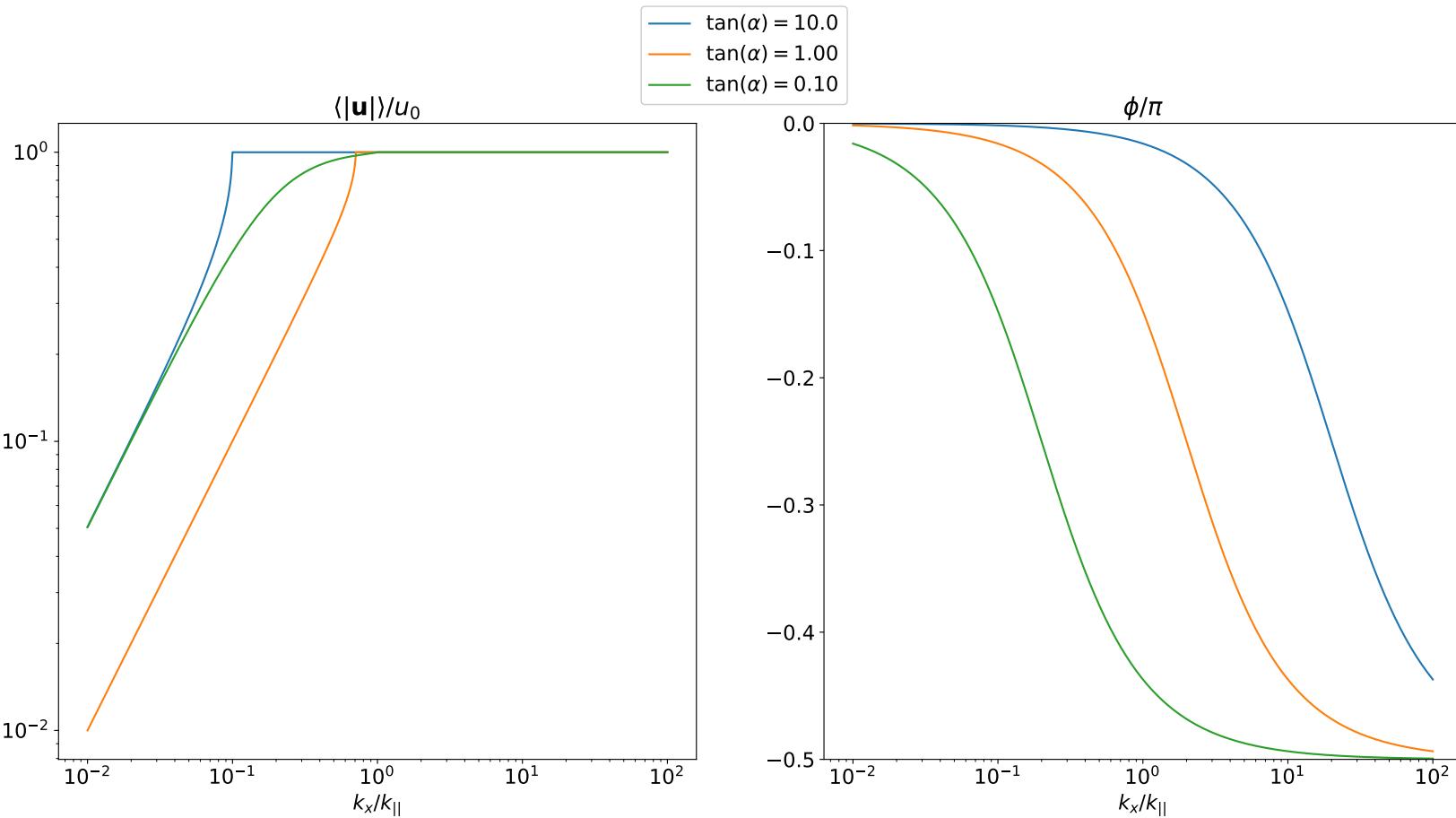


$$k_x/k_{crit} = 0.30$$

$$k_{crit} = k_{||}\cos(\alpha) \quad k_{||} = 2\pi/L_0$$
$$\alpha = 0.167\pi \quad b_0 = B_0 u_0 / v_{A0}$$

Real part  
Imag part

# Reflected Alfvén wave



# Summary

- Fast wave energy  $\rightarrow 0$  as  $k_x \rightarrow \infty$
- Change in polarisation  $\rightarrow 0$  as  $k_x \rightarrow \infty$
- $\therefore$  Boundary layers have a minimal impact on resonance absorption

# Structure

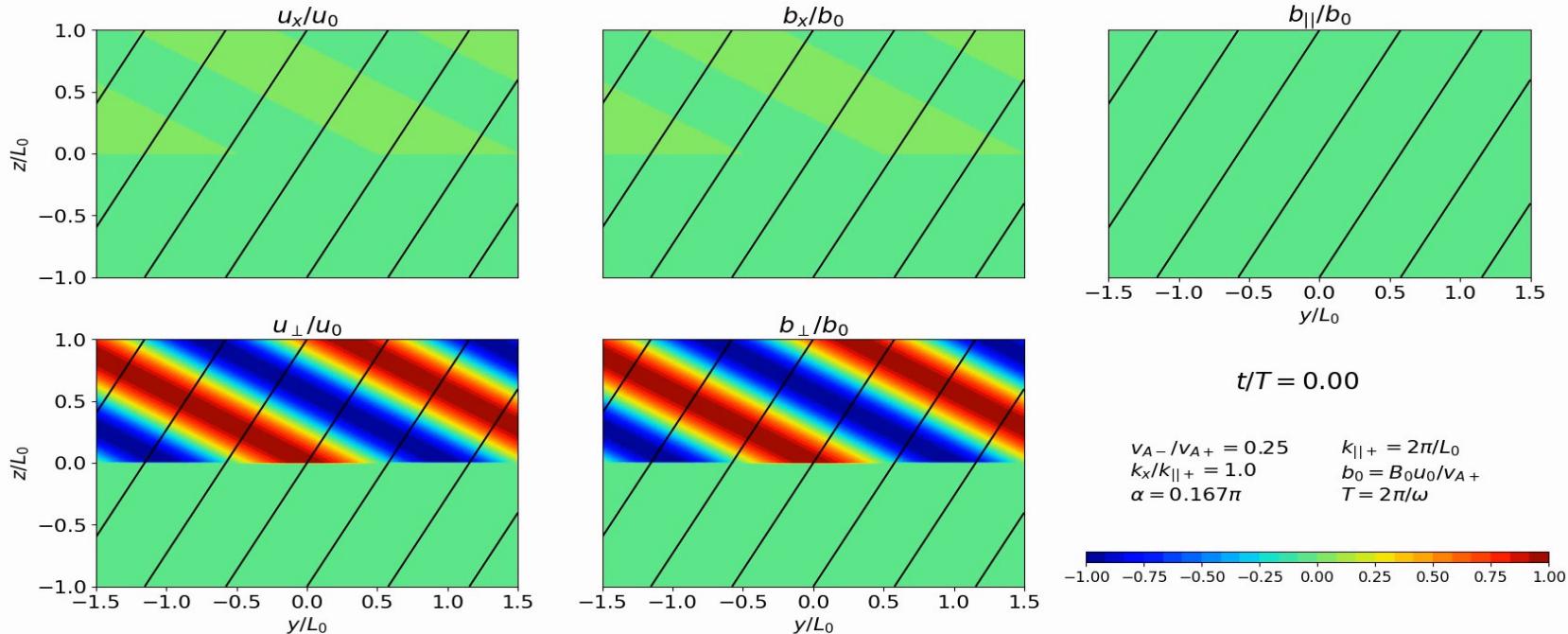
- Background
- Model 1:
  - Line-tied, pulse
- Model 2:
  - Line-tied, normal mode
- **Model 3:**
  - **Chromosphere, normal mode**
- Summary and conclusions

- $\Omega^2 / v_A^2 = k_x^2 + k_y^2 + k_z^2$
- Therefore, assuming line-tied boundary conditions forces the waves in the chromosphere to be chromospheric.

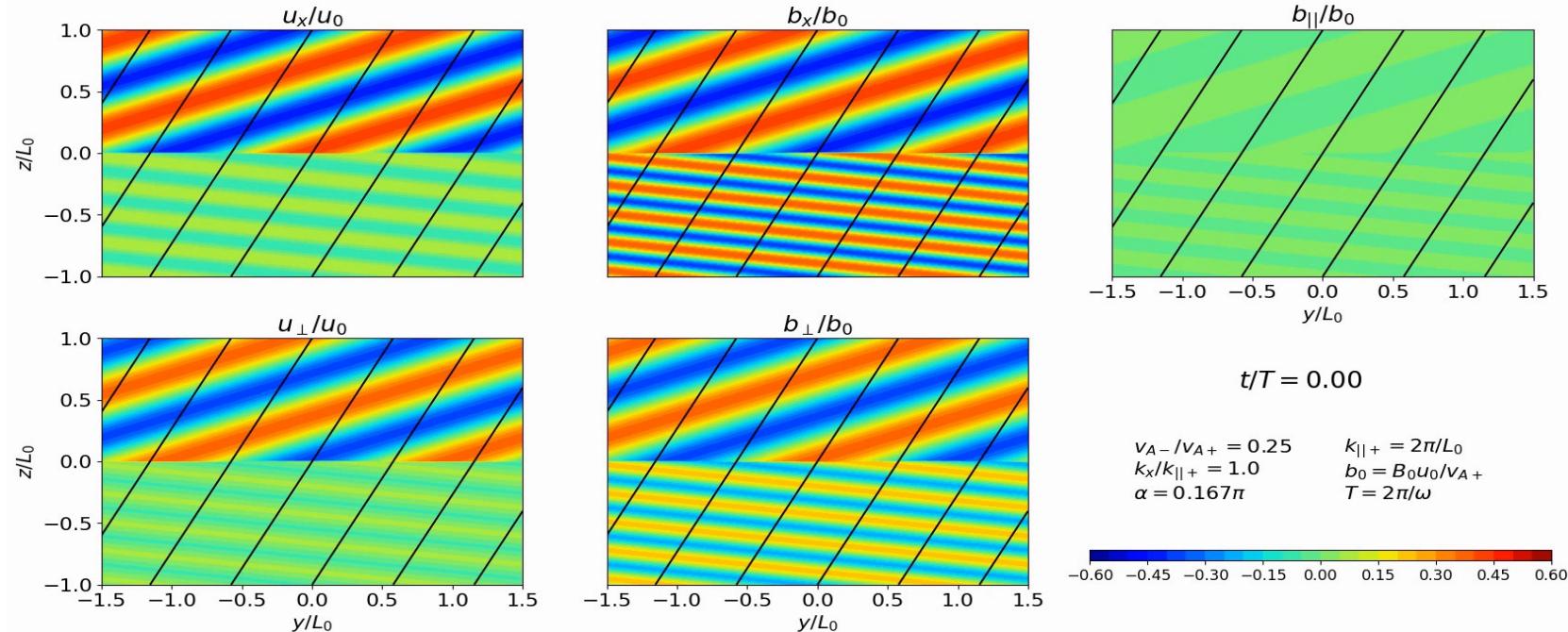
# Model

- $V_a = (v_{a+} z > 0 \text{ and } v_{a-} z < 0)$
- Implies  $k_{\text{par-}} \gg k_{\text{par+}}$
- Implies  $k_{\text{crit-}} \gg k_{\text{crit+}}$
- Impose continuity of  $u$  and  $b$

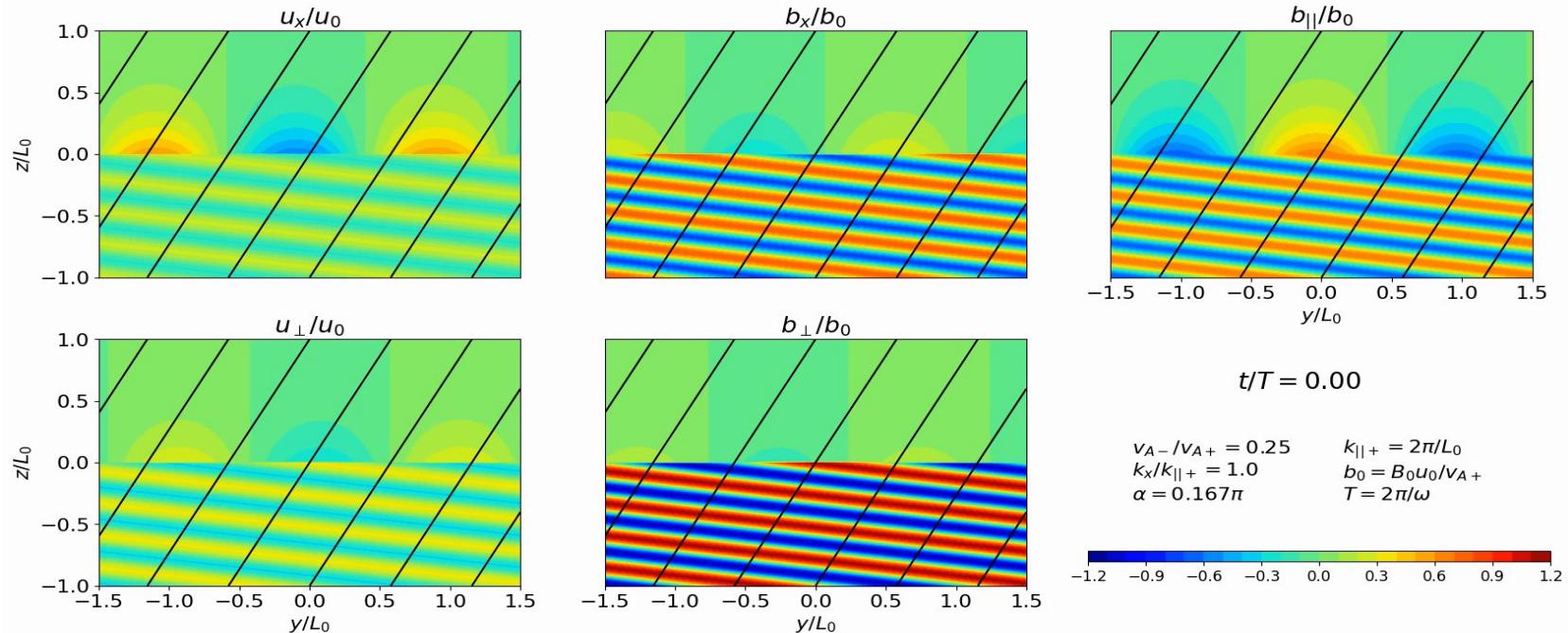
# Incident wave



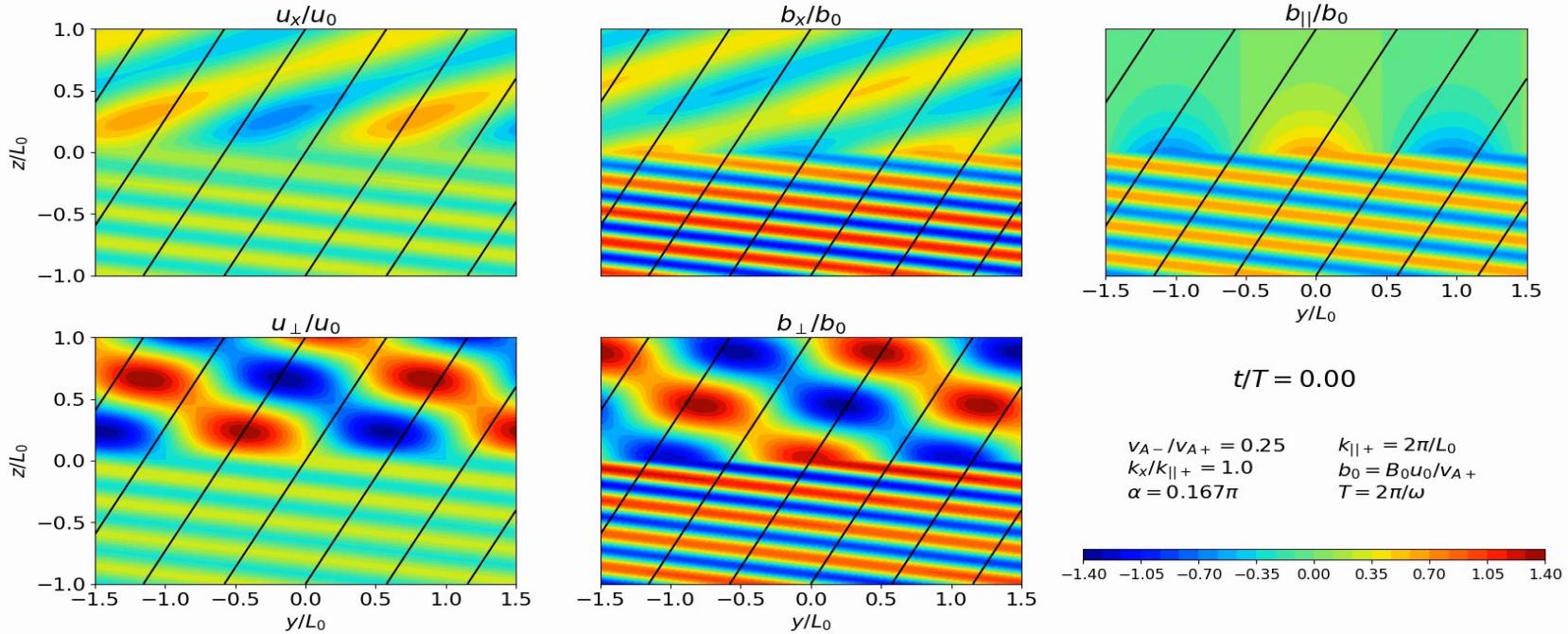
# Reflected + Transmitted Alfvén wave



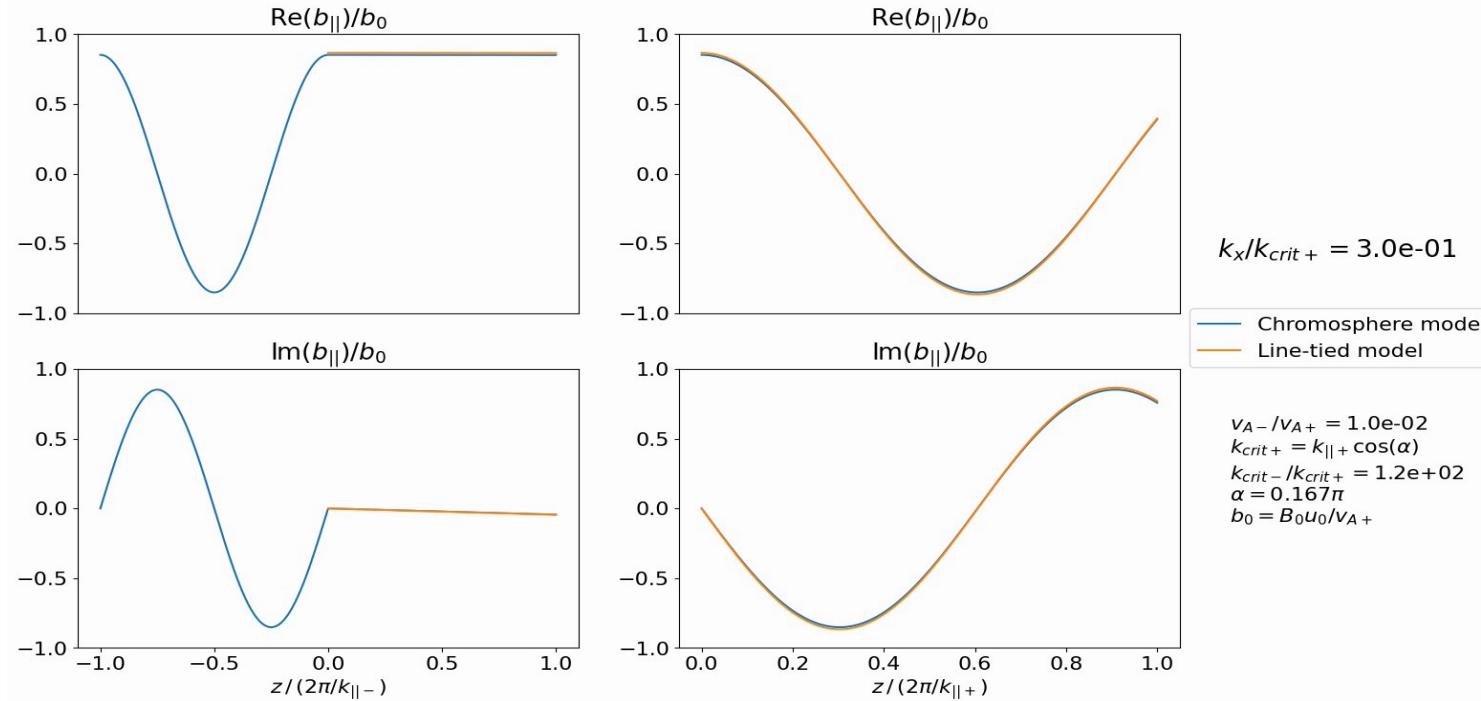
# Reflected + Transmitted Fast wave



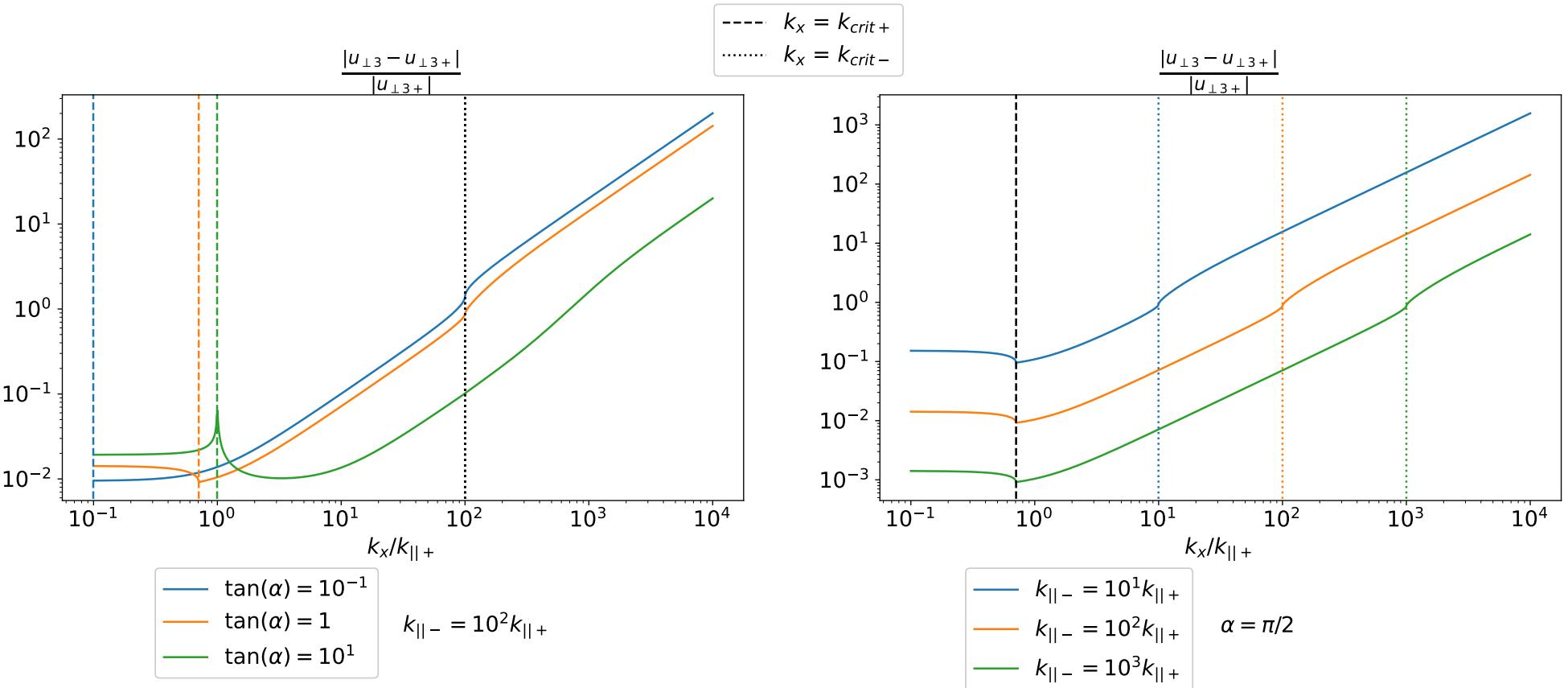
# Full solution



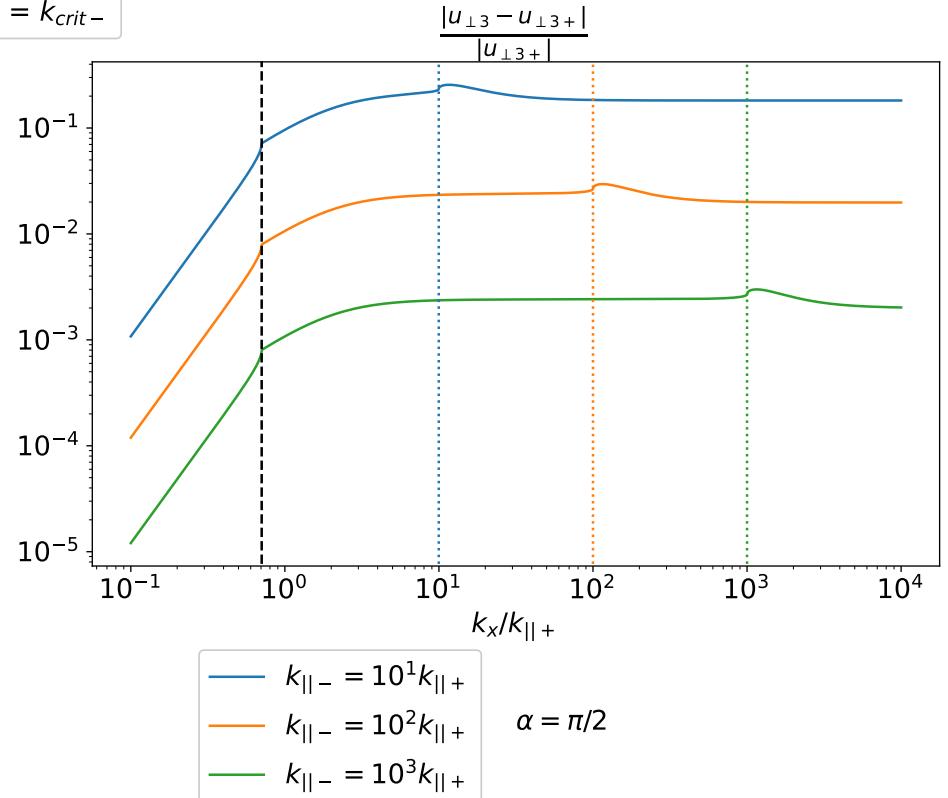
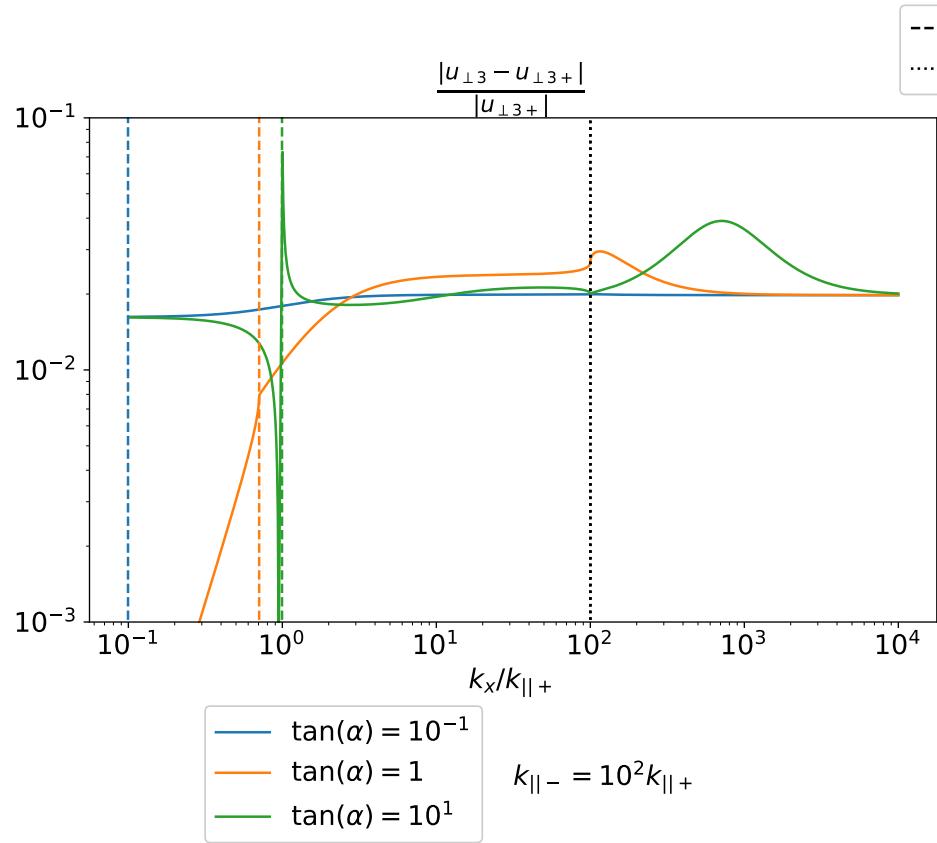
# Chromosphere vs. Line-tied model



# Fast wave error



# Alfvén wave error



# Summary + conclusions

- Alfvén waves couple to fast waves at TR
- Alfvén waves change polarisation when they reflect
- Line-tied BC's are usually a good approximation
- However, they generate non-physically large BL's if  $k_x > \sim k_{\parallel -}$