This document is used to assist with the Algebra in Appendix A.2

Note that ∇ has been replaced with  to ensure the code works okay.

> *restart*;

We normalise the velocity coefficents by u_0 and the field components by (B_0 * u_0 / v_{A+}).

We start by solving the Matrix equation given by Equation (57).

> $eqn1 := a_{11} \cdot x_1 + a_{12} \cdot x_2 + a_{13} \cdot x_3 + a_{14} \cdot x_4 = y_1$

$$eqn1 := a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 = y_1 \quad (1)$$

> $eqn2 := a_{21} \cdot x_1 + a_{22} \cdot x_2 + a_{23} \cdot x_3 + a_{24} \cdot x_4 = y_2$

$$eqn2 := a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + a_{24} x_4 = y_2 \quad (2)$$

> $eqn3 := a_{31} \cdot x_1 + a_{32} \cdot x_2 + a_{33} \cdot x_3 + a_{34} \cdot x_4 = y_3$

$$eqn3 := a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + a_{34} x_4 = y_3 \quad (3)$$

> $eqn4 := a_{41} \cdot x_1 + a_{42} \cdot x_2 + a_{43} \cdot x_3 + a_{44} \cdot x_4 = y_4$

$$eqn4 := a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4 = y_4 \quad (4)$$

> $solns := solve\left(\{eqn1, eqn2, eqn3, eqn4\}, \{x_1, x_2, x_3, x_4\}\right)$ :

> $sol1 := rhs(solns[1])$

$$sol1 := -\big(a_{12} a_{23} a_{34} y_4 - a_{12} a_{23} a_{44} y_3 - a_{12} a_{24} a_{33} y_4 + a_{12} a_{24} a_{43} y_3 + a_{12} a_{33} a_{44} y_2 - a_{12} a_{34} a_{43} y_2 - a_{13} a_{22} a_{34} y_4 + a_{13} a_{22} a_{44} y_3 + a_{13} a_{24} a_{32} y_4 - a_{13} a_{24} a_{42} y_3 - a_{13} a_{32} a_{44} y_2 + a_{13} a_{34} a_{42} y_2 + a_{14} a_{22} a_{33} y_4 - a_{14} a_{22} a_{43} y_3 - a_{14} a_{23} a_{32} y_4 + a_{14} a_{23} a_{42} y_3 + a_{14} a_{32} a_{43} y_2 - a_{14} a_{33} a_{42} y_2 - a_{22} a_{33} a_{44} y_1 + a_{22} a_{34} a_{43} y_1 + a_{23} a_{32} a_{44} y_1 - a_{23} a_{34} a_{42} y_1 - a_{24} a_{32} a_{43} y_1 + a_{24} a_{33} a_{42} y_1\big) \Big/ \big(a_{11} a_{22} a_{33} a_{44} - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} + a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} + a_{14} a_{22} a_{31} a_{43} - a_{14} a_{22} a_{33} a_{41} - a_{14} a_{23} a_{31} a_{42} + a_{14} a_{23} a_{32} a_{41}\big) \quad (5)$$

> $sol2 := rhs(solns[2])$

$$sol2 := \big(a_{11} a_{23} a_{34} y_4 - a_{11} a_{23} a_{44} y_3 - a_{11} a_{24} a_{33} y_4 + a_{11} a_{24} a_{43} y_3 + a_{11} a_{33} a_{44} y_2 - a_{11} a_{34} a_{43} y_2 - a_{13} a_{21} a_{34} y_4 + a_{13} a_{21} a_{44} y_3 + a_{13} a_{24} a_{31} y_4 - a_{13} a_{24} a_{41} y_3 - a_{13} a_{31} a_{44} y_2 + a_{13} a_{34} a_{41} y_2 + a_{14} a_{21} a_{33} y_4 - a_{14} a_{21} a_{43} y_3 - a_{14} a_{23} a_{31} y_4 + a_{14} a_{23} a_{41} y_3 + a_{14} a_{31} a_{43} y_2 - a_{14} a_{33} a_{41} y_2 - a_{21} a_{33} a_{44} y_1 + a_{21} a_{34} a_{43} y_1 + a_{23} a_{31} a_{44} y_1 - a_{23} a_{34} a_{41} y_1 - a_{24} a_{31} a_{43} y_1 + a_{24} a_{33} a_{41} y_1\big) \Big/ \big(a_{11} a_{22} a_{33} a_{44} - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} \quad (6)$$

$$+a_{13}a_{24}a_{31}a_{42}-a_{13}a_{24}a_{32}a_{41}-a_{14}a_{21}a_{32}a_{43}+a_{14}a_{21}a_{33}a_{42}+a_{14}a_{22}a_{31}a_{43}-a_{14}a_{22}a_{33}a_{41}-a_{14}a_{23}a_{31}a_{42}+a_{14}a_{23}a_{32}a_{41}\big)$$

> $sol3 := rhs(solns[3])$

$$sol3 := -\big(a_{11}a_{22}a_{34}y_4-a_{11}a_{22}a_{44}y_3-a_{11}a_{24}a_{32}y_4+a_{11}a_{24}a_{42}y_3+a_{11}a_{32}a_{44}y_2-a_{11}a_{34}a_{42}y_2-a_{12}a_{21}a_{34}y_4+a_{12}a_{21}a_{44}y_3+a_{12}a_{24}a_{31}y_4-a_{12}a_{24}a_{41}y_3-a_{12}a_{31}a_{44}y_2+a_{12}a_{34}a_{41}y_2+a_{14}a_{21}a_{32}y_4-a_{14}a_{21}a_{42}y_3-a_{14}a_{22}a_{31}y_4+a_{14}a_{22}a_{41}y_3+a_{14}a_{31}a_{42}y_2-a_{14}a_{32}a_{41}y_2-a_{21}a_{32}a_{44}y_1+a_{21}a_{34}a_{42}y_1+a_{22}a_{31}a_{44}y_1-a_{22}a_{34}a_{41}y_1-a_{24}a_{31}a_{42}y_1+a_{24}a_{32}a_{41}y_1\big)\Big/\big(a_{11}a_{22}a_{33}a_{44}-a_{11}a_{22}a_{34}a_{43}-a_{11}a_{23}a_{32}a_{44}+a_{11}a_{23}a_{34}a_{42}+a_{11}a_{24}a_{32}a_{43}-a_{11}a_{24}a_{33}a_{42}-a_{12}a_{21}a_{33}a_{44}+a_{12}a_{21}a_{34}a_{43}+a_{12}a_{23}a_{31}a_{44}-a_{12}a_{23}a_{34}a_{41}-a_{12}a_{24}a_{31}a_{43}+a_{12}a_{24}a_{33}a_{41}+a_{13}a_{21}a_{32}a_{44}-a_{13}a_{21}a_{34}a_{42}-a_{13}a_{22}a_{31}a_{44}+a_{13}a_{22}a_{34}a_{41}+a_{13}a_{24}a_{31}a_{42}-a_{13}a_{24}a_{32}a_{41}-a_{14}a_{21}a_{32}a_{43}+a_{14}a_{21}a_{33}a_{42}+a_{14}a_{22}a_{31}a_{43}-a_{14}a_{22}a_{33}a_{41}-a_{14}a_{23}a_{31}a_{42}+a_{14}a_{23}a_{32}a_{41}\big) \quad (7)$$

> $sol4 := rhs(solns[4])$

$$sol4 := \big(a_{11}a_{22}a_{33}y_4-a_{11}a_{22}a_{43}y_3-a_{11}a_{23}a_{32}y_4+a_{11}a_{23}a_{42}y_3+a_{11}a_{32}a_{43}y_2-a_{11}a_{33}a_{42}y_2-a_{12}a_{21}a_{33}y_4+a_{12}a_{21}a_{43}y_3+a_{12}a_{23}a_{31}y_4-a_{12}a_{23}a_{41}y_3-a_{12}a_{31}a_{43}y_2+a_{12}a_{33}a_{41}y_2+a_{13}a_{21}a_{32}y_4-a_{13}a_{21}a_{42}y_3-a_{13}a_{22}a_{31}y_4+a_{13}a_{22}a_{41}y_3+a_{13}a_{31}a_{42}y_2-a_{13}a_{32}a_{41}y_2-a_{21}a_{32}a_{43}y_1+a_{21}a_{33}a_{42}y_1+a_{22}a_{31}a_{43}y_1-a_{22}a_{33}a_{41}y_1-a_{23}a_{31}a_{42}y_1+a_{23}a_{32}a_{41}y_1\big)\Big/\big(a_{11}a_{22}a_{33}a_{44}-a_{11}a_{22}a_{34}a_{43}-a_{11}a_{23}a_{32}a_{44}+a_{11}a_{23}a_{34}a_{42}+a_{11}a_{24}a_{32}a_{43}-a_{11}a_{24}a_{33}a_{42}-a_{12}a_{21}a_{33}a_{44}+a_{12}a_{21}a_{34}a_{43}+a_{12}a_{23}a_{31}a_{44}-a_{12}a_{23}a_{34}a_{41}-a_{12}a_{24}a_{31}a_{43}+a_{12}a_{24}a_{33}a_{41}+a_{13}a_{21}a_{32}a_{44}-a_{13}a_{21}a_{34}a_{42}-a_{13}a_{22}a_{31}a_{44}+a_{13}a_{22}a_{34}a_{41}+a_{13}a_{24}a_{31}a_{42}-a_{13}a_{24}a_{32}a_{41}-a_{14}a_{21}a_{32}a_{43}+a_{14}a_{21}a_{33}a_{42}+a_{14}a_{22}a_{31}a_{43}-a_{14}a_{22}a_{33}a_{41}-a_{14}a_{23}a_{31}a_{42}+a_{14}a_{23}a_{32}a_{41}\big) \quad (8)$$

> $a_{11} := u_{x10-}$

$$a_{11} := u_{x10\text{-}} \quad (9)$$

> $a_{12} := u_{x40-}$

$$a_{12} := u_{x40\text{-}} \quad (10)$$

> $a_{13} := -u_{x20+}$

$$a_{13} := -u_{x20+} \quad (11)$$

> $a_{14} := -u_{x30+}$

$$a_{14} := -u_{x30+} \quad (12)$$

> $a_{21} := k_{z1-} \cdot u_{x10-}$

(13)

$$a_{21} := k_{z1-}\, u_{x10-} \quad (13)$$

> $a_{22} := k_{z4-} \cdot u_{x40-}$

$$a_{22} := k_{z4-}\, u_{x40-} \quad (14)$$

> $a_{23} := -k_{z2+} \cdot u_{x20+}$

$$a_{23} := -k_{z2+}\, u_{x20+} \quad (15)$$

> $a_{24} := -k_{z3+} \cdot u_{x30+}$

$$a_{24} := -k_{z3+}\, u_{x30+} \quad (16)$$

> $a_{31} := 1$

$$a_{31} := 1 \quad (17)$$

> $a_{32} := 1$

$$a_{32} := 1 \quad (18)$$

> $a_{33} := -1$

$$a_{33} := -1 \quad (19)$$

> $a_{34} := -1$

$$a_{34} := -1 \quad (20)$$

> $a_{41} := k_{z1-}$

$$a_{41} := k_{z1-} \quad (21)$$

> $a_{42} := k_{z4-}$

$$a_{42} := k_{z4-} \quad (22)$$

> $a_{43} := -k_{z2+}$

$$a_{43} := -k_{z2+} \quad (23)$$

> $a_{44} := -k_{z3+}$

$$a_{44} := -k_{z3+} \quad (24)$$

> $y_1 := u_{x10+}$

$$y_1 := u_{x10+} \quad (25)$$

> $y_2 := k_{z1+} \cdot u_{x10+}$

$$y_2 := k_{z1+}\, u_{x10+} \quad (26)$$

> $y_3 := 1$

$$y_3 := 1 \quad (27)$$

> $y_4 := k_{z1+}$

$$y_4 := k_{z1+} \quad (28)$$

Note that ux_0[n] denotes \hat{u}_{xn} and  u_x[n±] denotes u_{xn±}.

> $u_{x10-} := -\frac{\mathrm{i}\cdot k_x\cdot\Delta_{\perp 1-}}{L_{1-}-k_x^2}$

$$u_{x10-} := -\frac{\mathrm{I}\, k_x\, \Delta_{\perp 1-}}{-k_x^2+L_{1-}} \tag{29}$$

> $u_{x10+} := -\frac{\mathrm{i}\cdot k_x\cdot\Delta_{\perp 1+}}{L_{1+}-k_x^2}$

$$u_{x10+} := -\frac{\mathrm{I}\, k_x\, \Delta_{\perp 1+}}{-k_x^2+L_{1+}} \tag{30}$$

> $u_{x20-} := -\frac{\mathrm{i}\cdot k_x\cdot\Delta_{\perp 2-}}{L_{2-}-k_x^2}$

$$u_{x20-} := -\frac{\mathrm{I}\, k_x\, \Delta_{\perp 2-}}{-k_x^2+L_{2-}} \tag{31}$$

> $u_{x20+} := -\frac{\mathrm{i}\cdot k_x\cdot\Delta_{\perp 2+}}{L_{2+}-k_x^2}$

$$u_{x20+} := -\frac{\mathrm{I}\, k_x\, \Delta_{\perp 2+}}{-k_x^2+L_{2+}} \tag{32}$$

> $u_{x30-} := -\frac{\mathrm{i}\cdot k_x\cdot\Delta_{\perp 3-}}{L_{3-}-k_x^2}$

$$u_{x30-} := -\frac{\mathrm{I}\, k_x\, \Delta_{\perp 3-}}{-k_x^2+L_{3-}} \tag{33}$$

> $u_{x30+} := -\frac{\mathrm{i}\cdot k_x\cdot\Delta_{\perp 3+}}{L_{3+}-k_x^2}$

$$u_{x30+} := -\frac{\mathrm{I}\, k_x\, \Delta_{\perp 3+}}{-k_x^2+L_{3+}} \tag{34}$$

> $u_{x40-} := -\frac{\mathrm{i}\cdot k_x\cdot\Delta_{\perp 4-}}{L_{4-}-k_x^2}$

$$u_{x40-} := -\frac{\mathrm{I}\, k_x\, \Delta_{\perp 4-}}{-k_x^2+L_{4-}} \tag{35}$$

> $u_{x40+} := -\frac{i \cdot k_x \cdot \Delta_{\perp 4+}}{L_{4+} - k_x^2}$

$$u_{x40+} := -\frac{I\, k_x\, \Delta_{\perp 4+}}{-k_x^2 + L_{4+}} \quad (36)$$

> $L_{1-} := \Delta_{\|1-}^2 + k_{\|-}^2$

$$L_{1-} := k_{\|-}^2 + \Delta_{\|1-}^2 \quad (37)$$

> $L_{1+} := \Delta_{\|1+}^2 + k_{\|+}^2$

$$L_{1+} := k_{\|+}^2 + \Delta_{\|1+}^2 \quad (38)$$

> $L_{2-} := \Delta_{\|2-}^2 + k_{\|-}^2$

$$L_{2-} := k_{\|-}^2 + \Delta_{\|2-}^2 \quad (39)$$

> $L_{2+} := \Delta_{\|2+}^2 + k_{\|+}^2$

$$L_{2+} := k_{\|+}^2 + \Delta_{\|2+}^2 \quad (40)$$

> $L_{3-} := \Delta_{\|3-}^2 + k_{\|-}^2$

$$L_{3-} := k_{\|-}^2 + \Delta_{\|3-}^2 \quad (41)$$

> $L_{3+} := \Delta_{\|3+}^2 + k_{\|+}^2$

$$L_{3+} := k_{\|+}^2 + \Delta_{\|3+}^2 \quad (42)$$

> $L_{4-} := \Delta_{\|4-}^2 + k_{\|-}^2$

$$L_{4-} := k_{\|-}^2 + \Delta_{\|4-}^2 \quad (43)$$

> $L_{4+} := \Delta_{\|4+}^2 + k_{\|+}^2$

$$L_{4+} := k_{\|+}^2 + \Delta_{\|4+}^2 \quad (44)$$

> $\Delta_{\perp 1-} := i \cdot (k_y \cdot \cos(\alpha) - k_{z1-} \cdot \sin(\alpha))$

$$\Delta_{\perp 1-} := I\,(k_y \cos(\alpha) - k_{z1-} \sin(\alpha)) \quad (45)$$

> $\Delta_{\perp 1+} := i \cdot (k_y \cdot \cos(\alpha) - k_{z1+} \cdot \sin(\alpha))$

$$\Delta_{\perp 1+} := I\,(k_y \cos(\alpha) - k_{z1+} \sin(\alpha)) \quad (46)$$

> $\Delta_{\perp 2-} := i \cdot (k_y \cdot \cos(\alpha) - k_{z2-} \cdot \sin(\alpha))$

$$\Delta_{\perp 2-} := I\,(k_y \cos(\alpha) - k_{z2-} \sin(\alpha)) \quad (47)$$

> $\Delta_{\perp 2+} := i \cdot (k_y \cdot \cos(\alpha) - k_{z2+} \cdot \sin(\alpha))$

(48)

$$\Delta_{\perp 2+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z2+} \sin(\alpha)\right) \tag{48}$$

> $\Delta_{\perp 3-} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z3-}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 3\text{-}} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z3\text{-}} \sin(\alpha)\right) \tag{49}$$

> $\Delta_{\perp 3+} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z3+}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 3+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z3+} \sin(\alpha)\right) \tag{50}$$

> $\Delta_{\perp 4-} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z4-}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 4\text{-}} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z4\text{-}} \sin(\alpha)\right) \tag{51}$$

> $\Delta_{\perp 4+} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z4+}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 4+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z4+} \sin(\alpha)\right) \tag{52}$$

> $\Delta_{||1-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z1-}\cdot\cos(\alpha)\right)$

$$\Delta_{||1\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z1\text{-}} \cos(\alpha)\right) \tag{53}$$

> $\Delta_{||1+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z1+}\cdot\cos(\alpha)\right)$

$$\Delta_{||1+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z1+} \cos(\alpha)\right) \tag{54}$$

> $\Delta_{||2-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z2-}\cdot\cos(\alpha)\right)$

$$\Delta_{||2\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z2\text{-}} \cos(\alpha)\right) \tag{55}$$

> $\Delta_{||2+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z2+}\cdot\cos(\alpha)\right)$

$$\Delta_{||2+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z2+} \cos(\alpha)\right) \tag{56}$$

> $\Delta_{||3-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z3-}\cdot\cos(\alpha)\right)$

$$\Delta_{||3\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z3\text{-}} \cos(\alpha)\right) \tag{57}$$

> $\Delta_{||3+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z3+}\cdot\cos(\alpha)\right)$

$$\Delta_{||3+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z3+} \cos(\alpha)\right) \tag{58}$$

> $\Delta_{||4-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z4-}\cdot\cos(\alpha)\right)$

$$\Delta_{||4\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z4\text{-}} \cos(\alpha)\right) \tag{59}$$

> $\Delta_{||4+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z4+}\cdot\cos(\alpha)\right)$

$$\Delta_{||4+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z4+} \cos(\alpha)\right) \tag{60}$$

> $k_{z1-} := \dfrac{k_{||-}}{\cos(\alpha)} - k_y\cdot\tan(\alpha)$

$$k_{z1\text{-}} := \frac{k_{||\text{-}}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{61}$$

> $k_{z1+} := \dfrac{k_{||+}}{\cos(\alpha)} - k_y\cdot\tan(\alpha)$

$$k_{z1+} := \frac{k_{||+}}{\cos(\alpha)} - k_y \tan(\alpha) \quad \mathbf{(62)}$$

> $k_{z2-} := -\frac{k_{||-}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z2-} := -\frac{k_{||-}}{\cos(\alpha)} - k_y \tan(\alpha) \quad \mathbf{(63)}$$

> $k_{z2+} := -\frac{k_{||+}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z2+} := -\frac{k_{||+}}{\cos(\alpha)} - k_y \tan(\alpha) \quad \mathbf{(64)}$$

> $k_{z3-} := \mathrm{i} \cdot \sqrt{k_x^2 + k_y^2 - k_{||-}^2}$

$$k_{z3-} := \mathrm{I} \sqrt{k_x^2 + k_y^2 - k_{||-}^2} \quad \mathbf{(65)}$$

> $k_{z3+} := \mathrm{i} \cdot \sqrt{k_x^2 + k_y^2 - k_{||+}^2}$

$$k_{z3+} := \mathrm{I} \sqrt{k_x^2 + k_y^2 - k_{||+}^2} \quad \mathbf{(66)}$$

> $k_{z4-} := -\mathrm{i} \cdot \sqrt{k_x^2 + k_y^2 - k_{||-}^2}$

$$k_{z4-} := -\mathrm{I} \sqrt{k_x^2 + k_y^2 - k_{||-}^2} \quad \mathbf{(67)}$$

> $k_{z4+} := -\mathrm{i} \cdot \sqrt{k_x^2 + k_y^2 - k_{||+}^2}$

$$k_{z4+} := -\mathrm{I} \sqrt{k_x^2 + k_y^2 - k_{||+}^2} \quad \mathbf{(68)}$$

>

> $k_x := \frac{k_{||+}}{\epsilon_+}$

$$k_x := \frac{k_{||+}}{\epsilon_+} \quad \mathbf{(69)}$$

> $k_y := a_y \cdot k_{||+}$

$$k_y := a_y \, k_{||+} \quad \mathbf{(70)}$$

> $k_{||-} := \frac{k_x}{\epsilon_-}$

$$k_{||-} := \frac{k_{||+}}{\epsilon_+ \, \epsilon_-} \quad \mathbf{(71)}$$

Need to help maple to take limit as epsilon_- and epsilon_+ go to zero by taking terms that go to infinity out of the square root.

> $k_{z3-} := \dfrac{\mathrm{i}\cdot\sqrt{\epsilon_-^{\,2}\cdot k_{\|+}^{\,2} + \epsilon_-^{\,2}\cdot\epsilon_+^{\,2}\cdot k_y^{\,2} - k_{\|+}^{\,2}}}{\epsilon_-\cdot\epsilon_+}$

$$k_{z3-} := \frac{I\sqrt{a_y^2\,\epsilon_-^2\,\epsilon_+^2\,k_{\|+}^2 + \epsilon_-^2\,k_{\|+}^2 - k_{\|+}^2}}{\epsilon_-\,\epsilon_+} \tag{72}$$

> $k_{z3+} := \dfrac{\mathrm{i}\cdot\sqrt{k_{\|+}^{\,2} + \epsilon_+^{\,2}\cdot k_y^{\,2} - \epsilon_+^{\,2}\cdot k_{\|+}^{\,2}}}{\epsilon_+}$

$$k_{z3+} := \frac{I\sqrt{a_y^2\,\epsilon_+^2\,k_{\|+}^2 - \epsilon_+^2\,k_{\|+}^2 + k_{\|+}^2}}{\epsilon_+} \tag{73}$$

> $k_{z4-} := -\dfrac{\mathrm{i}\cdot\sqrt{\epsilon_-^{\,2}\cdot k_{\|+}^{\,2} + \epsilon_-^{\,2}\cdot\epsilon_+^{\,2}\cdot k_y^{\,2} - k_{\|+}^{\,2}}}{\epsilon_-\cdot\epsilon_+}$

$$k_{z4-} := -\frac{I\sqrt{a_y^2\,\epsilon_-^2\,\epsilon_+^2\,k_{\|+}^2 + \epsilon_-^2\,k_{\|+}^2 - k_{\|+}^2}}{\epsilon_-\,\epsilon_+} \tag{74}$$

> $k_{z4+} := -\dfrac{\mathrm{i}\cdot\sqrt{k_{\|+}^{\,2} + \epsilon_+^{\,2}\cdot k_y^{\,2} - \epsilon_+^{\,2}\cdot k_{\|+}^{\,2}}}{\epsilon_+}$

$$k_{z4+} := -\frac{I\sqrt{a_y^2\,\epsilon_+^2\,k_{\|+}^2 - \epsilon_+^2\,k_{\|+}^2 + k_{\|+}^2}}{\epsilon_+} \tag{75}$$

ux leading order terms

> $u_{x1-} := u_{x10-}\cdot sol1$ :

> $u_{x4-} := u_{x40-}\cdot sol2$ :

> $u_{x1+} := u_{x10+}$ :

> $u_{x2+} := u_{x20+}\cdot sol3$ :

> $u_{x3+} := u_{x30+}\cdot sol4$ :

> $simplify\left(mtaylor\left(u_{x1-}, [\epsilon_-, \epsilon_+], 3\right)\right)$

$$-2\,I\sin(\alpha)\,\epsilon_-\,\epsilon_+\,\mathrm{csgn}(k_{\|+}) \tag{76}$$

> $simplify\left(mtaylor\left(u_{x4-}, [\epsilon_-, \epsilon_+], 4\right)\right)$

$$-\frac{2\,\epsilon_-^2\,\epsilon_+\cos(\alpha)}{\sin(\alpha)} \tag{77}$$

> $simplify\left(mtaylor\left(u_{x1+}, [\epsilon_-, \epsilon_+], 2\right)\right)$

$$\frac{\epsilon_{+}\left(-a_{y}+\sin(\alpha)\right)}{\cos(\alpha)} \quad (78)$$

> $simplify\left(mtaylor\left(u_{x2+}, \left[\epsilon_{-}, \epsilon_{+}\right], 2\right)\right)$

$$\frac{\epsilon_{+}\left(a_{y}+\sin(\alpha)\right)}{\cos(\alpha)} \quad (79)$$

> $simplify\left(mtaylor\left(u_{x3+}, \left[\epsilon_{-}, \epsilon_{+}\right], 2\right)\right)$

$$-\frac{2\sin(\alpha)\,\epsilon_{+}}{\cos(\alpha)} \quad (80)$$

u_perp leading order terms

> $simplify\left(mtaylor\left(sol1, \left[\epsilon_{-}, \epsilon_{+}\right], 4\right)\right)$

$$-2\,\mathrm{I}\,\epsilon_{-}^{2}\,\epsilon_{+}\cos(\alpha)\,\mathrm{csgn}\left(k_{\|+}\right) \quad (81)$$

> $simplify\left(mtaylor\left(sol2, \left[\epsilon_{-}, \epsilon_{+}\right], 3\right)\right)$

$$\frac{2\,\mathrm{I}\cos(\alpha)\,\epsilon_{+}\,\epsilon_{-}\,k_{\|+}}{\sqrt{-k_{\|+}^{2}}} \quad (82)$$

> $simplify\left(mtaylor\left(sol3, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$-1 \quad (83)$$

> $simplify\left(mtaylor\left(sol4, \left[\epsilon_{-}, \epsilon_{+}\right], 2\right)\right)$

$$\frac{2\,\mathrm{I}\,\epsilon_{+}\,\mathrm{csgn}\left(k_{\|+}\right)\sin(\alpha)^{2}}{\cos(\alpha)} \quad (84)$$

bx leading order terms

> $b_{x1-} := \frac{\Delta_{\|1-} \cdot u_{x1-}}{\mathrm{i} \cdot k_{\|+}}:$

> $b_{x4-} := \frac{\Delta_{\|4-} \cdot u_{x4-}}{\mathrm{i} \cdot k_{\|+}}:$

> $b_{x1+} := \frac{\Delta_{\|1+} \cdot u_{x1+}}{\mathrm{i} \cdot k_{\|+}}:$

> $b_{x2+} := \frac{\Delta_{\|2+} \cdot u_{x2+}}{\mathrm{i} \cdot k_{\|+}}:$

> $b_{x3+} := \frac{\Delta_{\|3+} \cdot u_{x3+}}{\mathrm{i} \cdot k_{\|+}}:$

> $simplify(mtaylor(b_{x1-}, [\epsilon_-, \epsilon_+], 1))$

$$-2\,I\sin(\alpha)\,\mathrm{csgn}(k_{\|+}) \tag{85}$$

> $simplify(mtaylor(b_{x4-}, [\epsilon_-, \epsilon_+], 2))$

$$-\frac{2\,I\,k_{\|+}\,\epsilon_-\cos(\alpha)^2}{\sqrt{-k_{\|+}^2}\,\sin(\alpha)} \tag{86}$$

> $simplify(mtaylor(b_{x1+}, [\epsilon_-, \epsilon_+], 2))$

$$\frac{\epsilon_+\,(-a_y+\sin(\alpha))}{\cos(\alpha)} \tag{87}$$

> $simplify(mtaylor(b_{x2+}, [\epsilon_-, \epsilon_+], 2))$

$$-\frac{\epsilon_+\,(a_y+\sin(\alpha))}{\cos(\alpha)} \tag{88}$$

> $simplify(mtaylor(b_{x3+}, [\epsilon_-, \epsilon_+], 1))$

$$-2\,I\sin(\alpha)\,\mathrm{csgn}(k_{\|+}) \tag{89}$$

b_perp leading order terms

> $b_{\perp 1-} := \frac{\Delta_{\|1-}\cdot sol1}{\mathrm{i}\cdot k_{\|+}}:$

> $b_{\perp 4-} := \frac{\Delta_{\|4-}\cdot sol2}{\mathrm{i}\cdot k_{\|+}}:$

> $b_{\perp 1+} := \frac{\Delta_{\|1+}\cdot 1}{\mathrm{i}\cdot k_{\|+}}:$

> $b_{\perp 2+} := \frac{\Delta_{\|2+}\cdot sol3}{\mathrm{i}\cdot k_{\|+}}:$

> $b_{\perp 3+} := \frac{\Delta_{\|3+}\cdot sol4}{\mathrm{i}\cdot k_{\|+}}:$

> $simplify(mtaylor(b_{\perp 1-}, [\epsilon_-, \epsilon_+], 2))$

$$-2\,I\,\epsilon_-\cos(\alpha)\,\mathrm{csgn}(k_{\|+}) \tag{90}$$

> $simplify(mtaylor(b_{\perp 4-}, [\epsilon_-, \epsilon_+], 1))$

$$2\cos(\alpha)^2 \tag{91}$$

> $simplify(mtaylor(b_{\perp 1+}, [\epsilon_-, \epsilon_+], 5))$

$$1 \tag{92}$$

> $simplify(mtaylor(b_{\perp 2+}, [\epsilon_-, \epsilon_+], 1))$

$$1 \tag{93}$$

> $simplify\left(mtaylor\left(b_{\perp 3+}, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$-2 \sin(\alpha)^2 \quad \textbf{(94)}$$

b_par leading order terms

> $b_{||1-} := -\frac{\mathrm{i} \cdot k_x \cdot u_{x1-} + \Delta_{\perp 1-} \cdot sol1}{\mathrm{i} \cdot k_{||+}}$ :

> $b_{||4-} := -\frac{\mathrm{i} \cdot k_x \cdot u_{x4-} + \Delta_{\perp 4-} \cdot sol2}{\mathrm{i} \cdot k_{||+}}$ :

> $b_{||1+} := -\frac{\mathrm{i} \cdot k_x \cdot u_{x1+} + \Delta_{\perp 1+}}{\mathrm{i} \cdot k_{||+}}$ :

> $b_{||2+} := -\frac{\mathrm{i} \cdot k_x \cdot u_{x2+} + \Delta_{\perp 2+} \cdot sol3}{\mathrm{i} \cdot k_{||+}}$ :

> $b_{||3+} := -\frac{\mathrm{i} \cdot k_x \cdot u_{x3+} + \Delta_{\perp 3+} \cdot sol4}{\mathrm{i} \cdot k_{||+}}$ :

> $simplify\left(mtaylor\left(b_{||1-}, \left[\epsilon_{-}, \epsilon_{+}\right], 2\right)\right)$

$$0 \quad \textbf{(95)}$$

> $simplify\left(mtaylor\left(b_{||4-}, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$2 \cos(\alpha) \sin(\alpha) \quad \textbf{(96)}$$

> $simplify\left(mtaylor\left(b_{||1+}, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$0 \quad \textbf{(97)}$$

> $simplify\left(mtaylor\left(b_{||2+}, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$0 \quad \textbf{(98)}$$

> $simplify\left(mtaylor\left(b_{||3+}, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$2 \cos(\alpha) \sin(\alpha) \quad \textbf{(99)}$$