This document is used to assist with the Algebra in Appendix A.2
Note that ∇ has been replaced with to ensure the code works okay.
> *restart*;
We normalise the velocity coefficents by u_0 and the field components by (B_0 * u_0 / v_{A+}).
We start by solving the Matrix equation given by Equation (57).

> $eqn1 := a_{11} \cdot x_1 + a_{12} \cdot x_2 + a_{13} \cdot x_3 + a_{14} \cdot x_4 = y_1$

$$eqn1 := a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 = y_1 \qquad (1)$$

> $eqn2 := a_{21} \cdot x_1 + a_{22} \cdot x_2 + a_{23} \cdot x_3 + a_{24} \cdot x_4 = y_2$

$$eqn2 := a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + a_{24} x_4 = y_2 \qquad (2)$$

> $eqn3 := a_{31} \cdot x_1 + a_{32} \cdot x_2 + a_{33} \cdot x_3 + a_{34} \cdot x_4 = y_3$

$$eqn3 := a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + a_{34} x_4 = y_3 \qquad (3)$$

> $eqn4 := a_{41} \cdot x_1 + a_{42} \cdot x_2 + a_{43} \cdot x_3 + a_{44} \cdot x_4 = y_4$

$$eqn4 := a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4 = y_4 \qquad (4)$$

> $solns := solve(\{eqn1, eqn2, eqn3, eqn4\}, \{x_1, x_2, x_3, x_4\})$:

> $sol1 := rhs(solns[1])$

$$
\begin{aligned}
sol1 := &-\big(a_{12}a_{23}a_{34}y_4 - a_{12}a_{23}a_{44}y_3 - a_{12}a_{24}a_{33}y_4 + a_{12}a_{24}a_{43}y_3 + a_{12}a_{33}a_{44}y_2 \\
&- a_{12}a_{34}a_{43}y_2 - a_{13}a_{22}a_{34}y_4 + a_{13}a_{22}a_{44}y_3 + a_{13}a_{24}a_{32}y_4 - a_{13}a_{24}a_{42}y_3 \\
&- a_{13}a_{32}a_{44}y_2 + a_{13}a_{34}a_{42}y_2 + a_{14}a_{22}a_{33}y_4 - a_{14}a_{22}a_{43}y_3 - a_{14}a_{23}a_{32}y_4 \\
&+ a_{14}a_{23}a_{42}y_3 + a_{14}a_{32}a_{43}y_2 - a_{14}a_{33}a_{42}y_2 - a_{22}a_{33}a_{44}y_1 + a_{22}a_{34}a_{43}y_1 \\
&+ a_{23}a_{32}a_{44}y_1 - a_{23}a_{34}a_{42}y_1 - a_{24}a_{32}a_{43}y_1 + a_{24}a_{33}a_{42}y_1\big) \big/ \big(a_{11}a_{22}a_{33}a_{44} \\
&- a_{11}a_{22}a_{34}a_{43} - a_{11}a_{23}a_{32}a_{44} + a_{11}a_{23}a_{34}a_{42} + a_{11}a_{24}a_{32}a_{43} - a_{11}a_{24}a_{33}a_{42} \\
&- a_{12}a_{21}a_{33}a_{44} + a_{12}a_{21}a_{34}a_{43} + a_{12}a_{23}a_{31}a_{44} - a_{12}a_{23}a_{34}a_{41} - a_{12}a_{24}a_{31}a_{43} \\
&+ a_{12}a_{24}a_{33}a_{41} + a_{13}a_{21}a_{32}a_{44} - a_{13}a_{21}a_{34}a_{42} - a_{13}a_{22}a_{31}a_{44} + a_{13}a_{22}a_{34}a_{41} \\
&+ a_{13}a_{24}a_{31}a_{42} - a_{13}a_{24}a_{32}a_{41} - a_{14}a_{21}a_{32}a_{43} + a_{14}a_{21}a_{33}a_{42} + a_{14}a_{22}a_{31}a_{43} \\
&- a_{14}a_{22}a_{33}a_{41} - a_{14}a_{23}a_{31}a_{42} + a_{14}a_{23}a_{32}a_{41}\big)
\end{aligned}
\qquad (5)
$$

> $sol2 := rhs(solns[2])$

$$
\begin{aligned}
sol2 := &\big(a_{11}a_{23}a_{34}y_4 - a_{11}a_{23}a_{44}y_3 - a_{11}a_{24}a_{33}y_4 + a_{11}a_{24}a_{43}y_3 + a_{11}a_{33}a_{44}y_2 \\
&- a_{11}a_{34}a_{43}y_2 - a_{13}a_{21}a_{34}y_4 + a_{13}a_{21}a_{44}y_3 + a_{13}a_{24}a_{31}y_4 - a_{13}a_{24}a_{41}y_3 \\
&- a_{13}a_{31}a_{44}y_2 + a_{13}a_{34}a_{41}y_2 + a_{14}a_{21}a_{33}y_4 - a_{14}a_{21}a_{43}y_3 - a_{14}a_{23}a_{31}y_4 \\
&+ a_{14}a_{23}a_{41}y_3 + a_{14}a_{31}a_{43}y_2 - a_{14}a_{33}a_{41}y_2 - a_{21}a_{33}a_{44}y_1 + a_{21}a_{34}a_{43}y_1 \\
&+ a_{23}a_{31}a_{44}y_1 - a_{23}a_{34}a_{41}y_1 - a_{24}a_{31}a_{43}y_1 + a_{24}a_{33}a_{41}y_1\big) \big/ \big(a_{11}a_{22}a_{33}a_{44} \\
&- a_{11}a_{22}a_{34}a_{43} - a_{11}a_{23}a_{32}a_{44} + a_{11}a_{23}a_{34}a_{42} + a_{11}a_{24}a_{32}a_{43} - a_{11}a_{24}a_{33}a_{42} \\
&- a_{12}a_{21}a_{33}a_{44} + a_{12}a_{21}a_{34}a_{43} + a_{12}a_{23}a_{31}a_{44} - a_{12}a_{23}a_{34}a_{41} - a_{12}a_{24}a_{31}a_{43} \\
&+ a_{12}a_{24}a_{33}a_{41} + a_{13}a_{21}a_{32}a_{44} - a_{13}a_{21}a_{34}a_{42} - a_{13}a_{22}a_{31}a_{44} + a_{13}a_{22}a_{34}a_{41}
\end{aligned}
\qquad (6)
$$

$$+ a_{13}\,a_{24}\,a_{31}\,a_{42} - a_{13}\,a_{24}\,a_{32}\,a_{41} - a_{14}\,a_{21}\,a_{32}\,a_{43} + a_{14}\,a_{21}\,a_{33}\,a_{42} + a_{14}\,a_{22}\,a_{31}\,a_{43}$$
$$- a_{14}\,a_{22}\,a_{33}\,a_{41} - a_{14}\,a_{23}\,a_{31}\,a_{42} + a_{14}\,a_{23}\,a_{32}\,a_{41})$$

> $sol3 := rhs(solns[3])$

$$sol3 := -\big(a_{11}\,a_{22}\,a_{34}\,y_4 - a_{11}\,a_{22}\,a_{44}\,y_3 - a_{11}\,a_{24}\,a_{32}\,y_4 + a_{11}\,a_{24}\,a_{42}\,y_3 + a_{11}\,a_{32}\,a_{44}\,y_2 \qquad (7)$$
$$- a_{11}\,a_{34}\,a_{42}\,y_2 - a_{12}\,a_{21}\,a_{34}\,y_4 + a_{12}\,a_{21}\,a_{44}\,y_3 + a_{12}\,a_{24}\,a_{31}\,y_4 - a_{12}\,a_{24}\,a_{41}\,y_3$$
$$- a_{12}\,a_{31}\,a_{44}\,y_2 + a_{12}\,a_{34}\,a_{41}\,y_2 + a_{14}\,a_{21}\,a_{32}\,y_4 - a_{14}\,a_{21}\,a_{42}\,y_3 - a_{14}\,a_{22}\,a_{31}\,y_4$$
$$+ a_{14}\,a_{22}\,a_{41}\,y_3 + a_{14}\,a_{31}\,a_{42}\,y_2 - a_{14}\,a_{32}\,a_{41}\,y_2 - a_{21}\,a_{32}\,a_{44}\,y_1 + a_{21}\,a_{34}\,a_{42}\,y_1$$
$$+ a_{22}\,a_{31}\,a_{44}\,y_1 - a_{22}\,a_{34}\,a_{41}\,y_1 - a_{24}\,a_{31}\,a_{42}\,y_1 + a_{24}\,a_{32}\,a_{41}\,y_1\big) \Big/ \big(a_{11}\,a_{22}\,a_{33}\,a_{44}$$
$$- a_{11}\,a_{22}\,a_{34}\,a_{43} - a_{11}\,a_{23}\,a_{32}\,a_{44} + a_{11}\,a_{23}\,a_{34}\,a_{42} + a_{11}\,a_{24}\,a_{32}\,a_{43} - a_{11}\,a_{24}\,a_{33}\,a_{42}$$
$$- a_{12}\,a_{21}\,a_{33}\,a_{44} + a_{12}\,a_{21}\,a_{34}\,a_{43} + a_{12}\,a_{23}\,a_{31}\,a_{44} - a_{12}\,a_{23}\,a_{34}\,a_{41} - a_{12}\,a_{24}\,a_{31}\,a_{43}$$
$$+ a_{12}\,a_{24}\,a_{33}\,a_{41} + a_{13}\,a_{21}\,a_{32}\,a_{44} - a_{13}\,a_{21}\,a_{34}\,a_{42} - a_{13}\,a_{22}\,a_{31}\,a_{44} + a_{13}\,a_{22}\,a_{34}\,a_{41}$$
$$+ a_{13}\,a_{24}\,a_{31}\,a_{42} - a_{13}\,a_{24}\,a_{32}\,a_{41} - a_{14}\,a_{21}\,a_{32}\,a_{43} + a_{14}\,a_{21}\,a_{33}\,a_{42} + a_{14}\,a_{22}\,a_{31}\,a_{43}$$
$$- a_{14}\,a_{22}\,a_{33}\,a_{41} - a_{14}\,a_{23}\,a_{31}\,a_{42} + a_{14}\,a_{23}\,a_{32}\,a_{41}\big)$$

> $sol4 := rhs(solns[4])$

$$sol4 := \big(a_{11}\,a_{22}\,a_{33}\,y_4 - a_{11}\,a_{22}\,a_{43}\,y_3 - a_{11}\,a_{23}\,a_{32}\,y_4 + a_{11}\,a_{23}\,a_{42}\,y_3 + a_{11}\,a_{32}\,a_{43}\,y_2 \qquad (8)$$
$$- a_{11}\,a_{33}\,a_{42}\,y_2 - a_{12}\,a_{21}\,a_{33}\,y_4 + a_{12}\,a_{21}\,a_{43}\,y_3 + a_{12}\,a_{23}\,a_{31}\,y_4 - a_{12}\,a_{23}\,a_{41}\,y_3$$
$$- a_{12}\,a_{31}\,a_{43}\,y_2 + a_{12}\,a_{33}\,a_{41}\,y_2 + a_{13}\,a_{21}\,a_{32}\,y_4 - a_{13}\,a_{21}\,a_{42}\,y_3 - a_{13}\,a_{22}\,a_{31}\,y_4$$
$$+ a_{13}\,a_{22}\,a_{41}\,y_3 + a_{13}\,a_{31}\,a_{42}\,y_2 - a_{13}\,a_{32}\,a_{41}\,y_2 - a_{21}\,a_{32}\,a_{43}\,y_1 + a_{21}\,a_{33}\,a_{42}\,y_1$$
$$+ a_{22}\,a_{31}\,a_{43}\,y_1 - a_{22}\,a_{33}\,a_{41}\,y_1 - a_{23}\,a_{31}\,a_{42}\,y_1 + a_{23}\,a_{32}\,a_{41}\,y_1\big) \Big/ \big(a_{11}\,a_{22}\,a_{33}\,a_{44}$$
$$- a_{11}\,a_{22}\,a_{34}\,a_{43} - a_{11}\,a_{23}\,a_{32}\,a_{44} + a_{11}\,a_{23}\,a_{34}\,a_{42} + a_{11}\,a_{24}\,a_{32}\,a_{43} - a_{11}\,a_{24}\,a_{33}\,a_{42}$$
$$- a_{12}\,a_{21}\,a_{33}\,a_{44} + a_{12}\,a_{21}\,a_{34}\,a_{43} + a_{12}\,a_{23}\,a_{31}\,a_{44} - a_{12}\,a_{23}\,a_{34}\,a_{41} - a_{12}\,a_{24}\,a_{31}\,a_{43}$$
$$+ a_{12}\,a_{24}\,a_{33}\,a_{41} + a_{13}\,a_{21}\,a_{32}\,a_{44} - a_{13}\,a_{21}\,a_{34}\,a_{42} - a_{13}\,a_{22}\,a_{31}\,a_{44} + a_{13}\,a_{22}\,a_{34}\,a_{41}$$
$$+ a_{13}\,a_{24}\,a_{31}\,a_{42} - a_{13}\,a_{24}\,a_{32}\,a_{41} - a_{14}\,a_{21}\,a_{32}\,a_{43} + a_{14}\,a_{21}\,a_{33}\,a_{42} + a_{14}\,a_{22}\,a_{31}\,a_{43}$$
$$- a_{14}\,a_{22}\,a_{33}\,a_{41} - a_{14}\,a_{23}\,a_{31}\,a_{42} + a_{14}\,a_{23}\,a_{32}\,a_{41}\big)$$

> $a_{11} := u_{x10-}$

$$a_{11} := u_{x10\text{-}} \qquad (9)$$

> $a_{12} := u_{x40-}$

$$a_{12} := u_{x40\text{-}} \qquad (10)$$

> $a_{13} := -u_{x20+}$

$$a_{13} := -u_{x20+} \qquad (11)$$

> $a_{14} := -u_{x30+}$

$$a_{14} := -u_{x30+} \qquad (12)$$

> $a_{21} := k_{z1-} \cdot u_{x10-}$

$$(13)$$

$$a_{21} := k_{z1-}\, u_{x10-} \tag{13}$$

> $a_{22} := k_{z4-}\cdot u_{x40-}$

$$a_{22} := k_{z4-}\, u_{x40-} \tag{14}$$

> $a_{23} := -k_{z2+}\cdot u_{x20+}$

$$a_{23} := -k_{z2+}\, u_{x20+} \tag{15}$$

> $a_{24} := -k_{z3+}\cdot u_{x30+}$

$$a_{24} := -k_{z3+}\, u_{x30+} \tag{16}$$

> $a_{31} := 1$

$$a_{31} := 1 \tag{17}$$

> $a_{32} := 1$

$$a_{32} := 1 \tag{18}$$

> $a_{33} := -1$

$$a_{33} := -1 \tag{19}$$

> $a_{34} := -1$

$$a_{34} := -1 \tag{20}$$

> $a_{41} := k_{z1-}$

$$a_{41} := k_{z1-} \tag{21}$$

> $a_{42} := k_{z4-}$

$$a_{42} := k_{z4-} \tag{22}$$

> $a_{43} := -k_{z2+}$

$$a_{43} := -k_{z2+} \tag{23}$$

> $a_{44} := -k_{z3+}$

$$a_{44} := -k_{z3+} \tag{24}$$

> $y_1 := u_{x10+}$

$$y_1 := u_{x10+} \tag{25}$$

> $y_2 := k_{z1+}\cdot u_{x10+}$

$$y_2 := k_{z1+}\, u_{x10+} \tag{26}$$

> $y_3 := 1$

$$y_3 := 1 \tag{27}$$

> $y_4 := k_{z1+}$

$$y_4 := k_{z1+} \tag{28}$$

Note that ux_0[n] denotes \hat{u}_{xn} and u_x[n±] denotes u_{xn±}.

$$> \quad u_{x10-} := -\frac{i \cdot k_x \cdot \Delta_{\perp 1-}}{L_{1-} - k_x^2}$$

$$u_{x10-} := \frac{-I k_x \Delta_{\perp 1-}}{-k_x^2 + L_{1-}} \tag{29}$$

$$> \quad u_{x10+} := -\frac{i \cdot k_x \cdot \Delta_{\perp 1+}}{L_{1+} - k_x^2}$$

$$u_{x10+} := \frac{-I k_x \Delta_{\perp 1+}}{-k_x^2 + L_{1+}} \tag{30}$$

$$> \quad u_{x20-} := -\frac{i \cdot k_x \cdot \Delta_{\perp 2-}}{L_{2-} - k_x^2}$$

$$u_{x20-} := \frac{-I k_x \Delta_{\perp 2-}}{-k_x^2 + L_{2-}} \tag{31}$$

$$> \quad u_{x20+} := -\frac{i \cdot k_x \cdot \Delta_{\perp 2+}}{L_{2+} - k_x^2}$$

$$u_{x20+} := \frac{-I k_x \Delta_{\perp 2+}}{-k_x^2 + L_{2+}} \tag{32}$$

$$> \quad u_{x30-} := -\frac{i \cdot k_x \cdot \Delta_{\perp 3-}}{L_{3-} - k_x^2}$$

$$u_{x30-} := \frac{-I k_x \Delta_{\perp 3-}}{-k_x^2 + L_{3-}} \tag{33}$$

$$> \quad u_{x30+} := -\frac{i \cdot k_x \cdot \Delta_{\perp 3+}}{L_{3+} - k_x^2}$$

$$u_{x30+} := \frac{-I k_x \Delta_{\perp 3+}}{-k_x^2 + L_{3+}} \tag{34}$$

$$> \quad u_{x40-} := -\frac{i \cdot k_x \cdot \Delta_{\perp 4-}}{L_{4-} - k_x^2}$$

$$u_{x40-} := \frac{-I k_x \Delta_{\perp 4-}}{-k_x^2 + L_{4-}} \tag{35}$$

> $u_{x40+} := -\dfrac{i \cdot k_x \cdot \Delta_{\perp 4+}}{L_{4+} - k_x^2}$

$$u_{x40+} := \frac{-I\, k_x\, \Delta_{\perp 4+}}{-k_x^2 + L_{4+}} \tag{36}$$

> $L_{1-} := \Delta_{\|1-}^2 + k_{\|-}^2$

$$L_{1-} := k_{\|-}^2 + \Delta_{\|1-}^2 \tag{37}$$

> $L_{1+} := \Delta_{\|1+}^2 + k_{\|+}^2$

$$L_{1+} := k_{\|+}^2 + \Delta_{\|1+}^2 \tag{38}$$

> $L_{2-} := \Delta_{\|2-}^2 + k_{\|-}^2$

$$L_{2-} := k_{\|-}^2 + \Delta_{\|2-}^2 \tag{39}$$

> $L_{2+} := \Delta_{\|2+}^2 + k_{\|+}^2$

$$L_{2+} := k_{\|+}^2 + \Delta_{\|2+}^2 \tag{40}$$

> $L_{3-} := \Delta_{\|3-}^2 + k_{\|-}^2$

$$L_{3-} := k_{\|-}^2 + \Delta_{\|3-}^2 \tag{41}$$

> $L_{3+} := \Delta_{\|3+}^2 + k_{\|+}^2$

$$L_{3+} := k_{\|+}^2 + \Delta_{\|3+}^2 \tag{42}$$

> $L_{4-} := \Delta_{\|4-}^2 + k_{\|-}^2$

$$L_{4-} := k_{\|-}^2 + \Delta_{\|4-}^2 \tag{43}$$

> $L_{4+} := \Delta_{\|4+}^2 + k_{\|+}^2$

$$L_{4+} := k_{\|+}^2 + \Delta_{\|4+}^2 \tag{44}$$

> $\Delta_{\perp 1-} := i \cdot \left(k_y \cdot \cos(\alpha) - k_{z1-} \cdot \sin(\alpha) \right)$

$$\Delta_{\perp 1-} := I \left(k_y \cos(\alpha) - k_{z1-} \sin(\alpha) \right) \tag{45}$$

> $\Delta_{\perp 1+} := i \cdot \left(k_y \cdot \cos(\alpha) - k_{z1+} \cdot \sin(\alpha) \right)$

$$\Delta_{\perp 1+} := I \left(k_y \cos(\alpha) - k_{z1+} \sin(\alpha) \right) \tag{46}$$

> $\Delta_{\perp 2-} := i \cdot \left(k_y \cdot \cos(\alpha) - k_{z2-} \cdot \sin(\alpha) \right)$

$$\Delta_{\perp 2-} := I \left(k_y \cos(\alpha) - k_{z2-} \sin(\alpha) \right) \tag{47}$$

> $\Delta_{\perp 2+} := i \cdot \left(k_y \cdot \cos(\alpha) - k_{z2+} \cdot \sin(\alpha) \right)$

$$\tag{48}$$

$$\Delta_{\perp 2+} := I\left(k_y \cos(\alpha) - k_{z2+} \sin(\alpha)\right) \tag{48}$$

$> \quad \Delta_{\perp 3-} := i \cdot \left(k_y \cdot \cos(\alpha) - k_{z3-} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 3-} := I\left(k_y \cos(\alpha) - k_{z3-} \sin(\alpha)\right) \tag{49}$$

$> \quad \Delta_{\perp 3+} := i \cdot \left(k_y \cdot \cos(\alpha) - k_{z3+} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 3+} := I\left(k_y \cos(\alpha) - k_{z3+} \sin(\alpha)\right) \tag{50}$$

$> \quad \Delta_{\perp 4-} := i \cdot \left(k_y \cdot \cos(\alpha) - k_{z4-} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 4-} := I\left(k_y \cos(\alpha) - k_{z4-} \sin(\alpha)\right) \tag{51}$$

$> \quad \Delta_{\perp 4+} := i \cdot \left(k_y \cdot \cos(\alpha) - k_{z4+} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 4+} := I\left(k_y \cos(\alpha) - k_{z4+} \sin(\alpha)\right) \tag{52}$$

$> \quad \Delta_{\parallel 1-} := i \cdot \left(k_y \cdot \sin(\alpha) + k_{z1-} \cdot \cos(\alpha)\right)$

$$\Delta_{\parallel 1-} := I\left(k_y \sin(\alpha) + k_{z1-} \cos(\alpha)\right) \tag{53}$$

$> \quad \Delta_{\parallel 1+} := i \cdot \left(k_y \cdot \sin(\alpha) + k_{z1+} \cdot \cos(\alpha)\right)$

$$\Delta_{\parallel 1+} := I\left(k_y \sin(\alpha) + k_{z1+} \cos(\alpha)\right) \tag{54}$$

$> \quad \Delta_{\parallel 2-} := i \cdot \left(k_y \cdot \sin(\alpha) + k_{z2-} \cdot \cos(\alpha)\right)$

$$\Delta_{\parallel 2-} := I\left(k_y \sin(\alpha) + k_{z2-} \cos(\alpha)\right) \tag{55}$$

$> \quad \Delta_{\parallel 2+} := i \cdot \left(k_y \cdot \sin(\alpha) + k_{z2+} \cdot \cos(\alpha)\right)$

$$\Delta_{\parallel 2+} := I\left(k_y \sin(\alpha) + k_{z2+} \cos(\alpha)\right) \tag{56}$$

$> \quad \Delta_{\parallel 3-} := i \cdot \left(k_y \cdot \sin(\alpha) + k_{z3-} \cdot \cos(\alpha)\right)$

$$\Delta_{\parallel 3-} := I\left(k_y \sin(\alpha) + k_{z3-} \cos(\alpha)\right) \tag{57}$$

$> \quad \Delta_{\parallel 3+} := i \cdot \left(k_y \cdot \sin(\alpha) + k_{z3+} \cdot \cos(\alpha)\right)$

$$\Delta_{\parallel 3+} := I\left(k_y \sin(\alpha) + k_{z3+} \cos(\alpha)\right) \tag{58}$$

$> \quad \Delta_{\parallel 4-} := i \cdot \left(k_y \cdot \sin(\alpha) + k_{z4-} \cdot \cos(\alpha)\right)$

$$\Delta_{\parallel 4-} := I\left(k_y \sin(\alpha) + k_{z4-} \cos(\alpha)\right) \tag{59}$$

$> \quad \Delta_{\parallel 4+} := i \cdot \left(k_y \cdot \sin(\alpha) + k_{z4+} \cdot \cos(\alpha)\right)$

$$\Delta_{\parallel 4+} := I\left(k_y \sin(\alpha) + k_{z4+} \cos(\alpha)\right) \tag{60}$$

$> \quad k_{z1-} := \dfrac{k_{\parallel -}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z1-} := \frac{k_{\parallel -}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{61}$$

$> \quad k_{z1+} := \dfrac{k_{\parallel +}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z1+} := \frac{k_{\|+}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{62}$$

> $k_{z2-} := -\dfrac{k_{\|-}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z2-} := -\frac{k_{\|-}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{63}$$

> $k_{z2+} := -\dfrac{k_{\|+}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z2+} := -\frac{k_{\|+}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{64}$$

> $k_{z3-} := i \cdot \sqrt{k_x^2 + k_y^2 - k_{\|-}^2}$

$$k_{z3-} := I \sqrt{k_x^2 + k_y^2 - k_{\|-}^2} \tag{65}$$

> $k_{z3+} := i \cdot \sqrt{k_x^2 + k_y^2 - k_{\|+}^2}$

$$k_{z3+} := I \sqrt{k_x^2 + k_y^2 - k_{\|+}^2} \tag{66}$$

> $k_{z4-} := -i \cdot \sqrt{k_x^2 + k_y^2 - k_{\|-}^2}$

$$k_{z4-} := -I \sqrt{k_x^2 + k_y^2 - k_{\|-}^2} \tag{67}$$

> $k_{z4+} := -i \cdot \sqrt{k_x^2 + k_y^2 - k_{\|+}^2}$

$$k_{z4+} := -I \sqrt{k_x^2 + k_y^2 - k_{\|+}^2} \tag{68}$$

>
> $k_{\|+} := r \cdot k_{\|-}$

$$k_{\|+} := r\, k_{\|-} \tag{69}$$

> $k_y := a_y \cdot k_{\|+}$

$$k_y := a_y\, r\, k_{\|-} \tag{70}$$

> $k_x := \dfrac{k_{\|-}}{\epsilon}$

$$k_x := \frac{k_{\|-}}{\epsilon} \tag{71}$$

ux leading order terms
> $u_{x1-} := u_{x10-} \cdot sol1 :$
> $u_{x4-} := u_{x40-} \cdot sol2 :$
> $u_{x1+} := u_{x10+} :$

> $u_{x2+} := u_{x20+} \cdot sol3$:

> $u_{x3+} := u_{x30+} \cdot sol4$:

> $simplify(series(u_{x1-}, \epsilon, 2))$

$$\frac{2r\left(-a_y r + \sin(\alpha)\right)}{\cos(\alpha)\,(r+1)}\,\epsilon + O(\epsilon^2) \tag{72}$$

> $simplify(series(u_{x4-}, \epsilon, 3))$

$$\frac{I\sin(\alpha)\,r\,(r-1)\,\mathrm{csgn}\!\left(\dfrac{k_{\parallel -}}{\epsilon}\right)}{\cos(\alpha)^2}\,\epsilon^2 + O(\epsilon^3) \tag{73}$$

> $simplify(series(u_{x1+}, \epsilon, 3))$

$$\frac{r\left(-a_y + \sin(\alpha)\right)}{\cos(\alpha)}\,\epsilon + O(\epsilon^3) \tag{74}$$

> $simplify(series(u_{x2+}, \epsilon, 3))$

$$-\frac{r\,(r-1)\,\left(a_y + \sin(\alpha)\right)}{\cos(\alpha)\,(r+1)}\,\epsilon + O(\epsilon^3) \tag{75}$$

> $simplify(series(u_{x3+}, \epsilon, 3))$

$$\frac{I\sin(\alpha)\,r\,(r-1)\,\mathrm{csgn}\!\left(\dfrac{k_{\parallel -}}{\epsilon}\right)}{\cos(\alpha)^2}\,\epsilon^2 + O(\epsilon^3) \tag{76}$$

u_perp leading order terms

> $simplify(series(sol1, \epsilon, 3))$

$$\frac{2r}{r+1} + 2\,\frac{r^2\,(r-1)\,(\cos(\alpha)-1)\,(\cos(\alpha)+1)\,\left(a_y - \sin(\alpha)\right)}{\sin(\alpha)\,\cos(\alpha)^2\,(r+1)}\,\epsilon^2 + O(\epsilon^3) \tag{77}$$

> $simplify(series(sol2, \epsilon, 3))$

$$\frac{(\cos(\alpha)-1)\,(\cos(\alpha)+1)\,(r-1)\,r}{\cos(\alpha)^2}\,\epsilon^2 + O(\epsilon^3) \tag{78}$$

> $simplify(series(sol3, \epsilon, 2))$

$$\frac{r-1}{r+1} + O(\epsilon^2) \tag{79}$$

> $simplify(series(sol4, \epsilon, 4))$

$$-\frac{(\cos(\alpha)-1)\,(\cos(\alpha)+1)\,(r-1)\,r}{\cos(\alpha)^2}\,\epsilon^2 \tag{80}$$

$$+\frac{I\,r^2\,a_y\left(\cos(\alpha)^4 - 3\cos(\alpha)^2 + 2\right)(r-1)\,\mathrm{csgn}\!\left(\dfrac{k_{\parallel -}}{\epsilon}\right)}{\sin(\alpha)\,\cos(\alpha)^3}\,\epsilon^3 + O(\epsilon^4)$$

bx leading order terms

> $b_{x1-} := \dfrac{\Delta_{\|1-} \cdot u_{x1-}}{i \cdot k_{\|+}}$:

> $b_{x4-} := \dfrac{\Delta_{\|4-} \cdot u_{x4-}}{i \cdot k_{\|+}}$:

> $b_{x1+} := \dfrac{\Delta_{\|1+} \cdot u_{x1+}}{i \cdot k_{\|+}}$:

> $b_{x2+} := \dfrac{\Delta_{\|2+} \cdot u_{x2+}}{i \cdot k_{\|+}}$:

> $b_{x3+} := \dfrac{\Delta_{\|3+} \cdot u_{x3+}}{i \cdot k_{\|+}}$:

> $simplify(series(b_{x1-}, \epsilon, 2))$

$$\frac{-2\,a_y\,r + 2\sin(\alpha)}{\cos(\alpha)\,(r+1)}\,\epsilon + O(\epsilon^2) \tag{81}$$

> $simplify(series(b_{x4-}, \epsilon, 2))$

$$\frac{\sin(\alpha)\,(r-1)}{\cos(\alpha)}\,\epsilon + O(\epsilon^2) \tag{82}$$

> $simplify(series(b_{x1+}, \epsilon, 2))$

$$\frac{r\left(-a_y + \sin(\alpha)\right)}{\cos(\alpha)}\,\epsilon + O(\epsilon^3) \tag{83}$$

> $simplify(series(b_{x2+}, \epsilon, 2))$

$$\frac{r\,(r-1)\left(a_y + \sin(\alpha)\right)}{\cos(\alpha)\,(r+1)}\,\epsilon + O(\epsilon^2) \tag{84}$$

> $simplify(series(b_{x3+}, \epsilon, 2))$

$$-\frac{\sin(\alpha)\,(r-1)}{\cos(\alpha)}\,\epsilon + O(\epsilon^2) \tag{85}$$

b_perp leading order terms

> $b_{\perp1-} := \dfrac{\Delta_{\|1-} \cdot sol1}{i \cdot k_{\|+}}$:

> $b_{\perp4-} := \dfrac{\Delta_{\|4-} \cdot sol2}{i \cdot k_{\|+}}$:

> $b_{\perp1+} := \dfrac{\Delta_{\|1+} \cdot 1}{i \cdot k_{\|+}}$:

> $b_{\perp 2+} := \dfrac{\Delta_{\parallel 2+} \cdot sol3}{i \cdot k_{\parallel +}}$:

> $b_{\perp 3+} := \dfrac{\Delta_{\parallel 3+} \cdot sol4}{i \cdot k_{\parallel +}}$:

> $simplify(series(b_{\perp 1-}, \epsilon, 2))$

$$\frac{2}{r+1} + O(\epsilon^2) \tag{86}$$

> $simplify(series(b_{\perp 4-}, \epsilon, 2))$

$$\frac{-I\,(\cos(\alpha)+1)\,(\cos(\alpha)-1)\,(r-1)\,\operatorname{csgn}\!\left(\dfrac{k_{\parallel -}}{\epsilon}\right)}{\cos(\alpha)}\,\epsilon + O(\epsilon^2) \tag{87}$$

> $simplify(series(b_{\perp 1+}, \epsilon, 2))$

$$1 \tag{88}$$

> $simplify(series(b_{\perp 2+}, \epsilon, 2))$

$$\frac{-r+1}{r+1} + O(\epsilon^2) \tag{89}$$

> $simplify(series(b_{\perp 3+}, \epsilon, 2))$

$$O(\epsilon) \tag{90}$$

b_par leading order terms

> $b_{\parallel 1-} := -\dfrac{i \cdot k_x \cdot u_{x1-} + \Delta_{\perp 1-} \cdot sol1}{i \cdot k_{\parallel +}}$:

> $b_{\parallel 4-} := -\dfrac{i \cdot k_x \cdot u_{x4-} + \Delta_{\perp 4-} \cdot sol2}{i \cdot k_{\parallel +}}$:

> $b_{\parallel 1+} := -\dfrac{i \cdot k_x \cdot u_{x1+} + \Delta_{\perp 1+}}{i \cdot k_{\parallel +}}$:

> $b_{\parallel 2+} := -\dfrac{i \cdot k_x \cdot u_{x2+} + \Delta_{\perp 2+} \cdot sol3}{i \cdot k_{\parallel +}}$:

> $b_{\parallel 3+} := -\dfrac{i \cdot k_x \cdot u_{x3+} + \Delta_{\perp 3+} \cdot sol4}{i \cdot k_{\parallel +}}$:

> $simplify(b_{\parallel 1-})$

$$0 \tag{91}$$

> $simplify(series(expand(b_{\parallel 4-}), \epsilon, 3))$

$$-I\,(r-1)\,\sin(\alpha)\,\operatorname{csgn}\!\left(\dfrac{k_{\parallel -}}{\epsilon}\right)\epsilon + O(\epsilon^2) \tag{92}$$

> $simplify(b_{\parallel 1+})$

$$0 \tag{93}$$

> $simplify(b_{\|2+})$

$$0 \tag{94}$$

> $simplify(series(expand(b_{\|3+}), \epsilon, 3))$

$$-\mathrm{I}\,(r-1)\,\sin(\alpha)\,\mathrm{csgn}\!\left(\frac{k_{\|-}}{\epsilon}\right)\epsilon + \mathrm{O}(\epsilon^2) \tag{95}$$