This document is used to assist with the Algebra in Appendix A.2
Note that ∇ has been replaced with to ensure the code works okay.

> *restart*;

We normalise the velocity coefficents by u_0 and the field components by (B_0 * u_0 / v_{A+}).
We start by solving the Matrix equation given by Equation (57).

> *eqn1* $:= a_{11} \cdot x_1 + a_{12} \cdot x_2 + a_{13} \cdot x_3 + a_{14} \cdot x_4 = y_1$

$$eqn1 := a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 = y_1 \tag{1}$$

> *eqn2* $:= a_{21} \cdot x_1 + a_{22} \cdot x_2 + a_{23} \cdot x_3 + a_{24} \cdot x_4 = y_2$

$$eqn2 := a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + a_{24} x_4 = y_2 \tag{2}$$

> *eqn3* $:= a_{31} \cdot x_1 + a_{32} \cdot x_2 + a_{33} \cdot x_3 + a_{34} \cdot x_4 = y_3$

$$eqn3 := a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + a_{34} x_4 = y_3 \tag{3}$$

> *eqn4* $:= a_{41} \cdot x_1 + a_{42} \cdot x_2 + a_{43} \cdot x_3 + a_{44} \cdot x_4 = y_4$

$$eqn4 := a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4 = y_4 \tag{4}$$

> *solns* $:= solve\left(\{eqn1, eqn2, eqn3, eqn4\}, \{x_1, x_2, x_3, x_4\} \right):$

> *sol1* $:= rhs(solns[1])$

$$
\begin{aligned}
sol1 := & -\big(a_{12}\,a_{23}\,a_{34}\,y_4 - a_{12}\,a_{23}\,a_{44}\,y_3 - a_{12}\,a_{24}\,a_{33}\,y_4 + a_{12}\,a_{24}\,a_{43}\,y_3 + a_{12}\,a_{33}\,a_{44}\,y_2 \\
& - a_{12}\,a_{34}\,a_{43}\,y_2 - a_{13}\,a_{22}\,a_{34}\,y_4 + a_{13}\,a_{22}\,a_{44}\,y_3 + a_{13}\,a_{24}\,a_{32}\,y_4 - a_{13}\,a_{24}\,a_{42}\,y_3 \\
& - a_{13}\,a_{32}\,a_{44}\,y_2 + a_{13}\,a_{34}\,a_{42}\,y_2 + a_{14}\,a_{22}\,a_{33}\,y_4 - a_{14}\,a_{22}\,a_{43}\,y_3 - a_{14}\,a_{23}\,a_{32}\,y_4 \\
& + a_{14}\,a_{23}\,a_{42}\,y_3 + a_{14}\,a_{32}\,a_{43}\,y_2 - a_{14}\,a_{33}\,a_{42}\,y_2 - a_{22}\,a_{33}\,a_{44}\,y_1 + a_{22}\,a_{34}\,a_{43}\,y_1 \\
& + a_{23}\,a_{32}\,a_{44}\,y_1 - a_{23}\,a_{34}\,a_{42}\,y_1 - a_{24}\,a_{32}\,a_{43}\,y_1 + a_{24}\,a_{33}\,a_{42}\,y_1\big) \big/ \big(a_{11}\,a_{22}\,a_{33}\,a_{44} \\
& - a_{11}\,a_{22}\,a_{34}\,a_{43} - a_{11}\,a_{23}\,a_{32}\,a_{44} + a_{11}\,a_{23}\,a_{34}\,a_{42} + a_{11}\,a_{24}\,a_{32}\,a_{43} - a_{11}\,a_{24}\,a_{33}\,a_{42} \\
& - a_{12}\,a_{21}\,a_{33}\,a_{44} + a_{12}\,a_{21}\,a_{34}\,a_{43} + a_{12}\,a_{23}\,a_{31}\,a_{44} - a_{12}\,a_{23}\,a_{34}\,a_{41} - a_{12}\,a_{24}\,a_{31}\,a_{43} \\
& + a_{12}\,a_{24}\,a_{33}\,a_{41} + a_{13}\,a_{21}\,a_{32}\,a_{44} - a_{13}\,a_{21}\,a_{34}\,a_{42} - a_{13}\,a_{22}\,a_{31}\,a_{44} + a_{13}\,a_{22}\,a_{34}\,a_{41} \\
& + a_{13}\,a_{24}\,a_{31}\,a_{42} - a_{13}\,a_{24}\,a_{32}\,a_{41} - a_{14}\,a_{21}\,a_{32}\,a_{43} + a_{14}\,a_{21}\,a_{33}\,a_{42} + a_{14}\,a_{22}\,a_{31}\,a_{43} \\
& - a_{14}\,a_{22}\,a_{33}\,a_{41} - a_{14}\,a_{23}\,a_{31}\,a_{42} + a_{14}\,a_{23}\,a_{32}\,a_{41}\big)
\end{aligned}
\tag{5}
$$

> *sol2* $:= rhs(solns[2])$

$$
\begin{aligned}
sol2 := & \big(a_{11}\,a_{23}\,a_{34}\,y_4 - a_{11}\,a_{23}\,a_{44}\,y_3 - a_{11}\,a_{24}\,a_{33}\,y_4 + a_{11}\,a_{24}\,a_{43}\,y_3 + a_{11}\,a_{33}\,a_{44}\,y_2 \\
& - a_{11}\,a_{34}\,a_{43}\,y_2 - a_{13}\,a_{21}\,a_{34}\,y_4 + a_{13}\,a_{21}\,a_{44}\,y_3 + a_{13}\,a_{24}\,a_{31}\,y_4 - a_{13}\,a_{24}\,a_{41}\,y_3 \\
& - a_{13}\,a_{31}\,a_{44}\,y_2 + a_{13}\,a_{34}\,a_{41}\,y_2 + a_{14}\,a_{21}\,a_{33}\,y_4 - a_{14}\,a_{21}\,a_{43}\,y_3 - a_{14}\,a_{23}\,a_{31}\,y_4 \\
& + a_{14}\,a_{23}\,a_{41}\,y_3 + a_{14}\,a_{31}\,a_{43}\,y_2 - a_{14}\,a_{33}\,a_{41}\,y_2 - a_{21}\,a_{33}\,a_{44}\,y_1 + a_{21}\,a_{34}\,a_{43}\,y_1 \\
& + a_{23}\,a_{31}\,a_{44}\,y_1 - a_{23}\,a_{34}\,a_{41}\,y_1 - a_{24}\,a_{31}\,a_{43}\,y_1 + a_{24}\,a_{33}\,a_{41}\,y_1\big) \big/ \big(a_{11}\,a_{22}\,a_{33}\,a_{44} \\
& - a_{11}\,a_{22}\,a_{34}\,a_{43} - a_{11}\,a_{23}\,a_{32}\,a_{44} + a_{11}\,a_{23}\,a_{34}\,a_{42} + a_{11}\,a_{24}\,a_{32}\,a_{43} - a_{11}\,a_{24}\,a_{33}\,a_{42} \\
& - a_{12}\,a_{21}\,a_{33}\,a_{44} + a_{12}\,a_{21}\,a_{34}\,a_{43} + a_{12}\,a_{23}\,a_{31}\,a_{44} - a_{12}\,a_{23}\,a_{34}\,a_{41} - a_{12}\,a_{24}\,a_{31}\,a_{43} \\
& + a_{12}\,a_{24}\,a_{33}\,a_{41} + a_{13}\,a_{21}\,a_{32}\,a_{44} - a_{13}\,a_{21}\,a_{34}\,a_{42} - a_{13}\,a_{22}\,a_{31}\,a_{44} + a_{13}\,a_{22}\,a_{34}\,a_{41}
\end{aligned}
\tag{6}
$$

$$+ a_{13}\,a_{24}\,a_{31}\,a_{42} - a_{13}\,a_{24}\,a_{32}\,a_{41} - a_{14}\,a_{21}\,a_{32}\,a_{43} + a_{14}\,a_{21}\,a_{33}\,a_{42} + a_{14}\,a_{22}\,a_{31}\,a_{43}$$
$$- a_{14}\,a_{22}\,a_{33}\,a_{41} - a_{14}\,a_{23}\,a_{31}\,a_{42} + a_{14}\,a_{23}\,a_{32}\,a_{41})$$

> $sol3 := rhs(solns[3])$

$$sol3 := -\big(a_{11}\,a_{22}\,a_{34}\,y_4 - a_{11}\,a_{22}\,a_{44}\,y_3 - a_{11}\,a_{24}\,a_{32}\,y_4 + a_{11}\,a_{24}\,a_{42}\,y_3 + a_{11}\,a_{32}\,a_{44}\,y_2 \qquad (7)$$
$$- a_{11}\,a_{34}\,a_{42}\,y_2 - a_{12}\,a_{21}\,a_{34}\,y_4 + a_{12}\,a_{21}\,a_{44}\,y_3 + a_{12}\,a_{24}\,a_{31}\,y_4 - a_{12}\,a_{24}\,a_{41}\,y_3$$
$$- a_{12}\,a_{31}\,a_{44}\,y_2 + a_{12}\,a_{34}\,a_{41}\,y_2 + a_{14}\,a_{21}\,a_{32}\,y_4 - a_{14}\,a_{21}\,a_{42}\,y_3 - a_{14}\,a_{22}\,a_{31}\,y_4$$
$$+ a_{14}\,a_{22}\,a_{41}\,y_3 + a_{14}\,a_{31}\,a_{42}\,y_2 - a_{14}\,a_{32}\,a_{41}\,y_2 - a_{21}\,a_{32}\,a_{44}\,y_1 + a_{21}\,a_{34}\,a_{42}\,y_1$$
$$+ a_{22}\,a_{31}\,a_{44}\,y_1 - a_{22}\,a_{34}\,a_{41}\,y_1 - a_{24}\,a_{31}\,a_{42}\,y_1 + a_{24}\,a_{32}\,a_{41}\,y_1\big)\Big/\big(a_{11}\,a_{22}\,a_{33}\,a_{44}$$
$$- a_{11}\,a_{22}\,a_{34}\,a_{43} - a_{11}\,a_{23}\,a_{32}\,a_{44} + a_{11}\,a_{23}\,a_{34}\,a_{42} + a_{11}\,a_{24}\,a_{32}\,a_{43} - a_{11}\,a_{24}\,a_{33}\,a_{42}$$
$$- a_{12}\,a_{21}\,a_{33}\,a_{44} + a_{12}\,a_{21}\,a_{34}\,a_{43} + a_{12}\,a_{23}\,a_{31}\,a_{44} - a_{12}\,a_{23}\,a_{34}\,a_{41} - a_{12}\,a_{24}\,a_{31}\,a_{43}$$
$$+ a_{12}\,a_{24}\,a_{33}\,a_{41} + a_{13}\,a_{21}\,a_{32}\,a_{44} - a_{13}\,a_{21}\,a_{34}\,a_{42} - a_{13}\,a_{22}\,a_{31}\,a_{44} + a_{13}\,a_{22}\,a_{34}\,a_{41}$$
$$+ a_{13}\,a_{24}\,a_{31}\,a_{42} - a_{13}\,a_{24}\,a_{32}\,a_{41} - a_{14}\,a_{21}\,a_{32}\,a_{43} + a_{14}\,a_{21}\,a_{33}\,a_{42} + a_{14}\,a_{22}\,a_{31}\,a_{43}$$
$$- a_{14}\,a_{22}\,a_{33}\,a_{41} - a_{14}\,a_{23}\,a_{31}\,a_{42} + a_{14}\,a_{23}\,a_{32}\,a_{41}\big)$$

> $sol4 := rhs(solns[4])$

$$sol4 := \big(a_{11}\,a_{22}\,a_{33}\,y_4 - a_{11}\,a_{22}\,a_{43}\,y_3 - a_{11}\,a_{23}\,a_{32}\,y_4 + a_{11}\,a_{23}\,a_{42}\,y_3 + a_{11}\,a_{32}\,a_{43}\,y_2 \qquad (8)$$
$$- a_{11}\,a_{33}\,a_{42}\,y_2 - a_{12}\,a_{21}\,a_{33}\,y_4 + a_{12}\,a_{21}\,a_{43}\,y_3 + a_{12}\,a_{23}\,a_{31}\,y_4 - a_{12}\,a_{23}\,a_{41}\,y_3$$
$$- a_{12}\,a_{31}\,a_{43}\,y_2 + a_{12}\,a_{33}\,a_{41}\,y_2 + a_{13}\,a_{21}\,a_{32}\,y_4 - a_{13}\,a_{21}\,a_{42}\,y_3 - a_{13}\,a_{22}\,a_{31}\,y_4$$
$$+ a_{13}\,a_{22}\,a_{41}\,y_3 + a_{13}\,a_{31}\,a_{42}\,y_2 - a_{13}\,a_{32}\,a_{41}\,y_2 - a_{21}\,a_{32}\,a_{43}\,y_1 + a_{21}\,a_{33}\,a_{42}\,y_1$$
$$+ a_{22}\,a_{31}\,a_{43}\,y_1 - a_{22}\,a_{33}\,a_{41}\,y_1 - a_{23}\,a_{31}\,a_{42}\,y_1 + a_{23}\,a_{32}\,a_{41}\,y_1\big)\Big/\big(a_{11}\,a_{22}\,a_{33}\,a_{44}$$
$$- a_{11}\,a_{22}\,a_{34}\,a_{43} - a_{11}\,a_{23}\,a_{32}\,a_{44} + a_{11}\,a_{23}\,a_{34}\,a_{42} + a_{11}\,a_{24}\,a_{32}\,a_{43} - a_{11}\,a_{24}\,a_{33}\,a_{42}$$
$$- a_{12}\,a_{21}\,a_{33}\,a_{44} + a_{12}\,a_{21}\,a_{34}\,a_{43} + a_{12}\,a_{23}\,a_{31}\,a_{44} - a_{12}\,a_{23}\,a_{34}\,a_{41} - a_{12}\,a_{24}\,a_{31}\,a_{43}$$
$$+ a_{12}\,a_{24}\,a_{33}\,a_{41} + a_{13}\,a_{21}\,a_{32}\,a_{44} - a_{13}\,a_{21}\,a_{34}\,a_{42} - a_{13}\,a_{22}\,a_{31}\,a_{44} + a_{13}\,a_{22}\,a_{34}\,a_{41}$$
$$+ a_{13}\,a_{24}\,a_{31}\,a_{42} - a_{13}\,a_{24}\,a_{32}\,a_{41} - a_{14}\,a_{21}\,a_{32}\,a_{43} + a_{14}\,a_{21}\,a_{33}\,a_{42} + a_{14}\,a_{22}\,a_{31}\,a_{43}$$
$$- a_{14}\,a_{22}\,a_{33}\,a_{41} - a_{14}\,a_{23}\,a_{31}\,a_{42} + a_{14}\,a_{23}\,a_{32}\,a_{41}\big)$$

> $a_{11} := u_{x10-}$

$$a_{11} := u_{x10\text{-}} \qquad (9)$$

> $a_{12} := u_{x40-}$

$$a_{12} := u_{x40\text{-}} \qquad (10)$$

> $a_{13} := -u_{x20+}$

$$a_{13} := -u_{x20+} \qquad (11)$$

> $a_{14} := -u_{x30+}$

$$a_{14} := -u_{x30+} \qquad (12)$$

> $a_{21} := k_{z1-} \cdot u_{x10-}$

$$(13)$$

$$a_{21} := k_{z1-} \, u_{x10-} \tag{13}$$

> $a_{22} := k_{z4-} \cdot u_{x40-}$

$$a_{22} := k_{z4-} \, u_{x40-} \tag{14}$$

> $a_{23} := -k_{z2+} \cdot u_{x20+}$

$$a_{23} := -k_{z2+} \, u_{x20+} \tag{15}$$

> $a_{24} := -k_{z3+} \cdot u_{x30+}$

$$a_{24} := -k_{z3+} \, u_{x30+} \tag{16}$$

> $a_{31} := 1$

$$a_{31} := 1 \tag{17}$$

> $a_{32} := 1$

$$a_{32} := 1 \tag{18}$$

> $a_{33} := -1$

$$a_{33} := -1 \tag{19}$$

> $a_{34} := -1$

$$a_{34} := -1 \tag{20}$$

> $a_{41} := k_{z1-}$

$$a_{41} := k_{z1-} \tag{21}$$

> $a_{42} := k_{z4-}$

$$a_{42} := k_{z4-} \tag{22}$$

> $a_{43} := -k_{z2+}$

$$a_{43} := -k_{z2+} \tag{23}$$

> $a_{44} := -k_{z3+}$

$$a_{44} := -k_{z3+} \tag{24}$$

> $y_1 := u_{x10+}$

$$y_1 := u_{x10+} \tag{25}$$

> $y_2 := k_{z1+} \cdot u_{x10+}$

$$y_2 := k_{z1+} \, u_{x10+} \tag{26}$$

> $y_3 := 1$

$$y_3 := 1 \tag{27}$$

> $y_4 := k_{z1+}$

$$y_4 := k_{z1+} \tag{28}$$

Note that ux_0[n] denotes \hat{u}_{xn} and u_x[n±] denotes u_{xn±}.

> $u_{x10-} := -\dfrac{i \cdot k_x \cdot \Delta_{\perp 1-}}{L_{1-} - k_x^2}$

$$u_{x10-} := \frac{-I\, k_x\, \Delta_{\perp 1-}}{-k_x^2 + L_{1-}} \tag{29}$$

> $u_{x10+} := -\dfrac{i \cdot k_x \cdot \Delta_{\perp 1+}}{L_{1+} - k_x^2}$

$$u_{x10+} := \frac{-I\, k_x\, \Delta_{\perp 1+}}{-k_x^2 + L_{1+}} \tag{30}$$

> $u_{x20-} := -\dfrac{i \cdot k_x \cdot \Delta_{\perp 2-}}{L_{2-} - k_x^2}$

$$u_{x20-} := \frac{-I\, k_x\, \Delta_{\perp 2-}}{-k_x^2 + L_{2-}} \tag{31}$$

> $u_{x20+} := -\dfrac{i \cdot k_x \cdot \Delta_{\perp 2+}}{L_{2+} - k_x^2}$

$$u_{x20+} := \frac{-I\, k_x\, \Delta_{\perp 2+}}{-k_x^2 + L_{2+}} \tag{32}$$

> $u_{x30-} := -\dfrac{i \cdot k_x \cdot \Delta_{\perp 3-}}{L_{3-} - k_x^2}$

$$u_{x30-} := \frac{-I\, k_x\, \Delta_{\perp 3-}}{-k_x^2 + L_{3-}} \tag{33}$$

> $u_{x30+} := -\dfrac{i \cdot k_x \cdot \Delta_{\perp 3+}}{L_{3+} - k_x^2}$

$$u_{x30+} := \frac{-I\, k_x\, \Delta_{\perp 3+}}{-k_x^2 + L_{3+}} \tag{34}$$

> $u_{x40-} := -\dfrac{i \cdot k_x \cdot \Delta_{\perp 4-}}{L_{4-} - k_x^2}$

$$u_{x40-} := \frac{-I\, k_x\, \Delta_{\perp 4-}}{-k_x^2 + L_{4-}} \tag{35}$$

> $u_{x40+} := -\dfrac{i \cdot k_x \cdot \Delta_{\perp 4+}}{L_{4+} - k_x^2}$

$$u_{x40+} := \frac{-I\, k_x\, \Delta_{\perp 4+}}{-k_x^2 + L_{4+}} \tag{36}$$

> $L_{1-} := \Delta_{\parallel 1-}{}^2 + k_{\parallel -}{}^2$

$$L_{1-} := k_{\parallel -}{}^2 + \Delta_{\parallel 1-}{}^2 \tag{37}$$

> $L_{1+} := \Delta_{\parallel 1+}{}^2 + k_{\parallel +}{}^2$

$$L_{1+} := k_{\parallel +}{}^2 + \Delta_{\parallel 1+}{}^2 \tag{38}$$

> $L_{2-} := \Delta_{\parallel 2-}{}^2 + k_{\parallel -}{}^2$

$$L_{2-} := k_{\parallel -}{}^2 + \Delta_{\parallel 2-}{}^2 \tag{39}$$

> $L_{2+} := \Delta_{\parallel 2+}{}^2 + k_{\parallel +}{}^2$

$$L_{2+} := k_{\parallel +}{}^2 + \Delta_{\parallel 2+}{}^2 \tag{40}$$

> $L_{3-} := \Delta_{\parallel 3-}{}^2 + k_{\parallel -}{}^2$

$$L_{3-} := k_{\parallel -}{}^2 + \Delta_{\parallel 3-}{}^2 \tag{41}$$

> $L_{3+} := \Delta_{\parallel 3+}{}^2 + k_{\parallel +}{}^2$

$$L_{3+} := k_{\parallel +}{}^2 + \Delta_{\parallel 3+}{}^2 \tag{42}$$

> $L_{4-} := \Delta_{\parallel 4-}{}^2 + k_{\parallel -}{}^2$

$$L_{4-} := k_{\parallel -}{}^2 + \Delta_{\parallel 4-}{}^2 \tag{43}$$

> $L_{4+} := \Delta_{\parallel 4+}{}^2 + k_{\parallel +}{}^2$

$$L_{4+} := k_{\parallel +}{}^2 + \Delta_{\parallel 4+}{}^2 \tag{44}$$

> $\Delta_{\perp 1-} := i \cdot \left(k_y \cdot \cos(\alpha) - k_{z1-} \cdot \sin(\alpha) \right)$

$$\Delta_{\perp 1-} := I \left(k_y \cos(\alpha) - k_{z1-} \sin(\alpha) \right) \tag{45}$$

> $\Delta_{\perp 1+} := i \cdot \left(k_y \cdot \cos(\alpha) - k_{z1+} \cdot \sin(\alpha) \right)$

$$\Delta_{\perp 1+} := I \left(k_y \cos(\alpha) - k_{z1+} \sin(\alpha) \right) \tag{46}$$

> $\Delta_{\perp 2-} := i \cdot \left(k_y \cdot \cos(\alpha) - k_{z2-} \cdot \sin(\alpha) \right)$

$$\Delta_{\perp 2-} := I \left(k_y \cos(\alpha) - k_{z2-} \sin(\alpha) \right) \tag{47}$$

> $\Delta_{\perp 2+} := i \cdot \left(k_y \cdot \cos(\alpha) - k_{z2+} \cdot \sin(\alpha) \right)$

$$\tag{48}$$

$$\Delta_{\perp2+} := I\left(k_y \cos(\alpha) - k_{z2+}\sin(\alpha)\right) \tag{48}$$

> $\Delta_{\perp3-} := i\cdot\left(k_y\cdot\cos(\alpha) - k_{z3-}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp3-} := I\left(k_y \cos(\alpha) - k_{z3_-}\sin(\alpha)\right) \tag{49}$$

> $\Delta_{\perp3+} := i\cdot\left(k_y\cdot\cos(\alpha) - k_{z3+}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp3+} := I\left(k_y \cos(\alpha) - k_{z3+}\sin(\alpha)\right) \tag{50}$$

> $\Delta_{\perp4-} := i\cdot\left(k_y\cdot\cos(\alpha) - k_{z4-}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp4-} := I\left(k_y \cos(\alpha) - k_{z4_-}\sin(\alpha)\right) \tag{51}$$

> $\Delta_{\perp4+} := i\cdot\left(k_y\cdot\cos(\alpha) - k_{z4+}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp4+} := I\left(k_y \cos(\alpha) - k_{z4+}\sin(\alpha)\right) \tag{52}$$

> $\Delta_{\|1-} := i\cdot\left(k_y\cdot\sin(\alpha) + k_{z1-}\cdot\cos(\alpha)\right)$

$$\Delta_{\|1-} := I\left(k_y \sin(\alpha) + k_{z1_-}\cos(\alpha)\right) \tag{53}$$

> $\Delta_{\|1+} := i\cdot\left(k_y\cdot\sin(\alpha) + k_{z1+}\cdot\cos(\alpha)\right)$

$$\Delta_{\|1+} := I\left(k_y \sin(\alpha) + k_{z1+}\cos(\alpha)\right) \tag{54}$$

> $\Delta_{\|2-} := i\cdot\left(k_y\cdot\sin(\alpha) + k_{z2-}\cdot\cos(\alpha)\right)$

$$\Delta_{\|2-} := I\left(k_y \sin(\alpha) + k_{z2_-}\cos(\alpha)\right) \tag{55}$$

> $\Delta_{\|2+} := i\cdot\left(k_y\cdot\sin(\alpha) + k_{z2+}\cdot\cos(\alpha)\right)$

$$\Delta_{\|2+} := I\left(k_y \sin(\alpha) + k_{z2+}\cos(\alpha)\right) \tag{56}$$

> $\Delta_{\|3-} := i\cdot\left(k_y\cdot\sin(\alpha) + k_{z3-}\cdot\cos(\alpha)\right)$

$$\Delta_{\|3-} := I\left(k_y \sin(\alpha) + k_{z3_-}\cos(\alpha)\right) \tag{57}$$

> $\Delta_{\|3+} := i\cdot\left(k_y\cdot\sin(\alpha) + k_{z3+}\cdot\cos(\alpha)\right)$

$$\Delta_{\|3+} := I\left(k_y \sin(\alpha) + k_{z3+}\cos(\alpha)\right) \tag{58}$$

> $\Delta_{\|4-} := i\cdot\left(k_y\cdot\sin(\alpha) + k_{z4-}\cdot\cos(\alpha)\right)$

$$\Delta_{\|4-} := I\left(k_y \sin(\alpha) + k_{z4_-}\cos(\alpha)\right) \tag{59}$$

> $\Delta_{\|4+} := i\cdot\left(k_y\cdot\sin(\alpha) + k_{z4+}\cdot\cos(\alpha)\right)$

$$\Delta_{\|4+} := I\left(k_y \sin(\alpha) + k_{z4+}\cos(\alpha)\right) \tag{60}$$

> $k_{z1-} := \dfrac{k_{\|-}}{\cos(\alpha)} - k_y\cdot\tan(\alpha)$

$$k_{z1-} := \frac{k_{\|-}}{\cos(\alpha)} - k_y\tan(\alpha) \tag{61}$$

> $k_{z1+} := \dfrac{k_{\|+}}{\cos(\alpha)} - k_y\cdot\tan(\alpha)$

$$k_{z1+} := \frac{k_{\|+}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{62}$$

> $k_{z2-} := -\dfrac{k_{\|-}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z2-} := -\frac{k_{\|-}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{63}$$

> $k_{z2+} := -\dfrac{k_{\|+}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z2+} := -\frac{k_{\|+}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{64}$$

> $k_{z3-} := i \cdot \sqrt{k_x^2 + k_y^2 - k_{\|-}^2}$

$$k_{z3-} := I \sqrt{k_x^2 + k_y^2 - k_{\|-}^2} \tag{65}$$

> $k_{z3+} := i \cdot \sqrt{k_x^2 + k_y^2 - k_{\|+}^2}$

$$k_{z3+} := I \sqrt{k_x^2 + k_y^2 - k_{\|+}^2} \tag{66}$$

> $k_{z4-} := -i \cdot \sqrt{k_x^2 + k_y^2 - k_{\|-}^2}$

$$k_{z4-} := -I \sqrt{k_x^2 + k_y^2 - k_{\|-}^2} \tag{67}$$

> $k_{z4+} := -i \cdot \sqrt{k_x^2 + k_y^2 - k_{\|+}^2}$

$$k_{z4+} := -I \sqrt{k_x^2 + k_y^2 - k_{\|+}^2} \tag{68}$$

>

> $k_x := \dfrac{k_{\|+}}{\epsilon_+}$

$$k_x := \frac{k_{\|+}}{\epsilon_+} \tag{69}$$

> $k_y := a_y \cdot k_{\|+}$

$$k_y := a_y k_{\|+} \tag{70}$$

> $k_{\|-} := \dfrac{k_x}{\epsilon_-}$

$$k_{\|-} := \frac{k_{\|+}}{\epsilon_+ \epsilon_-} \tag{71}$$

Need to help maple to take limit as epsilon_- and epsilon_+ go to zero by taking terms that go to infinity out of the square root.

> $k_{z3-} := \dfrac{i \cdot \sqrt{\epsilon_-^{\,2} \cdot k_{\parallel+}^{\,2} + \epsilon_-^{\,2} \cdot \epsilon_+^{\,2} \cdot k_y^{\,2} - k_{\parallel+}^{\,2}}}{\epsilon_- \cdot \epsilon_+}$

$$k_{z3-} := \frac{I\sqrt{a_y^{\,2}\,\epsilon_-^{\,2}\,\epsilon_+^{\,2}\,k_{\parallel+}^{\,2} + \epsilon_-^{\,2}\,k_{\parallel+}^{\,2} - k_{\parallel+}^{\,2}}}{\epsilon_-\,\epsilon_+} \tag{72}$$

> $k_{z3+} := \dfrac{i \cdot \sqrt{k_{\parallel+}^{\,2} + \epsilon_+^{\,2} \cdot k_y^{\,2} - \epsilon_+^{\,2} \cdot k_{\parallel+}^{\,2}}}{\epsilon_+}$

$$k_{z3+} := \frac{I\sqrt{a_y^{\,2}\,\epsilon_+^{\,2}\,k_{\parallel+}^{\,2} - \epsilon_+^{\,2}\,k_{\parallel+}^{\,2} + k_{\parallel+}^{\,2}}}{\epsilon_+} \tag{73}$$

> $k_{z4-} := -\dfrac{i \cdot \sqrt{\epsilon_-^{\,2} \cdot k_{\parallel+}^{\,2} + \epsilon_-^{\,2} \cdot \epsilon_+^{\,2} \cdot k_y^{\,2} - k_{\parallel+}^{\,2}}}{\epsilon_- \cdot \epsilon_+}$

$$k_{z4-} := \frac{-I\sqrt{a_y^{\,2}\,\epsilon_-^{\,2}\,\epsilon_+^{\,2}\,k_{\parallel+}^{\,2} + \epsilon_-^{\,2}\,k_{\parallel+}^{\,2} - k_{\parallel+}^{\,2}}}{\epsilon_-\,\epsilon_+} \tag{74}$$

> $k_{z4+} := -\dfrac{i \cdot \sqrt{k_{\parallel+}^{\,2} + \epsilon_+^{\,2} \cdot k_y^{\,2} - \epsilon_+^{\,2} \cdot k_{\parallel+}^{\,2}}}{\epsilon_+}$

$$k_{z4+} := \frac{-I\sqrt{a_y^{\,2}\,\epsilon_+^{\,2}\,k_{\parallel+}^{\,2} - \epsilon_+^{\,2}\,k_{\parallel+}^{\,2} + k_{\parallel+}^{\,2}}}{\epsilon_+} \tag{75}$$

ux leading order terms
> $u_{x1-} := u_{x10-} \cdot sol1 :$
> $u_{x4-} := u_{x40-} \cdot sol2 :$
> $u_{x1+} := u_{x10+} :$
> $u_{x2+} := u_{x20+} \cdot sol3 :$
> $u_{x3+} := u_{x30+} \cdot sol4 :$
> $simplify(\,mtaylor(\,u_{x1-},\,[\epsilon_-,\epsilon_+\,],3\,)\,)$

$$-2\,I\sin(\alpha)\,\epsilon_-\,\epsilon_+\,csgn(k_{\parallel+}) \tag{76}$$

> $simplify(\,mtaylor(\,u_{x4-},\,[\epsilon_-,\epsilon_+\,],4\,)\,)$

$$-\frac{2\,\epsilon_-^{\,2}\,\epsilon_+\,\cos(\alpha)}{\sin(\alpha)} \tag{77}$$

> $simplify(\,mtaylor(\,u_{x1+},\,[\epsilon_-,\epsilon_+\,],2\,)\,)$

$$\frac{\epsilon_+ \left(-a_y + \sin(\alpha) \right)}{\cos(\alpha)} \tag{78}$$

> $simplify\left(mtaylor\left(u_{x2+}, \left[\epsilon_-, \epsilon_+ \right], 2 \right) \right)$

$$\frac{\epsilon_+ \left(a_y + \sin(\alpha) \right)}{\cos(\alpha)} \tag{79}$$

> $simplify\left(mtaylor\left(u_{x3+}, \left[\epsilon_-, \epsilon_+ \right], 2 \right) \right)$

$$-\frac{2 \sin(\alpha)\, \epsilon_+}{\cos(\alpha)} \tag{80}$$

u_perp leading order terms

> $simplify\left(mtaylor\left(sol1, \left[\epsilon_-, \epsilon_+ \right], 4 \right) \right)$

$$-2\, I\, \epsilon_-^2\, \epsilon_+ \cos(\alpha)\, csgn(k_{\|+}) \tag{81}$$

> $simplify\left(mtaylor\left(sol2, \left[\epsilon_-, \epsilon_+ \right], 3 \right) \right)$

$$\frac{2\, I \cos(\alpha)\, \epsilon_-\, \epsilon_+\, k_{\|+}}{\sqrt{-k_{\|+}^2}} \tag{82}$$

> $simplify\left(mtaylor\left(sol3, \left[\epsilon_-, \epsilon_+ \right], 1 \right) \right)$

$$-1 \tag{83}$$

> $simplify\left(mtaylor\left(sol4, \left[\epsilon_-, \epsilon_+ \right], 2 \right) \right)$

$$\frac{2\, I\, \epsilon_+\, csgn(k_{\|+})\, \sin(\alpha)^2}{\cos(\alpha)} \tag{84}$$

bx leading order terms

> $b_{x1-} := \dfrac{\Delta_{\|1-} \cdot u_{x1-}}{i \cdot k_{\|+}}$:

> $b_{x4-} := \dfrac{\Delta_{\|4-} \cdot u_{x4-}}{i \cdot k_{\|+}}$:

> $b_{x1+} := \dfrac{\Delta_{\|1+} \cdot u_{x1+}}{i \cdot k_{\|+}}$:

> $b_{x2+} := \dfrac{\Delta_{\|2+} \cdot u_{x2+}}{i \cdot k_{\|+}}$:

> $b_{x3+} := \dfrac{\Delta_{\|3+} \cdot u_{x3+}}{i \cdot k_{\|+}}$:

> $simplify\left(mtaylor\left(b_{x1-}, \left[\epsilon_-, \epsilon_+\right], 1\right)\right)$

$$-2\,I\sin(\alpha)\,\text{csgn}(k_{\|+}) \tag{85}$$

> $simplify\left(mtaylor\left(b_{x4-}, \left[\epsilon_-, \epsilon_+\right], 2\right)\right)$

$$\frac{-2\,I\,k_{\|+}\,\epsilon_-\cos(\alpha)^2}{\sqrt{-k_{\|+}^2}\,\sin(\alpha)} \tag{86}$$

> $simplify\left(mtaylor\left(b_{x1+}, \left[\epsilon_-, \epsilon_+\right], 2\right)\right)$

$$\frac{\epsilon_+\left(-a_y+\sin(\alpha)\right)}{\cos(\alpha)} \tag{87}$$

> $simplify\left(mtaylor\left(b_{x2+}, \left[\epsilon_-, \epsilon_+\right], 2\right)\right)$

$$-\frac{\epsilon_+\left(a_y+\sin(\alpha)\right)}{\cos(\alpha)} \tag{88}$$

> $simplify\left(mtaylor\left(b_{x3+}, \left[\epsilon_-, \epsilon_+\right], 1\right)\right)$

$$-2\,I\sin(\alpha)\,\text{csgn}(k_{\|+}) \tag{89}$$

b_perp leading order terms

> $b_{\perp1-} := \dfrac{\Delta_{\|1-}\cdot sol1}{i\cdot k_{\|+}}$:

> $b_{\perp4-} := \dfrac{\Delta_{\|4-}\cdot sol2}{i\cdot k_{\|+}}$:

> $b_{\perp1+} := \dfrac{\Delta_{\|1+}\cdot 1}{i\cdot k_{\|+}}$:

> $b_{\perp2+} := \dfrac{\Delta_{\|2+}\cdot sol3}{i\cdot k_{\|+}}$:

> $b_{\perp3+} := \dfrac{\Delta_{\|3+}\cdot sol4}{i\cdot k_{\|+}}$:

> $simplify\left(mtaylor\left(b_{\perp1-}, \left[\epsilon_-, \epsilon_+\right], 2\right)\right)$

$$-2\,I\epsilon_-\cos(\alpha)\,\text{csgn}(k_{\|+}) \tag{90}$$

> $simplify\left(mtaylor\left(b_{\perp4-}, \left[\epsilon_-, \epsilon_+\right], 1\right)\right)$

$$2\cos(\alpha)^2 \tag{91}$$

> $simplify\left(mtaylor\left(b_{\perp1+}, \left[\epsilon_-, \epsilon_+\right], 5\right)\right)$

$$1 \tag{92}$$

> $simplify\left(mtaylor\left(b_{\perp2+}, \left[\epsilon_-, \epsilon_+\right], 1\right)\right)$

$$1 \tag{93}$$

> $simplify\left(mtaylor\left(b_{\perp 3+}, \left[\epsilon_-, \epsilon_+\right], 1\right)\right)$

$$-2\sin(\alpha)^2 \qquad\qquad (94)$$

b_par leading order terms

> $b_{\parallel 1-} := -\dfrac{i \cdot k_x \cdot u_{x1-} + \Delta_{\perp 1-} \cdot sol1}{i \cdot k_{\parallel +}}$:

> $b_{\parallel 4-} := -\dfrac{i \cdot k_x \cdot u_{x4-} + \Delta_{\perp 4-} \cdot sol2}{i \cdot k_{\parallel +}}$:

> $b_{\parallel 1+} := -\dfrac{i \cdot k_x \cdot u_{x1+} + \Delta_{\perp 1+}}{i \cdot k_{\parallel +}}$:

> $b_{\parallel 2+} := -\dfrac{i \cdot k_x \cdot u_{x2+} + \Delta_{\perp 2+} \cdot sol3}{i \cdot k_{\parallel +}}$:

> $b_{\parallel 3+} := -\dfrac{i \cdot k_x \cdot u_{x3+} + \Delta_{\perp 3+} \cdot sol4}{i \cdot k_{\parallel +}}$:

> $simplify\left(mtaylor\left(b_{\parallel 1-}, \left[\epsilon_-, \epsilon_+\right], 2\right)\right)$

$$0 \qquad\qquad (95)$$

> $simplify\left(mtaylor\left(b_{\parallel 4-}, \left[\epsilon_-, \epsilon_+\right], 1\right)\right)$

$$2\sin(\alpha)\cos(\alpha) \qquad\qquad (96)$$

> $simplify\left(mtaylor\left(b_{\parallel 1+}, \left[\epsilon_-, \epsilon_+\right], 1\right)\right)$

$$0 \qquad\qquad (97)$$

> $simplify\left(mtaylor\left(b_{\parallel 2+}, \left[\epsilon_-, \epsilon_+\right], 1\right)\right)$

$$0 \qquad\qquad (98)$$

> $simplify\left(mtaylor\left(b_{\parallel 3+}, \left[\epsilon_-, \epsilon_+\right], 1\right)\right)$

$$2\sin(\alpha)\cos(\alpha) \qquad\qquad (99)$$