This document is used to assist the Algebra in Appendix A.1.

Note that ∇ has been replaced with '\Delta' to ensure the code works okay.

> *restart*;

Let a_y = k_y / k_{||+}, hence, k_y = a_y k_{||+}.
Let epsilon = k_{||+} / k_x, hence, k_x = k_{||+} / epsilon.

$$> k_z := \left[\left(\frac{k_{\|+}}{\cos(\alpha)} - a_y \cdot k_{\|+} \cdot \tan(\alpha)\right), \left(-\frac{k_{\|+}}{\cos(\alpha)} - a_y \cdot k_{\|+} \cdot \tan(\alpha)\right), \mathrm{i} \cdot k_{\|+} \cdot \sqrt{\frac{1}{\epsilon^2} + a_y^2 - 1}\right];$$

$$k_z := \left[\frac{k_{\|+}}{\cos(\alpha)} - a_y k_{\|+} \tan(\alpha), -\frac{k_{\|+}}{\cos(\alpha)} - a_y k_{\|+} \tan(\alpha), \mathrm{I}\, k_{\|+} \sqrt{\frac{1}{\epsilon^2} + a_y^2 - 1}\right] \tag{1}$$

Note that ux_0[n] denotes \hat{u}_{xn} and u_{x0}[n] denotes u_{xn}. Also we normalise the velocity coefficents by u_0 and the field components by (B_0 * u_0 / v_{A+}).

$$
\begin{aligned}
> \; & \Delta_\perp := [\;]: \\
& \Delta_{\|} := [\;]: \\
& L := [\;]: \\
& ux_0 := [\;]: \\
& \textbf{for } i \textbf{ from } 1 \textbf{ to } 3 \textbf{ do}: \\
& \quad \Delta_\perp := \left[op(\Delta_\perp), \mathrm{i} \cdot \left(a_y \cdot k_{\|+} \cdot \cos(\alpha) - k_z[i] \cdot \sin(\alpha)\right)\right]: \\
& \quad \Delta_{\|} := \left[op(\Delta_{\|}), \mathrm{i} \cdot \left(a_y \cdot k_{\|+} \cdot \sin(\alpha) + k_z[i] \cdot \cos(\alpha)\right)\right]: \\
& \quad L := \left[op(L), \Delta_{\|}[i]^2 + k_{\|+}^2\right]: \\
& \quad ux_0 := \left[op(ux_0), -\frac{\mathrm{i} \cdot \left(\frac{1}{\epsilon}\right) \cdot k_{\|+} \cdot \Delta_\perp[i]}{L[i] - \left(\frac{1}{\epsilon}\right)^2 \cdot k_{\|+}^2}\right]: \\
& \textbf{end do}:
\end{aligned}
$$

$$> u_{\perp 0} := \left[1, -\frac{ux_0[1] - ux_0[3]}{ux_0[2] - ux_0[3]}, \frac{ux_0[1] - ux_0[2]}{ux_0[2] - ux_0[3]}\right]:$$

$$
\begin{aligned}
> \; & u_{x0} := [\;]: \\
& b_{x0} := [\;]: \\
& b_{\perp 0} := [\;]: \\
& b_{\|0} := [\;]: \\
& \textbf{for } i \textbf{ from } 1 \textbf{ to } 3 \textbf{ do}: \\
& \quad u_{x0} := \left[op(u_{x0}), ux_0[i] \cdot u_{\perp 0}[i]\right]: \\
& \quad b_{x0} := \left[op(b_{x0}), \frac{\Delta_{\|}[i] \cdot u_{x0}[i]}{\mathrm{i} \cdot k_{\|+}}\right]:
\end{aligned}
$$

$$b_{\perp 0} := \left[op(b_{\perp 0}), \frac{\Delta_{||}[i] \cdot u_{\perp 0}[i]}{\mathrm{i} \cdot k_{||+}} \right]:$$

$$b_{||0} := \left[op(b_{||0}), -\frac{\left(\frac{\mathrm{i} \cdot k_{||+}}{\epsilon} \cdot u_{x0}[i] + \Delta_{\perp}[i] \cdot u_{\perp 0}[i] \right)}{\mathrm{i} \cdot k_{||+}} \right]:$$

end do:

ux leading order terms

> $simplify(series(expand(u_{x0}[1]), \epsilon, 3));$

$$\frac{-a_y + \sin(\alpha)}{\cos(\alpha)} \epsilon + O(\epsilon^3) \quad \textbf{(2)}$$

> $simplify(series(expand(u_{x0}[2]), \epsilon, 3));$

$$\frac{a_y + \sin(\alpha)}{\cos(\alpha)} \epsilon + O(\epsilon^2) \quad \textbf{(3)}$$

> $simplify(series(expand(u_{x0}[3]), \epsilon, 3));$

$$-\frac{2 \sin(\alpha)}{\cos(\alpha)} \epsilon + O(\epsilon^2) \quad \textbf{(4)}$$

u_perp leading order terms

> $simplify(u_{\perp 0}[1]);$

$$1 \quad \textbf{(5)}$$

> $simplify(series(expand(u_{\perp 0}[2]), \epsilon, 2));$

$$-1 + O(\epsilon) \quad \textbf{(6)}$$

> $simplify(series(expand(u_{\perp 0}[3]), \epsilon, 2));$

$$\frac{2\, I\, \mathrm{csgn}\left(\frac{1}{\epsilon}\right) \sin(\alpha)^2}{\cos(\alpha)} \epsilon + O(\epsilon^2) \quad \textbf{(7)}$$

>

b_x leading order terms

> $simplify(series(expand(b_{x0}[1]), \epsilon, 2));$

$$\frac{-a_y + \sin(\alpha)}{\cos(\alpha)} \epsilon + O(\epsilon^3) \quad \textbf{(8)}$$

> $simplify(series(expand(b_{x0}[2]), \epsilon, 3));$

$$\frac{-a_y - \sin(\alpha)}{\cos(\alpha)} \epsilon + \mathrm{O}(\epsilon^2) \tag{9}$$

> $simplify(series(expand(b_{x0}[3]), \epsilon, 2));$

$$-2\,\mathrm{I}\,\mathrm{csgn}\left(\frac{1}{\epsilon}\right) \sin(\alpha) + \mathrm{O}(\epsilon) \tag{10}$$

b_perp leading order terms

> $simplify(b_{\perp 0}[1]);$

$$1 \tag{11}$$

> $simplify(series(expand(b_{\perp 0}[2]), \epsilon, 2));$

$$1 + \mathrm{O}(\epsilon) \tag{12}$$

> $simplify(series(expand(b_{\perp 0}[3]), \epsilon, 1));$

$$-2 \sin(\alpha)^2 + \mathrm{O}(\epsilon) \tag{13}$$

b_par leading order terms

> $simplify(b_{\|0}[1]);$

$$0 \tag{14}$$

> $simplify(b_{\|0}[2]);$

$$0 \tag{15}$$

> $simplify(series(expand(b_{\|0}[3]), \epsilon, 2));$

$$2 \sin(\alpha) \cos(\alpha) + 2\,\mathrm{I}\,\mathrm{csgn}\left(\frac{1}{\epsilon}\right) \sin(\alpha)^2 \left(-a_y + \sin(\alpha)\right) \epsilon + \mathrm{O}(\epsilon^2) \tag{16}$$