

[This document is used to assist the Algebra in Appendix A.1.

[Note that ∇ has been replaced with ∇ to ensure the code works okay.

[> restart;

Let $a_y = k_y / k_{||+}$, hence, $k_y = a_y k_{||+}$.

Let $\epsilon = k_{||+} / k_x$, hence, $k_x = k_{||+} / \epsilon$.

$$\begin{aligned} > k_z := \left[\left(\frac{k_{||+}}{\cos(\alpha)} - a_y \cdot k_{||+} \cdot \tan(\alpha) \right), \left(-\frac{k_{||+}}{\cos(\alpha)} - a_y \cdot k_{||+} \cdot \tan(\alpha) \right), i \cdot k_{||+} \cdot \sqrt{\frac{1}{\epsilon^2} + a_y^2 - 1} \right] \\ & \quad \left[\frac{k_{||+}}{\cos(\alpha)} - a_y k_{||+} \tan(\alpha), -\frac{k_{||+}}{\cos(\alpha)} - a_y k_{||+} \tan(\alpha), i k_{||+} \sqrt{\frac{1}{\epsilon^2} + a_y^2 - 1} \right] \end{aligned} \quad (1)$$

Note that $ux_0[n]$ denotes \hat{u}_{xn} and $u_{x0}[n]$ denotes u_{xn} . Also we normalise the velocity coefficients by u_0 and the field components by $(B_0 * u_0 / v_{A+})$.

```
> Δ⊥ := [ ]:
Δ|| := [ ]:
L := [ ]:
ux0 := [ ]:
for i from 1 to 3 do:
  Δ⊥ := [ op(Δ⊥), i · (ay · k||+ · cos(α) - kz[i] · sin(α)) ]:
  Δ|| := [ op(Δ||), i · (ay · k||+ · sin(α) + kz[i] · cos(α)) ]:
  L := [ op(L), Δ||[i]2 + k||+2 ]:
  ux0 := [ op(ux0), -  $\frac{i \cdot \left(\frac{1}{\epsilon}\right) \cdot k_{||+} \cdot \Delta_{\perp}[i]}{L[i] - \left(\frac{1}{\epsilon}\right)^2 \cdot k_{||+}^2}$  ]:
end do:
```

$$> u_{\perp 0} := \left[1, -\frac{ux_0[1] - ux_0[3]}{ux_0[2] - ux_0[3]}, \frac{ux_0[1] - ux_0[2]}{ux_0[2] - ux_0[3]} \right]:$$

```
> ux0 := [ ]:
bx0 := [ ]:
b⊥0 := [ ]:
b||0 := [ ]:
for i from 1 to 3 do:
  ux0 := [ op(ux0), ux0[i] · u⊥0[i] ]:
  bx0 := [ op(bx0),  $\frac{\Delta_{||}[i] \cdot u_{x0}[i]}{i \cdot k_{||+}}$  ]:
end do:
```

$$\begin{aligned}
& b_{\perp 0} := \left[op(b_{\perp 0}), \frac{\Delta_{\parallel}[i] \cdot u_{\perp 0}[i]}{i \cdot k_{\parallel+}} \right]; \\
& b_{\parallel 0} := \left[op(b_{\parallel 0}), -\frac{\left(\frac{i \cdot k_{\parallel+}}{\epsilon} \cdot u_{x0}[i] + \Delta_{\perp}[i] \cdot u_{\perp 0}[i] \right)}{i \cdot k_{\parallel+}} \right]; \\
& \textbf{end do:}
\end{aligned}$$

ux leading order terms

$$\begin{aligned}
& \text{> } simplify(series(expand(u_{x0}[1]), \epsilon, 3)); \\
& \frac{-a_y + \sin(\alpha)}{\cos(\alpha)} \epsilon + O(\epsilon^3) \tag{2}
\end{aligned}$$

$$\begin{aligned}
& \text{> } simplify(series(expand(u_{x0}[2]), \epsilon, 3)); \\
& \frac{a_y + \sin(\alpha)}{\cos(\alpha)} \epsilon + O(\epsilon^2) \tag{3}
\end{aligned}$$

$$\begin{aligned}
& \text{> } simplify(series(expand(u_{x0}[3]), \epsilon, 3)); \\
& -\frac{2 \sin(\alpha)}{\cos(\alpha)} \epsilon + O(\epsilon^2) \tag{4}
\end{aligned}$$

u_perp leading order terms

$$\begin{aligned}
& \text{> } simplify(u_{\perp 0}[1]); \\
& 1 \tag{5}
\end{aligned}$$

$$\begin{aligned}
& \text{> } simplify(series(expand(u_{\perp 0}[2]), \epsilon, 2)); \\
& -1 + O(\epsilon) \tag{6}
\end{aligned}$$

$$\begin{aligned}
& \text{> } simplify(series(expand(u_{\perp 0}[3]), \epsilon, 2)); \\
& \frac{2 \operatorname{I} \operatorname{csgn}\left(\frac{1}{\epsilon}\right) \sin(\alpha)^2}{\cos(\alpha)} \epsilon + O(\epsilon^2) \tag{7}
\end{aligned}$$

>

b_x leading order terms

$$\begin{aligned}
& \text{> } simplify(series(expand(b_{x0}[1]), \epsilon, 2)); \\
& \frac{-a_y + \sin(\alpha)}{\cos(\alpha)} \epsilon + O(\epsilon^3) \tag{8}
\end{aligned}$$

$$\left[\begin{array}{l} \text{> simplify(series(expand(b_{x0}[2]), \epsilon, 3));} \\ \frac{-a_y - \sin(\alpha)}{\cos(\alpha)} \epsilon + O(\epsilon^2) \end{array} \right] \quad (9)$$

$$\left[\begin{array}{l} \text{> simplify(series(expand(b_{x0}[3]), \epsilon, 2));} \\ -2 \operatorname{Icsgn}\left(\frac{1}{\epsilon}\right) \sin(\alpha) + O(\epsilon) \end{array} \right] \quad (10)$$

b_perp leading order terms

$$\left[\begin{array}{l} \text{> simplify(b_{\perp 0}[1]);} \\ 1 \end{array} \right] \quad (11)$$

$$\left[\begin{array}{l} \text{> simplify(series(expand(b_{\perp 0}[2]), \epsilon, 2));} \\ 1 + O(\epsilon) \end{array} \right] \quad (12)$$

$$\left[\begin{array}{l} \text{> simplify(series(expand(b_{\perp 0}[3]), \epsilon, 1));} \\ -2 \sin(\alpha)^2 + O(\epsilon) \end{array} \right] \quad (13)$$

b_par leading order terms

$$\left[\begin{array}{l} \text{> simplify(b_{\parallel 0}[1]);} \\ 0 \end{array} \right] \quad (14)$$

$$\left[\begin{array}{l} \text{> simplify(b_{\parallel 0}[2]);} \\ 0 \end{array} \right] \quad (15)$$

$$\left[\begin{array}{l} \text{> simplify(series(expand(b_{\parallel 0}[3]), \epsilon, 2));} \\ 2 \sin(\alpha) \cos(\alpha) + O(\epsilon) \end{array} \right] \quad (16)$$