This document is used to assist with the Algebra in Appendix A.2

Note that ∇ has been replaced with '\Delta' to ensure the code works okay.

> *restart*;

We normalise the velocity coefficents by u_0 and the field components by (B_0 * u_0 / v_{A+}).

We start by solving the Matrix equation given by Equation (57).

> $eqn1 := a_{11} \cdot x_1 + a_{12} \cdot x_2 + a_{13} \cdot x_3 + a_{14} \cdot x_4 = y_1$

$$eqn1 := a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 = y_1 \tag{1}$$

> $eqn2 := a_{21} \cdot x_1 + a_{22} \cdot x_2 + a_{23} \cdot x_3 + a_{24} \cdot x_4 = y_2$

$$eqn2 := a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + a_{24} x_4 = y_2 \tag{2}$$

> $eqn3 := a_{31} \cdot x_1 + a_{32} \cdot x_2 + a_{33} \cdot x_3 + a_{34} \cdot x_4 = y_3$

$$eqn3 := a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + a_{34} x_4 = y_3 \tag{3}$$

> $eqn4 := a_{41} \cdot x_1 + a_{42} \cdot x_2 + a_{43} \cdot x_3 + a_{44} \cdot x_4 = y_4$

$$eqn4 := a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4 = y_4 \tag{4}$$

> $solns := solve(\{eqn1, eqn2, eqn3, eqn4\}, \{x_1, x_2, x_3, x_4\})$:

> $sol1 := rhs(solns[1])$

$$\begin{aligned} sol1 := -\big(&a_{12} a_{23} a_{34} y_4 - a_{12} a_{23} a_{44} y_3 - a_{12} a_{24} a_{33} y_4 + a_{12} a_{24} a_{43} y_3 \\ &+ a_{12} a_{33} a_{44} y_2 - a_{12} a_{34} a_{43} y_2 - a_{13} a_{22} a_{34} y_4 + a_{13} a_{22} a_{44} y_3 + a_{13} a_{24} a_{32} y_4 \\ &- a_{13} a_{24} a_{42} y_3 - a_{13} a_{32} a_{44} y_2 + a_{13} a_{34} a_{42} y_2 + a_{14} a_{22} a_{33} y_4 - a_{14} a_{22} a_{43} y_3 \\ &- a_{14} a_{23} a_{32} y_4 + a_{14} a_{23} a_{42} y_3 + a_{14} a_{32} a_{43} y_2 - a_{14} a_{33} a_{42} y_2 - a_{22} a_{33} a_{44} y_1 \\ &+ a_{22} a_{34} a_{43} y_1 + a_{23} a_{32} a_{44} y_1 - a_{23} a_{34} a_{42} y_1 - a_{24} a_{32} a_{43} y_1 + a_{24} a_{33} a_{42} y_1\big) \\ &\Big/ \big(a_{11} a_{22} a_{33} a_{44} - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} \\ &+ a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} \\ &+ a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} + a_{12} a_{24} a_{33} a_{41} \\ &+ a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} \\ &+ a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} \\ &+ a_{14} a_{22} a_{31} a_{43} - a_{14} a_{22} a_{33} a_{41} - a_{14} a_{23} a_{31} a_{42} + a_{14} a_{23} a_{32} a_{41}\big) \end{aligned} \tag{5}$$

> $sol2 := rhs(solns[2])$

$$\begin{aligned} sol2 := \big(&a_{11} a_{23} a_{34} y_4 - a_{11} a_{23} a_{44} y_3 - a_{11} a_{24} a_{33} y_4 + a_{11} a_{24} a_{43} y_3 \\ &+ a_{11} a_{33} a_{44} y_2 - a_{11} a_{34} a_{43} y_2 - a_{13} a_{21} a_{34} y_4 + a_{13} a_{21} a_{44} y_3 + a_{13} a_{24} a_{31} y_4 \\ &- a_{13} a_{24} a_{41} y_3 - a_{13} a_{31} a_{44} y_2 + a_{13} a_{34} a_{41} y_2 + a_{14} a_{21} a_{33} y_4 - a_{14} a_{21} a_{43} y_3 \\ &- a_{14} a_{23} a_{31} y_4 + a_{14} a_{23} a_{41} y_3 + a_{14} a_{31} a_{43} y_2 - a_{14} a_{33} a_{41} y_2 - a_{21} a_{33} a_{44} y_1 \\ &+ a_{21} a_{34} a_{43} y_1 + a_{23} a_{31} a_{44} y_1 - a_{23} a_{34} a_{41} y_1 - a_{24} a_{31} a_{43} y_1 + a_{24} a_{33} a_{41} y_1\big) \\ &\Big/ \big(a_{11} a_{22} a_{33} a_{44} - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} \\ &+ a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} \end{aligned} \tag{6}$$

$$+ a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} + a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} + a_{14} a_{22} a_{31} a_{43} - a_{14} a_{22} a_{33} a_{41} - a_{14} a_{23} a_{31} a_{42} + a_{14} a_{23} a_{32} a_{41})$$

> $sol3 := rhs(solns[3])$

$$sol3 := -(a_{11} a_{22} a_{34} y_4 - a_{11} a_{22} a_{44} y_3 - a_{11} a_{24} a_{32} y_4 + a_{11} a_{24} a_{42} y_3 + a_{11} a_{32} a_{44} y_2 - a_{11} a_{34} a_{42} y_2 - a_{12} a_{21} a_{34} y_4 + a_{12} a_{21} a_{44} y_3 + a_{12} a_{24} a_{31} y_4 - a_{12} a_{24} a_{41} y_3 - a_{12} a_{31} a_{44} y_2 + a_{12} a_{34} a_{41} y_2 + a_{14} a_{21} a_{32} y_4 - a_{14} a_{21} a_{42} y_3 - a_{14} a_{22} a_{31} y_4 + a_{14} a_{22} a_{41} y_3 + a_{14} a_{31} a_{42} y_2 - a_{14} a_{32} a_{41} y_2 - a_{21} a_{32} a_{44} y_1 + a_{21} a_{34} a_{42} y_1 + a_{22} a_{31} a_{44} y_1 - a_{22} a_{34} a_{41} y_1 - a_{24} a_{31} a_{42} y_1 + a_{24} a_{32} a_{41} y_1) \Big/ (a_{11} a_{22} a_{33} a_{44} - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} + a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} + a_{14} a_{22} a_{31} a_{43} - a_{14} a_{22} a_{33} a_{41} - a_{14} a_{23} a_{31} a_{42} + a_{14} a_{23} a_{32} a_{41}) \quad (7)$$

> $sol4 := rhs(solns[4])$

$$sol4 := (a_{11} a_{22} a_{33} y_4 - a_{11} a_{22} a_{43} y_3 - a_{11} a_{23} a_{32} y_4 + a_{11} a_{23} a_{42} y_3 + a_{11} a_{32} a_{43} y_2 - a_{11} a_{33} a_{42} y_2 - a_{12} a_{21} a_{33} y_4 + a_{12} a_{21} a_{43} y_3 + a_{12} a_{23} a_{31} y_4 - a_{12} a_{23} a_{41} y_3 - a_{12} a_{31} a_{43} y_2 + a_{12} a_{33} a_{41} y_2 + a_{13} a_{21} a_{32} y_4 - a_{13} a_{21} a_{42} y_3 - a_{13} a_{22} a_{31} y_4 + a_{13} a_{22} a_{41} y_3 + a_{13} a_{31} a_{42} y_2 - a_{13} a_{32} a_{41} y_2 - a_{21} a_{32} a_{43} y_1 + a_{21} a_{33} a_{42} y_1 + a_{22} a_{31} a_{43} y_1 - a_{22} a_{33} a_{41} y_1 - a_{23} a_{31} a_{42} y_1 + a_{23} a_{32} a_{41} y_1) \Big/ (a_{11} a_{22} a_{33} a_{44} - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} + a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} + a_{14} a_{22} a_{31} a_{43} - a_{14} a_{22} a_{33} a_{41} - a_{14} a_{23} a_{31} a_{42} + a_{14} a_{23} a_{32} a_{41}) \quad (8)$$

> $a_{11} := u_{x10-}$

$$a_{11} := u_{x10-} \quad (9)$$

> $a_{12} := u_{x40-}$

$$a_{12} := u_{x40-} \quad (10)$$

> $a_{13} := -u_{x20+}$

(11)

$$a_{13} := -u_{x20+} \quad (11)$$

> $a_{14} := -u_{x30+}$

$$a_{14} := -u_{x30+} \quad (12)$$

> $a_{21} := k_{z1-} \cdot u_{x10-}$

$$a_{21} := k_{z1-} \, u_{x10-} \quad (13)$$

> $a_{22} := k_{z4-} \cdot u_{x40-}$

$$a_{22} := k_{z4-} \, u_{x40-} \quad (14)$$

> $a_{23} := -k_{z2+} \cdot u_{x20+}$

$$a_{23} := -k_{z2+} \, u_{x20+} \quad (15)$$

> $a_{24} := -k_{z3+} \cdot u_{x30+}$

$$a_{24} := -k_{z3+} \, u_{x30+} \quad (16)$$

> $a_{31} := 1$

$$a_{31} := 1 \quad (17)$$

> $a_{32} := 1$

$$a_{32} := 1 \quad (18)$$

> $a_{33} := -1$

$$a_{33} := -1 \quad (19)$$

> $a_{34} := -1$

$$a_{34} := -1 \quad (20)$$

> $a_{41} := k_{z1-}$

$$a_{41} := k_{z1-} \quad (21)$$

> $a_{42} := k_{z4-}$

$$a_{42} := k_{z4-} \quad (22)$$

> $a_{43} := -k_{z2+}$

$$a_{43} := -k_{z2+} \quad (23)$$

> $a_{44} := -k_{z3+}$

$$a_{44} := -k_{z3+} \quad (24)$$

> $y_1 := u_{x10+}$

$$y_1 := u_{x10+} \quad (25)$$

> $y_2 := k_{z1+} \cdot u_{x10+}$

$$y_2 := k_{z1+} \, u_{x10+} \quad (26)$$

> $y_3 := 1$

$$y_3 := 1 \quad (27)$$

> $y_4 := k_{z1+}$

$$y_4 := k_{z1+} \tag{28}$$

Note that ux_0[n] denotes \hat{u}_{xn} and u_x[n±] denotes u_{xn±}.

> $u_{x10-} := -\frac{\mathrm{i}\cdot k_x\cdot\Delta_{\perp 1-}}{L_{1-} - k_x^2}$

$$u_{x10-} := \frac{-\mathrm{I}\, k_x \Delta_{\perp 1-}}{-k_x^2 + L_{1-}} \tag{29}$$

> $u_{x10+} := -\frac{\mathrm{i}\cdot k_x\cdot\Delta_{\perp 1+}}{L_{1+} - k_x^2}$

$$u_{x10+} := \frac{-\mathrm{I}\, k_x \Delta_{\perp 1+}}{-k_x^2 + L_{1+}} \tag{30}$$

> $u_{x20-} := -\frac{\mathrm{i}\cdot k_x\cdot\Delta_{\perp 2-}}{L_{2-} - k_x^2}$

$$u_{x20-} := \frac{-\mathrm{I}\, k_x \Delta_{\perp 2-}}{-k_x^2 + L_{2-}} \tag{31}$$

> $u_{x20+} := -\frac{\mathrm{i}\cdot k_x\cdot\Delta_{\perp 2+}}{L_{2+} - k_x^2}$

$$u_{x20+} := \frac{-\mathrm{I}\, k_x \Delta_{\perp 2+}}{-k_x^2 + L_{2+}} \tag{32}$$

> $u_{x30-} := -\frac{\mathrm{i}\cdot k_x\cdot\Delta_{\perp 3-}}{L_{3-} - k_x^2}$

$$u_{x30-} := \frac{-\mathrm{I}\, k_x \Delta_{\perp 3-}}{-k_x^2 + L_{3-}} \tag{33}$$

> $u_{x30+} := -\frac{\mathrm{i}\cdot k_x\cdot\Delta_{\perp 3+}}{L_{3+} - k_x^2}$

$$u_{x30+} := \frac{-\mathrm{I}\, k_x \Delta_{\perp 3+}}{-k_x^2 + L_{3+}} \tag{34}$$

> $u_{x40-} := -\frac{\mathrm{i}\cdot k_x\cdot\Delta_{\perp 4-}}{L_{4-} - k_x^2}$

$$u_{x40-} := \frac{-\mathrm{I}\, k_x \Delta_{\perp 4-}}{-k_x^2 + L_{4-}} \tag{35}$$

> $u_{x40+} := -\frac{\mathrm{i}\cdot k_x\cdot\Delta_{\perp 4+}}{L_{4+} - k_x^2}$

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$$u_{x40+} := \frac{-I\, k_x \Delta_{\perp 4+}}{-k_x^2 + L_{4+}} \tag{36}$$

> $L_{1-} := \Delta_{\parallel 1-}{}^2 + k_{\parallel -}{}^2$

$$L_{1-} := k_{\parallel -}{}^2 + \Delta_{\parallel 1-}{}^2 \tag{37}$$

> $L_{1+} := \Delta_{\parallel 1+}{}^2 + k_{\parallel +}{}^2$

$$L_{1+} := k_{\parallel +}{}^2 + \Delta_{\parallel 1+}{}^2 \tag{38}$$

> $L_{2-} := \Delta_{\parallel 2-}{}^2 + k_{\parallel -}{}^2$

$$L_{2-} := k_{\parallel -}{}^2 + \Delta_{\parallel 2-}{}^2 \tag{39}$$

> $L_{2+} := \Delta_{\parallel 2+}{}^2 + k_{\parallel +}{}^2$

$$L_{2+} := k_{\parallel +}{}^2 + \Delta_{\parallel 2+}{}^2 \tag{40}$$

> $L_{3-} := \Delta_{\parallel 3-}{}^2 + k_{\parallel -}{}^2$

$$L_{3-} := k_{\parallel -}{}^2 + \Delta_{\parallel 3-}{}^2 \tag{41}$$

> $L_{3+} := \Delta_{\parallel 3+}{}^2 + k_{\parallel +}{}^2$

$$L_{3+} := k_{\parallel +}{}^2 + \Delta_{\parallel 3+}{}^2 \tag{42}$$

> $L_{4-} := \Delta_{\parallel 4-}{}^2 + k_{\parallel -}{}^2$

$$L_{4-} := k_{\parallel -}{}^2 + \Delta_{\parallel 4-}{}^2 \tag{43}$$

> $L_{4+} := \Delta_{\parallel 4+}{}^2 + k_{\parallel +}{}^2$

$$L_{4+} := k_{\parallel +}{}^2 + \Delta_{\parallel 4+}{}^2 \tag{44}$$

> $\Delta_{\perp 1-} := \mathrm{i} \cdot (k_y \cdot \cos(\alpha) - k_{z1-} \cdot \sin(\alpha))$

$$\Delta_{\perp 1-} := I\,(k_y \cos(\alpha) - k_{z1-} \sin(\alpha)) \tag{45}$$

> $\Delta_{\perp 1+} := \mathrm{i} \cdot (k_y \cdot \cos(\alpha) - k_{z1+} \cdot \sin(\alpha))$

$$\Delta_{\perp 1+} := I\,(k_y \cos(\alpha) - k_{z1+} \sin(\alpha)) \tag{46}$$

> $\Delta_{\perp 2-} := \mathrm{i} \cdot (k_y \cdot \cos(\alpha) - k_{z2-} \cdot \sin(\alpha))$

$$\Delta_{\perp 2-} := I\,(k_y \cos(\alpha) - k_{z2-} \sin(\alpha)) \tag{47}$$

> $\Delta_{\perp 2+} := \mathrm{i} \cdot (k_y \cdot \cos(\alpha) - k_{z2+} \cdot \sin(\alpha))$

$$\Delta_{\perp 2+} := I\,(k_y \cos(\alpha) - k_{z2+} \sin(\alpha)) \tag{48}$$

> $\Delta_{\perp 3-} := \mathrm{i} \cdot (k_y \cdot \cos(\alpha) - k_{z3-} \cdot \sin(\alpha))$

$$\Delta_{\perp 3-} := I\,(k_y \cos(\alpha) - k_{z3-} \sin(\alpha)) \tag{49}$$

> $\Delta_{\perp 3+} := \mathrm{i} \cdot (k_y \cdot \cos(\alpha) - k_{z3+} \cdot \sin(\alpha))$

$$\Delta_{\perp 3+} := I\,(k_y \cos(\alpha) - k_{z3+} \sin(\alpha)) \tag{50}$$

> $\Delta_{\perp 4-} := \mathrm{i} \cdot (k_y \cdot \cos(\alpha) - k_{z4-} \cdot \sin(\alpha))$

$$\Delta_{\perp 4-} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z4-} \sin(\alpha)\right) \tag{51}$$

> $\Delta_{\perp 4+} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z4+}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 4+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z4+} \sin(\alpha)\right) \tag{52}$$

> $\Delta_{\parallel 1-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z1-}\cdot\cos(\alpha)\right)$

$$\Delta_{\parallel 1-} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z1-} \cos(\alpha)\right) \tag{53}$$

> $\Delta_{\parallel 1+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z1+}\cdot\cos(\alpha)\right)$

$$\Delta_{\parallel 1+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z1+} \cos(\alpha)\right) \tag{54}$$

> $\Delta_{\parallel 2-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z2-}\cdot\cos(\alpha)\right)$

$$\Delta_{\parallel 2-} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z2-} \cos(\alpha)\right) \tag{55}$$

> $\Delta_{\parallel 2+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z2+}\cdot\cos(\alpha)\right)$

$$\Delta_{\parallel 2+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z2+} \cos(\alpha)\right) \tag{56}$$

> $\Delta_{\parallel 3-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z3-}\cdot\cos(\alpha)\right)$

$$\Delta_{\parallel 3-} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z3-} \cos(\alpha)\right) \tag{57}$$

> $\Delta_{\parallel 3+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z3+}\cdot\cos(\alpha)\right)$

$$\Delta_{\parallel 3+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z3+} \cos(\alpha)\right) \tag{58}$$

> $\Delta_{\parallel 4-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z4-}\cdot\cos(\alpha)\right)$

$$\Delta_{\parallel 4-} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z4-} \cos(\alpha)\right) \tag{59}$$

> $\Delta_{\parallel 4+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z4+}\cdot\cos(\alpha)\right)$

$$\Delta_{\parallel 4+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z4+} \cos(\alpha)\right) \tag{60}$$

> $k_{z1-} := \dfrac{k_{\parallel -}}{\cos(\alpha)} - k_y\cdot\tan(\alpha)$

$$k_{z1-} := \frac{k_{\parallel -}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{61}$$

> $k_{z1+} := \dfrac{k_{\parallel +}}{\cos(\alpha)} - k_y\cdot\tan(\alpha)$

$$k_{z1+} := \frac{k_{\parallel +}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{62}$$

> $k_{z2-} := -\dfrac{k_{\parallel -}}{\cos(\alpha)} - k_y\cdot\tan(\alpha)$

$$k_{z2-} := -\frac{k_{\parallel -}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{63}$$

> $k_{z2+} := -\dfrac{k_{\parallel +}}{\cos(\alpha)} - k_y\cdot\tan(\alpha)$

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$$k_{z2+} := -\frac{k_{\|+}}{\cos(\alpha)} - k_y \tan(\alpha) \quad (64)$$

> $k_{z3-} := \mathrm{i}\cdot\sqrt{k_x^2 + k_y^2 - k_{\|-}^2}$

$$k_{z3-} := \mathrm{I}\sqrt{k_x^2 + k_y^2 - k_{\|-}^2} \quad (65)$$

> $k_{z3+} := \mathrm{i}\cdot\sqrt{k_x^2 + k_y^2 - k_{\|+}^2}$

$$k_{z3+} := \mathrm{I}\sqrt{k_x^2 + k_y^2 - k_{\|+}^2} \quad (66)$$

> $k_{z4-} := -\mathrm{i}\cdot\sqrt{k_x^2 + k_y^2 - k_{\|-}^2}$

$$k_{z4-} := -\mathrm{I}\sqrt{k_x^2 + k_y^2 - k_{\|-}^2} \quad (67)$$

> $k_{z4+} := -\mathrm{i}\cdot\sqrt{k_x^2 + k_y^2 - k_{\|+}^2}$

$$k_{z4+} := -\mathrm{I}\sqrt{k_x^2 + k_y^2 - k_{\|+}^2} \quad (68)$$

>

> $k_{\|+} := r\cdot k_{\|-}$

$$k_{\|+} := r\, k_{\|-} \quad (69)$$

> $k_y := a_y\cdot k_{\|+}$

$$k_y := a_y\, r\, k_{\|-} \quad (70)$$

> $k_x := \frac{k_{\|-}}{\epsilon}$

$$k_x := \frac{k_{\|-}}{\epsilon} \quad (71)$$

ux leading order terms

> $u_{x1-} := u_{x10-}\cdot sol1:$

> $u_{x4-} := u_{x40-}\cdot sol2:$

> $u_{x1+} := u_{x10+}:$

> $u_{x2+} := u_{x20+}\cdot sol3:$

> $u_{x3+} := u_{x30+}\cdot sol4:$

> $simplify(series(u_{x1-}, \epsilon, 2))$

$$\frac{2\, r\,(-a_y\, r + \sin(\alpha))}{\cos(\alpha)\,(r+1)}\,\epsilon + \mathrm{O}(\epsilon^2) \quad (72)$$

> $simplify(series(u_{x4-}, \epsilon, 3))$

$$\frac{\mathrm{I}\sin(\alpha)\, r\,(r-1)\,\mathrm{csgn}\left(\frac{k_{\|-}}{\epsilon}\right)}{\cos(\alpha)^2}\,\epsilon^2 + \mathrm{O}(\epsilon^3) \quad (73)$$

> $simplify(series(u_{x1+}, \epsilon, 3))$

$$\frac{r\left(-a_y+\sin(\alpha)\right)}{\cos(\alpha)}\epsilon+O(\epsilon^3) \qquad (74)$$

> $simplify(series(u_{x2+}, \epsilon, 3))$

$$-\frac{r\,(r-1)\,\left(a_y+\sin(\alpha)\right)}{\cos(\alpha)\,(r+1)}\epsilon+O(\epsilon^3) \qquad (75)$$

> $simplify(series(u_{x3+}, \epsilon, 3))$

$$\frac{\mathrm{I}\sin(\alpha)\,r\,(r-1)\,\mathrm{csgn}\left(\frac{k_{\|-}}{\epsilon}\right)}{\cos(\alpha)^2}\epsilon^2+O(\epsilon^3) \qquad (76)$$

u_perp leading order terms

> $simplify(series(sol1, \epsilon, 3))$

$$\frac{2\,r}{r+1}+2\,\frac{r^2\,(r-1)\,(\cos(\alpha)-1)\,(\cos(\alpha)+1)\,\left(a_y-\sin(\alpha)\right)}{\cos(\alpha)^2\sin(\alpha)\,(r+1)}\epsilon^2+O(\epsilon^3) \qquad (77)$$

> $simplify(series(sol2, \epsilon, 3))$

$$\frac{(\cos(\alpha)-1)\,(\cos(\alpha)+1)\,(r-1)\,r}{\cos(\alpha)^2}\epsilon^2+O(\epsilon^3) \qquad (78)$$

> $simplify(series(sol3, \epsilon, 2))$

$$\frac{r-1}{r+1}+O(\epsilon^2) \qquad (79)$$

> $simplify(series(sol4, \epsilon, 4))$

$$-\frac{(\cos(\alpha)-1)\,(\cos(\alpha)+1)\,(r-1)\,r}{\cos(\alpha)^2}\epsilon^2 + \frac{\mathrm{I}\,r^2\,a_y\left(\cos(\alpha)^4-3\cos(\alpha)^2+2\right)(r-1)\,\mathrm{csgn}\left(\frac{k_{\|-}}{\epsilon}\right)}{\sin(\alpha)\cos(\alpha)^3}\epsilon^3+O(\epsilon^4) \qquad (80)$$

bx leading order terms

> $b_{x1-} := \frac{\Delta_{\|1-}\cdot u_{x1-}}{\mathrm{i}\cdot k_{\|+}}:$

> $b_{x4-} := \frac{\Delta_{\|4-}\cdot u_{x4-}}{\mathrm{i}\cdot k_{\|+}}:$

> $b_{x1+} := \frac{\Delta_{\|1+}\cdot u_{x1+}}{\mathrm{i}\cdot k_{\|+}}:$

> $b_{x2+} := \frac{\Delta_{\parallel 2+} \cdot u_{x2+}}{\mathrm{i} \cdot k_{\parallel +}}$:

> $b_{x3+} := \frac{\Delta_{\parallel 3+} \cdot u_{x3+}}{\mathrm{i} \cdot k_{\parallel +}}$:

> $simplify(series(b_{x1-}, \epsilon, 2))$

$$\frac{-2\, a_y\, r + 2 \sin(\alpha)}{\cos(\alpha)\, (r+1)}\, \epsilon + \mathrm{O}(\epsilon^2) \tag{81}$$

> $simplify(series(b_{x4-}, \epsilon, 2))$

$$\frac{\sin(\alpha)\, (r-1)}{\cos(\alpha)}\, \epsilon + \mathrm{O}(\epsilon^2) \tag{82}$$

> $simplify(series(b_{x1+}, \epsilon, 2))$

$$\frac{r\, (-a_y + \sin(\alpha))}{\cos(\alpha)}\, \epsilon + \mathrm{O}(\epsilon^3) \tag{83}$$

> $simplify(series(b_{x2+}, \epsilon, 2))$

$$\frac{r\, (r-1)\, (a_y + \sin(\alpha))}{\cos(\alpha)\, (r+1)}\, \epsilon + \mathrm{O}(\epsilon^2) \tag{84}$$

> $simplify(series(b_{x3+}, \epsilon, 2))$

$$-\frac{\sin(\alpha)\, (r-1)}{\cos(\alpha)}\, \epsilon + \mathrm{O}(\epsilon^2) \tag{85}$$

b_perp leading order terms

> $b_{\perp 1-} := \frac{\Delta_{\parallel 1-} \cdot sol1}{\mathrm{i} \cdot k_{\parallel +}}$:

> $b_{\perp 4-} := \frac{\Delta_{\parallel 4-} \cdot sol2}{\mathrm{i} \cdot k_{\parallel +}}$:

> $b_{\perp 1+} := \frac{\Delta_{\parallel 1+} \cdot 1}{\mathrm{i} \cdot k_{\parallel +}}$:

> $b_{\perp 2+} := \frac{\Delta_{\parallel 2+} \cdot sol3}{\mathrm{i} \cdot k_{\parallel +}}$:

> $b_{\perp 3+} := \frac{\Delta_{\parallel 3+} \cdot sol4}{\mathrm{i} \cdot k_{\parallel +}}$:

> $simplify(series(b_{\perp 1-}, \epsilon, 2))$

$$\frac{2}{r+1} + \mathrm{O}(\epsilon^2) \tag{86}$$

> $simplify(series(b_{\perp 4-}, \epsilon, 2))$

$$\frac{-\mathrm{I}\,\mathrm{csgn}\left(\frac{k_{\parallel-}}{\epsilon}\right)(\cos(\alpha)-1)(r-1)(\cos(\alpha)+1)}{\cos(\alpha)}\epsilon+\mathrm{O}(\epsilon^2) \quad (87)$$

> $simplify(series(b_{\perp 1+}, \epsilon, 2))$

$$1 \quad (88)$$

> $simplify(series(b_{\perp 2+}, \epsilon, 2))$

$$\frac{-r+1}{r+1}+\mathrm{O}(\epsilon^2) \quad (89)$$

> $simplify(series(b_{\perp 3+}, \epsilon, 2))$

$$\mathrm{O}(\epsilon) \quad (90)$$

b_par leading order terms

> $b_{\parallel 1-} := -\frac{\mathrm{i}\cdot k_x\cdot u_{x1-}+\Delta_{\perp 1-}\cdot sol1}{\mathrm{i}\cdot k_{\parallel +}}:$

> $b_{\parallel 4-} := -\frac{\mathrm{i}\cdot k_x\cdot u_{x4-}+\Delta_{\perp 4-}\cdot sol2}{\mathrm{i}\cdot k_{\parallel +}}:$

> $b_{\parallel 1+} := -\frac{\mathrm{i}\cdot k_x\cdot u_{x1+}+\Delta_{\perp 1+}}{\mathrm{i}\cdot k_{\parallel +}}:$

> $b_{\parallel 2+} := -\frac{\mathrm{i}\cdot k_x\cdot u_{x2+}+\Delta_{\perp 2+}\cdot sol3}{\mathrm{i}\cdot k_{\parallel +}}:$

> $b_{\parallel 3+} := -\frac{\mathrm{i}\cdot k_x\cdot u_{x3+}+\Delta_{\perp 3+}\cdot sol4}{\mathrm{i}\cdot k_{\parallel +}}:$

> $simplify(b_{\parallel 1-})$

$$0 \quad (91)$$

> $simplify(series(expand(b_{\parallel 4-}), \epsilon, 3))$

$$-\mathrm{I}\,(r-1)\sin(\alpha)\,\mathrm{csgn}\left(\frac{k_{\parallel-}}{\epsilon}\right)\epsilon+\mathrm{O}(\epsilon^2) \quad (92)$$

> $simplify(b_{\parallel 1+})$

$$0 \quad (93)$$

> $simplify(b_{\parallel 2+})$

$$0 \quad (94)$$

> $simplify(series(expand(b_{\parallel 3+}), \epsilon, 3))$

$$-\mathrm{I}\,(r-1)\sin(\alpha)\,\mathrm{csgn}\left(\frac{k_{\parallel-}}{\epsilon}\right)\epsilon+\mathrm{O}(\epsilon^2) \quad (95)$$