This document is used to assist with the Algebra in Appendix A.2

Note that ∇ has been replaced with '\Delta' to ensure the code works okay.

> *restart*;

We normalise the velocity coefficents by u_0 and the field components by (B_0 * u_0 / v_{A+}).

We start by solving the Matrix equation given by Equation (57).

> $eqn1 := a_{11} \cdot x_1 + a_{12} \cdot x_2 + a_{13} \cdot x_3 + a_{14} \cdot x_4 = y_1$

$$eqn1 := a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 = y_1 \quad (1)$$

> $eqn2 := a_{21} \cdot x_1 + a_{22} \cdot x_2 + a_{23} \cdot x_3 + a_{24} \cdot x_4 = y_2$

$$eqn2 := a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + a_{24} x_4 = y_2 \quad (2)$$

> $eqn3 := a_{31} \cdot x_1 + a_{32} \cdot x_2 + a_{33} \cdot x_3 + a_{34} \cdot x_4 = y_3$

$$eqn3 := a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + a_{34} x_4 = y_3 \quad (3)$$

> $eqn4 := a_{41} \cdot x_1 + a_{42} \cdot x_2 + a_{43} \cdot x_3 + a_{44} \cdot x_4 = y_4$

$$eqn4 := a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4 = y_4 \quad (4)$$

> $solns := solve(\{eqn1, eqn2, eqn3, eqn4\}, \{x_1, x_2, x_3, x_4\})$:

> $sol1 := rhs(solns[1])$

$$\begin{aligned} sol1 := -\big(& a_{12} a_{23} a_{34} y_4 - a_{12} a_{23} a_{44} y_3 - a_{12} a_{24} a_{33} y_4 + a_{12} a_{24} a_{43} y_3 + a_{12} a_{33} a_{44} y_2 \\ & - a_{12} a_{34} a_{43} y_2 - a_{13} a_{22} a_{34} y_4 + a_{13} a_{22} a_{44} y_3 + a_{13} a_{24} a_{32} y_4 - a_{13} a_{24} a_{42} y_3 \\ & - a_{13} a_{32} a_{44} y_2 + a_{13} a_{34} a_{42} y_2 + a_{14} a_{22} a_{33} y_4 - a_{14} a_{22} a_{43} y_3 - a_{14} a_{23} a_{32} y_4 \\ & + a_{14} a_{23} a_{42} y_3 + a_{14} a_{32} a_{43} y_2 - a_{14} a_{33} a_{42} y_2 - a_{22} a_{33} a_{44} y_1 + a_{22} a_{34} a_{43} y_1 \\ & + a_{23} a_{32} a_{44} y_1 - a_{23} a_{34} a_{42} y_1 - a_{24} a_{32} a_{43} y_1 + a_{24} a_{33} a_{42} y_1\big) \big/ \big(a_{11} a_{22} a_{33} a_{44} \\ & - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} \\ & - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} \\ & + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} \\ & + a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} + a_{14} a_{22} a_{31} a_{43} \\ & - a_{14} a_{22} a_{33} a_{41} - a_{14} a_{23} a_{31} a_{42} + a_{14} a_{23} a_{32} a_{41}\big) \end{aligned} \quad (5)$$

> $sol2 := rhs(solns[2])$

$$\begin{aligned} sol2 := \big(& a_{11} a_{23} a_{34} y_4 - a_{11} a_{23} a_{44} y_3 - a_{11} a_{24} a_{33} y_4 + a_{11} a_{24} a_{43} y_3 + a_{11} a_{33} a_{44} y_2 \\ & - a_{11} a_{34} a_{43} y_2 - a_{13} a_{21} a_{34} y_4 + a_{13} a_{21} a_{44} y_3 + a_{13} a_{24} a_{31} y_4 - a_{13} a_{24} a_{41} y_3 \\ & - a_{13} a_{31} a_{44} y_2 + a_{13} a_{34} a_{41} y_2 + a_{14} a_{21} a_{33} y_4 - a_{14} a_{21} a_{43} y_3 - a_{14} a_{23} a_{31} y_4 \\ & + a_{14} a_{23} a_{41} y_3 + a_{14} a_{31} a_{43} y_2 - a_{14} a_{33} a_{41} y_2 - a_{21} a_{33} a_{44} y_1 + a_{21} a_{34} a_{43} y_1 \\ & + a_{23} a_{31} a_{44} y_1 - a_{23} a_{34} a_{41} y_1 - a_{24} a_{31} a_{43} y_1 + a_{24} a_{33} a_{41} y_1\big) \big/ \big(a_{11} a_{22} a_{33} a_{44} \\ & - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} \\ & - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} \\ & + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} \end{aligned}$$

$$+a_{13}a_{24}a_{31}a_{42}-a_{13}a_{24}a_{32}a_{41}-a_{14}a_{21}a_{32}a_{43}+a_{14}a_{21}a_{33}a_{42}+a_{14}a_{22}a_{31}a_{43}$$
$$-a_{14}a_{22}a_{33}a_{41}-a_{14}a_{23}a_{31}a_{42}+a_{14}a_{23}a_{32}a_{41}\Big)$$

> $sol3 := rhs(solns[3])$

$$sol3 := -\Big(a_{11}a_{22}a_{34}y_4-a_{11}a_{22}a_{44}y_3-a_{11}a_{24}a_{32}y_4+a_{11}a_{24}a_{42}y_3+a_{11}a_{32}a_{44}y_2$$
$$-a_{11}a_{34}a_{42}y_2-a_{12}a_{21}a_{34}y_4+a_{12}a_{21}a_{44}y_3+a_{12}a_{24}a_{31}y_4-a_{12}a_{24}a_{41}y_3$$
$$-a_{12}a_{31}a_{44}y_2+a_{12}a_{34}a_{41}y_2+a_{14}a_{21}a_{32}y_4-a_{14}a_{21}a_{42}y_3-a_{14}a_{22}a_{31}y_4$$
$$+a_{14}a_{22}a_{41}y_3+a_{14}a_{31}a_{42}y_2-a_{14}a_{32}a_{41}y_2-a_{21}a_{32}a_{44}y_1+a_{21}a_{34}a_{42}y_1$$
$$+a_{22}a_{31}a_{44}y_1-a_{22}a_{34}a_{41}y_1-a_{24}a_{31}a_{42}y_1+a_{24}a_{32}a_{41}y_1\Big)\Big/\Big(a_{11}a_{22}a_{33}a_{44}$$
$$-a_{11}a_{22}a_{34}a_{43}-a_{11}a_{23}a_{32}a_{44}+a_{11}a_{23}a_{34}a_{42}+a_{11}a_{24}a_{32}a_{43}-a_{11}a_{24}a_{33}a_{42}$$
$$-a_{12}a_{21}a_{33}a_{44}+a_{12}a_{21}a_{34}a_{43}+a_{12}a_{23}a_{31}a_{44}-a_{12}a_{23}a_{34}a_{41}-a_{12}a_{24}a_{31}a_{43}$$
$$+a_{12}a_{24}a_{33}a_{41}+a_{13}a_{21}a_{32}a_{44}-a_{13}a_{21}a_{34}a_{42}-a_{13}a_{22}a_{31}a_{44}+a_{13}a_{22}a_{34}a_{41}$$
$$+a_{13}a_{24}a_{31}a_{42}-a_{13}a_{24}a_{32}a_{41}-a_{14}a_{21}a_{32}a_{43}+a_{14}a_{21}a_{33}a_{42}+a_{14}a_{22}a_{31}a_{43}$$
$$-a_{14}a_{22}a_{33}a_{41}-a_{14}a_{23}a_{31}a_{42}+a_{14}a_{23}a_{32}a_{41}\Big) \tag{7}$$

> $sol4 := rhs(solns[4])$

$$sol4 := \Big(a_{11}a_{22}a_{33}y_4-a_{11}a_{22}a_{43}y_3-a_{11}a_{23}a_{32}y_4+a_{11}a_{23}a_{42}y_3+a_{11}a_{32}a_{43}y_2$$
$$-a_{11}a_{33}a_{42}y_2-a_{12}a_{21}a_{33}y_4+a_{12}a_{21}a_{43}y_3+a_{12}a_{23}a_{31}y_4-a_{12}a_{23}a_{41}y_3$$
$$-a_{12}a_{31}a_{43}y_2+a_{12}a_{33}a_{41}y_2+a_{13}a_{21}a_{32}y_4-a_{13}a_{21}a_{42}y_3-a_{13}a_{22}a_{31}y_4$$
$$+a_{13}a_{22}a_{41}y_3+a_{13}a_{31}a_{42}y_2-a_{13}a_{32}a_{41}y_2-a_{21}a_{32}a_{43}y_1+a_{21}a_{33}a_{42}y_1$$
$$+a_{22}a_{31}a_{43}y_1-a_{22}a_{33}a_{41}y_1-a_{23}a_{31}a_{42}y_1+a_{23}a_{32}a_{41}y_1\Big)\Big/\Big(a_{11}a_{22}a_{33}a_{44}$$
$$-a_{11}a_{22}a_{34}a_{43}-a_{11}a_{23}a_{32}a_{44}+a_{11}a_{23}a_{34}a_{42}+a_{11}a_{24}a_{32}a_{43}-a_{11}a_{24}a_{33}a_{42}$$
$$-a_{12}a_{21}a_{33}a_{44}+a_{12}a_{21}a_{34}a_{43}+a_{12}a_{23}a_{31}a_{44}-a_{12}a_{23}a_{34}a_{41}-a_{12}a_{24}a_{31}a_{43}$$
$$+a_{12}a_{24}a_{33}a_{41}+a_{13}a_{21}a_{32}a_{44}-a_{13}a_{21}a_{34}a_{42}-a_{13}a_{22}a_{31}a_{44}+a_{13}a_{22}a_{34}a_{41}$$
$$+a_{13}a_{24}a_{31}a_{42}-a_{13}a_{24}a_{32}a_{41}-a_{14}a_{21}a_{32}a_{43}+a_{14}a_{21}a_{33}a_{42}+a_{14}a_{22}a_{31}a_{43}$$
$$-a_{14}a_{22}a_{33}a_{41}-a_{14}a_{23}a_{31}a_{42}+a_{14}a_{23}a_{32}a_{41}\Big) \tag{8}$$

> $a_{11} := u_{x10-}$

$$a_{11} := u_{x10\text{-}} \tag{9}$$

> $a_{12} := u_{x40-}$

$$a_{12} := u_{x40\text{-}} \tag{10}$$

> $a_{13} := -u_{x20+}$

$$a_{13} := -u_{x20+} \tag{11}$$

> $a_{14} := -u_{x30+}$

$$a_{14} := -u_{x30+} \tag{12}$$

> $a_{21} := k_{z1-}\cdot u_{x10-}$

(13)

$$a_{21} := k_{z1-}\, u_{x10-} \quad (13)$$

> $a_{22} := k_{z4-} \cdot u_{x40-}$

$$a_{22} := k_{z4-}\, u_{x40-} \quad (14)$$

> $a_{23} := -k_{z2+} \cdot u_{x20+}$

$$a_{23} := -k_{z2+}\, u_{x20+} \quad (15)$$

> $a_{24} := -k_{z3+} \cdot u_{x30+}$

$$a_{24} := -k_{z3+}\, u_{x30+} \quad (16)$$

> $a_{31} := 1$

$$a_{31} := 1 \quad (17)$$

> $a_{32} := 1$

$$a_{32} := 1 \quad (18)$$

> $a_{33} := -1$

$$a_{33} := -1 \quad (19)$$

> $a_{34} := -1$

$$a_{34} := -1 \quad (20)$$

> $a_{41} := k_{z1-}$

$$a_{41} := k_{z1-} \quad (21)$$

> $a_{42} := k_{z4-}$

$$a_{42} := k_{z4-} \quad (22)$$

> $a_{43} := -k_{z2+}$

$$a_{43} := -k_{z2+} \quad (23)$$

> $a_{44} := -k_{z3+}$

$$a_{44} := -k_{z3+} \quad (24)$$

> $y_1 := u_{x10+}$

$$y_1 := u_{x10+} \quad (25)$$

> $y_2 := k_{z1+} \cdot u_{x10+}$

$$y_2 := k_{z1+}\, u_{x10+} \quad (26)$$

> $y_3 := 1$

$$y_3 := 1 \quad (27)$$

> $y_4 := k_{z1+}$

$$y_4 := k_{z1+} \quad (28)$$

Note that ux_0[n] denotes \hat{u}_{xn} and u_x[n±] denotes u_{xn±}.

> $u_{x10-} := -\frac{\mathrm{i} \cdot k_x \cdot \Delta_{\perp 1-}}{L_{1-} - k_x^2}$

$$u_{x10-} := \frac{-\mathrm{I}\, k_x\, \Delta_{\perp 1\text{-}}}{-k_x^2 + L_{1\text{-}}} \quad \textbf{(29)}$$

> $u_{x10+} := -\frac{\mathrm{i} \cdot k_x \cdot \Delta_{\perp 1+}}{L_{1+} - k_x^2}$

$$u_{x10+} := \frac{-\mathrm{I}\, k_x\, \Delta_{\perp 1+}}{-k_x^2 + L_{1+}} \quad \textbf{(30)}$$

> $u_{x20-} := -\frac{\mathrm{i} \cdot k_x \cdot \Delta_{\perp 2-}}{L_{2-} - k_x^2}$

$$u_{x20-} := \frac{-\mathrm{I}\, k_x\, \Delta_{\perp 2\text{-}}}{-k_x^2 + L_{2\text{-}}} \quad \textbf{(31)}$$

> $u_{x20+} := -\frac{\mathrm{i} \cdot k_x \cdot \Delta_{\perp 2+}}{L_{2+} - k_x^2}$

$$u_{x20+} := \frac{-\mathrm{I}\, k_x\, \Delta_{\perp 2+}}{-k_x^2 + L_{2+}} \quad \textbf{(32)}$$

> $u_{x30-} := -\frac{\mathrm{i} \cdot k_x \cdot \Delta_{\perp 3-}}{L_{3-} - k_x^2}$

$$u_{x30-} := \frac{-\mathrm{I}\, k_x\, \Delta_{\perp 3\text{-}}}{-k_x^2 + L_{3\text{-}}} \quad \textbf{(33)}$$

> $u_{x30+} := -\frac{\mathrm{i} \cdot k_x \cdot \Delta_{\perp 3+}}{L_{3+} - k_x^2}$

$$u_{x30+} := \frac{-\mathrm{I}\, k_x\, \Delta_{\perp 3+}}{-k_x^2 + L_{3+}} \quad \textbf{(34)}$$

> $u_{x40-} := -\frac{\mathrm{i} \cdot k_x \cdot \Delta_{\perp 4-}}{L_{4-} - k_x^2}$

$$u_{x40-} := \frac{-\mathrm{I}\, k_x\, \Delta_{\perp 4\text{-}}}{-k_x^2 + L_{4\text{-}}} \quad \textbf{(35)}$$

> $u_{x40+} := -\frac{\mathrm{i} \cdot k_x \cdot \Delta_{\perp 4+}}{L_{4+} - k_x^2}$

$$u_{x40+} := \frac{-\mathrm{I}\, k_x\, \Delta_{\perp 4+}}{-k_x^2 + L_{4+}} \tag{36}$$

> $L_{1-} := \Delta_{\|1-}^2 + k_{\|-}^2$

$$L_{1-} := k_{\|-}^2 + \Delta_{\|1-}^2 \tag{37}$$

> $L_{1+} := \Delta_{\|1+}^2 + k_{\|+}^2$

$$L_{1+} := k_{\|+}^2 + \Delta_{\|1+}^2 \tag{38}$$

> $L_{2-} := \Delta_{\|2-}^2 + k_{\|-}^2$

$$L_{2-} := k_{\|-}^2 + \Delta_{\|2-}^2 \tag{39}$$

> $L_{2+} := \Delta_{\|2+}^2 + k_{\|+}^2$

$$L_{2+} := k_{\|+}^2 + \Delta_{\|2+}^2 \tag{40}$$

> $L_{3-} := \Delta_{\|3-}^2 + k_{\|-}^2$

$$L_{3-} := k_{\|-}^2 + \Delta_{\|3-}^2 \tag{41}$$

> $L_{3+} := \Delta_{\|3+}^2 + k_{\|+}^2$

$$L_{3+} := k_{\|+}^2 + \Delta_{\|3+}^2 \tag{42}$$

> $L_{4-} := \Delta_{\|4-}^2 + k_{\|-}^2$

$$L_{4-} := k_{\|-}^2 + \Delta_{\|4-}^2 \tag{43}$$

> $L_{4+} := \Delta_{\|4+}^2 + k_{\|+}^2$

$$L_{4+} := k_{\|+}^2 + \Delta_{\|4+}^2 \tag{44}$$

> $\Delta_{\perp 1-} := \mathrm{i} \cdot \left(k_y \cdot \cos(\alpha) - k_{z1-} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 1-} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z1-} \sin(\alpha)\right) \tag{45}$$

> $\Delta_{\perp 1+} := \mathrm{i} \cdot \left(k_y \cdot \cos(\alpha) - k_{z1+} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 1+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z1+} \sin(\alpha)\right) \tag{46}$$

> $\Delta_{\perp 2-} := \mathrm{i} \cdot \left(k_y \cdot \cos(\alpha) - k_{z2-} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 2-} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z2-} \sin(\alpha)\right) \tag{47}$$

> $\Delta_{\perp 2+} := \mathrm{i} \cdot \left(k_y \cdot \cos(\alpha) - k_{z2+} \cdot \sin(\alpha)\right)$

(48)

$$\Delta_{\perp 2+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z2+} \sin(\alpha)\right) \quad \textbf{(48)}$$

> $\Delta_{\perp 3-} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z3-}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 3\text{-}} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z3\text{-}} \sin(\alpha)\right) \quad \textbf{(49)}$$

> $\Delta_{\perp 3+} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z3+}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 3+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z3+} \sin(\alpha)\right) \quad \textbf{(50)}$$

> $\Delta_{\perp 4-} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z4-}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 4\text{-}} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z4\text{-}} \sin(\alpha)\right) \quad \textbf{(51)}$$

> $\Delta_{\perp 4+} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z4+}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 4+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z4+} \sin(\alpha)\right) \quad \textbf{(52)}$$

> $\Delta_{\parallel 1-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z1-}\cdot\cos(\alpha)\right)$

$$\Delta_{\parallel 1\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z1\text{-}} \cos(\alpha)\right) \quad \textbf{(53)}$$

> $\Delta_{\parallel 1+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z1+}\cdot\cos(\alpha)\right)$

$$\Delta_{\parallel 1+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z1+} \cos(\alpha)\right) \quad \textbf{(54)}$$

> $\Delta_{\parallel 2-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z2-}\cdot\cos(\alpha)\right)$

$$\Delta_{\parallel 2\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z2\text{-}} \cos(\alpha)\right) \quad \textbf{(55)}$$

> $\Delta_{\parallel 2+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z2+}\cdot\cos(\alpha)\right)$

$$\Delta_{\parallel 2+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z2+} \cos(\alpha)\right) \quad \textbf{(56)}$$

> $\Delta_{\parallel 3-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z3-}\cdot\cos(\alpha)\right)$

$$\Delta_{\parallel 3\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z3\text{-}} \cos(\alpha)\right) \quad \textbf{(57)}$$

> $\Delta_{\parallel 3+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z3+}\cdot\cos(\alpha)\right)$

$$\Delta_{\parallel 3+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z3+} \cos(\alpha)\right) \quad \textbf{(58)}$$

> $\Delta_{\parallel 4-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z4-}\cdot\cos(\alpha)\right)$

$$\Delta_{\parallel 4\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z4\text{-}} \cos(\alpha)\right) \quad \textbf{(59)}$$

> $\Delta_{\parallel 4+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z4+}\cdot\cos(\alpha)\right)$

$$\Delta_{\parallel 4+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z4+} \cos(\alpha)\right) \quad \textbf{(60)}$$

> $k_{z1-} := \dfrac{k_{\parallel -}}{\cos(\alpha)} - k_y\cdot\tan(\alpha)$

$$k_{z1\text{-}} := \frac{k_{\parallel\text{-}}}{\cos(\alpha)} - k_y \tan(\alpha) \quad \textbf{(61)}$$

> $k_{z1+} := \dfrac{k_{\parallel +}}{\cos(\alpha)} - k_y\cdot\tan(\alpha)$

$$k_{z1+} := \frac{k_{||+}}{\cos(\alpha)} - k_y \tan(\alpha) \quad \textbf{(62)}$$

> $k_{z2-} := -\frac{k_{||-}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z2\text{-}} := -\frac{k_{||\text{-}}}{\cos(\alpha)} - k_y \tan(\alpha) \quad \textbf{(63)}$$

> $k_{z2+} := -\frac{k_{||+}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z2+} := -\frac{k_{||+}}{\cos(\alpha)} - k_y \tan(\alpha) \quad \textbf{(64)}$$

> $k_{z3-} := \mathrm{i} \cdot \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||-}^{\,2}}$

$$k_{z3\text{-}} := \mathrm{I} \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||\text{-}}^{\,2}} \quad \textbf{(65)}$$

> $k_{z3+} := \mathrm{i} \cdot \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||+}^{\,2}}$

$$k_{z3+} := \mathrm{I} \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||+}^{\,2}} \quad \textbf{(66)}$$

> $k_{z4-} := -\mathrm{i} \cdot \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||-}^{\,2}}$

$$k_{z4\text{-}} := -\mathrm{I} \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||\text{-}}^{\,2}} \quad \textbf{(67)}$$

> $k_{z4+} := -\mathrm{i} \cdot \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||+}^{\,2}}$

$$k_{z4+} := -\mathrm{I} \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||+}^{\,2}} \quad \textbf{(68)}$$

>

> $k_{||+} := r \cdot k_{||-}$

$$k_{||+} := r\, k_{||\text{-}} \quad \textbf{(69)}$$

> $k_y := a_y \cdot k_{||+}$

$$k_y := a_y\, r\, k_{||\text{-}} \quad \textbf{(70)}$$

> $k_x := \frac{k_{||-}}{\epsilon}$

$$k_x := \frac{k_{||\text{-}}}{\epsilon} \quad \textbf{(71)}$$

ux leading order terms

> $u_{x1-} := u_{x10-} \cdot sol1 :$

> $u_{x4-} := u_{x40-} \cdot sol2 :$

> $u_{x1+} := u_{x10+} :$

> $u_{x2+} := u_{x20+} \cdot sol3 :$

> $u_{x3+} := u_{x30+} \cdot sol4 :$

> $simplify(series(u_{x1-}, \epsilon, 2))$

$$\frac{2\,r\,(-a_y\,r+\sin(\alpha))}{\cos(\alpha)\,(r+1)}\,\epsilon+O(\epsilon^2) \tag{72}$$

> $simplify(series(u_{x4-}, \epsilon, 3))$

$$\frac{\mathrm{I}\sin(\alpha)\,r\,(r-1)\,\mathrm{csgn}\left(\frac{k_{\|-}}{\epsilon}\right)}{\cos(\alpha)^2}\,\epsilon^2+O(\epsilon^3) \tag{73}$$

> $simplify(series(u_{x1+}, \epsilon, 3))$

$$\frac{r\,(-a_y+\sin(\alpha))}{\cos(\alpha)}\,\epsilon+O(\epsilon^3) \tag{74}$$

> $simplify(series(u_{x2+}, \epsilon, 3))$

$$-\frac{r\,(r-1)\,(a_y+\sin(\alpha))}{\cos(\alpha)\,(r+1)}\,\epsilon+O(\epsilon^3) \tag{75}$$

> $simplify(series(u_{x3+}, \epsilon, 3))$

$$\frac{\mathrm{I}\sin(\alpha)\,r\,(r-1)\,\mathrm{csgn}\left(\frac{k_{\|-}}{\epsilon}\right)}{\cos(\alpha)^2}\,\epsilon^2+O(\epsilon^3) \tag{76}$$

u_perp leading order terms

> $simplify(series(sol1, \epsilon, 3))$

$$\frac{2\,r}{r+1}+2\,\frac{r^2\,(r-1)\,(\cos(\alpha)-1)\,(\cos(\alpha)+1)\,(a_y-\sin(\alpha))}{\cos(\alpha)^2\,\sin(\alpha)\,(r+1)}\,\epsilon^2+O(\epsilon^3) \tag{77}$$

> $simplify(series(sol2, \epsilon, 3))$

$$\frac{(\cos(\alpha)-1)\,(\cos(\alpha)+1)\,(r-1)\,r}{\cos(\alpha)^2}\,\epsilon^2+O(\epsilon^3) \tag{78}$$

> $simplify(series(sol3, \epsilon, 2))$

$$\frac{r-1}{r+1}+O(\epsilon^2) \tag{79}$$

> $simplify(series(sol4, \epsilon, 4))$

$$-\frac{(\cos(\alpha)-1)\,(\cos(\alpha)+1)\,(r-1)\,r}{\cos(\alpha)^2}\,\epsilon^2 + \frac{\mathrm{I}\,r^2\,a_y\,(\cos(\alpha)^4-3\cos(\alpha)^2+2)\,(r-1)\,\mathrm{csgn}\left(\frac{k_{\|-}}{\epsilon}\right)}{\sin(\alpha)\,\cos(\alpha)^3}\,\epsilon^3+O(\epsilon^4) \tag{80}$$

bx leading order terms

> $b_{x1-} := \dfrac{\Delta_{||1-} \cdot u_{x1-}}{\mathrm{i} \cdot k_{||+}}$:

> $b_{x4-} := \dfrac{\Delta_{||4-} \cdot u_{x4-}}{\mathrm{i} \cdot k_{||+}}$:

> $b_{x1+} := \dfrac{\Delta_{||1+} \cdot u_{x1+}}{\mathrm{i} \cdot k_{||+}}$:

> $b_{x2+} := \dfrac{\Delta_{||2+} \cdot u_{x2+}}{\mathrm{i} \cdot k_{||+}}$:

> $b_{x3+} := \dfrac{\Delta_{||3+} \cdot u_{x3+}}{\mathrm{i} \cdot k_{||+}}$:

> $simplify(series(b_{x1-}, \epsilon, 2))$

$$\frac{-2\, a_y\, r + 2 \sin(\alpha)}{\cos(\alpha)\, (r+1)}\, \epsilon + \mathrm{O}(\epsilon^2) \tag{81}$$

> $simplify(series(b_{x4-}, \epsilon, 2))$

$$\frac{\sin(\alpha)\, (r-1)}{\cos(\alpha)}\, \epsilon + \mathrm{O}(\epsilon^2) \tag{82}$$

> $simplify(series(b_{x1+}, \epsilon, 2))$

$$\frac{r\, \left(-a_y + \sin(\alpha)\right)}{\cos(\alpha)}\, \epsilon + \mathrm{O}(\epsilon^3) \tag{83}$$

> $simplify(series(b_{x2+}, \epsilon, 2))$

$$\frac{r\, (r-1)\, \left(a_y + \sin(\alpha)\right)}{\cos(\alpha)\, (r+1)}\, \epsilon + \mathrm{O}(\epsilon^2) \tag{84}$$

> $simplify(series(b_{x3+}, \epsilon, 2))$

$$-\frac{\sin(\alpha)\, (r-1)}{\cos(\alpha)}\, \epsilon + \mathrm{O}(\epsilon^2) \tag{85}$$

b_perp leading order terms

> $b_{\perp 1-} := \dfrac{\Delta_{||1-} \cdot sol1}{\mathrm{i} \cdot k_{||+}}$:

> $b_{\perp 4-} := \dfrac{\Delta_{||4-} \cdot sol2}{\mathrm{i} \cdot k_{||+}}$:

> $b_{\perp 1+} := \dfrac{\Delta_{||1+} \cdot 1}{\mathrm{i} \cdot k_{||+}}$:

> $b_{\perp 2+} := \dfrac{\Delta_{\|2+} \cdot sol3}{\mathrm{i} \cdot k_{\|+}}$:

> $b_{\perp 3+} := \dfrac{\Delta_{\|3+} \cdot sol4}{\mathrm{i} \cdot k_{\|+}}$:

> $simplify(series(b_{\perp 1-}, \epsilon, 2))$

$$\frac{2}{r+1} + \mathrm{O}(\epsilon^2) \tag{86}$$

> $simplify(series(b_{\perp 4-}, \epsilon, 2))$

$$\frac{-\mathrm{I}\,\mathrm{csgn}\left(\frac{k_{\|-}}{\epsilon}\right) (\cos(\alpha) - 1)\,(r-1)\,(\cos(\alpha) + 1)}{\cos(\alpha)}\,\epsilon + \mathrm{O}(\epsilon^2) \tag{87}$$

> $simplify(series(b_{\perp 1+}, \epsilon, 2))$

$$1 \tag{88}$$

> $simplify(series(b_{\perp 2+}, \epsilon, 2))$

$$\frac{-r+1}{r+1} + \mathrm{O}(\epsilon^2) \tag{89}$$

> $simplify(series(b_{\perp 3+}, \epsilon, 2))$

$$\mathrm{O}(\epsilon) \tag{90}$$

b_par leading order terms

> $b_{\|1-} := -\dfrac{\mathrm{i} \cdot k_x \cdot u_{x1-} + \Delta_{\perp 1-} \cdot sol1}{\mathrm{i} \cdot k_{\|+}}$:

> $b_{\|4-} := -\dfrac{\mathrm{i} \cdot k_x \cdot u_{x4-} + \Delta_{\perp 4-} \cdot sol2}{\mathrm{i} \cdot k_{\|+}}$:

> $b_{\|1+} := -\dfrac{\mathrm{i} \cdot k_x \cdot u_{x1+} + \Delta_{\perp 1+}}{\mathrm{i} \cdot k_{\|+}}$:

> $b_{\|2+} := -\dfrac{\mathrm{i} \cdot k_x \cdot u_{x2+} + \Delta_{\perp 2+} \cdot sol3}{\mathrm{i} \cdot k_{\|+}}$:

> $b_{\|3+} := -\dfrac{\mathrm{i} \cdot k_x \cdot u_{x3+} + \Delta_{\perp 3+} \cdot sol4}{\mathrm{i} \cdot k_{\|+}}$:

> $simplify(b_{\|1-})$

$$0 \tag{91}$$

> $simplify(series(expand(b_{\|4-}), \epsilon, 3))$

$$-\mathrm{I}\,(r-1)\,\sin(\alpha)\,\mathrm{csgn}\left(\frac{k_{\|-}}{\epsilon}\right)\epsilon + \mathrm{O}(\epsilon^2) \tag{92}$$

> $simplify(b_{\|1+})$

$$0 \tag{93}$$

> $simplify(b_{||2+})$

$$0 \tag{94}$$

> $simplify(series(expand(b_{||3+}), \epsilon, 3))$

$$-\mathrm{I}\,(r-1)\,\sin(\alpha)\,\mathrm{csgn}\left(\frac{k_{||-}}{\epsilon}\right)\epsilon + \mathrm{O}(\epsilon^2) \tag{95}$$