This document is used to assist with the Algebra in Appendix A.2

Note that ∇ has been replaced with to ensure the code works okay.

> *restart*;

We normalise the velocity coefficents by u_0 and the field components by (B_0 * u_0 / v_{A+}).

We start by solving the Matrix equation given by Equation (57).

> $eqn1 := a_{11} \cdot x_1 + a_{12} \cdot x_2 + a_{13} \cdot x_3 + a_{14} \cdot x_4 = y_1$

$$eqn1 := a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 = y_1 \tag{1}$$

> $eqn2 := a_{21} \cdot x_1 + a_{22} \cdot x_2 + a_{23} \cdot x_3 + a_{24} \cdot x_4 = y_2$

$$eqn2 := a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + a_{24} x_4 = y_2 \tag{2}$$

> $eqn3 := a_{31} \cdot x_1 + a_{32} \cdot x_2 + a_{33} \cdot x_3 + a_{34} \cdot x_4 = y_3$

$$eqn3 := a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + a_{34} x_4 = y_3 \tag{3}$$

> $eqn4 := a_{41} \cdot x_1 + a_{42} \cdot x_2 + a_{43} \cdot x_3 + a_{44} \cdot x_4 = y_4$

$$eqn4 := a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4 = y_4 \tag{4}$$

> $solns := solve\left(\{eqn1, eqn2, eqn3, eqn4\}, \{x_1, x_2, x_3, x_4\}\right):$

> $sol1 := rhs(solns[1])$

$$\begin{aligned} sol1 := -\big(& a_{12} a_{23} a_{34} y_4 - a_{12} a_{23} a_{44} y_3 - a_{12} a_{24} a_{33} y_4 + a_{12} a_{24} a_{43} y_3 + a_{12} a_{33} a_{44} y_2 \\ & - a_{12} a_{34} a_{43} y_2 - a_{13} a_{22} a_{34} y_4 + a_{13} a_{22} a_{44} y_3 + a_{13} a_{24} a_{32} y_4 - a_{13} a_{24} a_{42} y_3 \\ & - a_{13} a_{32} a_{44} y_2 + a_{13} a_{34} a_{42} y_2 + a_{14} a_{22} a_{33} y_4 - a_{14} a_{22} a_{43} y_3 - a_{14} a_{23} a_{32} y_4 \\ & + a_{14} a_{23} a_{42} y_3 + a_{14} a_{32} a_{43} y_2 - a_{14} a_{33} a_{42} y_2 - a_{22} a_{33} a_{44} y_1 + a_{22} a_{34} a_{43} y_1 \\ & + a_{23} a_{32} a_{44} y_1 - a_{23} a_{34} a_{42} y_1 - a_{24} a_{32} a_{43} y_1 + a_{24} a_{33} a_{42} y_1\big) \big/ \big(a_{11} a_{22} a_{33} a_{44} \\ & - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} \\ & - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} \\ & + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} \\ & + a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} + a_{14} a_{22} a_{31} a_{43} \\ & - a_{14} a_{22} a_{33} a_{41} - a_{14} a_{23} a_{31} a_{42} + a_{14} a_{23} a_{32} a_{41}\big) \end{aligned} \tag{5}$$

> $sol2 := rhs(solns[2])$

$$\begin{aligned} sol2 := \big(& a_{11} a_{23} a_{34} y_4 - a_{11} a_{23} a_{44} y_3 - a_{11} a_{24} a_{33} y_4 + a_{11} a_{24} a_{43} y_3 + a_{11} a_{33} a_{44} y_2 \\ & - a_{11} a_{34} a_{43} y_2 - a_{13} a_{21} a_{34} y_4 + a_{13} a_{21} a_{44} y_3 + a_{13} a_{24} a_{31} y_4 - a_{13} a_{24} a_{41} y_3 \\ & - a_{13} a_{31} a_{44} y_2 + a_{13} a_{34} a_{41} y_2 + a_{14} a_{21} a_{33} y_4 - a_{14} a_{21} a_{43} y_3 - a_{14} a_{23} a_{31} y_4 \\ & + a_{14} a_{23} a_{41} y_3 + a_{14} a_{31} a_{43} y_2 - a_{14} a_{33} a_{41} y_2 - a_{21} a_{33} a_{44} y_1 + a_{21} a_{34} a_{43} y_1 \\ & + a_{23} a_{31} a_{44} y_1 - a_{23} a_{34} a_{41} y_1 - a_{24} a_{31} a_{43} y_1 + a_{24} a_{33} a_{41} y_1\big) \big/ \big(a_{11} a_{22} a_{33} a_{44} \\ & - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} \\ & - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} \\ & + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} \end{aligned} \tag{6}$$

$$+a_{13}a_{24}a_{31}a_{42}-a_{13}a_{24}a_{32}a_{41}-a_{14}a_{21}a_{32}a_{43}+a_{14}a_{21}a_{33}a_{42}+a_{14}a_{22}a_{31}a_{43}-a_{14}a_{22}a_{33}a_{41}-a_{14}a_{23}a_{31}a_{42}+a_{14}a_{23}a_{32}a_{41}\Big)$$

> $sol3 := rhs(solns[3])$

$$sol3 := -\Big(a_{11}a_{22}a_{34}y_4-a_{11}a_{22}a_{44}y_3-a_{11}a_{24}a_{32}y_4+a_{11}a_{24}a_{42}y_3+a_{11}a_{32}a_{44}y_2-a_{11}a_{34}a_{42}y_2-a_{12}a_{21}a_{34}y_4+a_{12}a_{21}a_{44}y_3+a_{12}a_{24}a_{31}y_4-a_{12}a_{24}a_{41}y_3-a_{12}a_{31}a_{44}y_2+a_{12}a_{34}a_{41}y_2+a_{14}a_{21}a_{32}y_4-a_{14}a_{21}a_{42}y_3-a_{14}a_{22}a_{31}y_4+a_{14}a_{22}a_{41}y_3+a_{14}a_{31}a_{42}y_2-a_{14}a_{32}a_{41}y_2-a_{21}a_{32}a_{44}y_1+a_{21}a_{34}a_{42}y_1+a_{22}a_{31}a_{44}y_1-a_{22}a_{34}a_{41}y_1-a_{24}a_{31}a_{42}y_1+a_{24}a_{32}a_{41}y_1\Big)\Big/\Big(a_{11}a_{22}a_{33}a_{44}-a_{11}a_{22}a_{34}a_{43}-a_{11}a_{23}a_{32}a_{44}+a_{11}a_{23}a_{34}a_{42}+a_{11}a_{24}a_{32}a_{43}-a_{11}a_{24}a_{33}a_{42}-a_{12}a_{21}a_{33}a_{44}+a_{12}a_{21}a_{34}a_{43}+a_{12}a_{23}a_{31}a_{44}-a_{12}a_{23}a_{34}a_{41}-a_{12}a_{24}a_{31}a_{43}+a_{12}a_{24}a_{33}a_{41}+a_{13}a_{21}a_{32}a_{44}-a_{13}a_{21}a_{34}a_{42}-a_{13}a_{22}a_{31}a_{44}+a_{13}a_{22}a_{34}a_{41}+a_{13}a_{24}a_{31}a_{42}-a_{13}a_{24}a_{32}a_{41}-a_{14}a_{21}a_{32}a_{43}+a_{14}a_{21}a_{33}a_{42}+a_{14}a_{22}a_{31}a_{43}-a_{14}a_{22}a_{33}a_{41}-a_{14}a_{23}a_{31}a_{42}+a_{14}a_{23}a_{32}a_{41}\Big) \tag{7}$$

> $sol4 := rhs(solns[4])$

$$sol4 := \Big(a_{11}a_{22}a_{33}y_4-a_{11}a_{22}a_{43}y_3-a_{11}a_{23}a_{32}y_4+a_{11}a_{23}a_{42}y_3+a_{11}a_{32}a_{43}y_2-a_{11}a_{33}a_{42}y_2-a_{12}a_{21}a_{33}y_4+a_{12}a_{21}a_{43}y_3+a_{12}a_{23}a_{31}y_4-a_{12}a_{23}a_{41}y_3-a_{12}a_{31}a_{43}y_2+a_{12}a_{33}a_{41}y_2+a_{13}a_{21}a_{32}y_4-a_{13}a_{21}a_{42}y_3-a_{13}a_{22}a_{31}y_4+a_{13}a_{22}a_{41}y_3+a_{13}a_{31}a_{42}y_2-a_{13}a_{32}a_{41}y_2-a_{21}a_{32}a_{43}y_1+a_{21}a_{33}a_{42}y_1+a_{22}a_{31}a_{43}y_1-a_{22}a_{33}a_{41}y_1-a_{23}a_{31}a_{42}y_1+a_{23}a_{32}a_{41}y_1\Big)\Big/\Big(a_{11}a_{22}a_{33}a_{44}-a_{11}a_{22}a_{34}a_{43}-a_{11}a_{23}a_{32}a_{44}+a_{11}a_{23}a_{34}a_{42}+a_{11}a_{24}a_{32}a_{43}-a_{11}a_{24}a_{33}a_{42}-a_{12}a_{21}a_{33}a_{44}+a_{12}a_{21}a_{34}a_{43}+a_{12}a_{23}a_{31}a_{44}-a_{12}a_{23}a_{34}a_{41}-a_{12}a_{24}a_{31}a_{43}+a_{12}a_{24}a_{33}a_{41}+a_{13}a_{21}a_{32}a_{44}-a_{13}a_{21}a_{34}a_{42}-a_{13}a_{22}a_{31}a_{44}+a_{13}a_{22}a_{34}a_{41}+a_{13}a_{24}a_{31}a_{42}-a_{13}a_{24}a_{32}a_{41}-a_{14}a_{21}a_{32}a_{43}+a_{14}a_{21}a_{33}a_{42}+a_{14}a_{22}a_{31}a_{43}-a_{14}a_{22}a_{33}a_{41}-a_{14}a_{23}a_{31}a_{42}+a_{14}a_{23}a_{32}a_{41}\Big) \tag{8}$$

> $a_{11} := u_{x10-}$

$$a_{11} := u_{x10\text{-}} \tag{9}$$

> $a_{12} := u_{x40-}$

$$a_{12} := u_{x40\text{-}} \tag{10}$$

> $a_{13} := -u_{x20+}$

$$a_{13} := -u_{x20+} \tag{11}$$

> $a_{14} := -u_{x30+}$

$$a_{14} := -u_{x30+} \tag{12}$$

> $a_{21} := k_{z1-} \cdot u_{x10-}$

(13)

$$a_{21} := k_{z1-}\, u_{x10-} \tag{13}$$

> $a_{22} := k_{z4-} \cdot u_{x40-}$

$$a_{22} := k_{z4-}\, u_{x40-} \tag{14}$$

> $a_{23} := -k_{z2+} \cdot u_{x20+}$

$$a_{23} := -k_{z2+}\, u_{x20+} \tag{15}$$

> $a_{24} := -k_{z3+} \cdot u_{x30+}$

$$a_{24} := -k_{z3+}\, u_{x30+} \tag{16}$$

> $a_{31} := 1$

$$a_{31} := 1 \tag{17}$$

> $a_{32} := 1$

$$a_{32} := 1 \tag{18}$$

> $a_{33} := -1$

$$a_{33} := -1 \tag{19}$$

> $a_{34} := -1$

$$a_{34} := -1 \tag{20}$$

> $a_{41} := k_{z1-}$

$$a_{41} := k_{z1-} \tag{21}$$

> $a_{42} := k_{z4-}$

$$a_{42} := k_{z4-} \tag{22}$$

> $a_{43} := -k_{z2+}$

$$a_{43} := -k_{z2+} \tag{23}$$

> $a_{44} := -k_{z3+}$

$$a_{44} := -k_{z3+} \tag{24}$$

> $y_1 := u_{x10+}$

$$y_1 := u_{x10+} \tag{25}$$

> $y_2 := k_{z1+} \cdot u_{x10+}$

$$y_2 := k_{z1+}\, u_{x10+} \tag{26}$$

> $y_3 := 1$

$$y_3 := 1 \tag{27}$$

> $y_4 := k_{z1+}$

$$y_4 := k_{z1+} \tag{28}$$

Note that ux_0[n] denotes \hat{u}_{xn} and u_x[n±] denotes u_{xn±}.

> $u_{x10-} := -\frac{i \cdot k_x \cdot \Delta_{\perp 1-}}{L_{1-} - k_x^2}$

$$u_{x10\text{-}} := \frac{-I\, k_x\, \Delta_{\perp 1\text{-}}}{-k_x^2 + L_{1\text{-}}} \quad (29)$$

> $u_{x10+} := -\frac{i \cdot k_x \cdot \Delta_{\perp 1+}}{L_{1+} - k_x^2}$

$$u_{x10+} := \frac{-I\, k_x\, \Delta_{\perp 1+}}{-k_x^2 + L_{1+}} \quad (30)$$

> $u_{x20-} := -\frac{i \cdot k_x \cdot \Delta_{\perp 2-}}{L_{2-} - k_x^2}$

$$u_{x20\text{-}} := \frac{-I\, k_x\, \Delta_{\perp 2\text{-}}}{-k_x^2 + L_{2\text{-}}} \quad (31)$$

> $u_{x20+} := -\frac{i \cdot k_x \cdot \Delta_{\perp 2+}}{L_{2+} - k_x^2}$

$$u_{x20+} := \frac{-I\, k_x\, \Delta_{\perp 2+}}{-k_x^2 + L_{2+}} \quad (32)$$

> $u_{x30-} := -\frac{i \cdot k_x \cdot \Delta_{\perp 3-}}{L_{3-} - k_x^2}$

$$u_{x30\text{-}} := \frac{-I\, k_x\, \Delta_{\perp 3\text{-}}}{-k_x^2 + L_{3\text{-}}} \quad (33)$$

> $u_{x30+} := -\frac{i \cdot k_x \cdot \Delta_{\perp 3+}}{L_{3+} - k_x^2}$

$$u_{x30+} := \frac{-I\, k_x\, \Delta_{\perp 3+}}{-k_x^2 + L_{3+}} \quad (34)$$

> $u_{x40-} := -\frac{i \cdot k_x \cdot \Delta_{\perp 4-}}{L_{4-} - k_x^2}$

$$u_{x40\text{-}} := \frac{-I\, k_x\, \Delta_{\perp 4\text{-}}}{-k_x^2 + L_{4\text{-}}} \quad (35)$$

> $u_{x40+} := -\frac{\mathrm{i}\cdot k_x\cdot\Delta_{\perp 4+}}{L_{4+}-k_x^2}$

$$u_{x40+} := \frac{-\mathrm{I}\, k_x\, \Delta_{\perp 4+}}{-k_x^2 + L_{4+}} \quad (36)$$

> $L_{1-} := \Delta_{\|1-}^2 + k_{\|-}^2$

$$L_{1-} := k_{\|-}^2 + \Delta_{\|1-}^2 \quad (37)$$

> $L_{1+} := \Delta_{\|1+}^2 + k_{\|+}^2$

$$L_{1+} := k_{\|+}^2 + \Delta_{\|1+}^2 \quad (38)$$

> $L_{2-} := \Delta_{\|2-}^2 + k_{\|-}^2$

$$L_{2-} := k_{\|-}^2 + \Delta_{\|2-}^2 \quad (39)$$

> $L_{2+} := \Delta_{\|2+}^2 + k_{\|+}^2$

$$L_{2+} := k_{\|+}^2 + \Delta_{\|2+}^2 \quad (40)$$

> $L_{3-} := \Delta_{\|3-}^2 + k_{\|-}^2$

$$L_{3-} := k_{\|-}^2 + \Delta_{\|3-}^2 \quad (41)$$

> $L_{3+} := \Delta_{\|3+}^2 + k_{\|+}^2$

$$L_{3+} := k_{\|+}^2 + \Delta_{\|3+}^2 \quad (42)$$

> $L_{4-} := \Delta_{\|4-}^2 + k_{\|-}^2$

$$L_{4-} := k_{\|-}^2 + \Delta_{\|4-}^2 \quad (43)$$

> $L_{4+} := \Delta_{\|4+}^2 + k_{\|+}^2$

$$L_{4+} := k_{\|+}^2 + \Delta_{\|4+}^2 \quad (44)$$

> $\Delta_{\perp 1-} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z1-}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 1-} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z1-} \sin(\alpha)\right) \quad (45)$$

> $\Delta_{\perp 1+} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z1+}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 1+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z1+} \sin(\alpha)\right) \quad (46)$$

> $\Delta_{\perp 2-} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z2-}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 2-} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z2-} \sin(\alpha)\right) \quad (47)$$

> $\Delta_{\perp 2+} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z2+}\cdot\sin(\alpha)\right)$

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$$\Delta_{\perp 2+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z2+} \sin(\alpha)\right) \tag{48}$$

> $\Delta_{\perp 3-} := \mathrm{i} \cdot \left(k_y \cdot \cos(\alpha) - k_{z3-} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 3\text{-}} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z3\text{-}} \sin(\alpha)\right) \tag{49}$$

> $\Delta_{\perp 3+} := \mathrm{i} \cdot \left(k_y \cdot \cos(\alpha) - k_{z3+} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 3+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z3+} \sin(\alpha)\right) \tag{50}$$

> $\Delta_{\perp 4-} := \mathrm{i} \cdot \left(k_y \cdot \cos(\alpha) - k_{z4-} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 4\text{-}} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z4\text{-}} \sin(\alpha)\right) \tag{51}$$

> $\Delta_{\perp 4+} := \mathrm{i} \cdot \left(k_y \cdot \cos(\alpha) - k_{z4+} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 4+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z4+} \sin(\alpha)\right) \tag{52}$$

> $\Delta_{\|1-} := \mathrm{i} \cdot \left(k_y \cdot \sin(\alpha) + k_{z1-} \cdot \cos(\alpha)\right)$

$$\Delta_{\|1\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z1\text{-}} \cos(\alpha)\right) \tag{53}$$

> $\Delta_{\|1+} := \mathrm{i} \cdot \left(k_y \cdot \sin(\alpha) + k_{z1+} \cdot \cos(\alpha)\right)$

$$\Delta_{\|1+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z1+} \cos(\alpha)\right) \tag{54}$$

> $\Delta_{\|2-} := \mathrm{i} \cdot \left(k_y \cdot \sin(\alpha) + k_{z2-} \cdot \cos(\alpha)\right)$

$$\Delta_{\|2\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z2\text{-}} \cos(\alpha)\right) \tag{55}$$

> $\Delta_{\|2+} := \mathrm{i} \cdot \left(k_y \cdot \sin(\alpha) + k_{z2+} \cdot \cos(\alpha)\right)$

$$\Delta_{\|2+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z2+} \cos(\alpha)\right) \tag{56}$$

> $\Delta_{\|3-} := \mathrm{i} \cdot \left(k_y \cdot \sin(\alpha) + k_{z3-} \cdot \cos(\alpha)\right)$

$$\Delta_{\|3\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z3\text{-}} \cos(\alpha)\right) \tag{57}$$

> $\Delta_{\|3+} := \mathrm{i} \cdot \left(k_y \cdot \sin(\alpha) + k_{z3+} \cdot \cos(\alpha)\right)$

$$\Delta_{\|3+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z3+} \cos(\alpha)\right) \tag{58}$$

> $\Delta_{\|4-} := \mathrm{i} \cdot \left(k_y \cdot \sin(\alpha) + k_{z4-} \cdot \cos(\alpha)\right)$

$$\Delta_{\|4\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z4\text{-}} \cos(\alpha)\right) \tag{59}$$

> $\Delta_{\|4+} := \mathrm{i} \cdot \left(k_y \cdot \sin(\alpha) + k_{z4+} \cdot \cos(\alpha)\right)$

$$\Delta_{\|4+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z4+} \cos(\alpha)\right) \tag{60}$$

> $k_{z1-} := \dfrac{k_{\|-}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z1\text{-}} := \frac{k_{\|\text{-}}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{61}$$

> $k_{z1+} := \dfrac{k_{\|+}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z1+} := \frac{k_{||+}}{\cos(\alpha)} - k_y \tan(\alpha) \quad \textbf{(62)}$$

> $k_{z2-} := -\frac{k_{||-}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z2-} := -\frac{k_{||-}}{\cos(\alpha)} - k_y \tan(\alpha) \quad \textbf{(63)}$$

> $k_{z2+} := -\frac{k_{||+}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z2+} := -\frac{k_{||+}}{\cos(\alpha)} - k_y \tan(\alpha) \quad \textbf{(64)}$$

> $k_{z3-} := \mathrm{i} \cdot \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||-}^{\,2}}$

$$k_{z3-} := \mathrm{I} \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||-}^{\,2}} \quad \textbf{(65)}$$

> $k_{z3+} := \mathrm{i} \cdot \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||+}^{\,2}}$

$$k_{z3+} := \mathrm{I} \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||+}^{\,2}} \quad \textbf{(66)}$$

> $k_{z4-} := -\mathrm{i} \cdot \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||-}^{\,2}}$

$$k_{z4-} := -\mathrm{I} \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||-}^{\,2}} \quad \textbf{(67)}$$

> $k_{z4+} := -\mathrm{i} \cdot \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||+}^{\,2}}$

$$k_{z4+} := -\mathrm{I} \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||+}^{\,2}} \quad \textbf{(68)}$$

>

> $k_x := \frac{k_{||+}}{\epsilon_+}$

$$k_x := \frac{k_{||+}}{\epsilon_{`+`}} \quad \textbf{(69)}$$

> $k_y := a_y \cdot k_{||+}$

$$k_y := a_y \, k_{||+} \quad \textbf{(70)}$$

> $k_{||-} := \frac{k_x}{\epsilon_-}$

$$k_{||-} := \frac{k_{||+}}{\epsilon_{`+`} \, \epsilon_{`-`}} \quad \textbf{(71)}$$

Need to help maple to take limit as epsilon_- and epsilon_+ go to zero by taking terms that go to infinity out of the square root.

> $k_{z3-} := \frac{\mathrm{i} \cdot \sqrt{\epsilon_{-}^{2} \cdot k_{\|+}^{2} + \epsilon_{-}^{2} \cdot \epsilon_{+}^{2} \cdot k_{y}^{2} - k_{\|+}^{2}}}{\epsilon_{-} \cdot \epsilon_{+}}$

$$k_{z3\text{-}} := \frac{I \sqrt{a_y^2 \epsilon_{`-`}^2 \epsilon_{`+`}^2 k_{\|+}^2 + \epsilon_{`-`}^2 k_{\|+}^2 - k_{\|+}^2}}{\epsilon_{`-`} \epsilon_{`+`}} \quad \textbf{(72)}$$

> $k_{z3+} := \frac{\mathrm{i} \cdot \sqrt{k_{\|+}^{2} + \epsilon_{+}^{2} \cdot k_{y}^{2} - \epsilon_{+}^{2} \cdot k_{\|+}^{2}}}{\epsilon_{+}}$

$$k_{z3+} := \frac{I \sqrt{a_y^2 \epsilon_{`+`}^2 k_{\|+}^2 - \epsilon_{`+`}^2 k_{\|+}^2 + k_{\|+}^2}}{\epsilon_{`+`}} \quad \textbf{(73)}$$

> $k_{z4-} := -\frac{\mathrm{i} \cdot \sqrt{\epsilon_{-}^{2} \cdot k_{\|+}^{2} + \epsilon_{-}^{2} \cdot \epsilon_{+}^{2} \cdot k_{y}^{2} - k_{\|+}^{2}}}{\epsilon_{-} \cdot \epsilon_{+}}$

$$k_{z4\text{-}} := \frac{-I \sqrt{a_y^2 \epsilon_{`-`}^2 \epsilon_{`+`}^2 k_{\|+}^2 + \epsilon_{`-`}^2 k_{\|+}^2 - k_{\|+}^2}}{\epsilon_{`-`} \epsilon_{`+`}} \quad \textbf{(74)}$$

> $k_{z4+} := -\frac{\mathrm{i} \cdot \sqrt{k_{\|+}^{2} + \epsilon_{+}^{2} \cdot k_{y}^{2} - \epsilon_{+}^{2} \cdot k_{\|+}^{2}}}{\epsilon_{+}}$

$$k_{z4+} := \frac{-I \sqrt{a_y^2 \epsilon_{`+`}^2 k_{\|+}^2 - \epsilon_{`+`}^2 k_{\|+}^2 + k_{\|+}^2}}{\epsilon_{`+`}} \quad \textbf{(75)}$$

ux leading order terms

> $u_{x1-} := u_{x10-} \cdot sol1 :$

> $u_{x4-} := u_{x40-} \cdot sol2 :$

> $u_{x1+} := u_{x10+} :$

> $u_{x2+} := u_{x20+} \cdot sol3 :$

> $u_{x3+} := u_{x30+} \cdot sol4 :$

> $simplify\left(mtaylor\left(u_{x1-}, \left[\epsilon_{-}, \epsilon_{+}\right], 3\right)\right)$

$$-2\, I \sin(\alpha)\, \epsilon_{`-`} \epsilon_{`+`} \operatorname{csgn}(k_{\|+}) \quad \textbf{(76)}$$

> $simplify\left(mtaylor\left(u_{x4-}, \left[\epsilon_{-}, \epsilon_{+}\right], 4\right)\right)$

$$-\frac{2\, \epsilon_{`-`}^2 \epsilon_{`+`} \cos(\alpha)}{\sin(\alpha)} \quad \textbf{(77)}$$

> $simplify\left(mtaylor\left(u_{x1+}, \left[\epsilon_{-}, \epsilon_{+}\right], 2\right)\right)$

$$\frac{\epsilon_{`+`}\left(-a_y+\sin(\alpha)\right)}{\cos(\alpha)} \tag{78}$$

> $simplify\left(mtaylor\left(u_{x2+}, [\epsilon_{-}, \epsilon_{+}], 2\right)\right)$

$$\frac{\epsilon_{`+`}\left(a_y+\sin(\alpha)\right)}{\cos(\alpha)} \tag{79}$$

> $simplify\left(mtaylor\left(u_{x3+}, [\epsilon_{-}, \epsilon_{+}], 2\right)\right)$

$$-\frac{2\sin(\alpha)\,\epsilon_{`+`}}{\cos(\alpha)} \tag{80}$$

u_perp leading order terms

> $simplify\left(mtaylor\left(sol1, [\epsilon_{-}, \epsilon_{+}], 4\right)\right)$

$$-2\,\mathrm{I}\,\epsilon_{`-`}^{2}\,\epsilon_{`+`}\cos(\alpha)\,\mathrm{csgn}(k_{\parallel+}) \tag{81}$$

> $simplify\left(mtaylor\left(sol2, [\epsilon_{-}, \epsilon_{+}], 3\right)\right)$

$$\frac{2\,\mathrm{I}\cos(\alpha)\,\epsilon_{`-`}\,\epsilon_{`+`}\,k_{\parallel+}}{\sqrt{-k_{\parallel+}^{2}}} \tag{82}$$

> $simplify\left(mtaylor\left(sol3, [\epsilon_{-}, \epsilon_{+}], 1\right)\right)$

$$-1 \tag{83}$$

> $simplify\left(mtaylor\left(sol4, [\epsilon_{-}, \epsilon_{+}], 2\right)\right)$

$$\frac{2\,\mathrm{I}\,\epsilon_{`+`}\,\mathrm{csgn}(k_{\parallel+})\sin(\alpha)^{2}}{\cos(\alpha)} \tag{84}$$

bx leading order terms

> $b_{x1-} := \frac{\Delta_{\parallel 1-}\cdot u_{x1-}}{\mathrm{i}\cdot k_{\parallel+}}$:

> $b_{x4-} := \frac{\Delta_{\parallel 4-}\cdot u_{x4-}}{\mathrm{i}\cdot k_{\parallel+}}$:

> $b_{x1+} := \frac{\Delta_{\parallel 1+}\cdot u_{x1+}}{\mathrm{i}\cdot k_{\parallel+}}$:

> $b_{x2+} := \frac{\Delta_{\parallel 2+}\cdot u_{x2+}}{\mathrm{i}\cdot k_{\parallel+}}$:

> $b_{x3+} := \frac{\Delta_{\parallel 3+}\cdot u_{x3+}}{\mathrm{i}\cdot k_{\parallel+}}$:

> $simplify(mtaylor(b_{x1-}, [\epsilon_-, \epsilon_+], 1))$

$$-2\,\mathrm{I}\sin(\alpha)\,\mathrm{csgn}(k_{\|+}) \quad \mathbf{(85)}$$

> $simplify(mtaylor(b_{x4-}, [\epsilon_-, \epsilon_+], 2))$

$$-\frac{2\,\mathrm{I}\,k_{\|+}\,\epsilon_{`\_`}\cos(\alpha)^2}{\sqrt{-k_{\|+}^2}\,\sin(\alpha)} \quad \mathbf{(86)}$$

> $simplify(mtaylor(b_{x1+}, [\epsilon_-, \epsilon_+], 2))$

$$\frac{\epsilon_{`+`}(-a_y + \sin(\alpha))}{\cos(\alpha)} \quad \mathbf{(87)}$$

> $simplify(mtaylor(b_{x2+}, [\epsilon_-, \epsilon_+], 2))$

$$-\frac{\epsilon_{`+`}(a_y + \sin(\alpha))}{\cos(\alpha)} \quad \mathbf{(88)}$$

> $simplify(mtaylor(b_{x3+}, [\epsilon_-, \epsilon_+], 1))$

$$-2\,\mathrm{I}\sin(\alpha)\,\mathrm{csgn}(k_{\|+}) \quad \mathbf{(89)}$$

b_perp leading order terms

> $b_{\perp 1-} := \frac{\Delta_{\|1-}\cdot sol1}{\mathrm{i}\cdot k_{\|+}}$:

> $b_{\perp 4-} := \frac{\Delta_{\|4-}\cdot sol2}{\mathrm{i}\cdot k_{\|+}}$:

> $b_{\perp 1+} := \frac{\Delta_{\|1+}\cdot 1}{\mathrm{i}\cdot k_{\|+}}$:

> $b_{\perp 2+} := \frac{\Delta_{\|2+}\cdot sol3}{\mathrm{i}\cdot k_{\|+}}$:

> $b_{\perp 3+} := \frac{\Delta_{\|3+}\cdot sol4}{\mathrm{i}\cdot k_{\|+}}$:

> $simplify(mtaylor(b_{\perp 1-}, [\epsilon_-, \epsilon_+], 2))$

$$-2\,\mathrm{I}\,\epsilon_{`\_`}\cos(\alpha)\,\mathrm{csgn}(k_{\|+}) \quad \mathbf{(90)}$$

> $simplify(mtaylor(b_{\perp 4-}, [\epsilon_-, \epsilon_+], 1))$

$$2\cos(\alpha)^2 \quad \mathbf{(91)}$$

> $simplify(mtaylor(b_{\perp 1+}, [\epsilon_-, \epsilon_+], 5))$

$$1 \quad \mathbf{(92)}$$

> $simplify(mtaylor(b_{\perp 2+}, [\epsilon_-, \epsilon_+], 1))$

$$1 \quad \mathbf{(93)}$$

> $simplify\left(mtaylor\left(b_{\perp 3+}, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$-2\sin(\alpha)^2 \tag{94}$$

b_par leading order terms

> $b_{\|1-} := -\frac{\mathrm{i}\cdot k_x \cdot u_{x1-} + \Delta_{\perp 1-}\cdot sol1}{\mathrm{i}\cdot k_{\|+}}$:

> $b_{\|4-} := -\frac{\mathrm{i}\cdot k_x \cdot u_{x4-} + \Delta_{\perp 4-}\cdot sol2}{\mathrm{i}\cdot k_{\|+}}$:

> $b_{\|1+} := -\frac{\mathrm{i}\cdot k_x \cdot u_{x1+} + \Delta_{\perp 1+}}{\mathrm{i}\cdot k_{\|+}}$:

> $b_{\|2+} := -\frac{\mathrm{i}\cdot k_x \cdot u_{x2+} + \Delta_{\perp 2+}\cdot sol3}{\mathrm{i}\cdot k_{\|+}}$:

> $b_{\|3+} := -\frac{\mathrm{i}\cdot k_x \cdot u_{x3+} + \Delta_{\perp 3+}\cdot sol4}{\mathrm{i}\cdot k_{\|+}}$:

> $simplify\left(mtaylor\left(b_{\|1-}, \left[\epsilon_{-}, \epsilon_{+}\right], 2\right)\right)$

$$0 \tag{95}$$

> $simplify\left(mtaylor\left(b_{\|4-}, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$2\sin(\alpha)\cos(\alpha) \tag{96}$$

> $simplify\left(mtaylor\left(b_{\|1+}, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$0 \tag{97}$$

> $simplify\left(mtaylor\left(b_{\|2+}, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$0 \tag{98}$$

> $simplify\left(mtaylor\left(b_{\|3+}, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$2\sin(\alpha)\cos(\alpha) \tag{99}$$