Note that ∇ has been replaced with  to ensure the code works okay.

> *restart*;

We normalise the velocity coefficents by u_0 and the field components by (B_0 * u_0 / v_{A+}).

We start by solving the following Matrix equation:

> $eqn1 := a_{11} \cdot x_1 + a_{12} \cdot x_2 + a_{13} \cdot x_3 + a_{14} \cdot x_4 = y_1$

$$eqn1 := a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 = y_1 \quad (1)$$

> $eqn2 := a_{21} \cdot x_1 + a_{22} \cdot x_2 + a_{23} \cdot x_3 + a_{24} \cdot x_4 = y_2$

$$eqn2 := a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + a_{24} x_4 = y_2 \quad (2)$$

> $eqn3 := a_{31} \cdot x_1 + a_{32} \cdot x_2 + a_{33} \cdot x_3 + a_{34} \cdot x_4 = y_3$

$$eqn3 := a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + a_{34} x_4 = y_3 \quad (3)$$

> $eqn4 := a_{41} \cdot x_1 + a_{42} \cdot x_2 + a_{43} \cdot x_3 + a_{44} \cdot x_4 = y_4$

$$eqn4 := a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4 = y_4 \quad (4)$$

> $solns := solve\left(\{eqn1, eqn2, eqn3, eqn4\}, \{x_1, x_2, x_3, x_4\}\right):$

> $sol1 := rhs(solns[1])$

$$sol1 := -\big(a_{12} a_{23} a_{34} y_4 - a_{12} a_{23} a_{44} y_3 - a_{12} a_{24} a_{33} y_4 + a_{12} a_{24} a_{43} y_3 + a_{12} a_{33} a_{44} y_2 - a_{12} a_{34} a_{43} y_2 - a_{13} a_{22} a_{34} y_4 + a_{13} a_{22} a_{44} y_3 + a_{13} a_{24} a_{32} y_4 - a_{13} a_{24} a_{42} y_3 - a_{13} a_{32} a_{44} y_2 + a_{13} a_{34} a_{42} y_2 + a_{14} a_{22} a_{33} y_4 - a_{14} a_{22} a_{43} y_3 - a_{14} a_{23} a_{32} y_4 + a_{14} a_{23} a_{42} y_3 + a_{14} a_{32} a_{43} y_2 - a_{14} a_{33} a_{42} y_2 - a_{22} a_{33} a_{44} y_1 + a_{22} a_{34} a_{43} y_1 + a_{23} a_{32} a_{44} y_1 - a_{23} a_{34} a_{42} y_1 - a_{24} a_{32} a_{43} y_1 + a_{24} a_{33} a_{42} y_1\big) \Big/ \big(a_{11} a_{22} a_{33} a_{44} - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} + a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} + a_{14} a_{22} a_{31} a_{43} - a_{14} a_{22} a_{33} a_{41} - a_{14} a_{23} a_{31} a_{42} + a_{14} a_{23} a_{32} a_{41}\big) \quad (5)$$

> $sol2 := rhs(solns[2])$

$$sol2 := \big(a_{11} a_{23} a_{34} y_4 - a_{11} a_{23} a_{44} y_3 - a_{11} a_{24} a_{33} y_4 + a_{11} a_{24} a_{43} y_3 + a_{11} a_{33} a_{44} y_2 - a_{11} a_{34} a_{43} y_2 - a_{13} a_{21} a_{34} y_4 + a_{13} a_{21} a_{44} y_3 + a_{13} a_{24} a_{31} y_4 - a_{13} a_{24} a_{41} y_3 - a_{13} a_{31} a_{44} y_2 + a_{13} a_{34} a_{41} y_2 + a_{14} a_{21} a_{33} y_4 - a_{14} a_{21} a_{43} y_3 - a_{14} a_{23} a_{31} y_4 + a_{14} a_{23} a_{41} y_3 + a_{14} a_{31} a_{43} y_2 - a_{14} a_{33} a_{41} y_2 - a_{21} a_{33} a_{44} y_1 + a_{21} a_{34} a_{43} y_1 + a_{23} a_{31} a_{44} y_1 - a_{23} a_{34} a_{41} y_1 - a_{24} a_{31} a_{43} y_1 + a_{24} a_{33} a_{41} y_1\big) \Big/ \big(a_{11} a_{22} a_{33} a_{44} - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} + a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} + a_{14} a_{22} a_{31} a_{43} \quad (6)$$

$$-a_{14}\,a_{22}\,a_{33}\,a_{41}-a_{14}\,a_{23}\,a_{31}\,a_{42}+a_{14}\,a_{23}\,a_{32}\,a_{41}\Big)$$

> $sol3 := rhs(solns[3])$

$$sol3 := -\Big(a_{11}\,a_{22}\,a_{34}\,y_4-a_{11}\,a_{22}\,a_{44}\,y_3-a_{11}\,a_{24}\,a_{32}\,y_4+a_{11}\,a_{24}\,a_{42}\,y_3+a_{11}\,a_{32}\,a_{44}\,y_2 -a_{11}\,a_{34}\,a_{42}\,y_2-a_{12}\,a_{21}\,a_{34}\,y_4+a_{12}\,a_{21}\,a_{44}\,y_3+a_{12}\,a_{24}\,a_{31}\,y_4-a_{12}\,a_{24}\,a_{41}\,y_3 -a_{12}\,a_{31}\,a_{44}\,y_2+a_{12}\,a_{34}\,a_{41}\,y_2+a_{14}\,a_{21}\,a_{32}\,y_4-a_{14}\,a_{21}\,a_{42}\,y_3-a_{14}\,a_{22}\,a_{31}\,y_4 +a_{14}\,a_{22}\,a_{41}\,y_3+a_{14}\,a_{31}\,a_{42}\,y_2-a_{14}\,a_{32}\,a_{41}\,y_2-a_{21}\,a_{32}\,a_{44}\,y_1+a_{21}\,a_{34}\,a_{42}\,y_1 +a_{22}\,a_{31}\,a_{44}\,y_1-a_{22}\,a_{34}\,a_{41}\,y_1-a_{24}\,a_{31}\,a_{42}\,y_1+a_{24}\,a_{32}\,a_{41}\,y_1\Big)\Big/\Big(a_{11}\,a_{22}\,a_{33}\,a_{44} -a_{11}\,a_{22}\,a_{34}\,a_{43}-a_{11}\,a_{23}\,a_{32}\,a_{44}+a_{11}\,a_{23}\,a_{34}\,a_{42}+a_{11}\,a_{24}\,a_{32}\,a_{43}-a_{11}\,a_{24}\,a_{33}\,a_{42} -a_{12}\,a_{21}\,a_{33}\,a_{44}+a_{12}\,a_{21}\,a_{34}\,a_{43}+a_{12}\,a_{23}\,a_{31}\,a_{44}-a_{12}\,a_{23}\,a_{34}\,a_{41}-a_{12}\,a_{24}\,a_{31}\,a_{43} +a_{12}\,a_{24}\,a_{33}\,a_{41}+a_{13}\,a_{21}\,a_{32}\,a_{44}-a_{13}\,a_{21}\,a_{34}\,a_{42}-a_{13}\,a_{22}\,a_{31}\,a_{44}+a_{13}\,a_{22}\,a_{34}\,a_{41} +a_{13}\,a_{24}\,a_{31}\,a_{42}-a_{13}\,a_{24}\,a_{32}\,a_{41}-a_{14}\,a_{21}\,a_{32}\,a_{43}+a_{14}\,a_{21}\,a_{33}\,a_{42}+a_{14}\,a_{22}\,a_{31}\,a_{43} -a_{14}\,a_{22}\,a_{33}\,a_{41}-a_{14}\,a_{23}\,a_{31}\,a_{42}+a_{14}\,a_{23}\,a_{32}\,a_{41}\Big) \tag{7}$$

> $sol4 := rhs(solns[4])$

$$sol4 := \Big(a_{11}\,a_{22}\,a_{33}\,y_4-a_{11}\,a_{22}\,a_{43}\,y_3-a_{11}\,a_{23}\,a_{32}\,y_4+a_{11}\,a_{23}\,a_{42}\,y_3+a_{11}\,a_{32}\,a_{43}\,y_2 -a_{11}\,a_{33}\,a_{42}\,y_2-a_{12}\,a_{21}\,a_{33}\,y_4+a_{12}\,a_{21}\,a_{43}\,y_3+a_{12}\,a_{23}\,a_{31}\,y_4-a_{12}\,a_{23}\,a_{41}\,y_3 -a_{12}\,a_{31}\,a_{43}\,y_2+a_{12}\,a_{33}\,a_{41}\,y_2+a_{13}\,a_{21}\,a_{32}\,y_4-a_{13}\,a_{21}\,a_{42}\,y_3-a_{13}\,a_{22}\,a_{31}\,y_4 +a_{13}\,a_{22}\,a_{41}\,y_3+a_{13}\,a_{31}\,a_{42}\,y_2-a_{13}\,a_{32}\,a_{41}\,y_2-a_{21}\,a_{32}\,a_{43}\,y_1+a_{21}\,a_{33}\,a_{42}\,y_1 +a_{22}\,a_{31}\,a_{43}\,y_1-a_{22}\,a_{33}\,a_{41}\,y_1-a_{23}\,a_{31}\,a_{42}\,y_1+a_{23}\,a_{32}\,a_{41}\,y_1\Big)\Big/\Big(a_{11}\,a_{22}\,a_{33}\,a_{44} -a_{11}\,a_{22}\,a_{34}\,a_{43}-a_{11}\,a_{23}\,a_{32}\,a_{44}+a_{11}\,a_{23}\,a_{34}\,a_{42}+a_{11}\,a_{24}\,a_{32}\,a_{43}-a_{11}\,a_{24}\,a_{33}\,a_{42} -a_{12}\,a_{21}\,a_{33}\,a_{44}+a_{12}\,a_{21}\,a_{34}\,a_{43}+a_{12}\,a_{23}\,a_{31}\,a_{44}-a_{12}\,a_{23}\,a_{34}\,a_{41}-a_{12}\,a_{24}\,a_{31}\,a_{43} +a_{12}\,a_{24}\,a_{33}\,a_{41}+a_{13}\,a_{21}\,a_{32}\,a_{44}-a_{13}\,a_{21}\,a_{34}\,a_{42}-a_{13}\,a_{22}\,a_{31}\,a_{44}+a_{13}\,a_{22}\,a_{34}\,a_{41} +a_{13}\,a_{24}\,a_{31}\,a_{42}-a_{13}\,a_{24}\,a_{32}\,a_{41}-a_{14}\,a_{21}\,a_{32}\,a_{43}+a_{14}\,a_{21}\,a_{33}\,a_{42}+a_{14}\,a_{22}\,a_{31}\,a_{43} -a_{14}\,a_{22}\,a_{33}\,a_{41}-a_{14}\,a_{23}\,a_{31}\,a_{42}+a_{14}\,a_{23}\,a_{32}\,a_{41}\Big) \tag{8}$$

> $a_{11} := u_{x10-}$

$$a_{11} := u_{x10\text{-}} \tag{9}$$

> $a_{12} := u_{x40-}$

$$a_{12} := u_{x40\text{-}} \tag{10}$$

> $a_{13} := -u_{x20+}$

$$a_{13} := -u_{x20+} \tag{11}$$

> $a_{14} := -u_{x30+}$

$$a_{14} := -u_{x30+} \tag{12}$$

> $a_{21} := k_{z1-} \cdot u_{x10-}$

$$a_{21} := k_{z1\text{-}}\,u_{x10\text{-}} \tag{13}$$

> $a_{22} := k_{z4-} \cdot u_{x40-}$

$$a_{22} := k_{z4-}\, u_{x40-} \quad (14)$$

> $a_{23} := -k_{z2+} \cdot u_{x20+}$

$$a_{23} := -k_{z2+}\, u_{x20+} \quad (15)$$

> $a_{24} := -k_{z3+} \cdot u_{x30+}$

$$a_{24} := -k_{z3+}\, u_{x30+} \quad (16)$$

> $a_{31} := 1$

$$a_{31} := 1 \quad (17)$$

> $a_{32} := 1$

$$a_{32} := 1 \quad (18)$$

> $a_{33} := -1$

$$a_{33} := -1 \quad (19)$$

> $a_{34} := -1$

$$a_{34} := -1 \quad (20)$$

> $a_{41} := k_{z1-}$

$$a_{41} := k_{z1-} \quad (21)$$

> $a_{42} := k_{z4-}$

$$a_{42} := k_{z4-} \quad (22)$$

> $a_{43} := -k_{z2+}$

$$a_{43} := -k_{z2+} \quad (23)$$

> $a_{44} := -k_{z3+}$

$$a_{44} := -k_{z3+} \quad (24)$$

> $y_1 := u_{x10+}$

$$y_1 := u_{x10+} \quad (25)$$

> $y_2 := k_{z1+} \cdot u_{x10+}$

$$y_2 := k_{z1+}\, u_{x10+} \quad (26)$$

> $y_3 := 1$

$$y_3 := 1 \quad (27)$$

> $y_4 := k_{z1+}$

$$y_4 := k_{z1+} \quad (28)$$

Note that ux_0[n] denotes \hat{u}_{xn} and  u_x[n±] denotes u_{xn±}.

> $u_{x10-} := -\dfrac{\mathrm{i} \cdot k_x \cdot \Delta_{\perp 1-}}{L_{1-} - k_x^2}$

$$u_{x10\text{-}} := \frac{-I\, k_x\, \Delta_{\perp 1\text{-}}}{-k_x^{\,2} + L_{1\text{-}}} \quad (29)$$

> $u_{x10+} := -\frac{\mathrm{i}\cdot k_x \cdot \Delta_{\perp 1+}}{L_{1+} - k_x^{\,2}}$

$$u_{x10+} := \frac{-I\, k_x\, \Delta_{\perp 1+}}{-k_x^{\,2} + L_{1+}} \quad (30)$$

> $u_{x20-} := -\frac{\mathrm{i}\cdot k_x \cdot \Delta_{\perp 2-}}{L_{2-} - k_x^{\,2}}$

$$u_{x20\text{-}} := \frac{-I\, k_x\, \Delta_{\perp 2\text{-}}}{-k_x^{\,2} + L_{2\text{-}}} \quad (31)$$

> $u_{x20+} := -\frac{\mathrm{i}\cdot k_x \cdot \Delta_{\perp 2+}}{L_{2+} - k_x^{\,2}}$

$$u_{x20+} := \frac{-I\, k_x\, \Delta_{\perp 2+}}{-k_x^{\,2} + L_{2+}} \quad (32)$$

> $u_{x30-} := -\frac{\mathrm{i}\cdot k_x \cdot \Delta_{\perp 3-}}{L_{3-} - k_x^{\,2}}$

$$u_{x30\text{-}} := \frac{-I\, k_x\, \Delta_{\perp 3\text{-}}}{-k_x^{\,2} + L_{3\text{-}}} \quad (33)$$

> $u_{x30+} := -\frac{\mathrm{i}\cdot k_x \cdot \Delta_{\perp 3+}}{L_{3+} - k_x^{\,2}}$

$$u_{x30+} := \frac{-I\, k_x\, \Delta_{\perp 3+}}{-k_x^{\,2} + L_{3+}} \quad (34)$$

> $u_{x40-} := -\frac{\mathrm{i}\cdot k_x \cdot \Delta_{\perp 4-}}{L_{4-} - k_x^{\,2}}$

$$u_{x40\text{-}} := \frac{-I\, k_x\, \Delta_{\perp 4\text{-}}}{-k_x^{\,2} + L_{4\text{-}}} \quad (35)$$

> $u_{x40+} := -\frac{\mathrm{i}\cdot k_x \cdot \Delta_{\perp 4+}}{L_{4+} - k_x^{\,2}}$

(36)

$$u_{x40+} := \frac{-\mathrm{I}\, k_x \Delta_{\perp 4+}}{-k_x^2 + L_{4+}} \tag{36}$$

> $L_{1-} := \Delta_{||1-}^{\ 2} + k_{||-}^{\ 2}$

$$L_{1-} := k_{||-}^{\ 2} + \Delta_{||1-}^{\ 2} \tag{37}$$

> $L_{1+} := \Delta_{||1+}^{\ 2} + k_{||+}^{\ 2}$

$$L_{1+} := k_{||+}^{\ 2} + \Delta_{||1+}^{\ 2} \tag{38}$$

> $L_{2-} := \Delta_{||2-}^{\ 2} + k_{||-}^{\ 2}$

$$L_{2-} := k_{||-}^{\ 2} + \Delta_{||2-}^{\ 2} \tag{39}$$

> $L_{2+} := \Delta_{||2+}^{\ 2} + k_{||+}^{\ 2}$

$$L_{2+} := k_{||+}^{\ 2} + \Delta_{||2+}^{\ 2} \tag{40}$$

> $L_{3-} := \Delta_{||3-}^{\ 2} + k_{||-}^{\ 2}$

$$L_{3-} := k_{||-}^{\ 2} + \Delta_{||3-}^{\ 2} \tag{41}$$

> $L_{3+} := \Delta_{||3+}^{\ 2} + k_{||+}^{\ 2}$

$$L_{3+} := k_{||+}^{\ 2} + \Delta_{||3+}^{\ 2} \tag{42}$$

> $L_{4-} := \Delta_{||4-}^{\ 2} + k_{||-}^{\ 2}$

$$L_{4-} := k_{||-}^{\ 2} + \Delta_{||4-}^{\ 2} \tag{43}$$

> $L_{4+} := \Delta_{||4+}^{\ 2} + k_{||+}^{\ 2}$

$$L_{4+} := k_{||+}^{\ 2} + \Delta_{||4+}^{\ 2} \tag{44}$$

> $\Delta_{\perp 1-} := \mathrm{i} \cdot \left(k_y \cdot \cos(\alpha) - k_{z1-} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 1-} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z1-} \sin(\alpha)\right) \tag{45}$$

> $\Delta_{\perp 1+} := \mathrm{i} \cdot \left(k_y \cdot \cos(\alpha) - k_{z1+} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 1+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z1+} \sin(\alpha)\right) \tag{46}$$

> $\Delta_{\perp 2-} := \mathrm{i} \cdot \left(k_y \cdot \cos(\alpha) - k_{z2-} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 2-} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z2-} \sin(\alpha)\right) \tag{47}$$

> $\Delta_{\perp 2+} := \mathrm{i} \cdot \left(k_y \cdot \cos(\alpha) - k_{z2+} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 2+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z2+} \sin(\alpha)\right) \tag{48}$$

> $\Delta_{\perp 3-} := \mathrm{i} \cdot \left(k_y \cdot \cos(\alpha) - k_{z3-} \cdot \sin(\alpha)\right)$

(49)

$$\Delta_{\perp 3\text{-}} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z3\text{-}} \sin(\alpha)\right) \tag{49}$$

> $\Delta_{\perp 3+} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z3+}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 3+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z3+} \sin(\alpha)\right) \tag{50}$$

> $\Delta_{\perp 4-} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z4-}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 4\text{-}} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z4\text{-}} \sin(\alpha)\right) \tag{51}$$

> $\Delta_{\perp 4+} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z4+}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 4+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z4+} \sin(\alpha)\right) \tag{52}$$

> $\Delta_{\|1-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z1-}\cdot\cos(\alpha)\right)$

$$\Delta_{\|1\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z1\text{-}} \cos(\alpha)\right) \tag{53}$$

> $\Delta_{\|1+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z1+}\cdot\cos(\alpha)\right)$

$$\Delta_{\|1+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z1+} \cos(\alpha)\right) \tag{54}$$

> $\Delta_{\|2-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z2-}\cdot\cos(\alpha)\right)$

$$\Delta_{\|2\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z2\text{-}} \cos(\alpha)\right) \tag{55}$$

> $\Delta_{\|2+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z2+}\cdot\cos(\alpha)\right)$

$$\Delta_{\|2+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z2+} \cos(\alpha)\right) \tag{56}$$

> $\Delta_{\|3-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z3-}\cdot\cos(\alpha)\right)$

$$\Delta_{\|3\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z3\text{-}} \cos(\alpha)\right) \tag{57}$$

> $\Delta_{\|3+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z3+}\cdot\cos(\alpha)\right)$

$$\Delta_{\|3+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z3+} \cos(\alpha)\right) \tag{58}$$

> $\Delta_{\|4-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z4-}\cdot\cos(\alpha)\right)$

$$\Delta_{\|4\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z4\text{-}} \cos(\alpha)\right) \tag{59}$$

> $\Delta_{\|4+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z4+}\cdot\cos(\alpha)\right)$

$$\Delta_{\|4+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z4+} \cos(\alpha)\right) \tag{60}$$

> $k_{z1-} := \dfrac{k_{\|-}}{\cos(\alpha)} - k_y\cdot\tan(\alpha)$

$$k_{z1\text{-}} := \frac{k_{\|\text{-}}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{61}$$

> $k_{z1+} := \dfrac{k_{\|+}}{\cos(\alpha)} - k_y\cdot\tan(\alpha)$

$$k_{z1+} := \frac{k_{\|+}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{62}$$

> $k_{z2-} := -\dfrac{k_{\|-}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z2\text{-}} := -\frac{k_{\|\text{-}}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{63}$$

> $k_{z2+} := -\dfrac{k_{\|+}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z2+} := -\frac{k_{\|+}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{64}$$

> $k_{z3-} := \mathrm{i} \cdot \sqrt{k_x^2 + k_y^2 - k_{\|-}^2}$

$$k_{z3\text{-}} := \mathrm{I} \sqrt{k_x^2 + k_y^2 - k_{\|\text{-}}^2} \tag{65}$$

> $k_{z3+} := \mathrm{i} \cdot \sqrt{k_x^2 + k_y^2 - k_{\|+}^2}$

$$k_{z3+} := \mathrm{I} \sqrt{k_x^2 + k_y^2 - k_{\|+}^2} \tag{66}$$

> $k_{z4-} := -\mathrm{i} \cdot \sqrt{k_x^2 + k_y^2 - k_{\|-}^2}$

$$k_{z4\text{-}} := -\mathrm{I} \sqrt{k_x^2 + k_y^2 - k_{\|\text{-}}^2} \tag{67}$$

> $k_{z4+} := -\mathrm{i} \cdot \sqrt{k_x^2 + k_y^2 - k_{\|+}^2}$

$$k_{z4+} := -\mathrm{I} \sqrt{k_x^2 + k_y^2 - k_{\|+}^2} \tag{68}$$

>

> $k_x := \dfrac{k_{\|+}}{\epsilon_+}$

$$k_x := \frac{k_{\|+}}{\epsilon_+} \tag{69}$$

> $k_y := a_y \cdot k_{\|+}$

$$k_y := a_y k_{\|+} \tag{70}$$

> $k_{\|-} := \dfrac{k_x}{\epsilon_-}$

$$k_{\|\text{-}} := \frac{k_{\|+}}{\epsilon_+ \epsilon_\text{-}} \tag{71}$$

Need to help maple to take limit as epsilon_- and epsilon_+ go to zero by taking terms that go to infinity out of the square root.

> $k_{z3-} := \dfrac{\mathrm{i} \cdot \sqrt{\epsilon_-^2 \cdot k_{\|+}^2 + \epsilon_-^2 \cdot \epsilon_+^2 \cdot k_y^2 - k_{\|+}^2}}{\epsilon_- \cdot \epsilon_+}$

$$k_{z3-} := \frac{\mathrm{I}\sqrt{a_y^2\,\epsilon_-^2\,\epsilon_+^2\,k_{\|+}^2+\epsilon_-^2\,k_{\|+}^2-k_{\|+}^2}}{\epsilon_-\,\epsilon_+} \quad \textbf{(72)}$$

> $k_{z3+} := \frac{\mathrm{i}\cdot\sqrt{k_{\|+}^2+\epsilon_+^2\cdot k_y^2-\epsilon_+^2\cdot k_{\|+}^2}}{\epsilon_+}$

$$k_{z3+} := \frac{\mathrm{I}\sqrt{a_y^2\,\epsilon_+^2\,k_{\|+}^2-\epsilon_+^2\,k_{\|+}^2+k_{\|+}^2}}{\epsilon_+} \quad \textbf{(73)}$$

> $k_{z4-} := -\frac{\mathrm{i}\cdot\sqrt{\epsilon_-^2\cdot k_{\|+}^2+\epsilon_-^2\cdot\epsilon_+^2\cdot k_y^2-k_{\|+}^2}}{\epsilon_-\cdot\epsilon_+}$

$$k_{z4-} := \frac{-\mathrm{I}\sqrt{a_y^2\,\epsilon_-^2\,\epsilon_+^2\,k_{\|+}^2+\epsilon_-^2\,k_{\|+}^2-k_{\|+}^2}}{\epsilon_-\,\epsilon_+} \quad \textbf{(74)}$$

> $k_{z4+} := -\frac{\mathrm{i}\cdot\sqrt{k_{\|+}^2+\epsilon_+^2\cdot k_y^2-\epsilon_{`+`}^2\cdot k_{\|+}^2}}{\epsilon_+}$

$$k_{z4+} := \frac{-\mathrm{I}\sqrt{a_y^2\,\epsilon_+^2\,k_{\|+}^2-\mathit{epsilon\_\_}^2\,k_{\|+}^2+k_{\|+}^2}}{\epsilon_+} \quad \textbf{(75)}$$

ux leading order terms

> $u_{x1-} := u_{x10-}\cdot sol1:$

> $u_{x4-} := u_{x40-}\cdot sol2:$

> $u_{x1+} := u_{x10+}:$

> $u_{x2+} := u_{x20+}\cdot sol3:$

> $u_{x3+} := u_{x30+}\cdot sol4:$

> $simplify\left(mtaylor\left(u_{x1-}, \left[\epsilon_-, \epsilon_+\right], 3\right)\right)$

$$-2\,\mathrm{I}\sin(\alpha)\,\epsilon_-\,\epsilon_+\,\mathrm{csgn}(k_{\|+}) \quad \textbf{(76)}$$

> $simplify\left(mtaylor\left(u_{x4-}, \left[\epsilon_-, \epsilon_+\right], 4\right)\right)$

$$-\frac{2\,\epsilon_-^2\,\epsilon_+\cos(\alpha)}{\sin(\alpha)} \quad \textbf{(77)}$$

> $simplify\left(mtaylor\left(u_{x1+}, \left[\epsilon_-, \epsilon_+\right], 2\right)\right)$

$$\frac{\epsilon_+\left(-a_y+\sin(\alpha)\right)}{\cos(\alpha)} \quad \textbf{(78)}$$

> $simplify\left(mtaylor\left(u_{x2+}, \left[\epsilon_-, \epsilon_+\right], 2\right)\right)$

$$\frac{\epsilon_{+}\left(a_{y}+\sin(\alpha)\right)}{\cos(\alpha)} \tag{79}$$

> $simplify\left(mtaylor\left(u_{x3+}, \left[\epsilon_{-}, \epsilon_{+}\right], 2\right)\right)$

$$-\frac{2\sin(\alpha)\,\epsilon_{+}}{\cos(\alpha)} \tag{80}$$

u_perp leading order terms

> $simplify\left(mtaylor\left(sol1, \left[\epsilon_{-}, \epsilon_{+}\right], 4\right)\right)$

$$-2\,\mathrm{I}\,\epsilon_{-}^{2}\,\epsilon_{+}\cos(\alpha)\,\mathrm{csgn}\left(k_{\parallel +}\right) \tag{81}$$

> $simplify\left(mtaylor\left(sol2, \left[\epsilon_{-}, \epsilon_{+}\right], 3\right)\right)$

$$\frac{2\,\mathrm{I}\cos(\alpha)\,\epsilon_{+}\,\epsilon_{-}\,k_{\parallel +}}{\sqrt{-k_{\parallel +}^{2}}} \tag{82}$$

> $simplify\left(mtaylor\left(sol3, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$-1 \tag{83}$$

> $simplify\left(mtaylor\left(sol4, \left[\epsilon_{-}, \epsilon_{+}\right], 2\right)\right)$

$$\frac{2\,\mathrm{I}\,\epsilon_{+}\,\mathrm{csgn}\left(k_{\parallel +}\right)\sin(\alpha)^{2}}{\cos(\alpha)} \tag{84}$$

bx leading order terms

> $b_{x1-} := \dfrac{\Delta_{\parallel 1-} \cdot u_{x1-}}{\mathrm{i} \cdot k_{\parallel +}}$ :

> $b_{x4-} := \dfrac{\Delta_{\parallel 4-} \cdot u_{x4-}}{\mathrm{i} \cdot k_{\parallel +}}$ :

> $b_{x1+} := \dfrac{\Delta_{\parallel 1+} \cdot u_{x1+}}{\mathrm{i} \cdot k_{\parallel +}}$ :

> $b_{x2+} := \dfrac{\Delta_{\parallel 2+} \cdot u_{x2+}}{\mathrm{i} \cdot k_{\parallel +}}$ :

> $b_{x3+} := \dfrac{\Delta_{\parallel 3+} \cdot u_{x3+}}{\mathrm{i} \cdot k_{\parallel +}}$ :

> $simplify\left(mtaylor\left(b_{x1-}, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$-2\,\mathrm{I}\sin(\alpha)\,\mathrm{csgn}\left(k_{\parallel +}\right) \tag{85}$$

> $simplify\left(mtaylor\left(b_{x4-}, \left[\epsilon_{-}, \epsilon_{+}\right], 2\right)\right)$

$$\frac{-2\,\mathrm{I}\,k_{\parallel+}\,\epsilon_{-}\cos(\alpha)^2}{\sqrt{-k_{\parallel+}^{\;2}}\,\sin(\alpha)} \tag{86}$$

> $simplify\left(mtaylor\left(b_{x1+}, \left[\epsilon_{-}, \epsilon_{+}\right], 2\right)\right)$

$$\frac{\epsilon_{+}\left(-a_y+\sin(\alpha)\right)}{\cos(\alpha)} \tag{87}$$

> $simplify\left(mtaylor\left(b_{x2+}, \left[\epsilon_{-}, \epsilon_{+}\right], 2\right)\right)$

$$-\frac{\epsilon_{+}\left(a_y+\sin(\alpha)\right)}{\cos(\alpha)} \tag{88}$$

> $simplify\left(mtaylor\left(b_{x3+}, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$-2\,\mathrm{I}\sin(\alpha)\,\mathrm{csgn}(k_{\parallel+}) \tag{89}$$

b_perp leading order terms

> $b_{\perp 1-} := \frac{\Delta_{\parallel 1-}\cdot sol1}{\mathrm{i}\cdot k_{\parallel+}}$ :

> $b_{\perp 4-} := \frac{\Delta_{\parallel 4-}\cdot sol2}{\mathrm{i}\cdot k_{\parallel+}}$ :

> $b_{\perp 1+} := \frac{\Delta_{\parallel 1+}\cdot 1}{\mathrm{i}\cdot k_{\parallel+}}$ :

> $b_{\perp 2+} := \frac{\Delta_{\parallel 2+}\cdot sol3}{\mathrm{i}\cdot k_{\parallel+}}$ :

> $b_{\perp 3+} := \frac{\Delta_{\parallel 3+}\cdot sol4}{\mathrm{i}\cdot k_{\parallel+}}$ :

> $simplify\left(mtaylor\left(b_{\perp 1-}, \left[\epsilon_{-}, \epsilon_{+}\right], 2\right)\right)$

$$-2\,\mathrm{I}\,\epsilon_{-}\cos(\alpha)\,\mathrm{csgn}(k_{\parallel+}) \tag{90}$$

> $simplify\left(mtaylor\left(b_{\perp 4-}, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$2\cos(\alpha)^2 \tag{91}$$

> $simplify\left(mtaylor\left(b_{\perp 1+}, \left[\epsilon_{-}, \epsilon_{+}\right], 5\right)\right)$

$$1 \tag{92}$$

> $simplify\left(mtaylor\left(b_{\perp 2+}, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$1 \tag{93}$$

> $simplify\left(mtaylor\left(b_{\perp 3+}, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$-2\sin(\alpha)^2 \tag{94}$$

b_par leading order terms

> $b_{||1-} := -\frac{\mathrm{i} \cdot k_x \cdot u_{x1-} + \Delta_{\perp 1-} \cdot sol1}{\mathrm{i} \cdot k_{||+}}$ :

> $b_{||4-} := -\frac{\mathrm{i} \cdot k_x \cdot u_{x4-} + \Delta_{\perp 4-} \cdot sol2}{\mathrm{i} \cdot k_{||+}}$ :

> $b_{||1+} := -\frac{\mathrm{i} \cdot k_x \cdot u_{x1+} + \Delta_{\perp 1+}}{\mathrm{i} \cdot k_{||+}}$ :

> $b_{||2+} := -\frac{\mathrm{i} \cdot k_x \cdot u_{x2+} + \Delta_{\perp 2+} \cdot sol3}{\mathrm{i} \cdot k_{||+}}$ :

> $b_{||3+} := -\frac{\mathrm{i} \cdot k_x \cdot u_{x3+} + \Delta_{\perp 3+} \cdot sol4}{\mathrm{i} \cdot k_{||+}}$ :

> $simplify\left(mtaylor\left(b_{||1-}, \left[\epsilon_-, \epsilon_+\right], 2\right)\right)$

$$0 \quad \textbf{(95)}$$

> $simplify\left(mtaylor\left(b_{||4-}, \left[\epsilon_-, \epsilon_+\right], 1\right)\right)$

$$2 \sin(\alpha) \cos(\alpha) \quad \textbf{(96)}$$

> $simplify\left(mtaylor\left(b_{||1+}, \left[\epsilon_-, \epsilon_+\right], 1\right)\right)$

$$0 \quad \textbf{(97)}$$

> $simplify\left(mtaylor\left(b_{||2+}, \left[\epsilon_-, \epsilon_+\right], 1\right)\right)$

$$0 \quad \textbf{(98)}$$

> $simplify\left(mtaylor\left(b_{||3+}, \left[\epsilon_-, \epsilon_+\right], 1\right)\right)$

$$2 \sin(\alpha) \cos(\alpha) \quad \textbf{(99)}$$