Note that ∇ has been replaced with '\Delta' to ensure the code works okay.

> *restart*;

Let a_y = k_y / k_{||+}, hence, k_y = a_y k_{||+}.
Let epsilon = k_{||+} / k_x, hence, k_x = k_{||+} / epsilon.

$$> k_z := \left[\left(\frac{k_{||+}}{\cos(\alpha)} - a_y \cdot k_{||+} \cdot \tan(\alpha) \right), \left(-\frac{k_{||+}}{\cos(\alpha)} - a_y \cdot k_{||+} \cdot \tan(\alpha) \right), \mathrm{i} \cdot k_{||+} \cdot \sqrt{\frac{1}{\epsilon^2} + a_y^{\,2} - 1} \right];$$

$$k_z := \left[\frac{k_{||+}}{\cos(\alpha)} - a_y\, k_{||+} \tan(\alpha), -\frac{k_{||+}}{\cos(\alpha)} - a_y\, k_{||+} \tan(\alpha), \mathrm{I}\, k_{||+} \sqrt{\frac{1}{\epsilon^2} + a_y^{\,2} - 1} \right] \tag{1}$$

Note that ux_0[n] denotes \hat{u}_{xn} and u_{x0}[n] denotes u_{xn}. Also we normalise the velocity coefficents by u_0 and the field components by (B_0 * u_0 / v_{A+}).

$$> \Delta_\perp := [\ ]:$$
$$\Delta_{||} := [\ ]:$$
$$L := [\ ]:$$
$$ux_0 := [\ ]:$$
for *i* **from** 1 **to** 3 **do**:
$$\Delta_\perp := \left[op(\Delta_\perp), \mathrm{i} \cdot \left(a_y \cdot k_{||+} \cdot \cos(\alpha) - k_z[i] \cdot \sin(\alpha) \right) \right]:$$
$$\Delta_{||} := \left[op(\Delta_{||}), \mathrm{i} \cdot \left(a_y \cdot k_{||+} \cdot \sin(\alpha) + k_z[i] \cdot \cos(\alpha) \right) \right]:$$
$$L := \left[op(L), \Delta_{||}[i]^2 + k_{||+}^{\,2} \right]:$$
$$ux_0 := \left[op(ux_0), -\frac{\mathrm{i} \cdot \left(\frac{1}{\epsilon}\right) \cdot k_{||+} \cdot \Delta_\perp[i]}{L[i] - \left(\frac{1}{\epsilon}\right)^2 \cdot k_{||+}^{\,2}} \right]:$$
end do:

$$> u_{\perp 0} := \left[1, -\frac{ux_0[1] - ux_0[3]}{ux_0[2] - ux_0[3]}, \frac{ux_0[1] - ux_0[2]}{ux_0[2] - ux_0[3]} \right]:$$

$$> u_{x0} := [\ ]:$$
$$b_{x0} := [\ ]:$$
$$b_{\perp 0} := [\ ]:$$
$$b_{||0} := [\ ]:$$
for *i* **from** 1 **to** 3 **do**:
$$u_{x0} := \left[op(u_{x0}), ux_0[i] \cdot u_{\perp 0}[i] \right]:$$
$$b_{x0} := \left[op(b_{x0}), \frac{\Delta_{||}[i] \cdot u_{x0}[i]}{\mathrm{i} \cdot k_{||+}} \right]:$$

$$b_{\perp 0} := \left[op(b_{\perp 0}), \frac{\Delta_{\|}[i] \cdot u_{\perp 0}[i]}{\mathrm{i} \cdot k_{\| +}} \right]:$$

$$b_{\| 0} := \left[op(b_{\| 0}), - \frac{\left(\frac{\mathrm{i} \cdot k_{\| +}}{\epsilon} \cdot u_{x0}[i] + \Delta_{\perp}[i] \cdot u_{\perp 0}[i] \right)}{\mathrm{i} \cdot k_{\| +}} \right]:$$

end do:

ux leading order terms

> *simplify*(*series*(*expand*($u_{x0}[1]$), ϵ, 3));

$$\frac{-a_y + \sin(\alpha)}{\cos(\alpha)} \epsilon + \mathrm{O}(\epsilon^3) \tag{2}$$

> *simplify*(*series*(*expand*($u_{x0}[2]$), ϵ, 3));

$$\frac{a_y + \sin(\alpha)}{\cos(\alpha)} \epsilon + \mathrm{O}(\epsilon^2) \tag{3}$$

> *simplify*(*series*(*expand*($u_{x0}[3]$), ϵ, 3));

$$- \frac{2 \sin(\alpha)}{\cos(\alpha)} \epsilon + \mathrm{O}(\epsilon^2) \tag{4}$$

u_perp leading order terms

> *simplify*($u_{\perp 0}[1]$);

$$1 \tag{5}$$

> *simplify*(*series*(*expand*($u_{\perp 0}[2]$), ϵ, 2));

$$-1 + \mathrm{O}(\epsilon) \tag{6}$$

> *simplify*(*series*(*expand*($u_{\perp 0}[3]$), ϵ, 2));

$$\frac{2\,\mathrm{I}\,\mathrm{csgn}\left(\frac{1}{\epsilon}\right) \sin(\alpha)^2}{\cos(\alpha)} \epsilon + \mathrm{O}(\epsilon^2) \tag{7}$$

>

b_x leading order terms

> *simplify*(*series*(*expand*($b_{x0}[1]$), ϵ, 2));

$$\frac{-a_y + \sin(\alpha)}{\cos(\alpha)} \epsilon + \mathrm{O}(\epsilon^3) \tag{8}$$

> $simplify(series(expand(b_{x0}[2]), \epsilon, 3));$

$$\frac{-a_y - \sin(\alpha)}{\cos(\alpha)}\epsilon + O(\epsilon^2) \tag{9}$$

> $simplify(series(expand(b_{x0}[3]), \epsilon, 2));$

$$-2\,\mathrm{I}\,\mathrm{csgn}\left(\frac{1}{\epsilon}\right)\sin(\alpha) + O(\epsilon) \tag{10}$$

b_perp leading order terms

> $simplify(b_{\perp 0}[1]);$

$$1 \tag{11}$$

> $simplify(series(expand(b_{\perp 0}[2]), \epsilon, 2));$

$$1 + O(\epsilon) \tag{12}$$

> $simplify(series(expand(b_{\perp 0}[3]), \epsilon, 1));$

$$-2\sin(\alpha)^2 + O(\epsilon) \tag{13}$$

b_par leading order terms

> $simplify(b_{||0}[1]);$

$$0 \tag{14}$$

> $simplify(b_{||0}[2]);$

$$0 \tag{15}$$

> $simplify(series(expand(b_{||0}[3]), \epsilon, 2));$

$$2\sin(\alpha)\cos(\alpha) + O(\epsilon) \tag{16}$$