

$$\begin{aligned} &> \text{restart} \\ &> \text{eqn1} := \text{HankelH2}(0, \xi_0) \cdot c_2 = u_0 + c_4 \\ &\quad \text{eqn1} := \text{HankelH2}(0, \xi_0) \, c_2 = u_0 + c_4 \end{aligned} \tag{1}$$

$$\begin{aligned} &> \text{eqn2} := k_z \cdot \text{HankelH2}(1, \xi_0) \cdot c_2 = i \cdot k_z \cdot u_0 - i \cdot k_z \cdot c_4 \\ &\quad \text{eqn2} := k_z \text{HankelH2}(1, \xi_0) \, c_2 = I k_z u_0 - I k_z c_4 \end{aligned} \tag{2}$$

$$\begin{aligned} &> \text{solns} := \text{solve}(\{\text{eqn1}, \text{eqn2}\}, \{c_2, c_4\}) \\ \text{solns} &:= \left\{ c_2 = \frac{2 I u_0}{I \text{HankelH2}(0, \xi_0) + \text{HankelH2}(1, \xi_0)}, c_4 \right. \\ &\quad \left. = \frac{u_0 (I \text{HankelH2}(0, \xi_0) - \text{HankelH2}(1, \xi_0))}{I \text{HankelH2}(0, \xi_0) + \text{HankelH2}(1, \xi_0)} \right\} \end{aligned} \tag{3}$$

$$\begin{aligned} &> t := \text{simplify}(\text{solns}[1]) \\ t &:= c_2 = \frac{2 I u_0}{I \text{HankelH2}(0, \xi_0) + \text{HankelH2}(1, \xi_0)} \end{aligned} \tag{4}$$

$$\begin{aligned} &> r := \text{simplify}(\text{solns}[2]) \\ r &:= c_4 = \frac{u_0 (I \text{HankelH2}(0, \xi_0) - \text{HankelH2}(1, \xi_0))}{I \text{HankelH2}(0, \xi_0) + \text{HankelH2}(1, \xi_0)} \end{aligned} \tag{5}$$

$$\begin{aligned} &> H_0 := \text{BesselJ}(0, \xi_0) - i \cdot \text{BesselY}(0, \xi_0) \\ &\quad H_0 := \text{BesselJ}(0, \xi_0) - I \text{BesselY}(0, \xi_0) \end{aligned} \tag{6}$$

$$\begin{aligned} &> H_1 := \text{BesselJ}(1, \xi_0) - i \cdot \text{BesselY}(1, \xi_0) \\ &\quad H_1 := \text{BesselJ}(1, \xi_0) - I \text{BesselY}(1, \xi_0) \end{aligned} \tag{7}$$

$$\begin{aligned} &> R := \frac{|H_0|^2 + |H_1|^2 - \frac{4}{\pi \cdot \xi_0}}{|H_0|^2 + |H_1|^2 + \frac{4}{\pi \cdot \xi_0}} \\ R &:= \left(|\text{BesselJ}(0, \xi_0) - I \text{BesselY}(0, \xi_0)|^2 + |\text{BesselJ}(1, \xi_0) - I \text{BesselY}(1, \xi_0)|^2 \right. \\ &\quad \left. - \frac{4}{\pi \xi_0} \right) / \left(|\text{BesselJ}(0, \xi_0) - I \text{BesselY}(0, \xi_0)|^2 + |\text{BesselJ}(1, \xi_0) \right. \\ &\quad \left. - I \text{BesselY}(1, \xi_0)|^2 + \frac{4}{\pi \xi_0} \right) \end{aligned} \tag{8}$$

$$\begin{aligned} &> R := \text{evalc}(R) \\ &\tag{9} \end{aligned}$$

$$R := \frac{\text{BesselJ}(0, \xi_0)^2 + \text{BesselY}(0, \xi_0)^2 + \text{BesselJ}(1, \xi_0)^2 + \text{BesselY}(1, \xi_0)^2 - \frac{4}{\pi \xi_0}}{\text{BesselJ}(0, \xi_0)^2 + \text{BesselY}(0, \xi_0)^2 + \text{BesselJ}(1, \xi_0)^2 + \text{BesselY}(1, \xi_0)^2 + \frac{4}{\pi \xi_0}}$$

(9)

$$\text{> series}(R, \xi_0^2)$$

(10)

$$1 - 2 \pi \xi_0 + O(\xi_0^2)$$