

Numerical solution  
Analytic solution

$$\omega_i/\omega_r = 1.00\text{e-}04$$

$$a_0/L_z = 1.00$$

$$k_y/L_z = 2.00\pi$$

