Note that ∇ has been replaced with '\Delta' to ensure the code works okay.

> *restart*;

We normalise the velocity coefficents by u_0 and the field components by (B_0 * u_0 / v_{A+}).

We start by solving the following Matrix equation:

> $eqn1 := a_{11} \cdot x_1 + a_{12} \cdot x_2 + a_{13} \cdot x_3 + a_{14} \cdot x_4 = y_1$

$$eqn1 := a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 = y_1 \tag{1}$$

> $eqn2 := a_{21} \cdot x_1 + a_{22} \cdot x_2 + a_{23} \cdot x_3 + a_{24} \cdot x_4 = y_2$

$$eqn2 := a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + a_{24} x_4 = y_2 \tag{2}$$

> $eqn3 := a_{31} \cdot x_1 + a_{32} \cdot x_2 + a_{33} \cdot x_3 + a_{34} \cdot x_4 = y_3$

$$eqn3 := a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + a_{34} x_4 = y_3 \tag{3}$$

> $eqn4 := a_{41} \cdot x_1 + a_{42} \cdot x_2 + a_{43} \cdot x_3 + a_{44} \cdot x_4 = y_4$

$$eqn4 := a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4 = y_4 \tag{4}$$

> $solns := solve\left(\{eqn1, eqn2, eqn3, eqn4\}, \{x_1, x_2, x_3, x_4\}\right)$:

> $sol1 := rhs(solns[1])$

$$\begin{aligned} sol1 := -\big(& a_{12} a_{23} a_{34} y_4 - a_{12} a_{23} a_{44} y_3 - a_{12} a_{24} a_{33} y_4 + a_{12} a_{24} a_{43} y_3 + a_{12} a_{33} a_{44} y_2 \\ & - a_{12} a_{34} a_{43} y_2 - a_{13} a_{22} a_{34} y_4 + a_{13} a_{22} a_{44} y_3 + a_{13} a_{24} a_{32} y_4 - a_{13} a_{24} a_{42} y_3 \\ & - a_{13} a_{32} a_{44} y_2 + a_{13} a_{34} a_{42} y_2 + a_{14} a_{22} a_{33} y_4 - a_{14} a_{22} a_{43} y_3 - a_{14} a_{23} a_{32} y_4 \\ & + a_{14} a_{23} a_{42} y_3 + a_{14} a_{32} a_{43} y_2 - a_{14} a_{33} a_{42} y_2 - a_{22} a_{33} a_{44} y_1 + a_{22} a_{34} a_{43} y_1 \\ & + a_{23} a_{32} a_{44} y_1 - a_{23} a_{34} a_{42} y_1 - a_{24} a_{32} a_{43} y_1 + a_{24} a_{33} a_{42} y_1\big) \Big/ \big(a_{11} a_{22} a_{33} a_{44} \\ & - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} \\ & - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} \\ & + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} \\ & + a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} + a_{14} a_{22} a_{31} a_{43} \\ & - a_{14} a_{22} a_{33} a_{41} - a_{14} a_{23} a_{31} a_{42} + a_{14} a_{23} a_{32} a_{41}\big) \end{aligned} \tag{5}$$

> $sol2 := rhs(solns[2])$

$$\begin{aligned} sol2 := \big(& a_{11} a_{23} a_{34} y_4 - a_{11} a_{23} a_{44} y_3 - a_{11} a_{24} a_{33} y_4 + a_{11} a_{24} a_{43} y_3 + a_{11} a_{33} a_{44} y_2 \\ & - a_{11} a_{34} a_{43} y_2 - a_{13} a_{21} a_{34} y_4 + a_{13} a_{21} a_{44} y_3 + a_{13} a_{24} a_{31} y_4 - a_{13} a_{24} a_{41} y_3 \\ & - a_{13} a_{31} a_{44} y_2 + a_{13} a_{34} a_{41} y_2 + a_{14} a_{21} a_{33} y_4 - a_{14} a_{21} a_{43} y_3 - a_{14} a_{23} a_{31} y_4 \\ & + a_{14} a_{23} a_{41} y_3 + a_{14} a_{31} a_{43} y_2 - a_{14} a_{33} a_{41} y_2 - a_{21} a_{33} a_{44} y_1 + a_{21} a_{34} a_{43} y_1 \\ & + a_{23} a_{31} a_{44} y_1 - a_{23} a_{34} a_{41} y_1 - a_{24} a_{31} a_{43} y_1 + a_{24} a_{33} a_{41} y_1\big) \Big/ \big(a_{11} a_{22} a_{33} a_{44} \\ & - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} \\ & - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} \\ & + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} \\ & + a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} + a_{14} a_{22} a_{31} a_{43} \end{aligned} \tag{6}$$

$$-a_{14} a_{22} a_{33} a_{41} - a_{14} a_{23} a_{31} a_{42} + a_{14} a_{23} a_{32} a_{41} \Big)$$

> $sol3 := rhs(solns[3])$

$$sol3 := -\Big(a_{11} a_{22} a_{34} y_4 - a_{11} a_{22} a_{44} y_3 - a_{11} a_{24} a_{32} y_4 + a_{11} a_{24} a_{42} y_3 + a_{11} a_{32} a_{44} y_2 - a_{11} a_{34} a_{42} y_2 - a_{12} a_{21} a_{34} y_4 + a_{12} a_{21} a_{44} y_3 + a_{12} a_{24} a_{31} y_4 - a_{12} a_{24} a_{41} y_3 - a_{12} a_{31} a_{44} y_2 + a_{12} a_{34} a_{41} y_2 + a_{14} a_{21} a_{32} y_4 - a_{14} a_{21} a_{42} y_3 - a_{14} a_{22} a_{31} y_4 + a_{14} a_{22} a_{41} y_3 + a_{14} a_{31} a_{42} y_2 - a_{14} a_{32} a_{41} y_2 - a_{21} a_{32} a_{44} y_1 + a_{21} a_{34} a_{42} y_1 + a_{22} a_{31} a_{44} y_1 - a_{22} a_{34} a_{41} y_1 - a_{24} a_{31} a_{42} y_1 + a_{24} a_{32} a_{41} y_1\Big) \Big/ \Big(a_{11} a_{22} a_{33} a_{44} - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} + a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} + a_{14} a_{22} a_{31} a_{43} - a_{14} a_{22} a_{33} a_{41} - a_{14} a_{23} a_{31} a_{42} + a_{14} a_{23} a_{32} a_{41}\Big) \quad (7)$$

> $sol4 := rhs(solns[4])$

$$sol4 := \Big(a_{11} a_{22} a_{33} y_4 - a_{11} a_{22} a_{43} y_3 - a_{11} a_{23} a_{32} y_4 + a_{11} a_{23} a_{42} y_3 + a_{11} a_{32} a_{43} y_2 - a_{11} a_{33} a_{42} y_2 - a_{12} a_{21} a_{33} y_4 + a_{12} a_{21} a_{43} y_3 + a_{12} a_{23} a_{31} y_4 - a_{12} a_{23} a_{41} y_3 - a_{12} a_{31} a_{43} y_2 + a_{12} a_{33} a_{41} y_2 + a_{13} a_{21} a_{32} y_4 - a_{13} a_{21} a_{42} y_3 - a_{13} a_{22} a_{31} y_4 + a_{13} a_{22} a_{41} y_3 + a_{13} a_{31} a_{42} y_2 - a_{13} a_{32} a_{41} y_2 - a_{21} a_{32} a_{43} y_1 + a_{21} a_{33} a_{42} y_1 + a_{22} a_{31} a_{43} y_1 - a_{22} a_{33} a_{41} y_1 - a_{23} a_{31} a_{42} y_1 + a_{23} a_{32} a_{41} y_1\Big) \Big/ \Big(a_{11} a_{22} a_{33} a_{44} - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} + a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} + a_{14} a_{22} a_{31} a_{43} - a_{14} a_{22} a_{33} a_{41} - a_{14} a_{23} a_{31} a_{42} + a_{14} a_{23} a_{32} a_{41}\Big) \quad (8)$$

> $a_{11} := u_{x10-}$

$$a_{11} := u_{x10\text{-}} \quad (9)$$

> $a_{12} := u_{x40-}$

$$a_{12} := u_{x40\text{-}} \quad (10)$$

> $a_{13} := -u_{x20+}$

$$a_{13} := -u_{x20+} \quad (11)$$

> $a_{14} := -u_{x30+}$

$$a_{14} := -u_{x30+} \quad (12)$$

> $a_{21} := k_{z1-} \cdot u_{x10-}$

$$a_{21} := k_{z1\text{-}}\, u_{x10\text{-}} \quad (13)$$

> $a_{22} := k_{z4-} \cdot u_{x40-}$

$$a_{22} := k_{z4-}\, u_{x40-} \tag{14}$$

> $a_{23} := -k_{z2+} \cdot u_{x20+}$

$$a_{23} := -k_{z2+}\, u_{x20+} \tag{15}$$

> $a_{24} := -k_{z3+} \cdot u_{x30+}$

$$a_{24} := -k_{z3+}\, u_{x30+} \tag{16}$$

> $a_{31} := 1$

$$a_{31} := 1 \tag{17}$$

> $a_{32} := 1$

$$a_{32} := 1 \tag{18}$$

> $a_{33} := -1$

$$a_{33} := -1 \tag{19}$$

> $a_{34} := -1$

$$a_{34} := -1 \tag{20}$$

> $a_{41} := k_{z1-}$

$$a_{41} := k_{z1-} \tag{21}$$

> $a_{42} := k_{z4-}$

$$a_{42} := k_{z4-} \tag{22}$$

> $a_{43} := -k_{z2+}$

$$a_{43} := -k_{z2+} \tag{23}$$

> $a_{44} := -k_{z3+}$

$$a_{44} := -k_{z3+} \tag{24}$$

> $y_1 := u_{x10+}$

$$y_1 := u_{x10+} \tag{25}$$

> $y_2 := k_{z1+} \cdot u_{x10+}$

$$y_2 := k_{z1+}\, u_{x10+} \tag{26}$$

> $y_3 := 1$

$$y_3 := 1 \tag{27}$$

> $y_4 := k_{z1+}$

$$y_4 := k_{z1+} \tag{28}$$

Note that ux_0[n] denotes \hat{u}_{xn} and u_x[n±] denotes u_{xn±}.

> $u_{x10-} := -\dfrac{\mathrm{i} \cdot k_x \cdot \Delta_{\perp 1-}}{L_{1-} - k_x^2}$

$$u_{x10-} := \frac{-I\, k_x\, \Delta_{\perp 1-}}{-k_x^{\,2} + L_{1-}} \tag{29}$$

> $u_{x10+} := -\frac{\mathrm{i}\cdot k_x\cdot \Delta_{\perp 1+}}{L_{1+} - k_x^{\,2}}$

$$u_{x10+} := \frac{-I\, k_x\, \Delta_{\perp 1+}}{-k_x^{\,2} + L_{1+}} \tag{30}$$

> $u_{x20-} := -\frac{\mathrm{i}\cdot k_x\cdot \Delta_{\perp 2-}}{L_{2-} - k_x^{\,2}}$

$$u_{x20-} := \frac{-I\, k_x\, \Delta_{\perp 2-}}{-k_x^{\,2} + L_{2-}} \tag{31}$$

> $u_{x20+} := -\frac{\mathrm{i}\cdot k_x\cdot \Delta_{\perp 2+}}{L_{2+} - k_x^{\,2}}$

$$u_{x20+} := \frac{-I\, k_x\, \Delta_{\perp 2+}}{-k_x^{\,2} + L_{2+}} \tag{32}$$

> $u_{x30-} := -\frac{\mathrm{i}\cdot k_x\cdot \Delta_{\perp 3-}}{L_{3-} - k_x^{\,2}}$

$$u_{x30-} := \frac{-I\, k_x\, \Delta_{\perp 3-}}{-k_x^{\,2} + L_{3-}} \tag{33}$$

> $u_{x30+} := -\frac{\mathrm{i}\cdot k_x\cdot \Delta_{\perp 3+}}{L_{3+} - k_x^{\,2}}$

$$u_{x30+} := \frac{-I\, k_x\, \Delta_{\perp 3+}}{-k_x^{\,2} + L_{3+}} \tag{34}$$

> $u_{x40-} := -\frac{\mathrm{i}\cdot k_x\cdot \Delta_{\perp 4-}}{L_{4-} - k_x^{\,2}}$

$$u_{x40-} := \frac{-I\, k_x\, \Delta_{\perp 4-}}{-k_x^{\,2} + L_{4-}} \tag{35}$$

> $u_{x40+} := -\frac{\mathrm{i}\cdot k_x\cdot \Delta_{\perp 4+}}{L_{4+} - k_x^{\,2}}$

(36)

$$u_{x40+} := \frac{-\mathrm{I}\, k_x \, \Delta_{\perp 4+}}{-k_x^{\,2} + L_{4+}} \tag{36}$$

> $L_{1-} := \Delta_{\|1-}^{\;\;2} + k_{\|-}^{\;2}$

$$L_{1\text{-}} := k_{\|\text{-}}^{\;2} + \Delta_{\|1\text{-}}^{\;\;2} \tag{37}$$

> $L_{1+} := \Delta_{\|1+}^{\;\;2} + k_{\|+}^{\;2}$

$$L_{1+} := k_{\|+}^{\;2} + \Delta_{\|1+}^{\;\;2} \tag{38}$$

> $L_{2-} := \Delta_{\|2-}^{\;\;2} + k_{\|-}^{\;2}$

$$L_{2\text{-}} := k_{\|\text{-}}^{\;2} + \Delta_{\|2\text{-}}^{\;\;2} \tag{39}$$

> $L_{2+} := \Delta_{\|2+}^{\;\;2} + k_{\|+}^{\;2}$

$$L_{2+} := k_{\|+}^{\;2} + \Delta_{\|2+}^{\;\;2} \tag{40}$$

> $L_{3-} := \Delta_{\|3-}^{\;\;2} + k_{\|-}^{\;2}$

$$L_{3\text{-}} := k_{\|\text{-}}^{\;2} + \Delta_{\|3\text{-}}^{\;\;2} \tag{41}$$

> $L_{3+} := \Delta_{\|3+}^{\;\;2} + k_{\|+}^{\;2}$

$$L_{3+} := k_{\|+}^{\;2} + \Delta_{\|3+}^{\;\;2} \tag{42}$$

> $L_{4-} := \Delta_{\|4-}^{\;\;2} + k_{\|-}^{\;2}$

$$L_{4\text{-}} := k_{\|\text{-}}^{\;2} + \Delta_{\|4\text{-}}^{\;\;2} \tag{43}$$

> $L_{4+} := \Delta_{\|4+}^{\;\;2} + k_{\|+}^{\;2}$

$$L_{4+} := k_{\|+}^{\;2} + \Delta_{\|4+}^{\;\;2} \tag{44}$$

> $\Delta_{\perp 1-} := \mathrm{i} \cdot \left(k_y \cdot \cos(\alpha) - k_{z1-} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 1\text{-}} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z1\text{-}} \sin(\alpha)\right) \tag{45}$$

> $\Delta_{\perp 1+} := \mathrm{i} \cdot \left(k_y \cdot \cos(\alpha) - k_{z1+} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 1+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z1+} \sin(\alpha)\right) \tag{46}$$

> $\Delta_{\perp 2-} := \mathrm{i} \cdot \left(k_y \cdot \cos(\alpha) - k_{z2-} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 2\text{-}} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z2\text{-}} \sin(\alpha)\right) \tag{47}$$

> $\Delta_{\perp 2+} := \mathrm{i} \cdot \left(k_y \cdot \cos(\alpha) - k_{z2+} \cdot \sin(\alpha)\right)$

$$\Delta_{\perp 2+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z2+} \sin(\alpha)\right) \tag{48}$$

> $\Delta_{\perp 3-} := \mathrm{i} \cdot \left(k_y \cdot \cos(\alpha) - k_{z3-} \cdot \sin(\alpha)\right)$

(49)

$$\Delta_{\perp 3\text{-}} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z3\text{-}} \sin(\alpha)\right) \tag{49}$$

> $\Delta_{\perp 3+} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z3+}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 3+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z3+} \sin(\alpha)\right) \tag{50}$$

> $\Delta_{\perp 4-} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z4-}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 4\text{-}} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z4\text{-}} \sin(\alpha)\right) \tag{51}$$

> $\Delta_{\perp 4+} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z4+}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 4+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z4+} \sin(\alpha)\right) \tag{52}$$

> $\Delta_{\| 1-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z1-}\cdot\cos(\alpha)\right)$

$$\Delta_{\| 1\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z1\text{-}} \cos(\alpha)\right) \tag{53}$$

> $\Delta_{\| 1+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z1+}\cdot\cos(\alpha)\right)$

$$\Delta_{\| 1+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z1+} \cos(\alpha)\right) \tag{54}$$

> $\Delta_{\| 2-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z2-}\cdot\cos(\alpha)\right)$

$$\Delta_{\| 2\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z2\text{-}} \cos(\alpha)\right) \tag{55}$$

> $\Delta_{\| 2+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z2+}\cdot\cos(\alpha)\right)$

$$\Delta_{\| 2+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z2+} \cos(\alpha)\right) \tag{56}$$

> $\Delta_{\| 3-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z3-}\cdot\cos(\alpha)\right)$

$$\Delta_{\| 3\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z3\text{-}} \cos(\alpha)\right) \tag{57}$$

> $\Delta_{\| 3+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z3+}\cdot\cos(\alpha)\right)$

$$\Delta_{\| 3+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z3+} \cos(\alpha)\right) \tag{58}$$

> $\Delta_{\| 4-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z4-}\cdot\cos(\alpha)\right)$

$$\Delta_{\| 4\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z4\text{-}} \cos(\alpha)\right) \tag{59}$$

> $\Delta_{\| 4+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z4+}\cdot\cos(\alpha)\right)$

$$\Delta_{\| 4+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z4+} \cos(\alpha)\right) \tag{60}$$

> $k_{z1-} := \dfrac{k_{\|-}}{\cos(\alpha)} - k_y\cdot\tan(\alpha)$

$$k_{z1\text{-}} := \frac{k_{\|\text{-}}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{61}$$

> $k_{z1+} := \dfrac{k_{\|+}}{\cos(\alpha)} - k_y\cdot\tan(\alpha)$

$$k_{z1+} := \frac{k_{\|+}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{62}$$

> $k_{z2-} := -\dfrac{k_{||-}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z2\text{-}} := -\frac{k_{||\text{-}}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{63}$$

> $k_{z2+} := -\dfrac{k_{||+}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z2+} := -\frac{k_{||+}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{64}$$

> $k_{z3-} := \mathrm{i} \cdot \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||-}^{\,2}}$

$$k_{z3\text{-}} := \mathrm{I}\sqrt{k_x^{\,2} + k_y^{\,2} - k_{||\text{-}}^{\,2}} \tag{65}$$

> $k_{z3+} := \mathrm{i} \cdot \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||+}^{\,2}}$

$$k_{z3+} := \mathrm{I}\sqrt{k_x^{\,2} + k_y^{\,2} - k_{||+}^{\,2}} \tag{66}$$

> $k_{z4-} := -\mathrm{i} \cdot \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||-}^{\,2}}$

$$k_{z4\text{-}} := -\mathrm{I}\sqrt{k_x^{\,2} + k_y^{\,2} - k_{||\text{-}}^{\,2}} \tag{67}$$

> $k_{z4+} := -\mathrm{i} \cdot \sqrt{k_x^{\,2} + k_y^{\,2} - k_{||+}^{\,2}}$

$$k_{z4+} := -\mathrm{I}\sqrt{k_x^{\,2} + k_y^{\,2} - k_{||+}^{\,2}} \tag{68}$$

>

> $k_{||+} := r \cdot k_{||-}$

$$k_{||+} := r\, k_{||\text{-}} \tag{69}$$

> $k_y := a_y \cdot k_{||+}$

$$k_y := a_y\, r\, k_{||\text{-}} \tag{70}$$

> $k_x := \dfrac{k_{||-}}{\epsilon}$

$$k_x := \frac{k_{||\text{-}}}{\epsilon} \tag{71}$$

ux leading order terms

> $u_{x1-} := u_{x10-} \cdot sol1 :$

> $u_{x4-} := u_{x40-} \cdot sol2 :$

> $u_{x1+} := u_{x10+} :$

> $u_{x2+} := u_{x20+} \cdot sol3 :$

> $u_{x3+} := u_{x30+} \cdot sol4 :$

> simplify(series(u_{x1-}, ε, 2))

$$\frac{2\, r\, \left(-a_y\, r + \sin(\alpha)\right)}{\cos(\alpha)\, (r+1)}\, \epsilon + O(\epsilon^2) \tag{72}$$

> simplify(series(u_{x4-}, ε, 3))

$$\frac{I \sin(\alpha)\, r\, (r-1)\, \mathrm{csgn}\left(\frac{k_{||-}}{\epsilon}\right)}{\cos(\alpha)^2}\, \epsilon^2 + O(\epsilon^3) \tag{73}$$

> simplify(series(u_{x1+}, ε, 3))

$$\frac{r\, \left(-a_y + \sin(\alpha)\right)}{\cos(\alpha)}\, \epsilon + O(\epsilon^3) \tag{74}$$

> simplify(series(u_{x2+}, ε, 3))

$$-\frac{r\, (r-1)\, \left(a_y + \sin(\alpha)\right)}{\cos(\alpha)\, (r+1)}\, \epsilon + O(\epsilon^3) \tag{75}$$

> simplify(series(u_{x3+}, ε, 3))

$$\frac{I \sin(\alpha)\, r\, (r-1)\, \mathrm{csgn}\left(\frac{k_{||-}}{\epsilon}\right)}{\cos(\alpha)^2}\, \epsilon^2 + O(\epsilon^3) \tag{76}$$

u_perp leading order terms

> simplify(series(sol1, ε, 3))

$$\frac{2\, r}{r+1} + 2\, \frac{r^2\, (r-1)\, (\cos(\alpha)-1)\, (\cos(\alpha)+1)\, \left(a_y - \sin(\alpha)\right)}{\sin(\alpha)\, \cos(\alpha)^2\, (r+1)}\, \epsilon^2 + O(\epsilon^3) \tag{77}$$

> simplify(series(sol2, ε, 3))

$$\frac{(\cos(\alpha)-1)\, (\cos(\alpha)+1)\, (r-1)\, r}{\cos(\alpha)^2}\, \epsilon^2 + O(\epsilon^3) \tag{78}$$

> simplify(series(sol3, ε, 2))

$$\frac{r-1}{r+1} + O(\epsilon^2) \tag{79}$$

> simplify(series(sol4, ε, 4))

$$-\frac{(\cos(\alpha)-1)\, (\cos(\alpha)+1)\, (r-1)\, r}{\cos(\alpha)^2}\, \epsilon^2 + \frac{I\, r^2\, a_y \left(\cos(\alpha)^4 - 3\cos(\alpha)^2 + 2\right)\, (r-1)\, \mathrm{csgn}\left(\frac{k_{||-}}{\epsilon}\right)}{\sin(\alpha)\, \cos(\alpha)^3}\, \epsilon^3 + O(\epsilon^4) \tag{80}$$

bx leading order terms

> $b_{x1-} := \dfrac{\Delta_{||1-} \cdot u_{x1-}}{\mathrm{i} \cdot k_{||+}}$:

> $b_{x4-} := \dfrac{\Delta_{||4-} \cdot u_{x4-}}{\mathrm{i} \cdot k_{||+}}$:

> $b_{x1+} := \dfrac{\Delta_{||1+} \cdot u_{x1+}}{\mathrm{i} \cdot k_{||+}}$:

> $b_{x2+} := \dfrac{\Delta_{||2+} \cdot u_{x2+}}{\mathrm{i} \cdot k_{||+}}$:

> $b_{x3+} := \dfrac{\Delta_{||3+} \cdot u_{x3+}}{\mathrm{i} \cdot k_{||+}}$:

> $simplify(series(b_{x1-}, \epsilon, 2))$

$$\frac{-2\, a_y\, r + 2 \sin(\alpha)}{\cos(\alpha)\, (r+1)}\, \epsilon + \mathrm{O}(\epsilon^2) \qquad \textbf{(81)}$$

> $simplify(series(b_{x4-}, \epsilon, 2))$

$$\frac{\sin(\alpha)\, (r-1)}{\cos(\alpha)}\, \epsilon + \mathrm{O}(\epsilon^2) \qquad \textbf{(82)}$$

> $simplify(series(b_{x1+}, \epsilon, 2))$

$$\frac{r\, (-a_y + \sin(\alpha))}{\cos(\alpha)}\, \epsilon + \mathrm{O}(\epsilon^3) \qquad \textbf{(83)}$$

> $simplify(series(b_{x2+}, \epsilon, 2))$

$$\frac{r\, (r-1)\, (a_y + \sin(\alpha))}{\cos(\alpha)\, (r+1)}\, \epsilon + \mathrm{O}(\epsilon^2) \qquad \textbf{(84)}$$

> $simplify(series(b_{x3+}, \epsilon, 2))$

$$-\frac{\sin(\alpha)\, (r-1)}{\cos(\alpha)}\, \epsilon + \mathrm{O}(\epsilon^2) \qquad \textbf{(85)}$$

b_perp leading order terms

> $b_{\perp 1-} := \dfrac{\Delta_{||1-} \cdot sol1}{\mathrm{i} \cdot k_{||+}}$:

> $b_{\perp 4-} := \dfrac{\Delta_{||4-} \cdot sol2}{\mathrm{i} \cdot k_{||+}}$:

> $b_{\perp 1+} := \dfrac{\Delta_{||1+} \cdot 1}{\mathrm{i} \cdot k_{||+}}$:

> $b_{\perp 2+} := \dfrac{\Delta_{\parallel 2+} \cdot sol3}{\mathrm{i} \cdot k_{\parallel +}}$:

> $b_{\perp 3+} := \dfrac{\Delta_{\parallel 3+} \cdot sol4}{\mathrm{i} \cdot k_{\parallel +}}$:

> $simplify(series(b_{\perp 1-}, \epsilon, 2))$

$$\frac{2}{r+1} + \mathrm{O}(\epsilon^2) \tag{86}$$

> $simplify(series(b_{\perp 4-}, \epsilon, 2))$

$$\frac{-\mathrm{I}\,\mathrm{csgn}\left(\frac{k_{\parallel -}}{\epsilon}\right) (\cos(\alpha) - 1)\,(r-1)\,(\cos(\alpha) + 1)}{\cos(\alpha)}\,\epsilon + \mathrm{O}(\epsilon^2) \tag{87}$$

> $simplify(series(b_{\perp 1+}, \epsilon, 2))$

$$1 \tag{88}$$

> $simplify(series(b_{\perp 2+}, \epsilon, 2))$

$$\frac{-r+1}{r+1} + \mathrm{O}(\epsilon^2) \tag{89}$$

> $simplify(series(b_{\perp 3+}, \epsilon, 2))$

$$\mathrm{O}(\epsilon) \tag{90}$$

b_par leading order terms

> $b_{\parallel 1-} := -\dfrac{\mathrm{i} \cdot k_x \cdot u_{x1-} + \Delta_{\perp 1-} \cdot sol1}{\mathrm{i} \cdot k_{\parallel +}}$:

> $b_{\parallel 4-} := -\dfrac{\mathrm{i} \cdot k_x \cdot u_{x4-} + \Delta_{\perp 4-} \cdot sol2}{\mathrm{i} \cdot k_{\parallel +}}$:

> $b_{\parallel 1+} := -\dfrac{\mathrm{i} \cdot k_x \cdot u_{x1+} + \Delta_{\perp 1+}}{\mathrm{i} \cdot k_{\parallel +}}$:

> $b_{\parallel 2+} := -\dfrac{\mathrm{i} \cdot k_x \cdot u_{x2+} + \Delta_{\perp 2+} \cdot sol3}{\mathrm{i} \cdot k_{\parallel +}}$:

> $b_{\parallel 3+} := -\dfrac{\mathrm{i} \cdot k_x \cdot u_{x3+} + \Delta_{\perp 3+} \cdot sol4}{\mathrm{i} \cdot k_{\parallel +}}$:

> $simplify(b_{\parallel 1-})$

$$0 \tag{91}$$

> $simplify(series(expand(b_{\parallel 4-}), \epsilon, 3))$

$$-\mathrm{I}\,(r-1)\,\sin(\alpha)\,\mathrm{csgn}\left(\frac{k_{\parallel -}}{\epsilon}\right)\epsilon + \mathrm{O}(\epsilon^2) \tag{92}$$

> $simplify(b_{\parallel 1+})$

$$0 \tag{93}$$

> $simplify(b_{||2+})$

$$0 \tag{94}$$

> $simplify(series(expand(b_{||3+}), \epsilon, 3))$

$$-\mathrm{I}\,(r-1)\,\sin(\alpha)\,\mathrm{csgn}\left(\frac{k_{||-}}{\epsilon}\right)\epsilon+\mathrm{O}(\epsilon^2) \tag{95}$$