Note that ∇ has been replaced with to ensure the code works okay.

> *restart*;

We normalise the velocity coefficents by u_0 and the field components by (B_0 * u_0 / v_{A+}).

We start by solving the following Matrix equation:

> $eqn1 := a_{11}\cdot x_1 + a_{12}\cdot x_2 + a_{13}\cdot x_3 + a_{14}\cdot x_4 = y_1$

$$eqn1 := a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 = y_1 \quad (1)$$

> $eqn2 := a_{21}\cdot x_1 + a_{22}\cdot x_2 + a_{23}\cdot x_3 + a_{24}\cdot x_4 = y_2$

$$eqn2 := a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + a_{24} x_4 = y_2 \quad (2)$$

> $eqn3 := a_{31}\cdot x_1 + a_{32}\cdot x_2 + a_{33}\cdot x_3 + a_{34}\cdot x_4 = y_3$

$$eqn3 := a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + a_{34} x_4 = y_3 \quad (3)$$

> $eqn4 := a_{41}\cdot x_1 + a_{42}\cdot x_2 + a_{43}\cdot x_3 + a_{44}\cdot x_4 = y_4$

$$eqn4 := a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4 = y_4 \quad (4)$$

> $solns := solve\left(\{eqn1, eqn2, eqn3, eqn4\}, \{x_1, x_2, x_3, x_4\}\right) :$

> $sol1 := rhs(solns[1])$

$$sol1 := -\big(a_{12} a_{23} a_{34} y_4 - a_{12} a_{23} a_{44} y_3 - a_{12} a_{24} a_{33} y_4 + a_{12} a_{24} a_{43} y_3 + a_{12} a_{33} a_{44} y_2 - a_{12} a_{34} a_{43} y_2 - a_{13} a_{22} a_{34} y_4 + a_{13} a_{22} a_{44} y_3 + a_{13} a_{24} a_{32} y_4 - a_{13} a_{24} a_{42} y_3 - a_{13} a_{32} a_{44} y_2 + a_{13} a_{34} a_{42} y_2 + a_{14} a_{22} a_{33} y_4 - a_{14} a_{22} a_{43} y_3 - a_{14} a_{23} a_{32} y_4 + a_{14} a_{23} a_{42} y_3 + a_{14} a_{32} a_{43} y_2 - a_{14} a_{33} a_{42} y_2 - a_{22} a_{33} a_{44} y_1 + a_{22} a_{34} a_{43} y_1 + a_{23} a_{32} a_{44} y_1 - a_{23} a_{34} a_{42} y_1 - a_{24} a_{32} a_{43} y_1 + a_{24} a_{33} a_{42} y_1\big) \Big/ \big(a_{11} a_{22} a_{33} a_{44} - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} + a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} + a_{14} a_{22} a_{31} a_{43} - a_{14} a_{22} a_{33} a_{41} - a_{14} a_{23} a_{31} a_{42} + a_{14} a_{23} a_{32} a_{41}\big) \quad (5)$$

> $sol2 := rhs(solns[2])$

$$sol2 := \big(a_{11} a_{23} a_{34} y_4 - a_{11} a_{23} a_{44} y_3 - a_{11} a_{24} a_{33} y_4 + a_{11} a_{24} a_{43} y_3 + a_{11} a_{33} a_{44} y_2 - a_{11} a_{34} a_{43} y_2 - a_{13} a_{21} a_{34} y_4 + a_{13} a_{21} a_{44} y_3 + a_{13} a_{24} a_{31} y_4 - a_{13} a_{24} a_{41} y_3 - a_{13} a_{31} a_{44} y_2 + a_{13} a_{34} a_{41} y_2 + a_{14} a_{21} a_{33} y_4 - a_{14} a_{21} a_{43} y_3 - a_{14} a_{23} a_{31} y_4 + a_{14} a_{23} a_{41} y_3 + a_{14} a_{31} a_{43} y_2 - a_{14} a_{33} a_{41} y_2 - a_{21} a_{33} a_{44} y_1 + a_{21} a_{34} a_{43} y_1 + a_{23} a_{31} a_{44} y_1 - a_{23} a_{34} a_{41} y_1 - a_{24} a_{31} a_{43} y_1 + a_{24} a_{33} a_{41} y_1\big) \Big/ \big(a_{11} a_{22} a_{33} a_{44} - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} + a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} + a_{14} a_{22} a_{31} a_{43} \quad (6)$$

$$- a_{14} a_{22} a_{33} a_{41} - a_{14} a_{23} a_{31} a_{42} + a_{14} a_{23} a_{32} a_{41} \Big)$$

> $sol3 := rhs(solns[3])$

$$sol3 := - \Big(a_{11} a_{22} a_{34} y_4 - a_{11} a_{22} a_{44} y_3 - a_{11} a_{24} a_{32} y_4 + a_{11} a_{24} a_{42} y_3 + a_{11} a_{32} a_{44} y_2 - a_{11} a_{34} a_{42} y_2 - a_{12} a_{21} a_{34} y_4 + a_{12} a_{21} a_{44} y_3 + a_{12} a_{24} a_{31} y_4 - a_{12} a_{24} a_{41} y_3 - a_{12} a_{31} a_{44} y_2 + a_{12} a_{34} a_{41} y_2 + a_{14} a_{21} a_{32} y_4 - a_{14} a_{21} a_{42} y_3 - a_{14} a_{22} a_{31} y_4 + a_{14} a_{22} a_{41} y_3 + a_{14} a_{31} a_{42} y_2 - a_{14} a_{32} a_{41} y_2 - a_{21} a_{32} a_{44} y_1 + a_{21} a_{34} a_{42} y_1 + a_{22} a_{31} a_{44} y_1 - a_{22} a_{34} a_{41} y_1 - a_{24} a_{31} a_{42} y_1 + a_{24} a_{32} a_{41} y_1 \Big) \Big/ \Big(a_{11} a_{22} a_{33} a_{44} - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} + a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} + a_{14} a_{22} a_{31} a_{43} - a_{14} a_{22} a_{33} a_{41} - a_{14} a_{23} a_{31} a_{42} + a_{14} a_{23} a_{32} a_{41} \Big) \quad \textbf{(7)}$$

> $sol4 := rhs(solns[4])$

$$sol4 := \Big(a_{11} a_{22} a_{33} y_4 - a_{11} a_{22} a_{43} y_3 - a_{11} a_{23} a_{32} y_4 + a_{11} a_{23} a_{42} y_3 + a_{11} a_{32} a_{43} y_2 - a_{11} a_{33} a_{42} y_2 - a_{12} a_{21} a_{33} y_4 + a_{12} a_{21} a_{43} y_3 + a_{12} a_{23} a_{31} y_4 - a_{12} a_{23} a_{41} y_3 - a_{12} a_{31} a_{43} y_2 + a_{12} a_{33} a_{41} y_2 + a_{13} a_{21} a_{32} y_4 - a_{13} a_{21} a_{42} y_3 - a_{13} a_{22} a_{31} y_4 + a_{13} a_{22} a_{41} y_3 + a_{13} a_{31} a_{42} y_2 - a_{13} a_{32} a_{41} y_2 - a_{21} a_{32} a_{43} y_1 + a_{21} a_{33} a_{42} y_1 + a_{22} a_{31} a_{43} y_1 - a_{22} a_{33} a_{41} y_1 - a_{23} a_{31} a_{42} y_1 + a_{23} a_{32} a_{41} y_1 \Big) \Big/ \Big(a_{11} a_{22} a_{33} a_{44} - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} + a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} + a_{14} a_{22} a_{31} a_{43} - a_{14} a_{22} a_{33} a_{41} - a_{14} a_{23} a_{31} a_{42} + a_{14} a_{23} a_{32} a_{41} \Big) \quad \textbf{(8)}$$

> $a_{11} := u_{x10-}$

$$a_{11} := u_{x10\text{-}} \quad \textbf{(9)}$$

> $a_{12} := u_{x40-}$

$$a_{12} := u_{x40\text{-}} \quad \textbf{(10)}$$

> $a_{13} := -u_{x20+}$

$$a_{13} := -u_{x20+} \quad \textbf{(11)}$$

> $a_{14} := -u_{x30+}$

$$a_{14} := -u_{x30+} \quad \textbf{(12)}$$

> $a_{21} := k_{z1-} \cdot u_{x10-}$

$$a_{21} := k_{z1\text{-}} \, u_{x10\text{-}} \quad \textbf{(13)}$$

> $a_{22} := k_{z4-} \cdot u_{x40-}$

$$a_{22} := k_{z4-}\, u_{x40-} \tag{14}$$

> $a_{23} := -k_{z2+} \cdot u_{x20+}$

$$a_{23} := -k_{z2+}\, u_{x20+} \tag{15}$$

> $a_{24} := -k_{z3+} \cdot u_{x30+}$

$$a_{24} := -k_{z3+}\, u_{x30+} \tag{16}$$

> $a_{31} := 1$

$$a_{31} := 1 \tag{17}$$

> $a_{32} := 1$

$$a_{32} := 1 \tag{18}$$

> $a_{33} := -1$

$$a_{33} := -1 \tag{19}$$

> $a_{34} := -1$

$$a_{34} := -1 \tag{20}$$

> $a_{41} := k_{z1-}$

$$a_{41} := k_{z1-} \tag{21}$$

> $a_{42} := k_{z4-}$

$$a_{42} := k_{z4-} \tag{22}$$

> $a_{43} := -k_{z2+}$

$$a_{43} := -k_{z2+} \tag{23}$$

> $a_{44} := -k_{z3+}$

$$a_{44} := -k_{z3+} \tag{24}$$

> $y_1 := u_{x10+}$

$$y_1 := u_{x10+} \tag{25}$$

> $y_2 := k_{z1+} \cdot u_{x10+}$

$$y_2 := k_{z1+}\, u_{x10+} \tag{26}$$

> $y_3 := 1$

$$y_3 := 1 \tag{27}$$

> $y_4 := k_{z1+}$

$$y_4 := k_{z1+} \tag{28}$$

Note that ux_0[n] denotes \hat{u}_{xn} and u_x[n±] denotes u_{xn±}.

> $u_{x10-} := -\frac{\mathrm{i} \cdot k_x \cdot \Delta_{\perp 1-}}{L_{1-} - k_x^2}$

$$u_{x10-} := \frac{-I\, k_x\, \Delta_{\perp 1-}}{-k_x^2 + L_{1-}} \quad (29)$$

> $u_{x10+} := -\frac{i \cdot k_x \cdot \Delta_{\perp 1+}}{L_{1+} - k_x^2}$

$$u_{x10+} := \frac{-I\, k_x\, \Delta_{\perp 1+}}{-k_x^2 + L_{1+}} \quad (30)$$

> $u_{x20-} := -\frac{i \cdot k_x \cdot \Delta_{\perp 2-}}{L_{2-} - k_x^2}$

$$u_{x20-} := \frac{-I\, k_x\, \Delta_{\perp 2-}}{-k_x^2 + L_{2-}} \quad (31)$$

> $u_{x20+} := -\frac{i \cdot k_x \cdot \Delta_{\perp 2+}}{L_{2+} - k_x^2}$

$$u_{x20+} := \frac{-I\, k_x\, \Delta_{\perp 2+}}{-k_x^2 + L_{2+}} \quad (32)$$

> $u_{x30-} := -\frac{i \cdot k_x \cdot \Delta_{\perp 3-}}{L_{3-} - k_x^2}$

$$u_{x30-} := \frac{-I\, k_x\, \Delta_{\perp 3-}}{-k_x^2 + L_{3-}} \quad (33)$$

> $u_{x30+} := -\frac{i \cdot k_x \cdot \Delta_{\perp 3+}}{L_{3+} - k_x^2}$

$$u_{x30+} := \frac{-I\, k_x\, \Delta_{\perp 3+}}{-k_x^2 + L_{3+}} \quad (34)$$

> $u_{x40-} := -\frac{i \cdot k_x \cdot \Delta_{\perp 4-}}{L_{4-} - k_x^2}$

$$u_{x40-} := \frac{-I\, k_x\, \Delta_{\perp 4-}}{-k_x^2 + L_{4-}} \quad (35)$$

> $u_{x40+} := -\frac{i \cdot k_x \cdot \Delta_{\perp 4+}}{L_{4+} - k_x^2}$

(36)

$$u_{x40+} := \frac{-\mathrm{I}\,k_x\,\Delta_{\perp 4+}}{-k_x^2 + L_{4+}} \tag{36}$$

> $L_{1-} := \Delta_{||1-}^{\;2} + k_{||-}^{\;2}$

$$L_{1-} := k_{||-}^{\;2} + \Delta_{||1-}^{\;2} \tag{37}$$

> $L_{1+} := \Delta_{||1+}^{\;2} + k_{||+}^{\;2}$

$$L_{1+} := k_{||+}^{\;2} + \Delta_{||1+}^{\;2} \tag{38}$$

> $L_{2-} := \Delta_{||2-}^{\;2} + k_{||-}^{\;2}$

$$L_{2-} := k_{||-}^{\;2} + \Delta_{||2-}^{\;2} \tag{39}$$

> $L_{2+} := \Delta_{||2+}^{\;2} + k_{||+}^{\;2}$

$$L_{2+} := k_{||+}^{\;2} + \Delta_{||2+}^{\;2} \tag{40}$$

> $L_{3-} := \Delta_{||3-}^{\;2} + k_{||-}^{\;2}$

$$L_{3-} := k_{||-}^{\;2} + \Delta_{||3-}^{\;2} \tag{41}$$

> $L_{3+} := \Delta_{||3+}^{\;2} + k_{||+}^{\;2}$

$$L_{3+} := k_{||+}^{\;2} + \Delta_{||3+}^{\;2} \tag{42}$$

> $L_{4-} := \Delta_{||4-}^{\;2} + k_{||-}^{\;2}$

$$L_{4-} := k_{||-}^{\;2} + \Delta_{||4-}^{\;2} \tag{43}$$

> $L_{4+} := \Delta_{||4+}^{\;2} + k_{||+}^{\;2}$

$$L_{4+} := k_{||+}^{\;2} + \Delta_{||4+}^{\;2} \tag{44}$$

> $\Delta_{\perp 1-} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z1-}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 1-} := \mathrm{I}\left(k_y\cos(\alpha) - k_{z1-}\sin(\alpha)\right) \tag{45}$$

> $\Delta_{\perp 1+} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z1+}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 1+} := \mathrm{I}\left(k_y\cos(\alpha) - k_{z1+}\sin(\alpha)\right) \tag{46}$$

> $\Delta_{\perp 2-} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z2-}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 2-} := \mathrm{I}\left(k_y\cos(\alpha) - k_{z2-}\sin(\alpha)\right) \tag{47}$$

> $\Delta_{\perp 2+} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z2+}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 2+} := \mathrm{I}\left(k_y\cos(\alpha) - k_{z2+}\sin(\alpha)\right) \tag{48}$$

> $\Delta_{\perp 3-} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z3-}\cdot\sin(\alpha)\right)$

(49)

$$\Delta_{\perp 3\text{-}} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z3\text{-}} \sin(\alpha)\right) \tag{49}$$

> $\Delta_{\perp 3+} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z3+}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 3+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z3+} \sin(\alpha)\right) \tag{50}$$

> $\Delta_{\perp 4-} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z4-}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 4\text{-}} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z4\text{-}} \sin(\alpha)\right) \tag{51}$$

> $\Delta_{\perp 4+} := \mathrm{i}\cdot\left(k_y\cdot\cos(\alpha) - k_{z4+}\cdot\sin(\alpha)\right)$

$$\Delta_{\perp 4+} := \mathrm{I}\left(k_y \cos(\alpha) - k_{z4+} \sin(\alpha)\right) \tag{52}$$

> $\Delta_{||1-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z1-}\cdot\cos(\alpha)\right)$

$$\Delta_{||1\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z1\text{-}} \cos(\alpha)\right) \tag{53}$$

> $\Delta_{||1+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z1+}\cdot\cos(\alpha)\right)$

$$\Delta_{||1+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z1+} \cos(\alpha)\right) \tag{54}$$

> $\Delta_{||2-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z2-}\cdot\cos(\alpha)\right)$

$$\Delta_{||2\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z2\text{-}} \cos(\alpha)\right) \tag{55}$$

> $\Delta_{||2+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z2+}\cdot\cos(\alpha)\right)$

$$\Delta_{||2+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z2+} \cos(\alpha)\right) \tag{56}$$

> $\Delta_{||3-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z3-}\cdot\cos(\alpha)\right)$

$$\Delta_{||3\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z3\text{-}} \cos(\alpha)\right) \tag{57}$$

> $\Delta_{||3+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z3+}\cdot\cos(\alpha)\right)$

$$\Delta_{||3+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z3+} \cos(\alpha)\right) \tag{58}$$

> $\Delta_{||4-} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z4-}\cdot\cos(\alpha)\right)$

$$\Delta_{||4\text{-}} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z4\text{-}} \cos(\alpha)\right) \tag{59}$$

> $\Delta_{||4+} := \mathrm{i}\cdot\left(k_y\cdot\sin(\alpha) + k_{z4+}\cdot\cos(\alpha)\right)$

$$\Delta_{||4+} := \mathrm{I}\left(k_y \sin(\alpha) + k_{z4+} \cos(\alpha)\right) \tag{60}$$

> $k_{z1-} := \dfrac{k_{||-}}{\cos(\alpha)} - k_y\cdot\tan(\alpha)$

$$k_{z1\text{-}} := \frac{k_{||\text{-}}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{61}$$

> $k_{z1+} := \dfrac{k_{||+}}{\cos(\alpha)} - k_y\cdot\tan(\alpha)$

$$k_{z1+} := \frac{k_{||+}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{62}$$

> $k_{z2-} := -\frac{k_{\|-}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z2-} := -\frac{k_{\|-}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{63}$$

> $k_{z2+} := -\frac{k_{\|+}}{\cos(\alpha)} - k_y \cdot \tan(\alpha)$

$$k_{z2+} := -\frac{k_{\|+}}{\cos(\alpha)} - k_y \tan(\alpha) \tag{64}$$

> $k_{z3-} := \mathrm{i} \cdot \sqrt{k_x^2 + k_y^2 - k_{\|-}^2}$

$$k_{z3-} := \mathrm{I} \sqrt{k_x^2 + k_y^2 - k_{\|-}^2} \tag{65}$$

> $k_{z3+} := \mathrm{i} \cdot \sqrt{k_x^2 + k_y^2 - k_{\|+}^2}$

$$k_{z3+} := \mathrm{I} \sqrt{k_x^2 + k_y^2 - k_{\|+}^2} \tag{66}$$

> $k_{z4-} := -\mathrm{i} \cdot \sqrt{k_x^2 + k_y^2 - k_{\|-}^2}$

$$k_{z4-} := -\mathrm{I} \sqrt{k_x^2 + k_y^2 - k_{\|-}^2} \tag{67}$$

> $k_{z4+} := -\mathrm{i} \cdot \sqrt{k_x^2 + k_y^2 - k_{\|+}^2}$

$$k_{z4+} := -\mathrm{I} \sqrt{k_x^2 + k_y^2 - k_{\|+}^2} \tag{68}$$

>

> $k_x := \frac{k_{\|+}}{\epsilon_+}$

$$k_x := \frac{k_{\|+}}{\epsilon_{`+`}} \tag{69}$$

> $k_y := a_y \cdot k_{\|+}$

$$k_y := a_y k_{\|+} \tag{70}$$

> $k_{\|-} := \frac{k_x}{\epsilon_-}$

$$k_{\|-} := \frac{k_{\|+}}{\epsilon_{`+`} \epsilon_{`-`}} \tag{71}$$

Need to help maple to take limit as epsilon_- and epsilon_+ go to zero by taking terms that go to infinity out of the square root.

> $k_{z3-} := \frac{\mathrm{i} \cdot \sqrt{\epsilon_-^2 \cdot k_{\|+}^2 + \epsilon_-^2 \cdot \epsilon_+^2 \cdot k_y^2 - k_{\|+}^2}}{\epsilon_- \cdot \epsilon_+}$

$$k_{z3-} := \frac{I\sqrt{a_y^2\,\epsilon_{`-`}^2\,\epsilon_{`+`}^2\,k_{||+}^2+\epsilon_{`-`}^2\,k_{||+}^2-k_{||+}^2}}{\epsilon_{`-`}\,\epsilon_{`+`}} \tag{72}$$

> $k_{z3+} := \frac{i\cdot\sqrt{k_{||+}^2+\epsilon_+^2\cdot k_y^2-\epsilon_+^2\cdot k_{||+}^2}}{\epsilon_+}$

$$k_{z3+} := \frac{I\sqrt{a_y^2\,\epsilon_{`+`}^2\,k_{||+}^2-\epsilon_{`+`}^2\,k_{||+}^2+k_{||+}^2}}{\epsilon_{`+`}} \tag{73}$$

> $k_{z4-} := -\frac{i\cdot\sqrt{\epsilon_-^2\cdot k_{||+}^2+\epsilon_-^2\cdot\epsilon_+^2\cdot k_y^2-k_{||+}^2}}{\epsilon_-\cdot\epsilon_+}$

$$k_{z4-} := \frac{-I\sqrt{a_y^2\,\epsilon_{`-`}^2\,\epsilon_{`+`}^2\,k_{||+}^2+\epsilon_{`-`}^2\,k_{||+}^2-k_{||+}^2}}{\epsilon_{`-`}\,\epsilon_{`+`}} \tag{74}$$

> $k_{z4+} := -\frac{i\cdot\sqrt{k_{||+}^2+\epsilon_+^2\cdot k_y^2-\epsilon_{`+`}^2\cdot k_{||+}^2}}{\epsilon_+}$

$$k_{z4+} := \frac{-I\sqrt{a_y^2\,\epsilon_{`+`}^2\,k_{||+}^2-epsilon\_\_^2\,k_{||+}^2+k_{||+}^2}}{\epsilon_{`+`}} \tag{75}$$

ux leading order terms

> $u_{x1-} := u_{x10-}\cdot sol1:$

> $u_{x4-} := u_{x40-}\cdot sol2:$

> $u_{x1+} := u_{x10+}:$

> $u_{x2+} := u_{x20+}\cdot sol3:$

> $u_{x3+} := u_{x30+}\cdot sol4:$

> $simplify\left(mtaylor\left(u_{x1-},[\epsilon_-,\epsilon_+],3\right)\right)$

$$-2\,I\sin(\alpha)\,\epsilon_{`-`}\,\epsilon_{`+`}\,\mathrm{csgn}(k_{||+}) \tag{76}$$

> $simplify\left(mtaylor\left(u_{x4-},[\epsilon_-,\epsilon_+],4\right)\right)$

$$-\frac{2\,\epsilon_{`-`}^2\,\epsilon_{`+`}\cos(\alpha)}{\sin(\alpha)} \tag{77}$$

> $simplify\left(mtaylor\left(u_{x1+},[\epsilon_-,\epsilon_+],2\right)\right)$

$$\frac{\epsilon_{`+`}\left(-a_y+\sin(\alpha)\right)}{\cos(\alpha)} \tag{78}$$

> $simplify\left(mtaylor\left(u_{x2+}, \left[\epsilon_{-}, \epsilon_{+}\right], 2\right)\right)$

$$\frac{\epsilon_{`+`}\left(a_y + \sin(\alpha)\right)}{\cos(\alpha)} \quad \textbf{(79)}$$

> $simplify\left(mtaylor\left(u_{x3+}, \left[\epsilon_{-}, \epsilon_{+}\right], 2\right)\right)$

$$-\frac{2\sin(\alpha)\,\epsilon_{`+`}}{\cos(\alpha)} \quad \textbf{(80)}$$

u_perp leading order terms

> $simplify\left(mtaylor\left(sol1, \left[\epsilon_{-}, \epsilon_{+}\right], 4\right)\right)$

$$-2\,\mathrm{I}\,\epsilon_{`-`}^{2}\,\epsilon_{`+`}\cos(\alpha)\,\mathrm{csgn}\left(k_{\parallel +}\right) \quad \textbf{(81)}$$

> $simplify\left(mtaylor\left(sol2, \left[\epsilon_{-}, \epsilon_{+}\right], 3\right)\right)$

$$\frac{2\,\mathrm{I}\cos(\alpha)\,\epsilon_{`-`}\,\epsilon_{`+`}\,k_{\parallel +}}{\sqrt{-k_{\parallel +}^{2}}} \quad \textbf{(82)}$$

> $simplify\left(mtaylor\left(sol3, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$-1 \quad \textbf{(83)}$$

> $simplify\left(mtaylor\left(sol4, \left[\epsilon_{-}, \epsilon_{+}\right], 2\right)\right)$

$$\frac{2\,\mathrm{I}\,\epsilon_{`+`}\,\mathrm{csgn}\left(k_{\parallel +}\right)\sin(\alpha)^{2}}{\cos(\alpha)} \quad \textbf{(84)}$$

bx leading order terms

> $b_{x1-} := \frac{\Delta_{\parallel 1-} \cdot u_{x1-}}{\mathrm{i} \cdot k_{\parallel +}}$:

> $b_{x4-} := \frac{\Delta_{\parallel 4-} \cdot u_{x4-}}{\mathrm{i} \cdot k_{\parallel +}}$:

> $b_{x1+} := \frac{\Delta_{\parallel 1+} \cdot u_{x1+}}{\mathrm{i} \cdot k_{\parallel +}}$:

> $b_{x2+} := \frac{\Delta_{\parallel 2+} \cdot u_{x2+}}{\mathrm{i} \cdot k_{\parallel +}}$:

> $b_{x3+} := \frac{\Delta_{\parallel 3+} \cdot u_{x3+}}{\mathrm{i} \cdot k_{\parallel +}}$:

> $simplify\left(mtaylor\left(b_{x1-}, \left[\epsilon_{-}, \epsilon_{+}\right], 1\right)\right)$

$$-2\,\mathrm{I}\sin(\alpha)\,\mathrm{csgn}\left(k_{\parallel +}\right) \quad \textbf{(85)}$$

> $simplify(mtaylor(b_{x4-}, [\epsilon_-, \epsilon_+], 2))$

$$\frac{-2\,\mathrm{I}\,k_{\|+}\,\epsilon_{`-`}\cos(\alpha)^2}{\sqrt{-k_{\|+}^2}\,\sin(\alpha)} \quad \textbf{(86)}$$

> $simplify(mtaylor(b_{x1+}, [\epsilon_-, \epsilon_+], 2))$

$$\frac{\epsilon_{`+`}\,(-a_y + \sin(\alpha))}{\cos(\alpha)} \quad \textbf{(87)}$$

> $simplify(mtaylor(b_{x2+}, [\epsilon_-, \epsilon_+], 2))$

$$-\frac{\epsilon_{`+`}\,(a_y + \sin(\alpha))}{\cos(\alpha)} \quad \textbf{(88)}$$

> $simplify(mtaylor(b_{x3+}, [\epsilon_-, \epsilon_+], 1))$

$$-2\,\mathrm{I}\sin(\alpha)\,\mathrm{csgn}(k_{\|+}) \quad \textbf{(89)}$$

b_perp leading order terms

> $b_{\perp 1-} := \frac{\Delta_{\|1-} \cdot sol1}{\mathrm{i} \cdot k_{\|+}}$:

> $b_{\perp 4-} := \frac{\Delta_{\|4-} \cdot sol2}{\mathrm{i} \cdot k_{\|+}}$:

> $b_{\perp 1+} := \frac{\Delta_{\|1+} \cdot 1}{\mathrm{i} \cdot k_{\|+}}$:

> $b_{\perp 2+} := \frac{\Delta_{\|2+} \cdot sol3}{\mathrm{i} \cdot k_{\|+}}$:

> $b_{\perp 3+} := \frac{\Delta_{\|3+} \cdot sol4}{\mathrm{i} \cdot k_{\|+}}$:

> $simplify(mtaylor(b_{\perp 1-}, [\epsilon_-, \epsilon_+], 2))$

$$-2\,\mathrm{I}\,\epsilon_{`-`}\cos(\alpha)\,\mathrm{csgn}(k_{\|+}) \quad \textbf{(90)}$$

> $simplify(mtaylor(b_{\perp 4-}, [\epsilon_-, \epsilon_+], 1))$

$$2\cos(\alpha)^2 \quad \textbf{(91)}$$

> $simplify(mtaylor(b_{\perp 1+}, [\epsilon_-, \epsilon_+], 5))$

$$1 \quad \textbf{(92)}$$

> $simplify(mtaylor(b_{\perp 2+}, [\epsilon_-, \epsilon_+], 1))$

$$1 \quad \textbf{(93)}$$

> $simplify(mtaylor(b_{\perp 3+}, [\epsilon_-, \epsilon_+], 1))$

(94)

$$-2 \sin(\alpha)^2 \tag{94}$$

b_par leading order terms

> $b_{||1-} := -\frac{\mathrm{i} \cdot k_x \cdot u_{x1-} + \Delta_{\perp 1-} \cdot sol1}{\mathrm{i} \cdot k_{||+}}$:

> $b_{||4-} := -\frac{\mathrm{i} \cdot k_x \cdot u_{x4-} + \Delta_{\perp 4-} \cdot sol2}{\mathrm{i} \cdot k_{||+}}$:

> $b_{||1+} := -\frac{\mathrm{i} \cdot k_x \cdot u_{x1+} + \Delta_{\perp 1+}}{\mathrm{i} \cdot k_{||+}}$:

> $b_{||2+} := -\frac{\mathrm{i} \cdot k_x \cdot u_{x2+} + \Delta_{\perp 2+} \cdot sol3}{\mathrm{i} \cdot k_{||+}}$:

> $b_{||3+} := -\frac{\mathrm{i} \cdot k_x \cdot u_{x3+} + \Delta_{\perp 3+} \cdot sol4}{\mathrm{i} \cdot k_{||+}}$:

> $simplify\left(mtaylor\left(b_{||1-}, \left[\epsilon_{-}, \epsilon_{`+`}\right], 2\right)\right)$

$$0 \tag{95}$$

> $simplify\left(mtaylor\left(b_{||4-}, \left[\epsilon_{-}, \epsilon_{`+`}\right], 1\right)\right)$

$$\begin{aligned}&\left(4 \left(\mathrm{I} \left(\frac{\sin(\alpha)}{2} + a_y \left(\cos(\alpha)^2 - \frac{1}{2}\right)\right) \epsilon_{`+`} \sqrt{\left(1 + \left(a_y^2 - 1\right) \epsilon_{`+`}^2\right) k_{||+}^2} + \left(\left(\frac{1}{2}\right.\right.\right.\\ &\quad \left.\left.\left. + \left(a_y^2 - \frac{1}{2}\right) \epsilon_{`+`}^2\right) \sin(\alpha) - \frac{a_y \epsilon_{`+`}^2}{2}\right) \cos(\alpha)\, k_{||+}\right) \sin(\alpha)\Bigg) \Bigg/ \Bigg(\mathrm{I} \cos(\alpha)\, \epsilon_{`+`} \left(a_y\right.\\ &\quad \left. + \sin(\alpha)\right) \sqrt{\left(1 + \left(a_y^2 - 1\right) \epsilon_{`+`}^2\right) k_{||+}^2} + \left(\left(1 + \left(a_y^2 - 1\right) \epsilon_{`+`}^2\right) \sin(\alpha)\right.\\ &\quad \left. - a_y \epsilon_{`+`}^2 \cos(\alpha)^2\right) k_{||+}\Bigg)\end{aligned} \tag{96}$$

> $simplify\left(mtaylor\left(b_{||1+}, \left[\epsilon_{-}, \epsilon_{`+`}\right], 1\right)\right)$

$$0 \tag{97}$$

> $simplify\left(mtaylor\left(b_{||2+}, \left[\epsilon_{-}, \epsilon_{`+`}\right], 1\right)\right)$

$$0 \tag{98}$$

> $simplify\left(mtaylor\left(b_{||3+}, \left[\epsilon_{-}, \epsilon_{`+`}\right], 1\right)\right)$

$$\begin{aligned}&\Bigg(8 \sin(\alpha) \Bigg(\mathrm{I} \sin(\alpha) \left(\left(\frac{1}{4} + \left(a_y^2 - \frac{1}{4}\right) \epsilon_{`+`}^2\right) \cos(\alpha)^2 + \epsilon_{`+`}^2 \left(-\frac{a_y^2}{4}\right.\right.\\ &\quad \left.\left. + \frac{1}{4}\right)\right) \sqrt{\left(1 + \left(a_y^2 - 1\right) \epsilon_{`+`}^2\right) k_{||+}^2} - \cos(\alpha) \left(\left(\frac{3}{4} + \left(a_y^2 - \frac{3}{4}\right) \epsilon_{`+`}^2\right) \cos(\alpha)^2\right.\\ &\quad \left. - \frac{1}{2} + \left(-\frac{3 a_y^2}{4} + \frac{3}{4}\right) \epsilon_{`+`}^2\right) k_{||+} \epsilon_{`+`} a_y\Bigg)\Bigg) \Bigg/ \Bigg(2 \mathrm{I} \cos(\alpha) \left(\sin(\alpha)\, a_y^2 \epsilon_{`+`}^2\right.\end{aligned} \tag{99}$$

$$-a_y\,\epsilon_{`+`}^{\,2}\cos(\alpha)^2 + a_y\,\epsilon_{`+`}^{\,2} + \frac{\sin(\alpha)}{2}\Big)\sqrt{\left(1+\left(a_y^{\,2}-1\right)\epsilon_{`+`}^{\,2}\right)k_{\|+}^{\,2}}$$

$$-2\left(\left(\left(a_y^{\,3}\,\epsilon_{`+`}^{\,2}+\sin(\alpha)\,a_y^{\,2}\,\epsilon_{`+`}^{\,2}-\frac{a_y\,\epsilon_{`+`}^{\,2}}{2}-\frac{\sin(\alpha)\,\epsilon_{`+`}^{\,2}}{2}+a_y\right.\right.\right.$$

$$\left.\left.\left.+\frac{\sin(\alpha)}{2}\right)\cos(\alpha)^2-\frac{\left(1+\left(a_y^{\,2}-1\right)\epsilon_{`+`}^{\,2}\right)\left(a_y+\sin(\alpha)\right)}{2}\right)k_{\|+}\,\epsilon_{`+`}\right)$$