

Note that ∇ has been replaced with \cdot to ensure the code works okay.

> restart;

We normalise the velocity coefficients by u_0 and the field components by $(B_0 * u_0 / v_{\{A\}})$.

We start by solving the following Matrix equation:

$$\begin{aligned} > \text{eqn1} := a_{11} \cdot x_1 + a_{12} \cdot x_2 + a_{13} \cdot x_3 + a_{14} \cdot x_4 = y_1 \\ & \text{eqn1} := a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 = y_1 \end{aligned} \quad (1)$$

$$\begin{aligned} > \text{eqn2} := a_{21} \cdot x_1 + a_{22} \cdot x_2 + a_{23} \cdot x_3 + a_{24} \cdot x_4 = y_2 \\ & \text{eqn2} := a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + a_{24} x_4 = y_2 \end{aligned} \quad (2)$$

$$\begin{aligned} > \text{eqn3} := a_{31} \cdot x_1 + a_{32} \cdot x_2 + a_{33} \cdot x_3 + a_{34} \cdot x_4 = y_3 \\ & \text{eqn3} := a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + a_{34} x_4 = y_3 \end{aligned} \quad (3)$$

$$\begin{aligned} > \text{eqn4} := a_{41} \cdot x_1 + a_{42} \cdot x_2 + a_{43} \cdot x_3 + a_{44} \cdot x_4 = y_4 \\ & \text{eqn4} := a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4 = y_4 \end{aligned} \quad (4)$$

> solns := solve({eqn1, eqn2, eqn3, eqn4}, {x1, x2, x3, x4}) :

$$\begin{aligned} > \text{sol1} := \text{rhs}(\text{solns}[1]) \\ \text{sol1} := & - (a_{12} a_{23} a_{34} y_4 - a_{12} a_{23} a_{44} y_3 - a_{12} a_{24} a_{33} y_4 + a_{12} a_{24} a_{43} y_3 + a_{12} a_{33} a_{44} y_2 \end{aligned} \quad (5)$$

$$\begin{aligned} & - a_{12} a_{34} a_{43} y_2 - a_{13} a_{22} a_{34} y_4 + a_{13} a_{22} a_{44} y_3 + a_{13} a_{24} a_{32} y_4 - a_{13} a_{24} a_{42} y_3 \\ & - a_{13} a_{32} a_{44} y_2 + a_{13} a_{34} a_{42} y_2 + a_{14} a_{22} a_{33} y_4 - a_{14} a_{22} a_{43} y_3 - a_{14} a_{23} a_{32} y_4 \\ & + a_{14} a_{23} a_{42} y_3 + a_{14} a_{32} a_{43} y_2 - a_{14} a_{33} a_{42} y_2 - a_{22} a_{33} a_{44} y_1 + a_{22} a_{34} a_{43} y_1 \\ & + a_{23} a_{32} a_{44} y_1 - a_{23} a_{34} a_{42} y_1 - a_{24} a_{32} a_{43} y_1 + a_{24} a_{33} a_{42} y_1) / (a_{11} a_{22} a_{33} a_{44} \\ & - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} \\ & - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} \\ & + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} \\ & + a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} + a_{14} a_{22} a_{31} a_{43} \\ & - a_{14} a_{22} a_{33} a_{41} - a_{14} a_{23} a_{31} a_{42} + a_{14} a_{23} a_{32} a_{41}) \end{aligned}$$

$$\begin{aligned} > \text{sol2} := \text{rhs}(\text{solns}[2]) \\ \text{sol2} := & (a_{11} a_{23} a_{34} y_4 - a_{11} a_{23} a_{44} y_3 - a_{11} a_{24} a_{33} y_4 + a_{11} a_{24} a_{43} y_3 + a_{11} a_{33} a_{44} y_2 \end{aligned} \quad (6)$$

$$\begin{aligned} & - a_{11} a_{34} a_{43} y_2 - a_{13} a_{21} a_{34} y_4 + a_{13} a_{21} a_{44} y_3 + a_{13} a_{24} a_{31} y_4 - a_{13} a_{24} a_{41} y_3 \\ & - a_{13} a_{31} a_{44} y_2 + a_{13} a_{34} a_{41} y_2 + a_{14} a_{21} a_{33} y_4 - a_{14} a_{21} a_{43} y_3 - a_{14} a_{23} a_{31} y_4 \\ & + a_{14} a_{23} a_{41} y_3 + a_{14} a_{31} a_{43} y_2 - a_{14} a_{33} a_{41} y_2 - a_{21} a_{33} a_{44} y_1 + a_{21} a_{34} a_{43} y_1 \\ & + a_{23} a_{31} a_{44} y_1 - a_{23} a_{34} a_{41} y_1 - a_{24} a_{31} a_{43} y_1 + a_{24} a_{33} a_{41} y_1) / (a_{11} a_{22} a_{33} a_{44} \\ & - a_{11} a_{22} a_{34} a_{43} - a_{11} a_{23} a_{32} a_{44} + a_{11} a_{23} a_{34} a_{42} + a_{11} a_{24} a_{32} a_{43} - a_{11} a_{24} a_{33} a_{42} \\ & - a_{12} a_{21} a_{33} a_{44} + a_{12} a_{21} a_{34} a_{43} + a_{12} a_{23} a_{31} a_{44} - a_{12} a_{23} a_{34} a_{41} - a_{12} a_{24} a_{31} a_{43} \\ & + a_{12} a_{24} a_{33} a_{41} + a_{13} a_{21} a_{32} a_{44} - a_{13} a_{21} a_{34} a_{42} - a_{13} a_{22} a_{31} a_{44} + a_{13} a_{22} a_{34} a_{41} \\ & + a_{13} a_{24} a_{31} a_{42} - a_{13} a_{24} a_{32} a_{41} - a_{14} a_{21} a_{32} a_{43} + a_{14} a_{21} a_{33} a_{42} + a_{14} a_{22} a_{31} a_{43} \end{aligned}$$

$$-a_{14}a_{22}a_{33}a_{41}-a_{14}a_{23}a_{31}a_{42}+a_{14}a_{23}a_{32}a_{41})$$

> sol3 := rhs(solns[3])

$$\begin{aligned} \text{sol3} := & - \left(a_{11}a_{22}a_{34}y_4 - a_{11}a_{22}a_{44}y_3 - a_{11}a_{24}a_{32}y_4 + a_{11}a_{24}a_{42}y_3 + a_{11}a_{32}a_{44}y_2 \right. \\ & - a_{11}a_{34}a_{42}y_2 - a_{12}a_{21}a_{34}y_4 + a_{12}a_{21}a_{44}y_3 + a_{12}a_{24}a_{31}y_4 - a_{12}a_{24}a_{41}y_3 \\ & - a_{12}a_{31}a_{44}y_2 + a_{12}a_{34}a_{41}y_2 + a_{14}a_{21}a_{32}y_4 - a_{14}a_{21}a_{42}y_3 - a_{14}a_{22}a_{31}y_4 \\ & + a_{14}a_{22}a_{41}y_3 + a_{14}a_{31}a_{42}y_2 - a_{14}a_{32}a_{41}y_2 - a_{21}a_{32}a_{44}y_1 + a_{21}a_{34}a_{42}y_1 \\ & + a_{22}a_{31}a_{44}y_1 - a_{22}a_{34}a_{41}y_1 - a_{24}a_{31}a_{42}y_1 + a_{24}a_{32}a_{41}y_1 \Big) / \left(a_{11}a_{22}a_{33}a_{44} \right. \\ & - a_{11}a_{22}a_{34}a_{43} - a_{11}a_{23}a_{32}a_{44} + a_{11}a_{23}a_{34}a_{42} + a_{11}a_{24}a_{32}a_{43} - a_{11}a_{24}a_{33}a_{42} \\ & - a_{12}a_{21}a_{33}a_{44} + a_{12}a_{21}a_{34}a_{43} + a_{12}a_{23}a_{31}a_{44} - a_{12}a_{23}a_{34}a_{41} - a_{12}a_{24}a_{31}a_{43} \\ & + a_{12}a_{24}a_{33}a_{41} + a_{13}a_{21}a_{32}a_{44} - a_{13}a_{21}a_{34}a_{42} - a_{13}a_{22}a_{31}a_{44} + a_{13}a_{22}a_{34}a_{41} \\ & + a_{13}a_{24}a_{31}a_{42} - a_{13}a_{24}a_{32}a_{41} - a_{14}a_{21}a_{32}a_{43} + a_{14}a_{21}a_{33}a_{42} + a_{14}a_{22}a_{31}a_{43} \\ & \left. - a_{14}a_{22}a_{33}a_{41} - a_{14}a_{23}a_{31}a_{42} + a_{14}a_{23}a_{32}a_{41} \right) \end{aligned}$$

(7)

> sol4 := rhs(solns[4])

$$\begin{aligned} \text{sol4} := & \left(a_{11}a_{22}a_{33}y_4 - a_{11}a_{22}a_{43}y_3 - a_{11}a_{23}a_{32}y_4 + a_{11}a_{23}a_{42}y_3 + a_{11}a_{32}a_{43}y_2 \right. \\ & - a_{11}a_{33}a_{42}y_2 - a_{12}a_{21}a_{33}y_4 + a_{12}a_{21}a_{43}y_3 + a_{12}a_{23}a_{31}y_4 - a_{12}a_{23}a_{41}y_3 \\ & - a_{12}a_{31}a_{43}y_2 + a_{12}a_{33}a_{41}y_2 + a_{13}a_{21}a_{32}y_4 - a_{13}a_{21}a_{42}y_3 - a_{13}a_{22}a_{31}y_4 \\ & + a_{13}a_{22}a_{41}y_3 + a_{13}a_{31}a_{42}y_2 - a_{13}a_{32}a_{41}y_2 - a_{21}a_{32}a_{43}y_1 + a_{21}a_{33}a_{42}y_1 \\ & + a_{22}a_{31}a_{43}y_1 - a_{22}a_{33}a_{41}y_1 - a_{23}a_{31}a_{42}y_1 + a_{23}a_{32}a_{41}y_1 \Big) / \left(a_{11}a_{22}a_{33}a_{44} \right. \\ & - a_{11}a_{22}a_{34}a_{43} - a_{11}a_{23}a_{32}a_{44} + a_{11}a_{23}a_{34}a_{42} + a_{11}a_{24}a_{32}a_{43} - a_{11}a_{24}a_{33}a_{42} \\ & - a_{12}a_{21}a_{33}a_{44} + a_{12}a_{21}a_{34}a_{43} + a_{12}a_{23}a_{31}a_{44} - a_{12}a_{23}a_{34}a_{41} - a_{12}a_{24}a_{31}a_{43} \\ & + a_{12}a_{24}a_{33}a_{41} + a_{13}a_{21}a_{32}a_{44} - a_{13}a_{21}a_{34}a_{42} - a_{13}a_{22}a_{31}a_{44} + a_{13}a_{22}a_{34}a_{41} \\ & + a_{13}a_{24}a_{31}a_{42} - a_{13}a_{24}a_{32}a_{41} - a_{14}a_{21}a_{32}a_{43} + a_{14}a_{21}a_{33}a_{42} + a_{14}a_{22}a_{31}a_{43} \\ & \left. - a_{14}a_{22}a_{33}a_{41} - a_{14}a_{23}a_{31}a_{42} + a_{14}a_{23}a_{32}a_{41} \right) \end{aligned}$$

(8)

> a₁₁ := u_{x10}-

$$a_{11} := u_{x10}-$$

(9)

> a₁₂ := u_{x40}-

$$a_{12} := u_{x40}-$$

(10)

> a₁₃ := -u_{x20}+

$$a_{13} := -u_{x20}+$$

(11)

> a₁₄ := -u_{x30}+

$$a_{14} := -u_{x30}+$$

(12)

> a₂₁ := k_{z1}-·u_{x10}-

$$a_{21} := k_{z1}-u_{x10}-$$

(13)

$$\begin{array}{l} \textcolor{red}{>} a_{22} := k_{z4-} \cdot u_{x40-} \\ \textcolor{blue}{a_{22} := k_{z4-} u_{x40-}} \end{array} \quad (14)$$

$$\begin{array}{l} \textcolor{red}{>} a_{23} := -k_{z2+} \cdot u_{x20+} \\ \textcolor{blue}{a_{23} := -k_{z2+} u_{x20+}} \end{array} \quad (15)$$

$$\begin{array}{l} \textcolor{red}{>} a_{24} := -k_{z3+} \cdot u_{x30+} \\ \textcolor{blue}{a_{24} := -k_{z3+} u_{x30+}} \end{array} \quad (16)$$

$$\begin{array}{l} \textcolor{red}{>} a_{31} := 1 \\ \textcolor{blue}{a_{31} := 1} \end{array} \quad (17)$$

$$\begin{array}{l} \textcolor{red}{>} a_{32} := 1 \\ \textcolor{blue}{a_{32} := 1} \end{array} \quad (18)$$

$$\begin{array}{l} \textcolor{red}{>} a_{33} := -1 \\ \textcolor{blue}{a_{33} := -1} \end{array} \quad (19)$$

$$\begin{array}{l} \textcolor{red}{>} a_{34} := -1 \\ \textcolor{blue}{a_{34} := -1} \end{array} \quad (20)$$

$$\begin{array}{l} \textcolor{red}{>} a_{41} := k_{z1-} \\ \textcolor{blue}{a_{41} := k_{z1-}} \end{array} \quad (21)$$

$$\begin{array}{l} \textcolor{red}{>} a_{42} := k_{z4-} \\ \textcolor{blue}{a_{42} := k_{z4-}} \end{array} \quad (22)$$

$$\begin{array}{l} \textcolor{red}{>} a_{43} := -k_{z2+} \\ \textcolor{blue}{a_{43} := -k_{z2+}} \end{array} \quad (23)$$

$$\begin{array}{l} \textcolor{red}{>} a_{44} := -k_{z3+} \\ \textcolor{blue}{a_{44} := -k_{z3+}} \end{array} \quad (24)$$

$$\begin{array}{l} \textcolor{red}{>} y_1 := u_{x10+} \\ \textcolor{blue}{y_1 := u_{x10+}} \end{array} \quad (25)$$

$$\begin{array}{l} \textcolor{red}{>} y_2 := k_{z1+} \cdot u_{x10+} \\ \textcolor{blue}{y_2 := k_{z1+} u_{x10+}} \end{array} \quad (26)$$

$$\begin{array}{l} \textcolor{red}{>} y_3 := 1 \\ \textcolor{blue}{y_3 := 1} \end{array} \quad (27)$$

$$\begin{array}{l} \textcolor{red}{>} y_4 := k_{z1+} \\ \textcolor{blue}{y_4 := k_{z1+}} \end{array} \quad (28)$$

Note that $u_{x0[n]}$ denotes $\hat{u}_{\{x\}_n}$ and $u_{x[n\pm]}$ denotes $u_{\{x\}_n}$.

$$\textcolor{red}{>} u_{x10-} := -\frac{i \cdot k_x \cdot \Delta_{\perp 1-}}{L_{1-} - k_x^2}$$

$$u_{x10-} := \frac{-\mathrm{I} k_x \Delta_{\perp 1-}}{-k_x^2 + L_{1-}} \quad (29)$$

$$> u_{x10+} := -\frac{\mathrm{i} \cdot k_x \cdot \Delta_{\perp 1+}}{L_{1+} - k_x^2}$$

$$u_{x10+} := \frac{-\mathrm{I} k_x \Delta_{\perp 1+}}{-k_x^2 + L_{1+}} \quad (30)$$

$$> u_{x20-} := -\frac{\mathrm{i} \cdot k_x \cdot \Delta_{\perp 2-}}{L_{2-} - k_x^2}$$

$$u_{x20-} := \frac{-\mathrm{I} k_x \Delta_{\perp 2-}}{-k_x^2 + L_{2-}} \quad (31)$$

$$> u_{x20+} := -\frac{\mathrm{i} \cdot k_x \cdot \Delta_{\perp 2+}}{L_{2+} - k_x^2}$$

$$u_{x20+} := \frac{-\mathrm{I} k_x \Delta_{\perp 2+}}{-k_x^2 + L_{2+}} \quad (32)$$

$$> u_{x30-} := -\frac{\mathrm{i} \cdot k_x \cdot \Delta_{\perp 3-}}{L_{3-} - k_x^2}$$

$$u_{x30-} := \frac{-\mathrm{I} k_x \Delta_{\perp 3-}}{-k_x^2 + L_{3-}} \quad (33)$$

$$> u_{x30+} := -\frac{\mathrm{i} \cdot k_x \cdot \Delta_{\perp 3+}}{L_{3+} - k_x^2}$$

$$u_{x30+} := \frac{-\mathrm{I} k_x \Delta_{\perp 3+}}{-k_x^2 + L_{3+}} \quad (34)$$

$$> u_{x40-} := -\frac{\mathrm{i} \cdot k_x \cdot \Delta_{\perp 4-}}{L_{4-} - k_x^2}$$

$$u_{x40-} := \frac{-\mathrm{I} k_x \Delta_{\perp 4-}}{-k_x^2 + L_{4-}} \quad (35)$$

$$> u_{x40+} := -\frac{\mathrm{i} \cdot k_x \cdot \Delta_{\perp 4+}}{L_{4+} - k_x^2}$$

$$(36)$$

$$u_{x40+} := \frac{-I k_x \Delta_{\perp 4+}}{-k_x^2 + L_{4+}} \quad (36)$$

$$> L_{I-} := \Delta_{\parallel I-}^2 + k_{\parallel-}^2$$

$$L_{I-} := k_{\parallel-}^2 + \Delta_{\parallel I-}^2 \quad (37)$$

$$> L_{I+} := \Delta_{\parallel I+}^2 + k_{\parallel+}^2$$

$$L_{I+} := k_{\parallel+}^2 + \Delta_{\parallel I+}^2 \quad (38)$$

$$> L_{2-} := \Delta_{\parallel 2-}^2 + k_{\parallel-}^2$$

$$L_{2-} := k_{\parallel-}^2 + \Delta_{\parallel 2-}^2 \quad (39)$$

$$> L_{2+} := \Delta_{\parallel 2+}^2 + k_{\parallel+}^2$$

$$L_{2+} := k_{\parallel+}^2 + \Delta_{\parallel 2+}^2 \quad (40)$$

$$> L_{3-} := \Delta_{\parallel 3-}^2 + k_{\parallel-}^2$$

$$L_{3-} := k_{\parallel-}^2 + \Delta_{\parallel 3-}^2 \quad (41)$$

$$> L_{3+} := \Delta_{\parallel 3+}^2 + k_{\parallel+}^2$$

$$L_{3+} := k_{\parallel+}^2 + \Delta_{\parallel 3+}^2 \quad (42)$$

$$> L_{4-} := \Delta_{\parallel 4-}^2 + k_{\parallel-}^2$$

$$L_{4-} := k_{\parallel-}^2 + \Delta_{\parallel 4-}^2 \quad (43)$$

$$> L_{4+} := \Delta_{\parallel 4+}^2 + k_{\parallel+}^2$$

$$L_{4+} := k_{\parallel+}^2 + \Delta_{\parallel 4+}^2 \quad (44)$$

$$> \Delta_{\perp I-} := i \cdot (k_y \cdot \cos(\alpha) - k_{zI-} \cdot \sin(\alpha))$$

$$\Delta_{\perp I-} := I (k_y \cos(\alpha) - k_{zI-} \sin(\alpha)) \quad (45)$$

$$> \Delta_{\perp I+} := i \cdot (k_y \cdot \cos(\alpha) - k_{zI+} \cdot \sin(\alpha))$$

$$\Delta_{\perp I+} := I (k_y \cos(\alpha) - k_{zI+} \sin(\alpha)) \quad (46)$$

$$> \Delta_{\perp 2-} := i \cdot (k_y \cdot \cos(\alpha) - k_{z2-} \cdot \sin(\alpha))$$

$$\Delta_{\perp 2-} := I (k_y \cos(\alpha) - k_{z2-} \sin(\alpha)) \quad (47)$$

$$> \Delta_{\perp 2+} := i \cdot (k_y \cdot \cos(\alpha) - k_{z2+} \cdot \sin(\alpha))$$

$$\Delta_{\perp 2+} := I (k_y \cos(\alpha) - k_{z2+} \sin(\alpha)) \quad (48)$$

$$> \Delta_{\perp 3-} := i \cdot (k_y \cdot \cos(\alpha) - k_{z3-} \cdot \sin(\alpha))$$

$$(49)$$

$$\Delta_{\perp 3-} := I \left(k_y \cos(\alpha) - k_{z3-} \sin(\alpha) \right) \quad (49)$$

$$\begin{aligned} &> \Delta_{\perp 3+} := i \cdot \left(k_y \cdot \cos(\alpha) - k_{z3+} \cdot \sin(\alpha) \right) \\ &\Delta_{\perp 3+} := I \left(k_y \cos(\alpha) - k_{z3+} \sin(\alpha) \right) \end{aligned} \quad (50)$$

$$\begin{aligned} &> \Delta_{\perp 4-} := i \cdot \left(k_y \cdot \cos(\alpha) - k_{z4-} \cdot \sin(\alpha) \right) \\ &\Delta_{\perp 4-} := I \left(k_y \cos(\alpha) - k_{z4-} \sin(\alpha) \right) \end{aligned} \quad (51)$$

$$\begin{aligned} &> \Delta_{\perp 4+} := i \cdot \left(k_y \cdot \cos(\alpha) - k_{z4+} \cdot \sin(\alpha) \right) \\ &\Delta_{\perp 4+} := I \left(k_y \cos(\alpha) - k_{z4+} \sin(\alpha) \right) \end{aligned} \quad (52)$$

$$\begin{aligned} &> \Delta_{\parallel 1-} := i \cdot \left(k_y \cdot \sin(\alpha) + k_{z1-} \cdot \cos(\alpha) \right) \\ &\Delta_{\parallel 1-} := I \left(k_y \sin(\alpha) + k_{z1-} \cos(\alpha) \right) \end{aligned} \quad (53)$$

$$\begin{aligned} &> \Delta_{\parallel 1+} := i \cdot \left(k_y \cdot \sin(\alpha) + k_{z1+} \cdot \cos(\alpha) \right) \\ &\Delta_{\parallel 1+} := I \left(k_y \sin(\alpha) + k_{z1+} \cos(\alpha) \right) \end{aligned} \quad (54)$$

$$\begin{aligned} &> \Delta_{\parallel 2-} := i \cdot \left(k_y \cdot \sin(\alpha) + k_{z2-} \cdot \cos(\alpha) \right) \\ &\Delta_{\parallel 2-} := I \left(k_y \sin(\alpha) + k_{z2-} \cos(\alpha) \right) \end{aligned} \quad (55)$$

$$\begin{aligned} &> \Delta_{\parallel 2+} := i \cdot \left(k_y \cdot \sin(\alpha) + k_{z2+} \cdot \cos(\alpha) \right) \\ &\Delta_{\parallel 2+} := I \left(k_y \sin(\alpha) + k_{z2+} \cos(\alpha) \right) \end{aligned} \quad (56)$$

$$\begin{aligned} &> \Delta_{\parallel 3-} := i \cdot \left(k_y \cdot \sin(\alpha) + k_{z3-} \cdot \cos(\alpha) \right) \\ &\Delta_{\parallel 3-} := I \left(k_y \sin(\alpha) + k_{z3-} \cos(\alpha) \right) \end{aligned} \quad (57)$$

$$\begin{aligned} &> \Delta_{\parallel 3+} := i \cdot \left(k_y \cdot \sin(\alpha) + k_{z3+} \cdot \cos(\alpha) \right) \\ &\Delta_{\parallel 3+} := I \left(k_y \sin(\alpha) + k_{z3+} \cos(\alpha) \right) \end{aligned} \quad (58)$$

$$\begin{aligned} &> \Delta_{\parallel 4-} := i \cdot \left(k_y \cdot \sin(\alpha) + k_{z4-} \cdot \cos(\alpha) \right) \\ &\Delta_{\parallel 4-} := I \left(k_y \sin(\alpha) + k_{z4-} \cos(\alpha) \right) \end{aligned} \quad (59)$$

$$\begin{aligned} &> \Delta_{\parallel 4+} := i \cdot \left(k_y \cdot \sin(\alpha) + k_{z4+} \cdot \cos(\alpha) \right) \\ &\Delta_{\parallel 4+} := I \left(k_y \sin(\alpha) + k_{z4+} \cos(\alpha) \right) \end{aligned} \quad (60)$$

$$\begin{aligned} &> k_{z1-} := \frac{k_{\parallel -}}{\cos(\alpha)} - k_y \cdot \tan(\alpha) \\ &k_{z1-} := \frac{k_{\parallel -}}{\cos(\alpha)} - k_y \tan(\alpha) \end{aligned} \quad (61)$$

$$\begin{aligned} &> k_{z1+} := \frac{k_{\parallel +}}{\cos(\alpha)} - k_y \cdot \tan(\alpha) \\ &k_{z1+} := \frac{k_{\parallel +}}{\cos(\alpha)} - k_y \tan(\alpha) \end{aligned} \quad (62)$$

$$\begin{aligned} &> k_{z2-} := -\frac{k_{||-}}{\cos(\alpha)} - k_y \cdot \tan(\alpha) \\ & \qquad \qquad \qquad k_{z2-} := -\frac{k_{||-}}{\cos(\alpha)} - k_y \tan(\alpha) \end{aligned} \tag{63}$$

$$\begin{aligned} &> k_{z2+} := -\frac{k_{||+}}{\cos(\alpha)} - k_y \cdot \tan(\alpha) \\ & \qquad \qquad \qquad k_{z2+} := -\frac{k_{||+}}{\cos(\alpha)} - k_y \tan(\alpha) \end{aligned} \tag{64}$$

$$\begin{aligned} &> k_{z3-} := i \cdot \sqrt{k_x^2 + k_y^2 - k_{||-}^2} \\ & \qquad \qquad \qquad k_{z3-} := I \sqrt{k_x^2 + k_y^2 - k_{||-}^2} \end{aligned} \tag{65}$$

$$\begin{aligned} &> k_{z3+} := i \cdot \sqrt{k_x^2 + k_y^2 - k_{||+}^2} \\ & \qquad \qquad \qquad k_{z3+} := I \sqrt{k_x^2 + k_y^2 - k_{||+}^2} \end{aligned} \tag{66}$$

$$\begin{aligned} &> k_{z4-} := -i \cdot \sqrt{k_x^2 + k_y^2 - k_{||-}^2} \\ & \qquad \qquad \qquad k_{z4-} := -I \sqrt{k_x^2 + k_y^2 - k_{||-}^2} \end{aligned} \tag{67}$$

$$\begin{aligned} &> k_{z4+} := -i \cdot \sqrt{k_x^2 + k_y^2 - k_{||+}^2} \\ & \qquad \qquad \qquad k_{z4+} := -I \sqrt{k_x^2 + k_y^2 - k_{||+}^2} \end{aligned} \tag{68}$$

$$\begin{aligned} &> k_{||+} := r \cdot k_{||-} \\ & \qquad \qquad \qquad k_{||+} := r k_{||-} \end{aligned} \tag{69}$$

$$\begin{aligned} &> k_y := a_y \cdot k_{||+} \\ & \qquad \qquad \qquad k_y := a_y r k_{||-} \end{aligned} \tag{70}$$

$$\begin{aligned} &> k_x := \frac{k_{||-}}{\epsilon} \\ & \qquad \qquad \qquad k_x := \frac{k_{||-}}{\epsilon} \end{aligned} \tag{71}$$

ux leading order terms

$$> u_{x1-} := u_{x10-} \cdot \text{sol1} :$$

$$> u_{x4-} := u_{x40-} \cdot \text{sol2} :$$

$$> u_{x1+} := u_{x10+} :$$

$$> u_{x2+} := u_{x20+} \cdot \text{sol3} :$$

$$> u_{x3+} := u_{x30+} \cdot \text{sol4} :$$

$$\begin{aligned} &> \text{simplify}(\text{series}(u_{x1-}, \epsilon, 2)) \\ &\quad \frac{2 r (-a_y r + \sin(\alpha))}{\cos(\alpha) (r+1)} \epsilon + O(\epsilon^2) \end{aligned} \quad (72)$$

$$\begin{aligned} &> \text{simplify}(\text{series}(u_{x4-}, \epsilon, 3)) \\ &\quad \frac{I \sin(\alpha) r (r-1) \operatorname{csign}\left(\frac{k_{||-}}{\epsilon}\right)}{\cos(\alpha)^2} \epsilon^2 + O(\epsilon^3) \end{aligned} \quad (73)$$

$$\begin{aligned} &> \text{simplify}(\text{series}(u_{x1+}, \epsilon, 3)) \\ &\quad \frac{r (-a_y + \sin(\alpha))}{\cos(\alpha)} \epsilon + O(\epsilon^3) \end{aligned} \quad (74)$$

$$\begin{aligned} &> \text{simplify}(\text{series}(u_{x2+}, \epsilon, 3)) \\ &\quad - \frac{r (r-1) (a_y + \sin(\alpha))}{\cos(\alpha) (r+1)} \epsilon + O(\epsilon^3) \end{aligned} \quad (75)$$

$$\begin{aligned} &> \text{simplify}(\text{series}(u_{x3+}, \epsilon, 3)) \\ &\quad \frac{I \sin(\alpha) r (r-1) \operatorname{csign}\left(\frac{k_{||+}}{\epsilon}\right)}{\cos(\alpha)^2} \epsilon^2 + O(\epsilon^3) \end{aligned} \quad (76)$$

u_perp leading order terms

$$\begin{aligned} &> \text{simplify}(\text{series}(\text{sol1}, \epsilon, 3)) \\ &\quad \frac{2 r}{r+1} + 2 \frac{r^2 (r-1) (\cos(\alpha) - 1) (\cos(\alpha) + 1) (a_y - \sin(\alpha))}{\sin(\alpha) \cos(\alpha)^2 (r+1)} \epsilon^2 + O(\epsilon^3) \end{aligned} \quad (77)$$

$$\begin{aligned} &> \text{simplify}(\text{series}(\text{sol2}, \epsilon, 3)) \\ &\quad \frac{(\cos(\alpha) - 1) (\cos(\alpha) + 1) (r-1) r}{\cos(\alpha)^2} \epsilon^2 + O(\epsilon^3) \end{aligned} \quad (78)$$

$$\begin{aligned} &> \text{simplify}(\text{series}(\text{sol3}, \epsilon, 2)) \\ &\quad \frac{r-1}{r+1} + O(\epsilon^2) \end{aligned} \quad (79)$$

$$\begin{aligned} &> \text{simplify}(\text{series}(\text{sol4}, \epsilon, 4)) \\ &\quad - \frac{(\cos(\alpha) - 1) (\cos(\alpha) + 1) (r-1) r}{\cos(\alpha)^2} \epsilon^2 \\ &\quad + \frac{I r^2 a_y (\cos(\alpha)^4 - 3 \cos(\alpha)^2 + 2) (r-1) \operatorname{csign}\left(\frac{k_{||+}}{\epsilon}\right)}{\sin(\alpha) \cos(\alpha)^3} \epsilon^3 + O(\epsilon^4) \end{aligned} \quad (80)$$

bx leading order terms

$$> b_{x1-} := \frac{\Delta_{||1-} \cdot u_{x1-}}{i \cdot k_{||+}} :$$

$$> b_{x4-} := \frac{\Delta_{||4-} \cdot u_{x4-}}{i \cdot k_{||+}} :$$

$$> b_{x1+} := \frac{\Delta_{||1+} \cdot u_{x1+}}{i \cdot k_{||+}} :$$

$$> b_{x2+} := \frac{\Delta_{||2+} \cdot u_{x2+}}{i \cdot k_{||+}} :$$

$$> b_{x3+} := \frac{\Delta_{||3+} \cdot u_{x3+}}{i \cdot k_{||+}} :$$

$$> \text{simplify}(\text{series}(b_{x1-}, \epsilon, 2))$$

$$\frac{-2 a_y r + 2 \sin(\alpha)}{\cos(\alpha) (r + 1)} \epsilon + O(\epsilon^2) \quad (81)$$

$$> \text{simplify}(\text{series}(b_{x4-}, \epsilon, 2))$$

$$\frac{\sin(\alpha) (r - 1)}{\cos(\alpha)} \epsilon + O(\epsilon^2) \quad (82)$$

$$> \text{simplify}(\text{series}(b_{x1+}, \epsilon, 2))$$

$$\frac{r (-a_y + \sin(\alpha))}{\cos(\alpha)} \epsilon + O(\epsilon^3) \quad (83)$$

$$> \text{simplify}(\text{series}(b_{x2+}, \epsilon, 2))$$

$$\frac{r (r - 1) (a_y + \sin(\alpha))}{\cos(\alpha) (r + 1)} \epsilon + O(\epsilon^2) \quad (84)$$

$$> \text{simplify}(\text{series}(b_{x3+}, \epsilon, 2))$$

$$- \frac{\sin(\alpha) (r - 1)}{\cos(\alpha)} \epsilon + O(\epsilon^2) \quad (85)$$

b_perp leading order terms

$$> b_{\perp 1-} := \frac{\Delta_{||1-} \cdot sol1}{i \cdot k_{||+}} :$$

$$> b_{\perp 4-} := \frac{\Delta_{||4-} \cdot sol2}{i \cdot k_{||+}} :$$

$$> b_{\perp 1+} := \frac{\Delta_{||1+} \cdot 1}{i \cdot k_{||+}} :$$

$$\begin{aligned}
& \text{> } b_{\perp 2+} := \frac{\Delta_{\parallel 2+} \cdot \text{sol3}}{i \cdot k_{\parallel+}} : \\
& \text{> } b_{\perp 3+} := \frac{\Delta_{\parallel 3+} \cdot \text{sol4}}{i \cdot k_{\parallel+}} : \\
& \text{> } \text{simplify}(\text{series}(b_{\perp 1-}, \epsilon, 2)) \\
& \qquad \qquad \qquad \frac{2}{r+1} + O(\epsilon^2) \tag{86}
\end{aligned}$$

$$\begin{aligned}
& \text{> } \text{simplify}(\text{series}(b_{\perp 4-}, \epsilon, 2)) \\
& \qquad \qquad \qquad \frac{-I \operatorname{csgn}\left(\frac{k_{\parallel-}}{\epsilon}\right) (\cos(\alpha) - 1) (r - 1) (\cos(\alpha) + 1)}{\cos(\alpha)} \epsilon + O(\epsilon^2) \tag{87}
\end{aligned}$$

$$\begin{aligned}
& \text{> } \text{simplify}(\text{series}(b_{\perp 1+}, \epsilon, 2)) \\
& \qquad \qquad \qquad 1 \tag{88}
\end{aligned}$$

$$\begin{aligned}
& \text{> } \text{simplify}(\text{series}(b_{\perp 2+}, \epsilon, 2)) \\
& \qquad \qquad \qquad \frac{-r+1}{r+1} + O(\epsilon^2) \tag{89}
\end{aligned}$$

$$\begin{aligned}
& \text{> } \text{simplify}(\text{series}(b_{\perp 3+}, \epsilon, 2)) \\
& \qquad \qquad \qquad O(\epsilon) \tag{90}
\end{aligned}$$

b_par leading order terms

$$\begin{aligned}
& \text{> } b_{\parallel 1-} := - \frac{i \cdot k_x \cdot u_{x1-} + \Delta_{\perp 1-} \cdot \text{sol1}}{i \cdot k_{\parallel+}} : \\
& \text{> } b_{\parallel 4-} := - \frac{i \cdot k_x \cdot u_{x4-} + \Delta_{\perp 4-} \cdot \text{sol2}}{i \cdot k_{\parallel+}} : \\
& \text{> } b_{\parallel 1+} := - \frac{i \cdot k_x \cdot u_{x1+} + \Delta_{\perp 1+}}{i \cdot k_{\parallel+}} : \\
& \text{> } b_{\parallel 2+} := - \frac{i \cdot k_x \cdot u_{x2+} + \Delta_{\perp 2+} \cdot \text{sol3}}{i \cdot k_{\parallel+}} : \\
& \text{> } b_{\parallel 3+} := - \frac{i \cdot k_x \cdot u_{x3+} + \Delta_{\perp 3+} \cdot \text{sol4}}{i \cdot k_{\parallel+}} : \\
& \text{> } \text{simplify}(b_{\parallel 1-}) \\
& \qquad \qquad \qquad 0 \tag{91}
\end{aligned}$$

$$\begin{aligned}
& \text{> } \text{simplify}(\text{series}(\text{expand}(b_{\parallel 4-}), \epsilon, 3)) \\
& \qquad \qquad \qquad -I (r - 1) \sin(\alpha) \operatorname{csgn}\left(\frac{k_{\parallel-}}{\epsilon}\right) \epsilon + O(\epsilon^2) \tag{92}
\end{aligned}$$

$$\begin{aligned}
& \text{> } \text{simplify}(b_{\parallel 1+}) \\
& \qquad \qquad \qquad 0 \tag{93}
\end{aligned}$$

$$\left[\begin{array}{l} \textcolor{red}{>} \textit{simplify}(b_{||2}+) \\ \textcolor{blue}{0} \end{array} \right] \qquad \qquad \qquad \textbf{(94)}$$

$$\left[\begin{array}{l} \textcolor{red}{>} \textit{simplify}(\textit{series}(\textit{expand}(b_{||3}+), \epsilon, 3)) \\ \textcolor{blue}{-I \, (r-1) \, \sin(\alpha) \, \text{csgn}\left(\frac{k_{||-}}{\epsilon}\right) \, \epsilon + O(\epsilon^2)} \end{array} \right] \qquad \qquad \qquad \textbf{(95)}$$