Analysis of Algorithms

Outline

1. Performance
2. Time Complexity
3. Space Complexity

Performance

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For example, if $f(n) = 31n^2 + 78n + 42$, then $T(n) = n^2$

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```
>_ ~/workspace/dsa/programs

$ cat ../data/1Kints.txt
 324110
-442472
...
 745942
$ /usr/bin/time --format='%e seconds' python3 threesum.py ../data/1Kints.txt
70
0.7 seconds
$ /usr/bin/time --format='%e seconds' python3 threesum.py ../data/2Kints.txt
528
5.9 seconds
```

Time Complexity

Time Complexity

threesum.py

```python
from instream import InStream
import stdio
import sys

def main():
    inStream = InStream(sys.argv[1])
    a = inStream.readAllInts()
    stdio.writeln(count(a))

def count(a):
    n = len(a)
    count = 0
    for i in range(0, n):
        for j in range(i + 1, n):
            for k in range(j + 1, n):
                if a[i] + a[j] + a[k] == 0:
                    count += 1
    return count

if __name__ == '__main__':
    main()
```

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1K	0.28s
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def count(a):
    n = len(a)
    count = 0                                  [A]
    for i in range(0, n):                      [B]
        for j in range(i + 1, n):              [C]
            for k in range(j + 1, n):          [D]
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Statement Block	Time	Frequency	Total Time
$[A]$	t_4	1	t_4
$[B]$	t_3	n	$t_3 n$
$[C]$	t_2	$\binom{n}{2}$[1] $= n^2/2 - n/2$	$t_2(n^2/2 - n/2)$
$[D]$	t_1	$\binom{n}{3} = n^3/6 - n^2/2 + n/3$	$t_1(n^3/6 - n^2/2 + n/3)$
$[E]$	t_0	x (depends on input)	$t_0 x$

[1] $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

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Grand total: $f(n) = (t_1/6)n^3 + (t_2/2 - t_1/2)n^2 + (t_1/3 - t_2/2 + t_3)n + t_4 + t_0 x$

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Grand total: $f(n) = (t_1/6)n^3 + (t_2/2 - t_1/2)n^2 + (t_1/3 - t_2/2 + t_3)n + t_4 + t_0 x$

Running time: $T(n) = n^3$

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Running time classifications

Name	$T(n)$	Code Description	Example
constant	1	statement	increment the ith element in an array
logarithmic	$\log n$	divide and discard	binary search
linear	n	loop	find the maximum
linearithmic	$n \log n$	divide and conquer	merge sort
quadratic	n^2	double loop	check all ordered pairs
cubic	n^3	triple loop	check all ordered triples
exponential	2^n	exhaustive search	check all subsets

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Sizes of built-in objects on a typical system

Object	Size in Bytes
integer	24
float	24
boolean	24
string of n characters	$40 + n$
list of n integers	$72 + 8n + 24n = 72 + 32n$
m-by-n list of integers	$72 + 8m + m(72 + 32n) = 72 + 80m + 32mn$
user-defined	hundreds of bytes, at least