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Project Mid-point Report: Model Predictive Control of Uncertain Systems

Project Description

Model Predictive Control (MPC) is a strategy to deal with infinite horizon control problems. At each time step, a finite horizon optimal control problem (FHOCP) is solved. The first calculated input is applied to the system and then the measured state at the next time step is used to solve another FHOCP. As the name suggests, MPC is model based and the accuracy of the model affects the quality of the results. It is often difficult or impossible to perfectly model the system we are trying to control, so this motivates the development of techniques to apply MPC to systems with uncertain models.

We are studying approaches to managing uncertainty in the context of MPC. After finalizing our literature review of existing algorithms that address this problem, we are implementing three approaches on an example system with uncertainty and disturbances to compare their performance. Our approach is to first show preliminary results by implementing the algorithms on a simplified linear system model and then apply them to a more complex and relevant model.

Overview of Progress

We completed a literature review of existing approaches to managing uncertainty within the context of Model Predictive Control and identified three approaches to further pursue: scenario-based MPC, tube-based MPC, and affine recourse.

Scenario-based MPC

In a stochastic system, the uncertainty is assumed to have a known distribution. Because the future states are not deterministic, MPC applied to a stochastic system must use the expected value in its objective function. Constraints can also be specified in terms of expected values. Another approach is to use chance constraints, which specify that a constraint must hold at least a certain percentage of the time. These problems are non-convex and are difficult to solve except in a few special cases.

Scenario MPC is an technique that approximates a stochastic MPC problem by taking samples of the uncertainty, which are called scenarios. The objective function of the deterministic problem is taken to be the average of the objective functions of each scenario. The solution is required to be robust with respect to the constraints imposed by each scenario. With linear system dynamics, the FHOCP can be turned into a convex optimization problem. Scenario MPC is attractive because it can handle a wide range of problems. It can deal with additive disturbances in the system dynamics and parameter uncertainty in the system matrices. Scenario MPC is applicable to disturbances of any distribution. In fact, the distribution does not need to be known explicitly - the only requirement is that samples of the uncertainty can be generated.

Scenario MPC can create a convex approximation of a stochastic problem but it may not be tractable because the numbers of variables and constraints grow with the number of scenarios. Intuitively, the probability of constraint satisfaction will improve as

the number of scenarios increases because the solution will be robust with respect to more realizations of the uncertainty. The key problem is then to find lower bounds on the required number of scenarios, called the sample complexity, to guarantee a specified constraint satisfaction probability with minimum computational complexity. According the Schildbach et al., previous theoretical lower bounds were conservative because they were higher than empirical sample complexities that gave the desired level of constraint satisfaction. They propose new tighter bounds that are functions of a property of each chance constraint in the problem, called the support rank.

We set to replicate the results of a numerical example given in Schildbach et al. to familiarize ourselves with the structure of Scenario MPC. The system dynamics were linear with two states and included parameter uncertainty in the A matrix and random additive disturbances. The objective was to minimize the norm of the states and inputs while keeping both states above 1. Based on a prescribed constraint violation level of 10%, the required sample complexity is K = 19.

The simulation was performed in Matlab using CVX to write the optimization problem, a quadratic program, that is solved at each iteration. After running the simulation for 2,500 closed-loop time steps, the empirical constraint violation level was 8.64%. The closed-loop stage cost had mean 3.79 and standard deviation 0.38. In the paper, the simulation was run for 10,000 closed-loop time steps producing an empirical constraint violation level of 9.87% and closed-loop stage costs with mean 3.78 and standard deviation 0.54.

Tube-based MPC

In the tube-based robust MPC approach, the nominal problem (system with no uncertainty) is solved independently of the uncertain system, while the error between the nominal and uncertain systems is bounded by a robust positively invariant set. Closed-loop feedback and the use of a terminal set ensures that the uncertain system remains within a tube surrounding the nominal state. This allows for the control of the nominal system while maintaining the guarantee that the uncertain system will satisfy the system constraints. The terminal set is calculated offline while the online problem is a quadratic program

We are currently working on replicating the results presented in Fleming et al. In this implementation, constraint feasibility is ensured for a system with multiplicative uncertainty by applying terminal constraints to the degrees of freedom available to the controller while constructing polytopic tube cross sections.

The paper present results for a linear system with multiplicative uncertainty and bounded states and inputs. Once we have replicated results consistent with those presented in the paper, we will then apply this approach to the system presented in Schildbach et al.

Affine Recourse

Affine recourse is a procedure for solving with multi-stage stochastic problems. The key idea is to realize that in many stochastic problems, some uncertainty becomes revealed at each time step. Affine recourse seeks, to exploit this fact by making the decision variables a causal, affine function of the uncertainty.

We decided to apply the concepts of affine recourse to the numerical example provided in Schildbach et al. Using ideas in Skaf and Boyd, we successfully reformulated the problem as a quadratic program in the input and state. We made the input an affine function of the state, which is in turn impacted by the uncertainty. To deal with the stochastic nature of the problem we approximated stochastic constraints with box constraints determined by the standard deviation of the distribution and by the expected value of the distribution. We excluded multiplicative noise from our derivation, so as to make it as simple as possible.

We implemented our system in MATLAB for five time steps. With the expected value approximation, we obtained reasonable results but still violate the constraints in the case of large adversity by particular realizations of noise. We are working on improving this result and analyzing the performance.

With the box constraint approximation, we bounded the noise by its standard deviation. We are currently not obtaining as good a result as we would have wanted but are working to improve the performance. We are also looking to implement the same example with multiplicative noise.

Next Steps

Our next step is to compare the performance of the three methods of robust control on the same system. We then plan to implement at least one of the algorithms on a more complex system.

References

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