USING CONVEX OPTIMIZATION TO CONTROL LINEAR SYSTEMS WITH DISTURBANCES

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DECEMBER 1ST, 2015

AFFINE RECOURSE

- An optimization technique used to manage uncertainty by making the decision variables a linear function of the uncertainty, i.e. linear feedback.
- Gain matrix, K can be computed using convex optimization and uncertainties can be modeled in expectation or robustly
- More concretely, in our case we seek to:

$$\min_{x,u0,K} E[\| u \|_2{}^2 + \| x \|^2{}_2]:$$

$$x > x_{low}$$

$$| u | < u_{\max}$$

$$x_{t+1} = Ax_t + Bu_t + w_t$$

$$u = u_0 + Kx$$

$$| w | < 3\sigma$$

MODEL PREDICTIVE CONTROL

- A control technique that uses numerical optimization to solve for the input, subject to state and input constraints
- Apply the first input, measure the state, and solve the problem again
- Stochastic version:

$$
\begin{aligned}
\min_{u} \quad & \sum_{i=0}^{N-1} \mathrm{E}[l(x_{i|t}, u_{i|t})] & \\
\text{subject to} \quad & x_{i+1|t} = f(x_{i|t}, u_{i|t}, \omega_{t+i}) & i = 0, \ldots N-1 \\
& x_{0|t} = x_t & \\
& \Pr(x_{i|t} \in \mathcal{X}) \geq 1 - \varepsilon & i = 1, \ldots N \\
& u_{i|t} \in \mathcal{U} & i = 0, \ldots, N-1
\end{aligned}
$$

TUBE-BASED MPC

- Uncertain system is decomposed into independent **nominal** and **error** systems:
  - LTI system with disturbance: $x_{k+1} = Ax_k + Bu_k + w_k$

    $$x_k = z_k + e_k, \quad u_k = Ke_k + c_k$$
  - Closed-loop decomposed system:

    $$z_{k+1} = Az_k + Bc_k$$
    $$e_{k+1} = (A + BK)e_k + w_k$$
  - Solve the nominal optimization problem (QP) over c_k (online)
  - Bound the error within robust positively invariant set (computed offline)
  - **Uncertain system evolves in a tube around the nominal system**

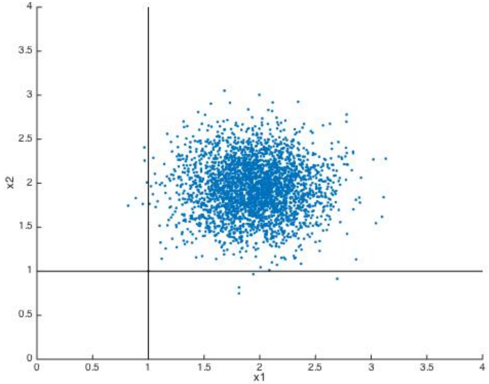
SCENARIO MPC

- Turn the stochastic problem into a deterministic one by sampling the uncertainty
- The solution is required to satisfy the constraints for all full-horizon samples, also called scenarios
- Bounds exist on the number of scenarios to guarantee chance constraint satisfaction

$$
\begin{aligned}
\min_{u} \quad & \sum_{k=1}^{K} \sum_{i=1}^{N-1} l({x_{i|t}}^{(k)},\ u_{i|t}) & \\
\text{subject to} \quad & {x_{i+1|t}}^{(k)} = f({x_{i|t}}^{(k)}, u_{i|t}, {\omega_{t+i}}^{(k)}) & i = 0, \dots N-1, k = 1, \dots, K \\
& {x_{0|t}}^{(k)} = x_t & k = 1, \dots, K \\
& {x_{i|t}}^{(k)} \in \mathcal{X} & i = 1, \dots N, k = 1, \dots, K \\
& u_{i|t} \in \mathcal{U} & i = 0, \dots, N-1
\end{aligned}
$$

SIMULATION RESULTS

Affine Recourse



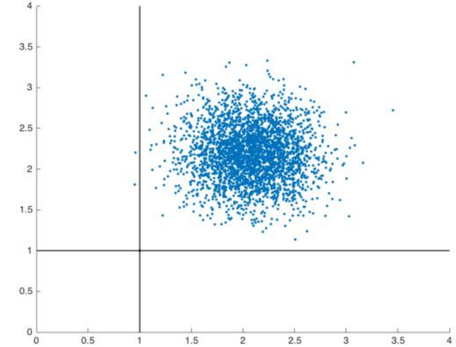
Single realization, 3000 time steps

- 0.20%, 0.13% constraint violation
- Cost: 2.8476 $*10^4$

200 trials, 6 time steps

- All % violation/time step = 0%
- Average cost = 35.99

Tube MPC



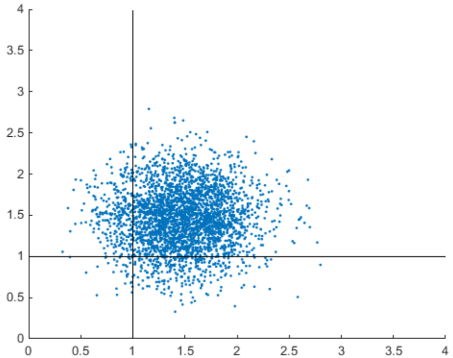
Single realization, 3000 time steps

- 0.07%, 0% constraint violation
- Cost: 3.3757 $*10^4$

200 trials, 6 time steps

- Highest % violation/time step = 0.5% t= 1; 0 %
- Average cost = 63.04

Scenario MPC



Single realization, 3000 time steps

- 9.87%, 10% constraint violation
- Cost: 1.7117 $*10^4$

200 trials, 6 time steps

- Highest % violation/time step = 10%, t = 6; 15.5%, t = 2
- Average cost = 32.73

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