Visibility Curve of an Object

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1 Introduction

This small computer program was built in order to compute the visibility curve of a sky object on a given day. The program outputs the object's rise, culmination and set times, minimum airmass, Alt. and Az. plots, the Sun's rise and set time and the beginning and end time for civil and astronomical twilight.

The resulting curves were confronted with other similar tools available online with good results.

2 Details on the Algorithm

NB: All non-time quantities are expressed in degrees where not otherwise specified.

The calculation follows the algorithm shown in Ch. 15 of "Astronomical Algorithms" with some details from "Explanatory Supplement to the Astronomical Almanac, 3rd Edition" and from the USNO.

The needed information is: Date of Observation, Object ICRS coordinates (α, δ), Observer latitude (ϕ) taken as positive in the northern hemisphere and negative in the southern, longitude (L) taken as positive eastward from the prime meridian and negative westward, and height above mean sea level (H_{msl}).

First the Greenwich Apparent Sidereal Time at 0^h UT (Θ_0) on the day of the observation is computed.
For this calculation an algorithm from USNO is used.
In the approximation $JD_{TT} \sim JD_{UT} \doteq JD$, at 0^h let $D = JD - 2451545.0$ be the number of days since Jan 1st 2000 at 12:00 and let $T = D/36525$ be the number of centuries since the year 2000.
The Greenwich Mean Sidereal Time (GMST) is given by

$$GMST = (6.697375 + 0.065709824279D + \\ + 0.0000258T^2)\%24$$

where "%" is the modulo operator.
A correction for the nutation in right ascension is needed to get the GAST:

$$\Theta_0 = (GMST + \Delta\psi \cos(\epsilon))\frac{360}{24} \tag{1}$$

where $\Delta\psi \simeq -0.000319 \sin(\Omega) - 0.000024 \sin(2L_S)$ is the nutation in longitude in hours, $\Omega = 125.04 - 0.052954D$ is the longitude of the ascending node of the Moon, $L_S = 280.47 + 0.98565D$ is the mean longitude of the Sun and
$\epsilon = 23.4393 - 0.0000004D$ is the obliquity.

Then the apparent RA and Dec of the body[1] at 0^h Dynamical Time[2] on day D-1, D and D+1 are needed, all expressed in degrees. These sets of coordinates will henceforth be indicated as (α_i,δ_i) with $i = 1, 2, 3$.
To convert from ICRS coordinates to the needed coordinates the python package Astropy is used.

A first guess on the rise (m_1), culmination (m_0) and set (m_2) times, expressed as fractions of day, is given by:

$$m_0 = \frac{\alpha_2 - L - \Theta_0}{360} \tag{2}$$

$$m_1 = m_0 - \frac{H_0}{360} \tag{3}$$

$$m_2 = m_0 + \frac{H_0}{360} \tag{4}$$

where H_0 is the hour angle of the body such that

$$cos(H_0) = \frac{\sin(h_0) - \sin(\phi)\sin(\delta_2)}{\cos(\phi)\cos(\delta_2)}$$

with h_0 being the geometric altitude of the body center at the time of apparent rising or setting.
If $\cos(H_0) > 1$ the object is always under the horizon on the day of the observation, similarly if $\cos(H_0) < -1$ the object is always visible.
The value of h_0 varies depending on the observed body ($-34/60°$ for a star/planet, $-50/60°$ for the Sun, $-6°$ for civil twilight, $-18°$ for astronomical twilight) and includes a correction for the observer's height above MSL ($-0.0353\sqrt{H_{msl}}$).

[1] "Apparent" coordinates are "True Equator True Equinox" coordinates
[2] "Dynamical Time" is used in the book as an abbreviated form of "Barycentric/Terrestrial Dynamical Time", TDT is needed and henceforth abbreviated TT for "Terrestrial Time"

An iterative correction on the three fractions is needed to get more precise estimates.
First the time difference between Terrestrial Time and UT is needed: $\Delta T = TT - UT$. This calculation follows from empirical formulae.
The following procedure is applied separately on each of the three m.
First the sidereal time at Greenwich is computed from $\theta_0 = \Theta_0 + 360.985647m$.
Then an estimate for the apparent RA and Dec at the day fraction is computed from

$$\alpha(m) = \alpha_2 + \frac{n}{2}(a + b + nc) \tag{5}$$

$$\delta(m) = \delta_2 + \frac{n}{2}(a + b + nc) \tag{6}$$

where $n = m + \Delta T/86400$, $a = \alpha_2 - \alpha 1$, $b = \alpha_3 - \alpha 2$, $c = b - a$ (and similar formulae for $\delta(m)$).
This estimate is needed to compute the local hour angle of the body $H = \theta_0 + L - \alpha(m)$ from which one derives the body Azimuth and Altitude angles as:

$$\tan(A(m)) = \frac{\sin(H)}{\cos(H)\sin(\phi) - \tan(\delta(m))\cos(\phi)} \tag{7}$$

$$\sin(h(m)) = \sin(\phi)\sin(\delta(m)) + \\ + \cos(\phi)\cos(\delta(m))\cos(H) \tag{8}$$

The correction is then given by

$$m = m + \Delta m \tag{9}$$

with

$$\Delta m = -H/360 \qquad \text{for } m_0 \tag{10}$$

$$\Delta m = \frac{h - h_0}{360\cos(\delta(m))\cos(\phi)\sin(H)} \qquad \text{for } m_{1/2} \tag{11}$$

The iterative procedure stops when the corrections are small.

At the end of the procedure the three m values are converted to times in order to be displayed.
The calculation from the estimate of θ_0 to that of $A(m)$ and $h(m)$ is repeated at steps of m between m_1 and m_2 in order to plot the Alt/Az curves.

3 References

- Meeus J., "Astronomical Algorithms" (Willmann-Bell)
- Urban S. E., Seidelmann P. K., "Explanatory Supplement to the Astronomical Almanac, 3rd Edition", (University Science Books)
- USNO, https://aa.usno.navy.mil/

4 Software Usage

The user can choose at the beginning ("Interactivity (y/n): »y") if he wants to enter all parameters such as observation date, object coordinates and location, otherwise ("Interactivity (y/n): »n") the current day, a standard object (M31) and location (Massa, MS) are chosen.

```
[Output]
Interactivity (y/n): >>y
Date of Observation:
        Year: >>2023
        Month: >>9
        Day: >>19
Object ICRS Coordinates:
        Object RA [deg]: >>101.28715533
        Object Dec [deg]: >>-16.71611586
Observer Coordinates:
        For Latitude + in the Northern Hemisphere, - in the Southern
        For Longitude + westwards from Prime Meridian, - eastwards
        Observer Latitude [deg]: >>44.007947
        Observer Longitude [deg]: >>10.099098
        Observer Height MSL [m]: >>0

Sun:
        Astro Twilight End:      2023-09-19 03:24:50 [UTC]
        Civil Twilight End:      2023-09-19 04:33:55 [UTC]
        Rising:                  2023-09-19 05:02:49 [UTC]       Az. from North 86.99 [deg]
        Transit:                 2023-09-19 11:13:29 [UTC]       Az. from North 180.00 [deg]
        Setting:                 2023-09-19 17:23:23 [UTC]       Az. from North 272.73 [deg]
        Civil Twilight Start:    2023-09-19 17:52:12 [UTC]
        Astro Twilight Start:    2023-09-19 19:01:01 [UTC]

Object:
        Rising:          2023-09-19 01:19:07 [UTC]       Alt. -0.57 [deg]        Az. from North 113.01 [deg]
        Transit:         2023-09-19 06:14:12 [UTC]       Alt. 29.26 [deg]        Az. from North 180.00 [deg]      Airmass 2.046
        Setting:         2023-09-19 11:09:16 [UTC]       Alt. -0.57 [deg]        Az. from North 246.99 [deg]
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